

Multi-Criteria Optimization as the Methodology of Ensuring Sustainable Development of Regions: Tula Region of the Russian Federation



Roman A. Zhukov^{1*}, Nadezhda O. Kozlova¹, Evgeny V. Manokhin¹, Elena B. Myasnikova²,
Elena A. Melay³

¹ Department of Mathematics and Informatics, Financial University under the Government of the Russian Federation (Tula Branch), Tula 300012, Russia

² Department of Finance and Accounting, Financial University under the Government of the Russian Federation (Tula Branch), Tula 300012, Russia

³ Department of Economics and Management, Financial University under the Government of the Russian Federation (Tula Branch), Tula 300012, Russia

Corresponding Author Email: pluszh@mail.ru

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ABSTRACT

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The effective ensure of sustainable development of regions, including the Tula region, characterized by an unfavorable demographic situation, the presence of environmental problems and stagnation of industrial production, without the use of economic and mathematical apparatus and tools for data analysis and decision-making is difficult to implement. At the same time, most decisions are made on the basis of expert assessments, and this is typical for most Russian regions. The development and implementation of modern decision-making methods based on multi-criteria optimization will increase the validity of such decisions at various levels of management, and which can be applied not only in the Tula region, but also in other regions to solve their problems. The purpose of the study is to develop and test a method for optimizing the results of the functioning of socio-ecological-economic systems on the example of the regions of the Central Federal District and the district as a whole on the basis of the author's methodology within the framework of a multi-level optimization approach. The results of the study show that due to changes in values of factors included in the optimization model of the socio-ecological-economic systems functioning, which was developed on the basis of the author's methodology. It is possible to improve the target indicators of the development of the Tula region. The presented methodology can be used for other regions, which expands the scope of its application.

1. INTRODUCTION

The key objective of the functioning of government at different levels is to ensure the sustainable development of the state of its administrative-territorial units – regions, which can be considered as hierarchical socio-ecological-economic systems (SEES), for example, "district– region". We will understand that the management of sustainable development by the region is public administration implemented at the federal and regional levels in order to ensure an ongoing process of positive social, environmental and economic changes in the state and functioning of the system, characterized by key evaluation indicators lying within the boundaries of the specified trajectories, without deterioration of other indicators of the elements and subsystems of SEES – changes aimed at ensuring well-being present and future generations and improving their quality of life. In the current unfavorable situation in economics, ecology, demography, lack of qualified labor resources, which are typical for most Russian regions, including the Tula region, one of the main problems is the adoption of informed management decisions, which are most often based on peer reviews. At the regional level, the modern economic and mathematical tools are rarely

used. It leads to a decrease in the efficiency and correctness of decisions, which should be based on the concept of sustainable development and the preservation of a favorable environment. The development and implementation into practice of administrations of modeling and decision-making methods based on formalized SEES models and optimization models that take into account the basic principles of the concept of sustainable development can become an excellent tool for improving the effectiveness of solving regional problems.

Modern approaches to the management of the sustainable development of regions include the stage of searching for optimal solutions, at which factors are determined that make it possible to improve the performance targets of the SEES, its subsystems and elements. At the same time, modeling methods and tools for assessing the functioning of the SEES allow to increase the level of validity of decisions made. At the same time, the use of the economic and mathematical apparatus for solving problems in the field of sustainable development is associated with the construction of models of the functioning and forecasting of the SEES, for example, Andrews et al. [1], Elhorst [2], the choice of evaluation indicators that allow for a correct assessment and analysis of different quality and multi-level SEES Gang and Zhu [3],

Wang et al. [4] (measurement of interaction through the indicator of coordination of interrelations), Cheng et al. [5] (economic growth), Miola and Schilts [6] (measurement of target indicators of sustainable development); creation of optimization models based on production functions, econometric models for decision-making, which make it possible to find changes in factors that bring the system into dynamic equilibrium. In the field of optimization, such works can be considered the works of Habib et al. [7], Manski [8], Mohammed et al. [9].

The existing models of the state and functioning of the SEES, including in the form of production functions (PF), are mainly used to develop forecasts and development scenarios and rarely act as a tool for building standards and optimizing the functioning of the SEES. The latter statement allows us to put forward a hypothesis about the possibility of expanding the scope of such models, confirming its right to exist by testing the methodology being developed on the example of the regions of the Central Federal District of the Russian Federation and Tula region.

The purpose of the study is to develop and test a method for optimizing the results of the functioning of SEES on the example of the regions of the Central Federal District (CFD) and the district as a whole on the basis of the author's methodology within the framework of a multi-level optimization approach.

The practical significance of the study lies in the possibility of using the results to develop measures aimed at ensuring the sustainable development of the regions. The remaining part of the paper is organized in the following manner. The next section presents a review of the literature on models and methods used in analyzing the functioning of SEES and decision-making. Next, we present the methodology of the multi-level optimization approach, which consists in finding factors that can improve the target indicators of the functioning of SEES. The next section is devoted to the models and data that were used in the study. The fifth section presents the main results of the study, which include the constructed models and the results of solving optimization problems. In the sixth section we discuss the results obtained. In the end, the paper presents theoretical and practical conclusions and limitations.

2. LITERATURE REVIEW

Modern conceptual and formalized descriptions of hierarchy of SEES are based on the classical works of Saati [10] and Mesarovich et al. [11], which make it possible to distinguish hierarchical systems from other structures and formally describe them in terms of set theory. On the basis of such structures, a number of optimization problems are solved, both with the optimization of hierarchical structures [12] and with the adoption of optimal solutions for one element or a group of level elements [13] on a variety of Pareto set [14].

Within the framework of the description of hierarchical structures, the classification of elements should be carried out, allowing them to be combined into groups of the same type (classes). Examples of such classifications are: classification of Belousova (general classification of socio-economic systems) [15]; Kleiner's spatial-temporal classification based on the new theory of economic systems [16]; Larionov (by the type of economic mechanism) [17] and others. With all the variety of classifications, in fact, such a procedure for combining elements makes it possible to "look" at the same

complex system (the same process or phenomenon) from one point of view or another, perceiving the totality of elements as a subsystem functioning, again, from the researcher's point of view, as a single whole. That is, the diversity of classifications is determined by the goals and views of the author, who accepts either the traditional or his own classification, the latter of which should be understandable not only to the author, but also to other researchers. In this sense, the procedure for coordinating research results can be solved by introducing a conjugacy table (matrix) that establishes the correspondence of the studied features in various classifications.

