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An Econophysical Approach for the Market Structure Determination of an Industry: An Extension of Robert Bishops Formula

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ABSTRACT

A precise understanding of the market structure of an industry forms the basis for any entrepreneurial decisions. The combination of Bishop's cross-price elasticity of demand and price elasticity of demand forms the mathematical basis for determining the market structure. However, these formulas are time-independent, which does not adequately capture the complex dynamics of today's economy. This paper attempts a systemic dynamic extension of the cross-price elasticity of demand and the price elasticity of demand by adding a temporal dimension to these formulas. This ensures that entrepreneurial and industry-specific decisions can be made more accurately by following a system-dynamic approach. This paper provides a methodological extension based on Estola's econophysical theory and combining it with Bishop's approach for determining the market structure of an industry. Econophysics ensures that the temporal dimension is taken into account in economic formulas and therefore providing a more accurate view on current economic issues.

1. INTRODUCTION

The highly dynamic business environment of today requires companies to constantly rethink how they operate in the market. State-of-the-art pricing models are a necessity, especially in markets that are highly competitive. In many markets, artificial intelligence-based (AI-based) pricing models are being used. Such models use AI to analyse large amounts of data and generate price proposals. The use of such AI models is an attempt to measure customer responsiveness by means of price elasticity and determine the profit-maximizing price. In many cases, however, pricing is based on time-independent neoclassical price models, which means the dynamic component of time is not considered at all. The development of effective and dynamic pricing models requires an in-depth knowledge of market structures and due consideration of the dynamic dimension.

A high degree of societal complexity characterises the modern-day globalised economy. The speed at which business dynamics change, new organisations are founded and new inter- and intraorganisational relationships are built is increasing. This increase raises the levels of complexity, which can result in higher levels of probability that mistakes are happening in organisation. Moreover, increasing complexity raises the risk of exogenous shocks, which can have a major impact on any kind of organisation. Of critical importance is the fact that a significant number of the theoretical models used in management science and economics lack adaptability. The adaptability of a model depends on the extent to which it incorporates dynamic variables. Unfortunately, most broad economic theories take a static approach to the different kinds of change that can occur in the socio-economic ecosystem. After all, the social ecosystem is dynamic, which makes it even more important to incorporate dynamic variables into existing economic theories [1].

The aim of this paper is to make a theoretical contribution to the field of system dynamics by taking an econophysical approach. Applying the econophysical method to economic theories, such as cross-price elasticity, will enable economists to consider temporal dimensions when they use these economic metrics. Econophysics offers a new and innovative approach to the classic economic theories.

Various economists have formulated theories on price elasticity of demand and cross price elasticity of demand. Their primary objective was to analyse the cross elasticity between different companies in a particular industry and draw conclusions about the market relationships, market dynamics and market power of companies. Cross elasticity is an economic formula that measures the market relationships between companies. This economic metric can be used to inform business and strategic decisions. For example, cross-price elasticity is helpful when new market participants enter the market since it provides relevant information for decision makers in organisation. They can use these pieces of information throughout the managerial decision-making process. For an entrepreneur, knowing how market share is divided among competitors can help them adapt their business strategies to a changing environment. Every competitor tries to defend its market share to guarantee future business success. Therefore, Francesco Sylos Labini identified certain defence mechanisms and provided a set or practical, market-based tools that entrepreneurs can use to prevent the entry of new market participants [2].

Moreover, to apply price and quantity policy strategies on the respective market, a central understanding of the market structure is important. Moreover, some competition policy decisions are made on the basis of cross-price elasticities using the test of small but significant and non-transitory increase in price (SSNIP) to define whether a specific company has significant market power or not. All the dimensions discussed in these introductory paragraphs highlight the importance of cross-price elasticity for policy maker and decision makers within an organisation [3].

However, the cross price elasticity of demand has a particular shortcoming: the cross price elasticity of demand remains constant throughout the analysis. This non-dynamical approach is insufficient to accurately describe the complexity of market relationships and the dynamics between different companies operating in specific markets. Cross-price elasticities is always analysed in a time-independent frame, resulting in a static analysis. Such an approach is not suited to the description of inter and intra-organisational market dynamics. Koutsoyiannis states in his introductory remarks, that economic theory aims to provide models and formulas to predict a certain economic behaviour of a specific economic unit. The quality of a model or a formula and its validity depends on several criteria such as the predictive power, the consistency and the assumptions on which the model or formula are based [4]. The primary objective of this paper is to further develop the classical theory of cross elasticity by incorporating a temporal dimension into the formula and therefore increase its validity. An additional objective is to demonstrate the existence of a temporal dimension that must be considered to accurately describe relationships and dynamics between companies operating in specific markets.

The paper is structured as follows: Following the introductory Section 1, Section 2 is an in-depth literature review that examines the theory of price elasticity and describes the status quo. In addition, a detailed review of the current literature on econophysics, a young field in economic research, presents the basis for the development of a model that considers the temporal dimension of cross-price elasticities. Based on recent research results from econophysics, I transfer proven physical models and concepts to the study of economics. This transference provides new and fundamental insights that the application of the classic economic models cannot deliver. Specifically, the application of Matti Estola's econophysical insights enables the integration of a temporal dimension into a model for the interpretation of cross-price elasticities. I discuss the newly developed model and provide specific use cases where the consideration of a temporal dimension is crucial for business decisions. Section 3 presents the mathematical model developed by the author. This section will describe the approach and provide the mathematical derivation. In Section 4, the practical value of the new formula, which can be applied in academia and business, is explored. The author draws a conclusion and considers the contribution made by this paper. In Section 5, the limitations of this paper are presented and avenues for future research are suggested.

