MODELING OF THE SEDIMENTATION PROCESS OF MONODISPERSE SUSPENSION

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ABSTRACT

During coagulation, particles with a flocculent shape and an irregular structure are formed in water. In the model sedimentation water presented in this study, three fractions of particles were distinguished and the method of calculating the sedimentation rate presented. Each fraction sediments in a different way due to different forces acting on particles of different shapes. The particles of fraction I are similar in shape to spherical particles, the particles of fraction II are non-spherical particles, and the particles of fraction III are porous agglomerates, for which the formula for liquid flow through a porous bed has been adopted. For the proposed theoretical sinking model for three types of suspensions composed of particles of each fraction, experimental tests were carried out, which confirmed the model predictions. The results for the sedimentation velocity of the monodisperse suspension of fractions I, II, and III are very similar to predictions: for fraction I, the discrepancy between theoretical and experimental results was 9%; for fraction II, 11%; and for fraction III, 17%. This proves a correctly selected methodology, and the proposed model can be used to calculate the sedimentation velocity of monodisperse suspensions of various shapes.

Keywords: flocculation, monodisperse suspension, sedimentation modeling.

1 INTRODUCTION

There is a growing interest in the use of geothermal water for technological purposes. The main limitation is the high content of iron in these waters. The iron compounds form a deposit on devices and often reduce their efficiency. Removal of iron compounds from water is typically done via chemical reaction in which iron forms sparingly soluble compounds, which are then removed in the process of sedimentation or filtration [2]. The production of this type of compounds takes place through saturation, changing the pH of the water or coagulation [1, 3]. Depending on the method of iron removal, solid particles that differ significantly in structure and shape are formed. As a result of changes in the pH of the water, iron forms solid particles with a size of several dozen μm, similar in shape to a sphere with a clearly porous structure (Fig. 1). They form a monodisperse mixture in which the particles sediment along rectilinear paths with a small number of collisions or aggregation of particles into larger agglomerates.

The addition of a coagulant causes small spherical particles to combine into larger particles of irregular shape, often oblong, which causes them to sediment along curvilinear paths (Fig. 2). The coagulant causes the formation of ions which flocculate the particles and form larger agglomerates. In continuous agitation flow devices, particles of this type reach a certain limit size above which, as a result of external forces, breakdown of larger agglomerates occurs. In flow devices, they form a monodisperse mixture.

During sedimentation, large agglomerates form in devices with standing liquid and settle to the bottom of the tank as sediment. Such agglomerates have macropores through which

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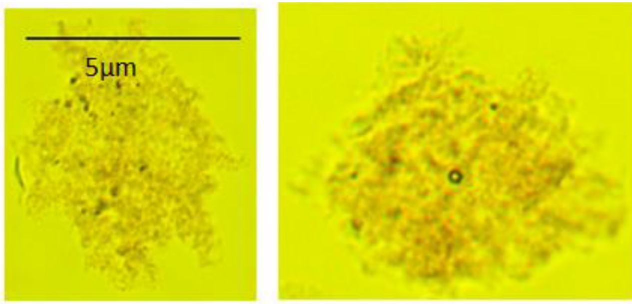

Figure 1: Particles formed by changing the pH of the water.

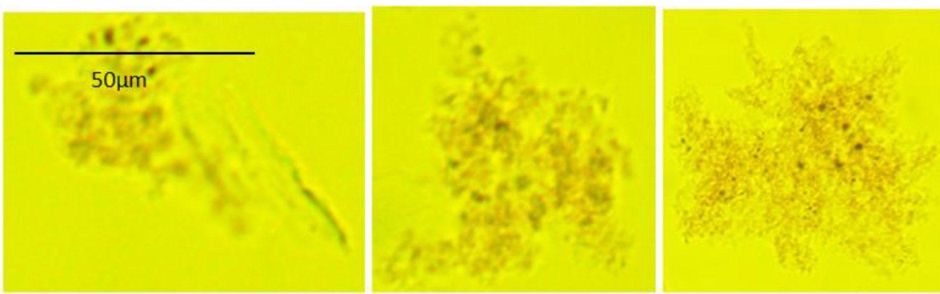

Figure 2: Particles formed by coagulation in flow devices.