If the object of research is identified – its structure is described (conceptually or formalized), classification is carried out – then the study (in this aspect, the assessment of the functioning of the SEES) can be carried out through its characteristic descriptions, which, in turn, are determined by the choice of appropriate indicators, which poses a certain problem for the researcher. The variety of classifications and goals of the conducted research give rise to a variety of both private and integral (for the totality of the elements of the SEES) evaluation indicators. Among such indicators, one can distinguish, for example: indicators of sustainable development [18]; indicators of the quality of life of the population [19]; indicators determined by the system of national accounts [20]; gross domestic product by regions (GDP by regions, most often used in regional studies when assessing the economic component of the SEES, as well as its modification – GDP by regions per capita income when constructing the affluence index) [21, 22]; indicators for assessing the functioning of certain areas of production of goods and services [23]; other macroeconomic indicators [24–26]. Along with the traditional indicators for assessing the functioning of the SEES, partial and integral indicators of effectiveness, efficiency and harmony can be used, the rationale for the use of which is reflected [27].

The presence of evaluation indicators allows you to compare the specified (normative, expected) results of the functioning of the SEES with their actual values, ensuring compliance with each other is the main task of management at various levels of the hierarchy. Therefore, in order to solve the management task aimed at such compliance – management of the development of hierarchical socio-economic systems – it is necessary to create such conditions due to the influences from the subject of management that would allow achieving the expected results – the goals of the functioning of the object of research. The formalization of the management problem, and in the most optimal (effective) way, determines the need to build models like the SEES itself (for example, using classical production functions [28]), and management models at different levels of the hierarchy. The following models are declining as models for evaluating SEES: linear models [29, 30]; quadratic models [31]; logarithmic models [32]; translogarithmic models based on the Cobb-Douglas function [33]; power multiplicative models, including taking into account the innovative component [34]; transcendental models [35] etc. The formation of a system of views on the process of managing complex systems, an overview of which is presented [36], determines the need to use both a techno-economic and a traditional (classical management) approach to the management of the SEES within the framework of general systems theory and system analysis [37].

In a number of modern publications related to the management of the development of socio-economic systems, management models are considered that allow formalizing

processes and forming sound management decisions. However, the emphasis is placed not so much on the construction of a management model, as on the description of management objects or management mechanisms. At the same time, models are presented graphically without reference to mathematical models, for example in the study [38]. Mathematical models act as a description of the control object [30]. The obvious relationship between the management of technical, techno-economic and socio-economic systems at various levels, for example, the situational approach to management and deviation management, is not explicitly taken into account when forming the management models of the SEES. A natural solution is the application of a well-developed mathematical apparatus used in the management of the technical and techno-economic systems to the tasks of state and municipal management.

Gorelik and Zolotova [39, 40] presents an approach to determining the principle of optimality when making a decision in conditions of incomplete information, based on game theory. The same authors considered the issues of optimal control based on the information theory of hierarchical systems [41]. The problem of finding the best approximate solutions in applied economic problems is discussed [42].

An overview of decision-making methods is presented in the study [43]. In the case of solving a multi-criteria optimization problem, when an objective function with weighting coefficients is compiled, these coefficients are selected by experts, which is not entirely correct, since such a procedure increases the influence of subjective estimates on the final result. This problem can be solved if the weighting coefficients are calculated in the process of solving optimization problems related to the field of mathematical programming and decision-making methods [44].

3. METHODOLOGY

3.1 Stages of the approach

The research was conducted in the Financial University under the Government of the Russian Federation (Tula Branch).

The used multilevel optimization approach to the management of the SEES involves the implementation of the following stages: 1. A formalized description of the SEES; 2. Identification of the results of the functioning of the SEES at each level of the hierarchy, as well as state and impact factors (control factors) characterizing the specific conditions of the activity of the object of study; 3. Construction of models of the relationship between the result features of elements, classes, levels of the SEES and the conditions by which the normative (expected) values are determined, which are the purpose of the functioning of the SEES; 4. Evaluation of performance indicators, efficiency and harmony of the functioning of the SEES, of the same type for each of the hierarchical levels; 5. Selection of result features that do not correspond to normative values; 6. Optimization of factor features, that is, the search for such values at which the considered result features characterizing the functioning of elements, classes, levels and the SEES as a whole would achieve (were within acceptable limits) their norms, that is, the goals of functioning would be achieved. Note that the values of the norms, if they are set for several time periods, can be associated with a given trajectory of the development of the SEES.

In fact, the last stage is the search for the optimal decision in managing the development of the SEES, which should be included in the appropriate feedback control based on the principle of deviation control (compensation) at each decision-making cycle. The control circuit can be represented in the form of cybernetic and process models.

3.2 Formalization of the approach

The formalization of this approach can be presented as follows.

1. Description SEES. Each element $k_{p,(p-1),v_p,s_q} \in L_{p,(p-1),s_q}$ of the set H of hierarchy $\langle H, R \rangle$, R is a relation of a non-strict order, starting from the second level, is numbered by four indexes: p is the number of the level; $(p-1)$ is the index of the element to which the element is subordinate; v_p is the number of this element; s_q is element class number. An example is a hierarchy: a state, a district, a region (a subject of the Russian Federation), enterprise. Classification can be carried out by dividing all elements into large social and economic classes, the latter of which can be divided according to spatial-temporal classification [45], sectoral classification [46] or in the traditional version according to sections of the Russian Classification of Economic Activities (NACE (OKVED)).