2. LITERATURE REVIEW

The aim of this section is to provide an overview of different theories about cross elasticity. These theories and their respective interpretations are heterogenous, and it is therefore important to describe the main thoughts of the economic theorists involved. A solid theoretical basis is crucial when deciding which theoretical interpretations are best suited to the development of a new model. An overview of the theory on price elasticity is presented first because it represents the basis on which the measure of cross-price elasticity of demand was calculated.

Nicholas Kaldor has written several papers on such topics as market imperfection and excess capacity, since these two concepts have an impact on the price setting behaviour of market participants. In his theoretical analysis of market imperfection, Kaldor identified several conditions that need to be met. First, many producers must be active and the products and services they provide must be slightly different. 'Slightly different' means that even the elasticity of demand is never great enough to induce producers to sell the entirety of their products and services at an identical price. This means that a minor price change would not reduce the demand for the products and services of higher-priced producers or service providers in its entirety. The second condition is that the preference structure and the income class must be homogenously distributed. Kaldor's third condition is that none of the producers has a so-called institutional monopoly (i.e., patents, copyrights, trademarks, trade names). This means that every producer can enter the market freely, without being hindered by mechanisms that prevent market entry, such as the strategies developed by Sylos Labini. The fourth condition is the existence of economies of scale. This means that the average long-term cost curve of all producers will fall until a certain level of output is reached. (Chamberlin defines this as the U-shaped cost curve. The mathematical turning point is where the X-inefficient part of the economic cost curve starts.) Based on this assumption, economic theory states that new producers/competitors will continue to enter the market as long as monopoly/oligopoly profits are made. This dynamic will continue, leading to an incremental reduction in the production output of existing producers and to a continuous increase in elasticity of wants until the elasticity becomes infinite and price is equal to average total cost. The final equilibrium is reached when none of the producers generates a profit. In the final equilibrium, marginal cost will be equal to marginal revenue on the one hand, and, on the other hand, average total cost will be equal to price. Once this market situation has been reached, no additional producer will enter the market [5].

The reasons for market imperfections based on Kaldor can be divided into three categories: (1) differences in the characteristics of the product; (2) differences in the geographical distribution of suppliers; (3) a certain threshold of price difference between two products or services is reached before the consumer switches to another supplier. Chamberlin assumed that preferences of consumers are evenly distributed. Consequently, an additional market entry would lead to a homogenous market share loss of the remaining market participants. However, Kaldor found that the assumption of evenly distributed preferences does not hold in real life [5, 6]. Nevertheless, in economic analysis it is necessary to rely on different kinds of assumptions to describe a specific economic dynamic.

One important factor that must be considered throughout the economic analysis and the price-elasticity analysis is the existence of market imperfections that stem from other dynamics, such as buyer inertia, the geographical location of suppliers and the institutional monopolisation of certain key business areas. These market imperfections complicate economic analysis. This complication is one of the primary reasons that mathematical economists defined a set of conditions to ensure that 'perfect competition' is the starting point of their subsequent analysis. These conditions include, for example, the perfect and infinite divisibility of assets, which leads to the absence of economies of scale. Furthermore, no forms of institutional monopolies exist because their existence would ultimately lead to market imperfection [5]. The main idea posited by this paper is that markets are imperfect and subject to specific dynamics (oscillation). This idea is in line with the hypothesis formulated at the outset, namely that a static view of the markets is not sufficient to describe market dynamics and thus make the right business decisions. The above discussion of imperfect markets is relevant since market imperfection has a significant impact on market structure.

The key role played by market structure is a feature of industrial economic analysis. The concept of market structure should therefore be discussed in more detail. Market structure is a generic term that includes certain aspects, such as the number of suppliers in a market, the degree of product differentiation, and cost structure, to develop a general overview and understanding of the market under investigation. A key variable when determining market structure is the number of suppliers operating in a market. In turn, this number has a significant influence on the cost structures and strategies of the companies involved. Market structure is also determined by the degree of product differentiation, one of the decisive variables when determining whether a product should be assigned to a specific product market. Therefore, the degree of homogeneity of a product also has an influence on market structure. Based on this result, the market assumes different forms, such as pure competition, pure oligopoly, differentiated competition, differentiated oligopoly and pure monopoly [7]. Clearly, market structure plays a central role in business decisions and is therefore the subject of further analysis throughout the paper. However, it is unclear how market structure can be determined analytically. One possible parameter with which to define market structure is cross-price elasticity, which is subject of subsequent analysis.

To analyse and define cross-price elasticity, it is necessary to fully understand the theory of price elasticity of demand. The reason is that the latter provides the theoretical foundation for such an analysis. Moreover, price elasticity of demand is one of the essential variables when an accurate definition of market structure is being sought. This is especially important since Bishop uses these two concepts (Price elasticity of demand and cross-price elasticity of demand) to analytically determine the market structure.

In the literature on elasticities, Alfred Marshall was the first to provide a definition of the elasticity of demand. In Book III, Chapter IV of P*rinciples of Economics*, Marshall describes the elasticity of wants as the extent to which the amount demanded increases (decreases) for a given percentage decrease (increase) of the price level of the observed commodity. This percentage increase or decrease can be small or great and is the basis for assessing whether the elasticity of wants is small or great. Marshall noticed a difference in elasticity of demand intensity depending on the price level. The elasticity of wants is great for high-priced commodities. The elasticity falls gradually as the price of the commodity falls [8].