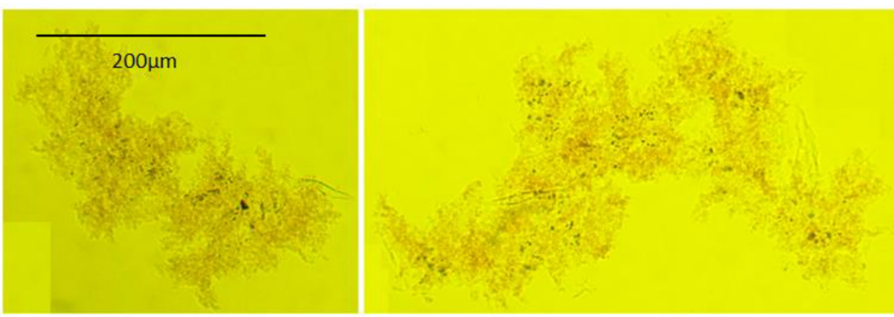

Figure 3: Particles formed by coagulation in still liquids.

water can flow (Fig. 3). The network of macropores creates a kind of network of channels through which the liquid can flow while the particle falls. This stabilizes the direction of sedimentation of the particles that fall along rectilinear paths.

The method of free fall of solid particles and the velocity with which they sediment largely depend on their shape. This requires the use of different mathematical models to calculate the sedimentation rate of this type of suspensions.

2 THE SEDIMENTATION PROCESS OF MONODISPERSE SUSPENSIONS

A typical course of the sedimentation process for a monodisperse suspension is shown in Fig. 4. The process begins when a homogeneous mixture is present in the entire volume. After some time, a layer of clean liquid forms in the upper part of the tank, while solid particles settle in the lower part, forming a layer of sediment. The middle part is the suspension. With time, the concentration of particles in the middle zone may increase. Over time, the volumes of the individual zones change, thus creating a boundary between the zones. This border does not have to be clear, often the border is blurred. The time shift of the interface is usually nonlinear. The interface curve is called the sedimentation curve. In turn, the curve defining the increase of the boundary between the sludge and the mixture is called the sludge growth curve. As shown in Fig. 4, after some time, the sedimentation curve and the sludge growth curve reach the same value, while the middle zone is completely reduced. From that moment on, the sludge thickens in accordance with the sedimentation curve.

The sedimentation rate is greatest in the initial phase of the process. During this time, the particles are compacted in space to the height of the sedimentation curve. This increases the concentration of particles in the suspension. The conclusion is that the concentration of particles has a direct influence on the sedimentation rate. This is a major difference compared to the free fall of solids, where the rate of sedimentation is constant over time.

The first mathematical models of sedimentation velocity were based on a modification of Stokes's law of the free fall of solids. The first modification was made by Robinson [4], and then in 1944, Steinour [5] proposed the formula that has been used until now.

$$v_s = \frac{d_e^2\left(\rho_s - \rho_l\right)g}{18\mu_Z}\left(1-\varnothing\right)^2\ 10^{-1.82\,\varnothing}, \tag{1}$$

where v_s is the sedimentation velocity, d_e is the equivalent particle diameter, ρs is the density of material, ρl is the liquid density, μZ is the dynamic viscosity of the suspension, g is the gravitational acceleration, and Ø is the concentration.

The equivalent particle diameter d_e, is the diameter of a sphere with a volume equal to that of the particle. To calculate the sedimentation velocity, it is necessary to know the suspension viscosity μZ. There are many models in the literature to determine the viscosity of the mixture [6].

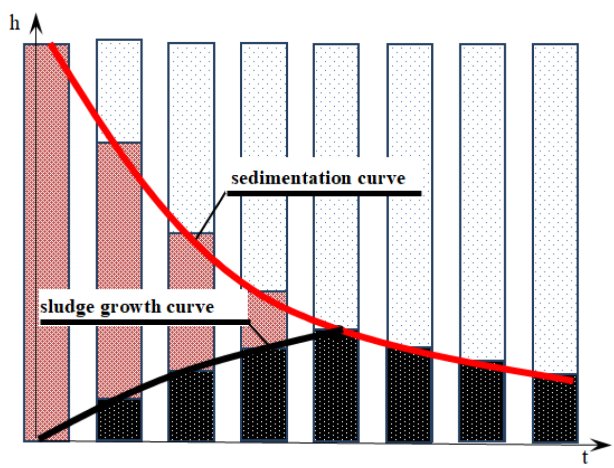

Figure 4: The sedimentation process of the monodisperse mixture.