2. Feature identification. Each of the elements is characterized by four types of features (it is allowed to study elements through their feature descriptions). These include: 1) performance feature $\hat{y}_{p,(p-1),v_p,s_q}(t)$ is the result of the functioning of the element in a period of time t , $t = 1, \dots, T$, T is number of periods (actual value); 2) state factors $x_{p,(p-1),v_p,s_q,j}(t)$ are element functioning conditions, $j = 1, \dots, J$, J is the number of state factors; 3) impact factors $z_{p,(p-1),v_p,s_q,u}(t)$, which characterize the control actions on the element, $u = 1, \dots, U$, U – number of impact factors; 4) normative performance feature $\hat{y}_{p,(p-1),v_p,s_q}(t)$ is normative (expected) result of the element functioning.

3. Relationship. Relationships between performance features and state and impact factors can be caused by non-random functions $f_{p,(p-1)}$, which we will call production functions (PF):

$$\hat{y}_{p,(p-1),v_p,s_q}(t) = f_{p,(p-1)}(x_{p,(p-1),v_p,s_q,j}(t), z_{p,(p-1),v_p,s_q,u}(t)) \quad (1)$$

The PF parameters can be determine using factorial analysis of dependencies from econometric equations of the form:

$$y_{p,(p-1),s_q} = \hat{y}_{p,(p-1),s_q} + \mathcal{E} \quad (2)$$

where, \mathcal{E} is stochastic random component (assumed to be a normal random variable in the first approximation).

Expression (1) characterizes the intra-level relationships of SEES elements. A formalized description of class relationships can be presented in the form of an aggregated PF:

$$\sqrt{\sum_{i_1=1}^I \sum_{i_2=1}^I \hat{r}_{p,i_1,i_2,s_q} \cdot \hat{y}_{p,i_1,v_p,s_q}(t) \cdot \hat{y}_{p,i_2,v_p,s_q}(t)} \quad (3)$$

where, \hat{r}_{p,i_1,i_2,s_q} are the values of the paired Pearson correlation coefficient between i_1 -th \hat{y}_{p,i_1,s_q} and i_2 -th \hat{y}_{p,i_2,s_q} variables ($i_1, i_2=1, \dots, I$).

Interlevel relationships are defined as the sum of the corresponding actual feature descriptions of elements or classes (for example, the average annual number of employees in the subjects of the Russian Federation and in the district (the sum of employees in all subjects of the Russian Federation that are part of the district)).

Models in the form (1) and (3) are constructed from statistical data for a set of v_p -th elements belonging to one element (formula (1)) or one class (formula (3)). An example of such data is statistical data for 17 regions of the Central Federal District (CFD) for several time periods. If we build a model from a set of partial and aggregated PF only based on data from one region (for example, the Tula region (TO)), then such a model $\tilde{y}_{p,(p-1),v_p,s_q}(t)$ (we will designate such models with the sign " \sim ", including for designating other indicators that are calculated only from data for the same subject) will characterize the functioning of only the investigated subject of the Russian Federation and will no longer act as a model for calculating the norm, but as a model for calculating the actual values of the results of its functioning with a certain degree of accuracy specified by this model. From the same positions, the constructed model will characterize the expected (normative) value for all subsystems (municipality (MO) or economic entities) that are part of the region under consideration (for this it is enough to substitute the actual values of the state factors and the impact of a particular MO).

This is one of the features of the approach, which consists in the construction of two types of models: the first - for a set of similar subsystems, the second - for one subsystem. With such a statement, it is important to choose the starting point: at which level to build the basic models, and then use them to determine the functioning of the elements (classes) of the SEES at other levels of the hierarchy. For example, to build models of the functioning of elements (classes) for a set of MO that are part of the region, and to identify the functioning of the region as the sum of the corresponding results and conditions obtained for municipalities. Within the framework of the study, it is proposed to choose a regional level as a starting point – a meso-level, which has some commonality of existing objects and processes, which allows using a number of models in the form of production functions to describe its functioning, which is consistent with the ideas of A.A. Bogdanov, who is rightfully recognized as one of the founders of a systematic approach to the knowledge of the world [47]. He believed that the study of the nature of things should begin with the middle – meso-level, and through this level to look for approaches to understanding the mega-, macro- and microstructure of the world.

4. Evaluation of the functioning of the SEES at different levels of the hierarchy can be carried out with the help of partial and integral indicators of performance, efficiency and harmony, which form a system of indicators that allow a comprehensive study of the object of research. The In general, partial and integral performance indicators can be defined as:

$$\xi_{p,(p-1),v_p,s_q}(t) = \frac{y_{p,(p-1),v_p,s_q}(t)}{\hat{y}_{p,(p-1),v_p,s_q}(t)} \quad (4)$$

$$\xi_{p,v_p,s_q}(t) = \frac{\sqrt{\sum_{i_1=1}^I \sum_{i_2=1}^I r_{p,i_1,i_2,s_q} \cdot y_{p,i_1,v_p,s_q}^0(t) \cdot y_{p,i_2,v_p,s_q}^0(t)}}{\sqrt{\sum_{i_1=1}^I \sum_{i_2=1}^I \hat{r}_{p,i_1,i_2,s_q} \cdot \hat{y}_{p,i_1,v_p,s_q}^0(t) \cdot \hat{y}_{p,i_2,v_p,s_q}^0(t)}} \quad (5)$$

where, the index "0" characterizes that the values are reduced to a scale from 0 to 1 after the standardization procedure (the random variable will have zero mathematical expectation and unit variance, such random variables will be denoted by the index "*"); r_{p,i_1,i_2,s_q} are values of the Pearson pair correlation coefficient (similar to (3)). If expressions (4) and (5) are close to one, then we can talk about the compliance of the functioning of the SEES with the expected (normative) results.

The assessment of balance, in the sense of compliance with the set of performance indicators with normative values, can be carried out using the coefficient of harmony, which is determined by the formulas:

$$H_{Ap}(t) = 1 - \frac{\sigma(\xi_{p,i}(t))}{M(\xi_{p,i}(t))} \quad (6)$$

$$H_{Ap,s_q}(t) = 1 - \frac{\sigma(\xi_{p,i,s_q}(t))}{M(\xi_{p,i,s_q}(t))}$$

where, $M(\xi_{p,i}(t))$, $M(\xi_{p,i,s_q}(t))$ are means; $\sigma(\xi_{p,i}(t))$, $\sigma(\xi_{p,i,s_q}(t))$ are standard deviations; $i = 1, \dots, Q$, Q the number of integral or partial (for class s_q) performance indicators correspondingly.