Furthermore, Marshall noticed that the elasticity of demand for different commodities is greatly dependent on the available budget of a household. Marshall classified the population in three classes: working class, middle class and the rich. Every social class has a different elasticity. Members of the working class were more sensitive to price changes for certain goods, such as meat, milk and butter, wool, tobacco, imported fruits and ordinary medical attendance. These behavioural differences can be partially explained by the Engel curve, also known as Engel's law. The Engel curve depicts income elasticity as a concave curve indicating that the percentage of total household income spent on food declines with increasing household income [9]. Based on this law, one can conclude that the smaller the percentage of total household income spent on food, the smaller a household's elasticity of demand (i.e., the more inelastic the demand becomes). In addition, elasticity of demand is dependent on the type of economic good the consumer is considering. In other words, elasticity of demand varies whether the good under consideration is a normal good (luxury or necessary good) or an inferior and/or Giffen good (A Giffen Good is a special type of good where the demand for these good increases when prices increase.). The working class is likely to be highly elastic when considering luxury goods, which means that a slight increase in price could result in a sharp decline in the quantity demanded. The different reactions can be illustrated graphically: The economic good can either have a concave, convex or linear demand curve, depending on its type. An inferior or Giffen good is likely to be inelastic in its response to price changes.

Additionally, it is important to note that geographical location and local habits can have a major impact on whether elasticity of demand is high or low. Marshall used the example of salt. In England, on the one hand, the price is so low that elasticity of demand is small. In India, on the other hand, the price of salt is comparatively high, and consequently demand is highly elastic there compared to England. Another crucial factor that influences individual elasticity of demand is the consumer's individual preferences; in other words, what characteristics must the good possess and what level of quality does the consumer require. These factors may have a significant impact on the degree to which a consumer is sensitive to price changes.

Lastly, and given the particular focus of this paper, time is one of the most important aspects to take into account. Marshall stated that time is one of the most difficult elements to study in economics and statistics. Rarely are statistics trustworthy when the numbers remain unchanged when observed over longer periods of time. While the numbers may stay the same over a short period, this observation would not provide an accurate representation of real economic dynamics. This challenge becomes even greater when one considers the concrete magnitude of the effects and how these effects are spread out, since they rarely all occur simultaneously. In economics, an asynchronous dynamic is usually experienced. This means that an exogenous shock is first observed, followed by the economic impact at a later stage. Since these events seldom occur at the same time, the time dimension is a crucial variable that must not be underestimated in the study of in economics. The importance of the temporal dimension, which is the subject of this paper, is therefore confirmed. The time effect can be represented best when observing the purchasing power of money, which changes constantly. One can therefore never assume that the value of money remains constant over time. Another example cited by Marshall is that habits and familiarity with different kinds of goods considered as substitutes evolves over time.

Based on all the factors discussed above, one can conclude that elasticity of demand is certainly not a static dimension or variable but a highly dynamic one. Time is therefore an important parameter that dare not be neglected in economic analysis [8]. However, price elasticity of demand refers to a single product, while this paper focuses on cross-price elasticity of demand. Nevertheless, price elasticity provides the mathematical basis for determining the cross-price elasticity between two products. Moreover, elasticity of demand is used by Bishop as an additional variable to define the market structure. Furthermore, cross-price elasticity is an economic formula that allows for the relationship between two different products to be observed. Such observations enable scholars to make various socio-economic interpretations and provide a sound economic basis for defining market structure.

The seminal thoughts on cross-price elasticity were first articulated by Kaldor in his commentary on Joan Robinson's book, *The Accumulation of Capital*. Kaldor stated that different vendors do not sell products that are either completely homogeneous or heterogeneous. Products are largely identical, with only minor differences. As a result, the demand for certain products is not completely elastic or inelastic in relation to price changes introduced by competitors. Products from different firms selling largely identical goods therefore have a substitutive character. Consequently, the demand for the products of different firms is similarly elastic in response to price variations introduced by competitors. It is noteworthy that the degree of cross-price elasticity of demand in response to price variations introduced by competitors is not identical for each firm. The demand for products of a particular firm may be affected to a greater degree by competitors' price changes than that of other firms [6].

In classic economic theory, two economists in particular, Triffin and Chamberlin, took a contrary view of cross-price elasticity. On the one hand, Triffin argued that the cross-price elasticity E_{ij} is infinite for pure competition, positive and finite for heterogeneous competition, and zero for pure monopoly. On the other hand, Chamberlin argued that only two market structures exist: non-oligopolistic (firms with a cross-price elasticity of zero) and oligopolistic (firms with a cross-price elasticity greater than zero). Chamberlin also differentiated between heterogeneous and homogeneous goods. The former has a finite price elasticity of demand, while the latter has an infinite price elasticity of demand. These distinctions result in different economic interpretations which, according to Bishop, have certain economic shortcomings in regards to the economic interpretation. Below, the paper will discuss these interpretations and the economic logic of Bishop [10-12].

Two types of cross elasticity have been defined: cross-price elasticity and cross-quantity elasticity. These two cross elasticities do not have a reciprocal relationship because different assumptions underlie them. The formal interpretation of a cross-price elasticity is the ratio of a percentage change in the j^{th} quantity demanded due to an infinitesimal percentage change in the i^{th} price, considering that all prices remain constant (the ceteris paribus condition). The formal mathematical representations can be found in the annexure. Cross-price elasticity is positive between competitors and negative between complementary firms.

As stated above, the concept of cross-price elasticity is used in particular to determine the structure of the market. It is important to mention that the demand function in economics is subject to a ceteris paribus interpretation. However, as Bishop correctly stated, a ceteris paribus interpretation is not realistic. The reason is that a market participant will always react to changes in the price or quantity of a competitor by changing its price or quantity. However, the ceteris paribus interpretation can be helpful to determine market structure, the use of a ceteris paribus interpretation is helpful. One of the key points made by Bishop is that the number of competitors alone is not sufficient to define whether a market is structured as a polypoly, oligopoly or monopoly. Another condition to consider when defining market structure is the degree of either homogeneity or heterogeneity of the products under consideration. Bishop argued that cross-price elasticity tends to infinity when the degree of homogeneity of two competing products increases. Consequently, cross-price elasticity can be infinite when perfectly homogeneous products are offered by a few market participants. It is therefore paradoxical to say that high but finite cross-price elasticity per se signals an oligopolistic interdependent relationship. According to Bishop, this paradox can only be resolved by linking two mathematical functions: cross-price elasticity (E_{ji}) and elasticity (E_{ii}). Only if E_{ji} is high compared to E_{ii} can it be said with the necessary certainty that an oligopoly exists. This formula provides only a reliable value and is a more sensitive instrument when the observed goods are imperfect substitutes, which is most commonly the case in the economic context. To solve the paradox, Bishop developed the following formula:

$$E_{ji} = -\frac{E_{ii}}{n-1} = \frac{E_{ji}}{E_{ii}} = -\frac{1}{n-1} \; and \; E_{ji} = -\frac{E_{ii}}{n_i - 1}$$