Cellular models are another popular solution [7, 8]. They assume that model cells are evenly distributed in the liquid. Each of the cells consists of a fluid with solid particles inside it. This model takes into account the volume fraction of solid particles in the volume of the fluid. The basic equation describing the relationship between the velocity of free-falling particles and the sedimentation velocity is as follows:

$$v_s = v_o \frac{1}{1 + n'\phi^{1/3}}, \tag{2}$$

where is the free-fall velocity of a single particle and n' is the coefficient depending on the type of suspension.

The value of n' equal to 1.5 was initially proposed by Happel and Epstein [9], but Barnea and Mizrahi [7] specified that this value should be in the range of 1–2.1, depending on the type of suspension.

Moreover, Barnea and Mizrahi [7], based on the analysis of the literature, stated that the existing solutions assumed that:

- The suspension is monodisperse and homogeneous throughout its entire volume.
- The particles are spherical.
- There is no interaction between the molecules other than hydrostatic.
- The particles do not disintegrate and do not agglomerate when moving.
- The influence of the vessel side walls is negligible.
- The flow is Newtonian.
- Each particle moves under the influence of the forces acting directly on it.

They found that increasing the concentration of a solid in the suspension causes hydrostatic and viscous effects of the liquid stream and the frictional effect. After taking into account the above factors, they presented their own corrected cell model of the sedimentation process. The sedimentation velocity is calculated from this model on the basis of the relationship:

$$v_s = v_o \frac{(1-\phi)^2}{\left(1+\phi^{1/3}\right) exp\left[\frac{5 \bullet \phi}{3 \bullet (1-\phi)}\right]}. \tag{3}$$

In many solutions, it is necessary to know the free-fall velocity of the particles. The method of free fall of solid particles and the velocity with which they sediment largely depend on their shape. In terms of shape, the particles can be divided into spherical (Fig. 1), non-spherical (Fig. 2), and porous agglomerates (Fig. 3).

3 FREE-FALL MODELS

The velocity of free fall of a single particle with a shape similar to a sphere can be determined on the basis of the balance of forces acting on the particle during its movement (Fig. 5). The main forces acting on such a particle are the force of gravity and the drag force of the medium.

The force of gravity F_g is defined by the Archimedes law:

$$F_g = (\rho_s - \rho_c) g V_s (1 - \varepsilon_S), \tag{4}$$

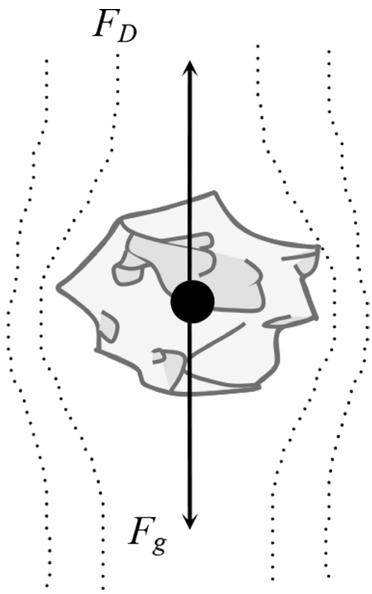

Figure 5: Forces acting on a spherical particle sediment in a fluid.

where ε*s* is the porosity of material and V_s is the volume of particle.

The general relation to the drag force *FD* is given by the equation:

$$F_D = C_D A_s \rho_c \frac{v_o |v_o|}{2} \tag{5}$$

where *As* is the particle area and *CD* is the drag coefficient.

The shape of particles moving in a liquid has a direct impact on the velocity of its sedimentation. It is expressed by the drag coefficient *CD*. Correctly determining the value of the drag coefficient is not easy, because it depends on many factors. Since the sedimentation velocities of the particles are very low, the relationship is used to determine the drag coefficient [10]:

$$C_D = \frac{24}{Re}. \tag{6}$$

This solution is valid for Reynolds numbers less than 0.3. The Reynolds number is calculated for the equivalent particle diameter:

$$\mathrm{Re} = \frac{v_o d_e \rho_c}{\mu_L}, \tag{7}$$

where *μL* is the dynamic viscosity of the liquid.

From the balance of forces acting on the falling particle and the dependence on the drag coefficient, a formula for the velocity of a free-falling particle with a shape similar to a sphere can be derived, taking into account the porosity of the material:

$$v_o = \frac{(\rho_s - \rho_c) g d_e^2 (1 - \varepsilon_S)}{18 \mu_L}. \tag{8}$$

Non-spherical particles rotate around their axis during movement. This creates a Magnus *FM* force that acts perpendicular to the direction of motion. For low velocities of movement, the irregular shape of the particles causes the particles to flow in a direction transverse to the falling direction. These forces cause the particles to move in a spiral path. The irregular shape of the particles means that the velocity of rotation of the particle about its axis is not constant. Hence, the particles sediment along an irregular path. The influence of the lateral force causes the particle to fall not along a rectilinear path. Hence, the sense of the resistance force will change (Fig. 6). For this reason, the average sedimentation velocity *vo* is most often determined.