The closer $H_{Ap}(t)$, $H_{Ap,s_q}(t)$ are to one, the more balanced (harmonious) the functioning of the object of study under consideration.

5. The criteria on the basis of which a decision is made on the need for optimization can be partial and integral performance indicators, as well as the coefficient of harmony. Their normative values are assumed to be equal to one and are the target indicators of the functioning of the SEES:

$$\widehat{\xi}_{p,(p-1),v_p,s_q}(t) = \widehat{\xi}_{p,v_p,s_q}(t) = \widehat{H}_{Ap} = 1 \quad (7)$$

Let's form the following conditions:

$$\xi_{p,(p-1),v_p,s_q}(t) \in \widehat{\xi}_{p,(p-1),v_p,s_q}(t) \pm \delta_{p,(p-1),v_p,s_q}(t),$$

$$\xi_{p,v_p,s_q}(t) \in \widehat{\xi}_{p,v_p,s_q}(t) \pm \delta_{pv_p,s_q}(t), \quad (8)$$

$$H_{Ap}(t) \in \widehat{H}_{Ap}(t) - \delta_{Ap}(t)$$

where, $\delta_{p,(p-1),v_p,s_q}(t)$, $\delta_{p,v_p,s_q}(t)$ and $\delta_{Ap}(t)$ are confidence intervals that can be set a priori (1%, 5%, 10% etc.) or determined by the formula:

$$\delta = t_{1-\alpha;n-p-1} \cdot s \cdot \sqrt{1 + A_0^T \cdot (A \cdot A^T)^{-1} A_0} \quad (9)$$

where, $t_{1-\alpha;n-p-1}$ is confidence level (determined from the student distribution table); α is significant level; n is the number of observations; p is the number of model parameters,

s is standard error; A is matrix of actual values of state and impact factors; A^0 is vector of expected values of state and impact factors.

If the conditions (8) are met, the functioning of the SEES is considered satisfactory. Otherwise, optimization is necessary.

6. Optimization procedure. Let the goal at the hierarchy level $L_p, p = 1, \dots, h$ be determined by some indicator ξ_p , which is an element of the set of characteristic descriptions D_g of the object (element) k_p (in this case, the indicator displays the result of the functioning of the SEES on L_p). If an element k_p of a set is an object consisting of a set of objects (that is, it itself is a subset) $k_{p,i,q} \in L_p, i = 1, \dots, N_p$ (N_p is the number of elements of the set at a given hierarchy level) correlated to elements of a set of class names (numbers) $s_q \in S$, then the goal for $\forall k_{p,i,q}$ can be matched to an indicator $\xi_{k,p,q}$, which is an element of a set of feature descriptions $D_{\xi_{k,p,q}}$ of the object $k_{p,i,q} \in L_p, q \in L_p, L_{p,q} \rightarrow s_q \in S$.

We introduce a function $\omega_p \div \xi_{p,q} \rightarrow \xi_p, q = 1, \dots, s$ (s is the number of classes), which will make sense of aggregating goals of $k_{p,i,q}$ at a given hierarchy level p . Then $\xi_p = \omega_p(\xi_{p,q})$ will determine the goal on L_p .

In the simplest case

$$\xi_p = \sum_{q=1}^s \omega_{p,q} \cdot \xi_{p,q} \quad (10)$$

$$F_{v_p}(t) = \frac{\mu \cdot (y_{p,v_p,s_q}^0(t) - \bar{y}_{p,v_p,s_q}^0(t)) + \sum_{i=1}^{(p-1)} \mu_i \cdot (y_{p,i,v_p,s_q}^0(t) - \bar{y}_{p,i,v_p,s_q}^0(t))}{\sum_{i=1}^{(p-1)} \sum_{j=1}^J \omega_{i,j} \cdot |\Delta x_{p,i,v_p,s_q,j}^0(t)| + \sum_{i=1}^{(p-1)} \sum_{u=1}^U \eta_{i,u} \cdot |\Delta z_{p,i,v_p,s_q,u}^0(t)|} \rightarrow \max \quad (12)$$

where, $\mu, \mu_i, \omega_{i,j}, \eta_{i,s}$ are the corresponding weighting coefficients of generalized and partial performance indicators, as well as factors of condition and impact; $y_{p,(p-1),v_p,s_q}(t)$ is determined using formula (11). In this ratio, the increments of factor features will be considered normalized, that is, reduced to the interval from 0 to 1. The numerator is the sum of the differences between the model and normative values for the generalized and particular performance indicators v_p -th the element (class, subsystem) that should tend to the maximum. The denominator is essentially a requirement to minimize the costs of implementing measures aimed at achieving the normative values of the functioning of the complex system under consideration.

In the simplest case, it is possible to optimize particular performance indicators separately, that is, solve problems of linear (if the production functions are linear) and nonlinear programming with constraints. As tools, you can use the "EFRA" software package [50] or the Python [51].

4. MODELS AND DATA

We will also consider SEES as open territorial socio-economic systems with a mixed type of economic relations. The elements of the levels can be grouped within the sectoral [46] and spatial-temporal classifications, the latter of which is based on the system paradigm [45]. Their correspondence is shown in Table 1.

where, $\sum_{q=1}^s \omega_{p,q} = 1$ and $\omega_{p,q}$ is weight function showing priority relative to the goal $\xi_{p,q}$.

Such a formal description makes it possible to identify the goals of modeling the development of SEES for each level of the hierarchy.

Next, we will set the following general problem. It is required to find such values of increments of state factors $\Delta x_{p,(p-1),v_p,s_q,j}(t)$ and impact factors $\Delta z_{p,(p-1),v_p,s_q,u}(t)$ for which the objective function (10) and its components approaches one.