It is noteworthy that Bishop deliberately decided against taking costs into account after Papandreou had attempted to include them in the calculation. The difficulty of including costs stems from the fact that elasticities can theoretically take on values in the interval $[-\infty; 0]$ and the output outlay elasticity can take on all values in the interval $[0; +\infty]$. In particular, since Bishop emphasises that market relationships can be described using only price elasticities of demand and cross-price elasticities of demand, without explicitly taking the cost function of organizations into account. This is possible because the elasticities implicitly account for cost structures [7].

Table 1 summarizes the following thoughts. Perfectly homogeneous goods produce positive outputs at an identical price level p. In this scenario, E_{ii} takes the value $-\infty$. This means that, ceteris paribus, an infinitesimal change in p_i leads to a significant change in the quantity demanded. Consequently, E_{ji} takes the following value: $+\infty$. However, this concept does not apply to heterogeneous goods because the price elasticity of demand assumes a finite value. Consequently, the cross-price elasticity also assumes a finite value.

Bishop summarised the results as follows:

Table 1. Market structure summary

	Nature of the product	
Numbers or 'Numbers Equivalent' of Other Suppliers	Near-Homogeneous $-E_{ii} \to \infty$	Significantly differentiated $-E_{ii} < \infty$ $(significantly)$
$-\frac{E_{ii}}{E_{ji}}$ $large$	Near-Pure Competition	Significantly Differentiated Competition or Pure Monopoly
$-\frac{E_{ii}}{E_{ji}}$ $small$	Near- Pure Oligopoly	Significantly Differentiated Oligopoly

Table 1 is differentiated in terms of the homogeneity of the product and the number of rival suppliers. Here, the (1) relative product heterogeneity is taken as a benchmark and (2) the independence or interdependence of price-output decisions between two competing firms. This table shows that the interaction of a high degree of product homogeneity and a high degree of interdependence between the price-output decisions of competing firms assumes the market structure of ‘near-pure competition’. In contrast, if product homogeneity is high and interdependence between firms is low, the market structure of near-pure oligopoly can be observed. However, if there is significant product heterogeneity and a high degree of interdependence of price-output decisions between competing firms, then the market structure of significantly differentiated competition or pure monopoly is observed. In the fourth case, when interdependence between price-output decisions between competing firms is low, the market structure of differentiated oligopoly is observed [7].

One can therefore deduce that the interaction of price elasticity and cross-price elasticity of demand is necessary to accurately and reliably identify market structure. Considering only cross-price elasticity of demand is not sufficient to make such an identification, according to Bishop. The question remains what the practical value is of Bishop’s insistence on considering both the interaction of price elasticity of demand and cross-price elasticity of demand. As mentioned above, the entry of new market participants poses a risk for existing companies. However, Bishop stresses that cross elasticity is not sufficient to determine the risk of a market entry. The reason is that the cross elasticity shows a discontinuity between the actual company i and the potential entrant j. A horizontal discontinuity also exists. The demand q_i as a function of the price p_i shows a critical price at which new market participants will suddenly enter the market. This is particularly important for existing market participants, since this assumption allows them to increase the market entry costs for new competitors by using certain defence mechanisms [7].

The Sylos postulate is one of the most common theories to describe these defence mechanisms. Established companies can raise their prices to the operating minimum (MC = AVC) and engage in a price war with the entering company in the short term. This leads to the market entrant having to account for higher losses due to its higher total average costs, which are primarily caused by the market entry costs. Consequently, the probability increases of the new participant leaving the market. Another possibility is for existing actors in the market to purchase all the available raw materials required to produce the product in question. The new market participant would then not be able to buy the necessary resources to produce its good. Another possibility presented by Sylos Labini is the poaching of qualified workers, thus removing specific knowledge from the new entrant. These mechanisms to raise barriers to entry and consequently secure market share will ultimately have an impact on market structure [2].

Having established the basis for determining price elasticity of demand and cross-price elasticity of demand, it is now possible to approach the focus of this paper: the dynamic component that is time. One of the central deficits in the determination of price elasticity and cross-price elasticity is the fact that time is not factored in. An exact identification of market structure in particular requires the factor of time to be considered. For this reason, the basis for a dynamic consideration of price elasticity of demand and cross-price elasticity of demand is presented below. This presentation needs to be linked to the ideas of Estola, whose work constitutes the primary literature at this point. Estola, who is an acknowledged expert in the new field of econophysics, includes the temporal dimension as a dynamic component to analyse microeconomic models more precisely [13].

The question that needs to be answered is the following: How can the issue of dynamisation be solved? A resolution is necessary because a purely static view is insufficient to describe the complex structure of economic relations. Since the classic models are static and do not take into account the speed of change in modern-day society and economies, a different approach is needed. Estola is one of the first scientists to link two disciplines, namely economics and physics, and his econophysical approach provides a good basis for incorporating the dynamic dimension. Econophysics is a relatively young and innovative discipline that uses models from physics to analyse and describe economic processes. In physics, the variable ‘time’ in often applied in the observation of physical phenomena. Physical phenomena are always dynamic, resulting in the use of time dimensions to describe their dynamic aspect, whereas in economics most economic phenomena are described in a time-independent manner. The econophysical approach ensures that economic formulas are used in a dynamic way by incorporating the time variable. This represents a major difference between the classic economic approach and the econophysical approach [13].