This problem can be reduced to one of the 7 types of multicriteria optimization problems [48], with the difference that instead of the actual values of the result features $y_{p,(p-1),v_p,s_q}(t)$, the value $\tilde{y}_{p,(p-1),v_p,s_q}(t)$ calculated from the data for only that v_p -th element or class is taken:

$$y_{p,(p-1),v_p,s_q}^0(t_0) = [\tilde{y}_{p,(p-1),v_p,s_q}^*(t_0) + \varepsilon_{p,(p-1),v_p,s_q}^*(t_0)]^0 \quad (11)$$

In order to eliminate the need to assign some values to the weighting coefficients in (10) with the help of expert assessments, it is possible to use an extension of the method based on the methodology of the data envelopment analysis (DEA) for socio-economic systems [49] and the Lagrange method, forming the objective function in the following form (when adding the harmony coefficient to the ratio, the corresponding term with the coefficient is added to the numerator):

Table 1. Classification of economic entities

Classes of the economic subsystem	Sections of the economic subsystem
Sector classification	
Commodity aggregate sector	A, B (A); B(C)
Manufacturing aggregate sector	C(D)
Construction aggregate sector	F(F)
Aggregate sector of market services	G(G); I(H); H,J(I); K(J); L,M,N(K)
Aggregate sector of non-market services	D,E(E); O(L); P(M); Q(N); R,S(O)
Spatial-temporal classification	
Object	A (A, B); B(C); C(D); D,E(E)
Environment	I(H); L,M,N(K); O(L); Q(N); R,S(O)
Process	H,J (I); P(M)
Project	F(F); G(G); K(J)

Note: () – NACE Rev. 1.1 sections

The classifications used make it possible to group economic entities operating in accordance with the Russian Classification of Economic Activities (NACE (OKVED)), which was used in the Russian Federation till 2015 [52] into a more general group, thereby making it possible to consider the totality of elements as a subsystem of the SEES and to study the state and functioning of such a subsystem using the same

characteristic descriptions. The sectoral and spatial-temporal classifications differ from each other by grouping the economic entities elements that are part of the economic subsystem of the SEES. The sectoral classification is used to study the specialization of regions, and the spatial-temporal classification is used to assess their system balance of the economy.

The choice of effective signs of the functioning of the SEES was due to the following reasons. The first of them is the informational sufficiency of statistical data. The official publications of the Federal State Statistics Service (Rosstat) for 2007-2018, as well as other statistical information posted on the official website [53], were selected as the information base for the study. Based on this information, the results of the economic activity of the regions are presented in the form of indicators such as the volume of products shipped for sections C, D, E (NACE 1); agricultural products (Section A); the scope of work on the types of economic activity "Construction" (Section F); characteristics of transport, communications, information and communication technologies (ICT), retail turnover, etc. It can be seen that the indicators are heterogeneous and can be used as indicative descriptions when studying individual areas of the functioning of the SEES based on the developed methodology.

Secondly, in order to understand the holistic picture of the results of the functioning of the subjects of the Russian Federation, their economic component requires a generalized indicator, which is the GDP by regions. GDP by regions is used in two variants. The first of them is based on a comparative analysis of the dynamics of GDP by regions in different regions. The second is based on the construction of models linking the volume of GDP by regions (or its components by type of economic activity) and factors characterizing the state and functioning of regions [54-56]. This is the option used in the study. Elements representing a set of economic units – institutional units–residents of the region (in the terminology of the SNA), whose activities correspond to one of the sections of the Russian classifier of

economic activities, are selected as elements of the SEES related to the economic subsystem. Since 2017, the results of the operation have already been presented according to the new version of the NACE rev. 2, which required the coordination of the characteristic descriptions relating to the periods before and after 2017 and bringing them to the same section of the NACE rev. 1.1.

Preliminary factor analysis of the dependencies between the effective signs and factors of the state (conditions of functioning of the elements) and impact factors (signs characterizing the control effects that change the state of the element) allowed us to identify significant (statistically significant) factors from them. Thus, a set of resultative and factorial features was formed, used in the study to assess the results of the functioning of the SEES and its elements.

We will consider the regional level of SEES and renumber the resultative and factor features in accordance with the algorithm presented in 3.1.

Since the characteristic descriptions are the same at different levels of the hierarchy, for convenience, we omit the level indices and the belonging of elements to the same element of a higher level, assuming that the totality of elements – regions (subjects of the Russian Federation) – is subordinated to the district in accordance with the administrative-territorial division.

Let's denote the economic subsystem as *econ*. Then each of the features will be characterized by two indexes: the feature number and the class to which the studied element (region) belongs.

The GDP by regions by economic activities of the economic subsystem is selected as the resultative features of functioning.

As part of the study, we will use sectoral and spatial-temporal classification, their relationship with the sections of the NACE is presented in Table 1. Generalized features will be designated by the class number. The description of the resultative features is presented in Table 2.

Tables 3 and 4 describe the state and impact factors for the economic subsystem.

Table 2. Description of the resultative features of the economic subsystem of the SEES

No.	Variables of the economic subsystem	Description of variables
1	y_{econ}	Generalized (integral) economic indicator
2	$y_1 / y_{1,s1} / y_{1,t1}$	Section A. Agriculture, forestry and fishing (A, B)
3	$y_2 / y_{2,s1} / y_{2,t1}$	Section B. Mining and quarrying (C)
4	$y_3 / y_{3,s2} / y_{3,t1}$	Section C. Manufacturing (D)
5	$y_4 / y_{4,s5} / y_{4,t1}$	Section D. Electricity, gas, steam and air conditioning supply. Section E. Water supply, sewerage, waste management and remediation activities (E)
6	$y_5 / y_{5,s3} / y_{5,t4}$	Section F. Construction (F)
7	$y_6 / y_{6,s4} / y_{6,t4}$	Section G. Wholesale and retail trade; repair of motor vehicles and motorcycles (G)
8	$y_7 / y_{7,s4} / y_{7,t2}$	Section I. Accommodation and food service activities (H)
9	$y_8 / y_{8,s4} / y_{8,t3}$	Section H. Transporting and storage. Section J. Information and communication (I)
10	$y_9 / y_{9,s4} / y_{9,t4}$	Section K. Financial and insurance activities (J)
11	$y_{10} / y_{10,s4} / y_{8,t2}$	Section L. Real estate activities. Section M. Professional, scientific and technical activities. Section N. Administrative and support service activities (K)
12	$y_{11} / y_{11,s5} / y_{8,t2}$	Section O. Public administration and defense; compulsory social security (L)
13	$y_{12} / y_{12,s5} / y_{12,t3}$	Section P. Education (M)
14	$y_{13} / y_{13,s5} / y_{13,t2}$	Section Q. Human health and social work activities (N)
15	$y_{14} / y_{14,s5} / y_{8,t2}$	Section R. Arts, entertainment and recreation. Section S. Other services activities (O)

Note: () – NACE Rev. 1.1; -/-/- – NACE / sector classification / space-time classification; s1 – commodity aggregate sector; s2 – manufacturing aggregate sector; s3 – construction aggregate sector; s4 – aggregate sector of market services; s5 – aggregate sector of non-market services; t1 – object subsystem; t2 – environment subsystem; t3 – process subsystem; t4 – project subsystem; all cost indicators are adjusted for the inflation rate and are given by 2007. The state and impact factors were selected based on the meaningful meaning, as well as on the basis of a preliminary correlation analysis of 309 factors reflected in the statistical collections of Rosstat.

Table 3. Description of variables of the economic subsystem (state factors)

No.	Variables of the economic subsystem	Description of variables	No.	Variables of the economic subsystem	Description of variables
1	The cost of fixed production assets at full accounting value at the end of the year by types of economic activity (FPA) (x_i)		2.4	$x_{4,2}$	Sections D,E (E)
1.1	$x_{1,1}$	Section A (A, B)	2.5	$x_{5,2}$	Section F (F)
1.2	$x_{2,1}$	Section B (C)	2.6	$x_{6,2}$	Section G (G)
1.3	$x_{3,1}$	Section C (D)	2.7	$x_{7,2}$	Section I (H)
1.4	$x_{4,1}$	Sections D,E (E)	2.8	$x_{8,2}$	Sections H, J (I)
1.5	$x_{5,1}$	Section F (F)	2.9	$x_{10,2}$	Sections L, M, N (K)
1.6	$x_{6,1}$	Section G (G)	2.10	$x_{11,2}$	Section P (M)
1.7	$x_{8,1}$	Sections H, J (I)	3	x_2	Average annual population
2	Average annual number of persons employed by types of economic activities (AN) ($x_{9,2}$)		4		Transport
2.1	$x_{1,2}$	Section A (A, B)	4.1	$x_{8,3}$	Passenger turnover of public buses
2.2	$x_{2,2}$	Section B (C)	4.2	$x_{8,4}$	Departure of passengers by public railway transport
2.3	$x_{3,2}$	Section C (D)	5	$x_{13,1}$	Morbidity per 1,000 of population, registered diseases diagnosed in patients for the first time in life

Note: the 1st index is number of the resultative feature, the second number is the number of the factor, except for the average annual population, since it is used in models for several sections of the NACE.

Table 4. Description of variables of the economic subsystem (impact factors)

No.	Variables of the economic subsystem	Description of variables	No.	Variables of the economic subsystem	Description of variables
1	Investments in fixed capital by kinds of economic activities		1.4	$z_{10,1}$	Sections L, M, N (K)
1.1	$z_{3,1}$	Section D (D)	2	Consolidated budget expenditures (by object)	
1.2	$z_{6,1}$	Section G (G)	2.1	z_2	Social policy
1.3	$z_{7,1}$	Section I (H)	2.2	$z_{12,2}$	Education

Note: The 1st index is the number of the resultative feature, the second index is the number of the factor, with the exception of social policy expenditures, since it is used in models for several sections of the NACE.

Within the framework of the study, a power multiplicative functional form of production functions was chosen, since they have proven themselves well for studying processes at the meso-level, to which the regions belong.

$$\hat{y}_i = C_{i,0} \cdot \prod_{j=1}^J x_{i,j}^{C_{i,j}} \cdot \prod_{u=1}^U z_{i,u}^{D_{i,u}} \quad (13)$$

Since factors and resultative features can be expressed in different units of measurement, it is advisable to build models for standardized variables (designation "*") and present them in a linearized form:

$$(\ln(\hat{y}_i))^* = C_{i,j}^* \cdot \sum_{j=1}^J (\ln(x_{i,j}))^* + D_{i,u}^* \cdot \sum_{u=1}^U (\ln(z_{i,u}))^* \quad (14)$$

5. RESULTS

The models constructed according to the data for 2007-2020 for a set of regions (excluding Moscow), as well as models constructed for a set of districts, proved to be adequate (the Fisher criterion was used to analyze the quality of models and the student's criterion to assess the significance of model parameters at the level of 5 %). For sections A, B, K and N, the $\exp(\lambda t)$ component was included in the model, characterizing scientific and technological progress and which indirectly takes into account the innovative component of SEES.

For 17 regions of the Central Federal District and the Central Federal District as a whole, the values of state and impact factors were calculated in two variants. In the first variant, only normative models (\hat{y} -models) were used, based on data for a set of regions of the Central Federal District for 2007-2020. Models were selected from models that were built for different time periods: 2007-2020, 2008-2020, etc. In the second variant is \tilde{y} -models and $\tilde{\tilde{y}}$ -models, the latter of which were built according to the data for Tula region.

The following tools were used: the EFRA software package, which implements the main stages of evaluating the results of the SEES functioning and decision-making within the framework of the proposed multilevel optimization approach and author's python program module integrated in EFRA, using SLSQP-method of Python. Optimization was carried out for those regions where the values of partial and integral performance indicators were less than one.

Optimization was carried out according to individual factors of condition and impact. During optimization, those factors of condition and impact were selected that lead to a greater change in the generalized performance indicator compared to other factors.