The driving force behind the way in which an economic unit acts on an economic quantity is described by the equation ‘pleasure minus pain’. Pleasure and pain are opposing forces. Equilibrium corresponds with the optimal state, when the opposing forces cancel each other out. Economic forces are therefore of particular importance since they impact significantly on the economic quantities that are observable. However, if a variable exists that in turn has an influence on the acting force, and this variable can be adjusted by an external person, then economic forces can be controlled. In this case, control theories can be used to model economic decisions. For example, one can argue that the easier an economic quantity can be changed, the more this variable will fluctuate. This is consistent with general economic observations on topics such as exchange rates, interest rates and stock prices, which vary daily. It is important to note, however, that physical laws are subject to constant parameters which is not the case in the context of economics. The reason is that economic forces vary over time. In general, it is possible to state that the longer the time dimension, the more difficult it is to provide an accurate forecast. This difficulty is caused by the fact that the many economic forces acting on each other weaken the accuracy of the forecast. Also noteworthy is the fact that physical laws are deterministic, whereas economic laws are probabilistic and subject to a certain statistical distribution. Consequently, one can argue that economic forces cause a change in the economic quantities. This leads to a certain probabilistic assumption that must be determined individually for each economic situation [13].

Dimensional analysis and classification are parts of an econophysical analysis. A dimensional analysis assumes that all units of measurement of the object under investigation belong to the same dimension. Furthermore, specific transformation rules apply by which certain quantities measured in a certain unit can be converted into another unit. When applied to an economic situation, all monetary values measured in a certain currency unit can be said to belong to the dimension of money. These monetary values can be converted at will and are therefore additive. Every science has certain elementary dimensions, also known as primary dimensions. In modern mechanical physics, the primary dimensions are mass, length and time. In economics, quantity, price and time can be regarded as the primary dimensions when describing economic facts. Every single primary dimension is described by a specific unit. The units of measurement for the primary dimensions in economics could be, for example, quantity, monetary unit (price) and seconds. De Jong [14] defined a fourth primary dimension for economic systems, the satisfaction level [14]. These four dimensions constitute the measurement variables of the economic system. It should also be noted that all parameters derived from the primary dimension are called secondary dimensions. Secondary dimensions are useful when quantifying specific economic measures [13].

The following question arises: Which primary and secondary dimensions are necessary to describe the economic phenomenon under consideration? As mentioned above, Estola has identified the primary dimensions of measurement for the economic system as the quantity of goods [R], the monetary value of goods [M] and the factor time [T]. To understand these primary dimensions, their main features should be explained. The measurement of the volume of goods (quantity of goods) can be in different units, such as kilogram, metre, meter^2, litre or real numbers. Of note is that adding a quantity of a certain good X in kilogram to a quantity of a certain good Y in litre is not appropriate. This means that even if the units used to measure the volume of goods belong to an identical dimension, they are not additive per se. However, as mentioned above, a dimension is a defined set of additive quantities. Different units of measurement for measuring the volume of goods belong to an identical dimension if a fixed transformation rule is used. Using these fixed transformation rules, certain units of measure belong to the same dimension, even if they are not directly additive [13].

Estola addressed the issue of how the value of goods can be determined. In economics, the market value of a good is defined by supply and demand: The higher the demand, popularity or scarcity of a good, the higher its value. Therefore, the value of a good is strongly dependent on the market in which it is offered. For a general understanding and to determine the dimension of monetary values, the common currencies Euro, US Dollar and British Pound are used. By using this universal dimension, monetary values can be 'exchanged' using a defined exchange rate. Based on this assumption, the dimension of money is an additive quantity, which is an important basis for carrying out the necessary mathematical transformations [13].

For a comprehensive economic evaluation of a situation, it is not sufficient to describe only the primary dimensions of the economy. The above description of the primary dimensions provides a basis for exploring the secondary dimensions. Secondary dimensions are the values that use primary dimensions to determine the corresponding value. For example, the value 'price' belongs to the secondary dimension [M/R] or €/Unit and is therefore a relative value [13].

The object of this paper is to identify and solve the temporal problem in economic models and formulate possible solutions. The temporal dimension can be described statistically using continuous or discrete variables. A discrete variable is a quantity that does not change continuously but rather by leaps and bounds. One of the main reasons for the discrete nature of a quantity is that the measurement of values is temporally fixed. Continuous variables that are dependent on time can be transformed into a discrete quantity at any time.

The consideration of continuous or discrete variables is of foundational importance, especially in the temporal dimension. Thus, discrete time variables are used in almost all economic analyses, involving a certain stocktaking that is carried out at a predefined time $(t_0, t_1, t_2, \dots, t_n)$. This process applies particularly to time series analyses. However, discrete time variables are insufficient to adequately describe the economic facts. Therefore, continuous time measures are used, which can be described mathematically as follows: $\Delta_t \rightarrow 0$. The unit of time with a length of zero can be determined and called 'time moment', and the continuous time dimension is constructed by linking it to the adjacent time moment. Mathematically, this phenomenon can be described as follows:

$$\Delta_t = (t_0 + \Delta_t) - t_0$$

In the following equation, t_0 is a fixed time moment, which is extended by an infinitesimal time difference. This infinitesimal time difference tends towards zero. For the consideration of the temporal dimension in a continuous manner, it is important to consider the central mathematical operations, which especially consider continuous time quantities measuring change. For further explanations regarding the temporal dimension in connection with cross-price elasticity, the instantaneous absolute change Eq. (1) and the instantaneous velocity or flow Eq. (2) formula are of central relevance:

$$lim_{\Delta t \rightarrow 0} [x * (t0 + \Delta t) - x(t0)] = dx| \, t = t_0 \tag{1}$$

$$lim_{\Delta t \rightarrow 0} \frac{x*(t0 + \Delta t) - x(t0)}{\Delta t} = \frac{dx}{dt} | \, t = t_0 \tag{2}$$

where, x is a variable that changes over time, t is time, and dx/dt is the time derivative of x. Using these formulas, we can describe the temporal dimension in a continuous manner by considering the central mathematical operations that measure change over time. Using these formulas, we can describe the temporal dimension in a continuous manner by considering the central mathematical operations that measure change over time.

From a mathematical point of view, the instantaneous absolute change of x and dx is regarded as a differential calculation of x and the instantaneous velocity of x as a function of time, dx/dt = x'(t), is called the time derivative of x. This equation provides the basis for including a temporal dimension in the formula of cross-price elasticity. As mentioned in the introduction, it is important to remove the static dimension from the mathematical operations to capture a realistic view of economic dynamics. In the next section, the formal mathematical approach by means of classical operations will be explained [13].

3. MATHEMATICAL DESCRIPTION

Mathematical proof requires that the definition range is defined in advance to set the frame. In the next step, the basic formulas according to Estola are presented. Finally, these formulas are applied to the classic cross-price elasticity formula to work out the temporal dimension. $\forall$ q, p, t $\in$ N^+.

The above mathematical condition, or the mathematically relevant definition range, states that q, p and t are part of the positive natural numbers. Also important is calculating the central mathematical formulas that are necessary for further derivation.

The formula that represents a certain quantity produced over time $(t_0;\ t_n)$ can be mathematically discussed as follows:

$$Q_k(t_0;\ t_n) = \lim_{\Delta t \to 0} \sum_{i=1}^{n} \frac{\Delta Q_k(t_0+i\Delta t)}{\Delta t} * \Delta t = \int_{t_0}^{t_n} \frac{dQ_k}{dt} * dt = \int_{t_0}^{t_n} Q'_k(t)dt = Q_k(t_n) - Q_k(t_0)$$

Consequently, this temporal formula is used to represent the change of a certain observation over time to accurately represent the dynamic aspect of an economic system. This equation can also be used to describe the quantity demanded by consumers, which means it can be used as an analogous equation for the cross-price elasticity of demand.

In a further step, the above mathematical equation will be integrated into the classic formula of cross-price elasticity. Using various arithmetic operations, a general equation/formula is derived to represent the temporal component of cross-price elasticity of demand and price elasticity of demand. Bishop indicated that an accurate identification of the market structure needs to include the cross-price elasticity of demand, the price elasticity of demand and the number of firms. The general formula is the following:

$$E_{ji} = -\frac{E_{ii}}{n-1} = \frac{E_{ji}}{E_{ii}} = -\frac{1}{n-1} \ and \ E_{ji} = -\frac{E_{ii}}{n_i - 1}$$

Incorporation the temporal dimension into Bishop's formula, we get:

$$E_{ji}(t) = -\frac{E_{ii}(t)}{n-1} = \frac{E_{ji}(t)}{E_{ii}(t)} = -\frac{1}{n-1} \ and \ E_{ji}(t) = -\frac{E_{ii}(t)}{n_i - 1}$$

where, E_{ji} and E_{ii} now depend on time, and their values change over time as market conditions and economic factors change.

Condition: $\Delta_t = t_1 - t_0$

A] E_{ji}

(1) $E_{ji} = \frac{\partial q_j}{\partial p_i} * \frac{p_i}{q_j} = \frac{q_{j2} - q_{j1}}{P_{i2} - P_{i1}} * \frac{p_i}{q_j}$

(2) $E_{ji}(t) = \frac{\partial q_{j\,(t)}}{\partial p_{i\,(t)}} * \frac{p_i}{q_j}(t)$

(3) $E_{ji}(t) = \lim_{\Delta t \to 0} \frac{\sum_{i=1}^{n} \frac{Q_j(t_0+i*\Delta t)}{\Delta t} * \Delta t - \sum_{i=1}^{n} \frac{Q_j(t_0)}{\Delta_t} * \Delta_t}{\sum_{i=1}^{n} \frac{P_i(t_0+i*\Delta t)}{\Delta t} * \Delta t - \sum_{i=1}^{n} \frac{P_i(t_0)}{\Delta_t} * \Delta_t} * \frac{p_i}{q_j}(t_0)$

(4) $\leftrightarrow \lim_{\Delta t \to 0} \frac{\sum_{i=1}^{n} Q_j(t_0) + Q_j(i*\Delta t) - \sum_{i=1}^{n} Q_j(t_0)}{\sum_{i=1}^{n} P_i(t_0) + P_i(i*\Delta t) - \sum_{i=1}^{n} P_i(t_0)} * \frac{p_i}{q_j}(t_0)$

(5) $\leftrightarrow \lim_{\Delta t \to 0} \frac{\sum_{i=1}^{n} Q_j(i*\Delta_t)}{\sum_{i=1}^{n} P_i(i*\Delta_t)} * \frac{p_i}{q_j}(t_0)$

(6) $\leftrightarrow \frac{i}{i} \lim_{\Delta t \to 0} \frac{\sum_{i=1}^{n} Q_j(\Delta_t)}{\sum_{i=1}^{n} P_i(\Delta_t)} * \frac{p_i}{q_j}(t_0)$

(7) $\leftrightarrow \lim_{\Delta t \to 0} \frac{\sum_{i=1}^{n} Q_j(\Delta_t)}{\sum_{i=1}^{n} P_i(\Delta_t)} * \frac{p_i}{q_j}(t_0)$