In the second variant, the basic constraints imposed on changes in the values of the state and impact factors were deviations from their current (during the estimated period) values by 100 % in one direction or another. That is during the time period t_0 , $\Delta(\cdot)_{p,(p-1),v_p,s_q,j}(t_{-1})$ is changed factors in previous period.

$$\begin{aligned} \Delta x_{p,(p-1),v_p,s_q,j}(t_0) &\in x_{p,(p-1),v_p,s_q,j}(t_0) \\ \pm \Delta x_{p,(p-1),v_p,s_q,j}(t_{-1}) & \\ \Delta z_{p,(p-1),v_p,s_q,u}(t_0) &\in z_{p,(p-1),v_p,s_q,u} \\ \pm \Delta z_{p,(p-1),v_p,s_q,u}(t_{-1}) & \end{aligned} \quad (15)$$

The problem for the district level, the norm was built as the sum of the norms for the regions. The expression is used as the goal function:

$$F_i = \sum_{v_2=1}^{17} (\hat{y}_{2,i,v_2}^0 - [\tilde{y}_{2,i,v_2} + \varepsilon_{2,i,v_2}]^0) \rightarrow \min \quad (16)$$

where, v_2 is the number of region of CFD ($v_2=1..17$); i is the number of performance indicator, corresponding to the NACE section (a total of 14 indicators, Table 2); 2 is the number of the level (level region, $p=2$); "0" is the operation of bringing values to the interval from 0 to 1 after the standardization procedure; \tilde{y}_{2,i,v_2} is the value of the resultative feature calculated according to the i -th model construct according to data for a particular region, ε_{2,i,v_2} is the residual between the actual and calculated \hat{y}_{2,i,v_2} in a given esimated period (2020) before optimization; \hat{y}_{2,i,v_2}^0 is the normalized value of the resultative feature calculated according to the normative model based on data for a set of regions.

The optimization results for Tula region are presented in Appendix A. In addition, during optimization, the restriction was removed on not changing factor characteristics for regions in the case when the value of a particular performance indicator calculated from them is greater than one.

6. DISCUSSION

The developed methodology adds to the studies carried out in the studies by using two types of models [8, 57]. The first of them contains data for a set of objects of the same type (elements, subsystems of regions), the second set contains data for elements and subsystems of only one object (region). This approach is in line with studies [39-41], where the goal is determined a priori. We use models of the first type to determine the standards, the achievement of which is the purpose of the functioning of the SEES elements.

Also, in accordance with the multilevel optimization approach, the optimization model can be included in the SEES management model, which is an extension of the management models [58, 59]. This makes it possible to introduce optimization models into the management process, which will increase the validity of decisions made.

In addition, when solving optimization problems, the proposed algorithm for constructing the objective function extends the DEA method, which complements the study [60]. As a result, the weighting coefficients of the objective function can be found from the solution of the optimization problem, which fits well with the approaches in [61, 62].

Testing of models based on statistical criteria (t-statistical, Fisher criteria, 5%-significant) showed the possibility of their use for evaluation, analysis and decision-making aimed at

improving the target indicators of sustainable development of the Tula region.

The results presented in Table A.1 make it possible to determine how much it is possible to improve the targets selected as estimates of the sustainable development of regions by changing the factors presented in Table A.2. changing factors serve as the basis for the development of appropriate socio-ecological and economic measures.

Analysis of the results of optimization of the economic subsystem for the Tula region (Table A.1) showed that among the 14 areas of economic activity in accordance with NACE, several characteristic groups can be distinguished.

6.1 The first group

The first group includes five areas of activity, the development of which corresponds to the identified general trends for the Central Federal District. These areas include Manufacturing (C), wholesale and retail trade; repair of motor vehicles and motorcycles (G), Accommodation and food service activities (I), Real estate transactions, Professional, scientific and technical activities, Administrative and support service activities (L, M, N), Arts, entertainment and recreation, other services activities (R, S).

Despite the compliance of the industries of the first group with the general trends of economic development of the Central Federal District regions, the results of optimization of economic development indicate that even in these industries it is necessary to increase production efficiency, labor productivity, return on investment (Table A.2). Thus, for Manufacturing (C), it was found that for optimal production volume, excess fixed assets in the amount of 134.378 million rubles are involved in them, investments in the amount of 131.53 million rubles may lead to an increase in production, but this may be accompanied by additional environmental pollution. At the same time, the industry lacks 883 employees.

In the Wholesale and retail trade; repair of motor vehicles and motorcycles section (G), excess capacity in the amount of 65.763 million rubles and a shortage of 175 employees were also identified. In the sphere Accommodation and food service activities (I), in comparison with optimal estimates, a bias was revealed in the excess financing of investments in the amount of 10.135 million rubles, part of which should have been directed to the salaries of staff in the amount of 400 people, who are not enough in the industry to provide high-quality services. At the same time, in the L, M, N sections, an excessive number of employees (500 people) has been identified in comparison with the optimal one. At the same time, the industry should increase the volume of investments in the amount of 24.574 million rubles. As for the provision of R, S sections, there is no need for additional actions.

6.2 The second group

The second group includes five more areas of activity that are developing practically in accordance with general trends for the Central Federal District. This group includes Agriculture, forestry and fishing (A), Mining and quarrying (B), construction (F), Financial and insurance activities (K), Human health and social work activities (Q).

In the field of Agriculture, forestry and fishing (A), the efficiency of the use of fixed assets should be improved, since the excess value of fixed assets in the amount of 28.06 million rubles has been revealed compared to the optimal value. The

industry can additionally attract employees in the number of 31 people. On the contrary, the Mining and quarrying (B) has an excessive number of employees (100 people), which negatively affects labor productivity. In the industry, it is necessary to update fixed assets in the amount of 20.72 million rubles. The situation is similar in the Construction (F). The excess number of employed in the amount of 900 people and the shortage of fixed assets in the amount of 189.086 million rubles were revealed. The need for additional actions has not been identified for the Financial and insurance activities (K), as well as Human health and social work activities (Q).

6.3 The third group

The third group includes four areas of activity that require development adjustments in order to achieve compliance with the general development trends of the Central Federal District regions. This group includes the production and distribution of the Electricity, gas, steam and air conditioning supply, Water supply, sewerage, Waste management and remediation activities (D, E), Transporting and storage, Information and communication (H, J), Public administration and defense, Compulsory social security (O), Education (P).