B] E_{ii}

(1) $E_{ii} = \frac{\partial q_i}{\partial p_i} * \frac{p_i}{q_i} = \frac{q_{i2} - q_{i1}}{P_{i2} - P_{i1}} * \frac{p_i}{q_i}$

(2) $E_{ii}(t) = \frac{\partial q_{i\,(t)}}{\partial p_{i\,(t)}} * \frac{p_i}{q_i}(t)$

(3) $E_{ii}(t) = \lim_{\Delta t \to 0} \frac{\sum_{i=1}^{n} \frac{Q_i(t_0+i*\Delta t)}{\Delta t} * \Delta t - \sum_{i=1}^{n} \frac{Q_i(t_0)}{\Delta_t} * \Delta_t}{\sum_{i=1}^{n} \frac{P_i(t_0+i*\Delta t)}{\Delta t} * \Delta t - \sum_{i=1}^{n} \frac{P_i(t_0)}{\Delta_t} * \Delta_t} * \frac{p_i}{q_i}(t_0)$

(4) $\leftrightarrow \lim_{\Delta t \to 0} \frac{\sum_{i=1}^{n} Q_i(t_0) + Q_i(i*\Delta t) - \sum_{i=1}^{n} Q_i(t_0)}{\sum_{i=1}^{n} P_i(t_0) + P_i(i*\Delta t) - \sum_{i=1}^{n} P_i(t_0)} * \frac{p_i}{q_i}(t_0)$

(5) $\leftrightarrow \lim_{\Delta t \to 0} \frac{\sum_{i=1}^{n} Q_i(i*\Delta_t)}{\sum_{i=1}^{n} P_i(i*\Delta_t)} * \frac{p_i}{q_i}(t_0)$

(6) $\leftrightarrow \frac{i}{i} \lim_{\Delta t \to 0} \frac{\sum_{i=1}^{n} Q_i(\Delta_t)}{\sum_{i=1}^{n} P_i(\Delta_t)} * \frac{p_i}{q_i}(t_0)$

(7) $\leftrightarrow \lim_{\Delta t \to 0} \frac{\sum_{i=1}^{n} Q_i(\Delta_t)}{\sum_{i=1}^{n} P_i(\Delta_t)} * \frac{p_i}{q_i}(t_0)$

C] $\frac{E_{ji}}{E_{ii}} = -\frac{1}{n-1}$

(1) $\frac{\lim_{\Delta t \to 0} \frac{\sum_{i=1}^{n} Q_j(\Delta_t)}{\sum_{i=1}^{n} P_i(\Delta_t)} * \frac{p_i}{q_j}(t_0)}{\lim_{\Delta t \to 0} \frac{\sum_{i=1}^{n} Q_i(\Delta_t)}{\sum_{i=1}^{n} P_i(\Delta_t)} * \frac{p_i}{q_i}(t_0)} = -\frac{1}{n-1}$

This economic formula looks at the changes in quantity and price combinations with infinitesimal changes in the unit of time. The infinitesimal change in the unit of time is gaining in importance due to the growing algorithmicizing of business processes and the associated increase in process speeds. Due to this, a fluent examination of market structures is important and essential.

The present formula has taken up the already studied considerations of Robert Bishop and extended them by a time unit in order to increase the accuracy of the formula. The mathematical formula development on which this paper is based does not follow the marginalistic approach where marginal changes in quantity are considered based on a unit change in price. The paper rather had a look at the infinitesimal change of the time unit. The cross-price elasticity and price elasticity formulae could also have been derived according to time (d/dt). This would have resulted in multipliers, which in turn can be interpreted individually. However, this was not the primary aim of this paper. Matti Estola provided a hint on how to approach the marginalistic perspective and derive the formula according to time. This possible calculation can be consulted in the appendix. The key objective pursued in this paper is the practical and entrepreneurial application of this formula in everyday life by the decision-maker in organization. Small and medium-sized enterprises, which have very limited resources, are the target of a sustainable and easy application of this formula in the entrepreneurial context. The question that can be raised is whether the use of a specific integral and the accompanying narrowing of the time dimension could be helpful in the analytical implementation of this formula. This is also used in physics to be able to limit the scope of analysis. The next chapter discusses the main findings and puts them into perspective.

4. DISCUSSION

This section is structured as follows: The relevance of mathematical modelling and suitability of econometric physics is discussed in more detail. The benefits of this approach when compared to the classic formulas are presented to demonstrate that a dynamic consideration of these formulas is necessary. Finally, possible perspectives for future research are suggested.

The practical relevance of the mathematical approach needs to be illustrated by a concrete example. The industries that are suitable for such illustration are those that exhibit high price dynamics. This applies in particular to the industries that use various dynamic pricing models, for example, the airline industry. Airlines introduced the dynamic pricing of its airline tickets as early as the 1970s. The dynamics of airline ticket pricing was so high that prices were sometimes adjusted by the second. Precise observations were made of the ways in which a price increase affected own-ticket demand (price elasticity of demand) and the ticket demand of competitors. In this scenario, very short time intervals are considered to offer demand-oriented prices in the market at any time, thus maximising revenue. This is described by means of the condition that $\Delta t \rightarrow 0$ and thus infinitesimal time intervals are considered. Mathematically, this allows for a granular and detailed consideration logic of the quantities: Quantity and Price. A singular and static survey of price elasticity and cross-price elasticity of demand is not sufficient to act successfully in a hypercompetitive market environment, which is geared towards crowding out the competitor. This approach would result in the wrong business decisions being made. A dynamisation of the approach is therefore of major importance and has significant practical value for everyday business. This is especially the case because a precise understanding of the market structure or market structure is required in hypercompetitive market situations. Only then can business decisions be made efficiently and effectively. Consequently, accurate knowledge of the market structure significantly influences market behaviour and, subsequently, market outcomes. As discussed, a static analysis is insufficient to generate sufficient relevant knowledge about the market. This paper therefore aims to contribute meaningfully to the integration of dynamic components into classic formulas to increase the accuracy of analysis and effective decision-making in an institution-specific manner. The dynamic view of neoclassical models opens up new possibilities in the dynamic understanding of the economic status quo. In particular, this formula can make a new contribution to the application of dynamic market and price models. The combination and application of intelligent, dynamic and time-dependent economic models with machine learning-based software programmes offer new possibilities in the detection of economic states and the derivation of specific market and price strategies.

In summary, this paper intends to spark discussion on how to further develop static economic models so that time-dependent evaluations and analyses can be performed. The time factor is particularly important considering the increasing complexity of the modern economic system and the increasing decentralisation of individual units. These characteristics point to the additional acceleration of our economic relations. In physics, acceleration includes a time dimension. Consequently, economic formulas must also take time dimensions into account to increase their accuracy.

Not only is the consideration of the time dimension essential but, in industrial economic analysis, the exact determination of the market structure is necessary to make the right pricing decisions in hypercompetitive markets. Based on these findings, Bishop's formula, which defines market structure in a way that represents an interplay of cross-price elasticity of demand and price elasticity of demand, forms the basis of this paper. A time dimension is then added to this formula, allowing for a time-dependent investigation. Thus, this paper makes a first and important contribution to the dynamization of static economic formulas.

5. LIMITATIONS AND FUTURE AVENUES FOR RESEARCH

This paper has the shortcoming that the formulas developed have not yet been applied in practice. Their statistical robustness has therefore not yet been investigated in depth. Such investigation is a necessity to verify the validity of the mathematical approach. Future investigations should therefore apply the mathematical formula suggested in this paper in a practical case, which might result in the adaptation of the formula to increase its robustness. In addition, it should be investigated whether industry-specific characteristics exist that necessitates an adjustment of the mathematical formula, since industry-specific characteristics might influence the use of the formula.

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APPENDIX

The following calculation show a marginalistic application to the study variables: Price elasticity of demand and cross-price elasticity of demand. This is achieved by deriving the individual formulas according to time in order to obtain multipliers.

At first, we need a demand function of good 1 in respect to good 2:

$$q_1(t) = f(p_1(t), p_2(t), q_2\,(t))$$

The time dependent price-elasticity formula could be:

$$E_1(t) = \frac{\frac{\partial q_1(t)}{\partial p_1(t)}}{\frac{q_1(t)}{p_1(t)}} = \frac{\partial q_1(t)}{\partial p_1(t)}\,\frac{p_1(t)}{q_1(t)'}$$

The time dependent cross-price elasticity formula could be:

$$E_{12}(t) = \frac{\frac{\partial q_1(t)}{\partial p_2(t)}}{\frac{q_1(t)}{p_2(t)}} = \frac{\partial q_1(t)}{\partial p_2(t)}\,\frac{p_2(t)}{q_1(t)'}$$

Using the first time derivative for the price elasticity formula could be:

$$E_1(t)' = \frac{d}{dt}\left(\frac{\partial q_1(t)}{\partial p_1(t)}\right)\frac{p_1(t)}{q_1(t)'} + \frac{\partial q_1(t)}{\partial p_1(t)}\,\frac{d}{dt}\left(\frac{p_1(t)}{q_1(t)}\right)$$

$$= \frac{p_1(t)}{q_1(t)}\,\frac{d}{dt}\left(\frac{\partial q_1(t)}{\partial p_1(t)}\right) + \left(\frac{\partial q_1(t)}{\partial p_1(t)}\right)\left(\frac{p'_1(t)q_1(t) - p_1(t)q'_1(t)}{\left(q_1(t)\right)^2}\right)$$

$$= \frac{p_1(t)}{q_1(t)}\,\frac{d}{dt}\left(\frac{\partial q_1(t)}{\partial p_1(t)}\right) + \left(\frac{\partial q_1(t)}{\partial p_1(t)}\right)\frac{p_1(t)}{q_1(t)}\left(\frac{p'_1(t)}{p_1(t)} - \frac{q'_1(t)}{q_1(t)}\right)$$

$$= \frac{p_1(t)}{q_1(t)}\,\frac{d}{dt}\left(\frac{\partial q_1(t)}{\partial p_1(t)}\right)\left[\frac{\frac{d}{dt}\left(\frac{\partial q_1(t)}{\partial p_1(t)}\right)}{\frac{\partial q_1(t)}{\partial p_1(t)}} + \left(\frac{p'_1(t)}{p_1(t)} - \frac{q'_1(t)}{q_1(t)}\right)\right]$$

Using the first time derivative for the cross price elasticity formula could be:

$$E_{12}(t)' = \frac{d}{dt}\left(\frac{\partial q_1(t)}{\partial p_2(t)}\right)\frac{p_2(t)}{q_1(t)'} + \frac{\partial q_1(t)}{\partial p_2(t)}\,\frac{d}{dt}\left(\frac{p_2(t)}{q_1(t)}\right)$$

$$= \frac{p_2(t)}{q_1(t)}\,\frac{d}{dt}\left(\frac{\partial q_1(t)}{\partial p_2(t)}\right) + \left(\frac{\partial q_1(t)}{\partial p_2(t)}\right)\left(\frac{p'_2(t)q_1(t) - p_2(t)q'_1(t)}{\left(q_1(t)\right)^2}\right)$$

$$= \frac{p_2(t)}{q_1(t)}\,\frac{d}{dt}\left(\frac{\partial q_1(t)}{\partial p_2(t)}\right) + \left(\frac{\partial q_1(t)}{\partial p_2(t)}\right)\frac{p_2(t)}{q_1(t)}\left(\frac{p'_2(t)}{p_2(t)} - \frac{q'_1(t)}{q_1(t)}\right)$$