The industries assigned to the third group require more significant regulatory impacts, since they to a certain extent do not correspond to the general trends of sustainable development characteristic of the Central Federal District regions.

The resultative indicator of the Electricity, gas, steam and air conditioning supply, Water supply, sewerage, Waste management and remediation activities (D, E) does not correspond to the optimal one by 0.045. The optimal value should be achieved by updating fixed assets with an increase in their value by 474 thousand rubles. The number of employees in the industry corresponds to the optimal value. The Transporting and storage, Information and communication (H, J) in the Tula region, according to the simulation results, is the most problematic, its result features differs from the optimal value by 157,292. In the industry, both excess capacity in the amount of 1,349 million rubles and excess number of employed (300 people) compared to the optimal one have been identified. But at the same time, the passenger turnover of public buses by 134.02 million passenger-km and the departure of passengers by public railway transport by 51.787 thousand people are insufficient. compared to the optimal values. For the sphere of public administration and military security; social security (L, M, N) and education (P), due to a slight deviation from the optimal value of the effective attribute (0.0007 and 0.0005, respectively), the need for additional actions was not revealed.

Based on the optimization, several general conclusions can be drawn.

1. In some cases, it is not possible to fully balance the functioning of the SEES.

2. With strong discrepancies between the actual and normative values of the effective features for the selected elements, when optimizing, changes in factors shift to the boundaries of the corresponding system of restrictions.

3. An iterative optimization procedure is required for several cycles of deviation control.

The calculated changes in factors will improve the target indicators of the functioning of the Tula region, which will bring the region closer to the trajectory of sustainable development.

7. CONCLUSION

A methodology for optimizing the results of the functioning of socio-economic systems is presented. Using the example of the regions of the Central Federal District and the district as a whole, the totality of which is considered as a two-level hierarchical socio-economic system, the values of factors that will improve their target indicators are obtained. We have come to the conclusion that with an extensive path of development, while maintaining the current trend of increases in factor characteristics, targets are difficult to achieve for a number of areas of the Central Federal District, and a sharp change in factor characteristics can lead to a loss of stability of the SEES itself. This leads to the conclusion that in order to ensure the sustainable development of the region, it is necessary to step up the use of the available resource base and finances. Such intensification can be carried out through the renewal and modernization of fixed assets; training and advanced training of specialists; optimization of transfers from federal and regional budgets based on the results of the assessment of the functioning of the subjects of the Russian Federation for the reporting period; introduction of modern approaches to management, including using mathematical apparatus and decision support tools.

The presented methodology and the results of the study are limited to a set of resultative and factorial characteristics included in the model, and the available information base of the study, in this case, the regions of the Central Federal District. However, the model can be expanded by including additional indicators and factors that are used in assessing the sustainable development of regions, as well as applying the methodology for subjects of other districts. This is one of the promising directions for the development of the SEES optimization model.

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APPENDIX

Result of optimization of Tula region economic subsystem functioning.

Table A.1. Changes in performance characteristics (economic subsystem)

Model	Type model	Attribute value before optimization	Attribute value after optimization	Attribute gain
A(A, B)	mul	22757,904	22757,904	1,091E-11
C(C)	mul	1422,369	1422,369	1,299E-09
D(D)	mul	113505,045	113505,045	0
D,E (E)	mul	11663,426	11663,471	0,045

F(F)	mul	12516,847	12516,847	7,6538E-06
G(G)	mul	24464,747	24464,747	0
H (I)	mul	1706,843	1706,843	0
H,J (I)	mul	853,421	853,421	9,298E-08
K(J)	mul	19059,744	19217,036	157,292
L,M,N(K)	mul	40110,805	40110,805	0
O(L)	mul	11947,900	11947,900	0,0007
P(M)	mul	8249,740	8249,741	0,0005
Q(N)	mul	13085,795	13085,795	-1,819E-12
R,S(O)	mul	3129,212	3129,212	0

Notes: Model – Section NASE 2 (NASE 1), mul – multiplicative model.

Table A.2. Changes in factor characteristics (economic subsystem)

Factor	Value	Upper bound of factor change (+/-)	Factor change
FRA A (A, B), MRUR	38939,841	28,060	-28,060
AN A (A, B), people	38,100	2,100	0,031
FRA B (C), MRUR	4461,359	509,110	20,723
AN B (C), MRUR	2,700	0,100	-0,100
FRA C (D), MRUR	199973,412	207,605	-134,378
AN C (D), people	160,300	4,000	0,883
Investments C (D), MRUR	21706,308	14678,682	-131,530
FRA D, E (E), MRUR	50519,701	257,827	0,474
AN D, E (E), people	25,800	0,400	0,000
FRA F (F), MRUR	7298,639	591,649	189,086
AN F (F), people	58,000	0,900	-0,900
FRA G (G), MRUR	20610,257	701,625	-65,763
AN G (G), people	124,700	1,500	0,175
AN I (H), people	17,300	0,400	0,400
Investments I (H), MRUR	26,110	13,980	-10,135
FRA (common), MRUR	1071057,737	23747,983	0,001
Average annual population, people	1457,600	14,900	-14,9
FRA H, J (I), MRUR	111474,473	2007,998	-1,349
AN H, J (I), people	58,800	0,300	-0,300
Passenger turnover of public buses, million passenger-kilometers	840,000	291,000	134,020
Departure of passengers by public railway transport, people	2434,000	874,000	51,787
AN L, M, N (K), people	20,200	0,500	-0,500
Investments L, M, N (K), MRUR	291,824	2530,692	24,574
Consolidated budget expenditures (social policy), MRUR	10848,661	935,496	0,003
AN P (M), people	53,500	0,000	0,000
Consolidated budget expenditures (education), MRUR	11921,556	265,521	0,001
Morbidity per 1,000 of population, registered diseases diagnosed in patients for the first time in life, people	741,7	14	0,000

Notes: FRA – the cost of fixed production assets at full accounting value at the end of the year by types of economic activity; AN – average annual number of persons employed by types of economic activities; NACE2 (NACE 1).