For low velocities of motion, the Magnus force may be negligible, but the irregular shape of the particles causes their motion to be curvilinear anyway. The reason is that the particle skids or rolls in a direction transverse to the vertical axis. A mathematical description of all the phenomena that occur during such a movement is impossible in practice, due to the fact that the shapes of the particles are too different. In practice, the Magnus coefficient is often determined *CM*, which is an average value determined experimentally. It takes into account both the influence of particle rotation and the slippage caused by the irregularity of the particle shape.

For such a model, the vertical sedimentation velocity of the particle depends on the balance of forces acting on it. Since the individual forces act in different directions, the descent rate is mainly influenced by two forces. The drop drag force *FD* which is the resultant of the drag force *Fv* and the Magnus force *FM* is calculated as follows:

$$F_D = \sqrt{F_v^2 + F_M^2}. \tag{9}$$

In order to calculate the value of the drag force for non-spherical particles, it is necessary to know the drag coefficient *CD*. Calculating the drag coefficient for non-spherical particles is not easy [11, 12, 13]. Similarly, to calculate the Magnus force, it is necessary to know the Magnus coefficient, which depends on the angular velocity of the rotating particle [12, 14, 15]. The drag force can be expressed using the relationship:

$$F_D = \frac{\pi d_e^2}{4} \rho_c \frac{{v_o}^2}{2} \sqrt{C_D^2 + C_M^2}. \tag{10}$$

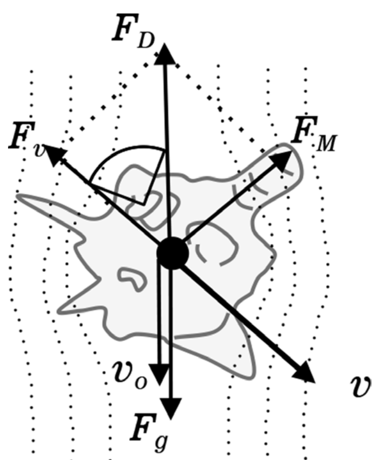

Figure 6: Forces acting on a non-spherical particle falling in a fluid.

Gravity is calculated from formula (4). Thus, on the basis of the balance of gravity *Fg* and the drag force of the medium *FD*, the sedimentation velocity of the particles is calculated:

$$v_o = \sqrt{\frac{4}{3}\frac{(\rho_s - \rho_c) g d_e (1-\varepsilon_S)}{\rho_c \sqrt{C_D^2 + C_M^2}}}. \tag{11}$$

In practice, the determination of the *CM* coefficient for particles of various shapes is often impossible. Also, determining the average rotational velocity is very difficult due to the lack of simple measurement methods allowing this type of measurement. Hence, a practical solution is to determine the total $CD^2 + CM^2$ coefficient experimentally for the selected sedimentation process. Due to this approach, the value of this coefficient will also include the influence of other phenomena accompanying the nonlinear particle sediment.

The third type of particles is agglomerates, which are formed as a result of joining together from a few to a dozen non-spherical particles. Their characteristic feature is large closed spaces, in the form of macropores, on which a complex resistance force acts (Fig. 7).

The sedimentation velocity of such particles is influenced by the force of gravity and the drag force caused by the flow of the particle by the liquid and the force related to the drag of the liquid that flows through the porous bed *Fp*. The force of gravity of the *FG* is determined similarly as shown by the dependence (4), taking into account the porosity of the agglomerate.

$$F_g = (\rho_s - \rho_c) g V_s (1-\varepsilon_S)(1-\varepsilon). \tag{12}$$

Since the liquid flows through the macropores, it encounters some drag, it should also be taken into account in the drag force, which is influenced by the drag force associated with flowing around the particle *FD* less the area of the pores through which the liquid flows and the force related to the drag of liquid flow through the macropores *Fp*. The drag force associated with the flow of the particle through the liquid is calculated from the relationship:

$$F_D = C_D A_s \rho_c \frac{v_o |v_o|}{2}(1-\varepsilon). \tag{13}$$

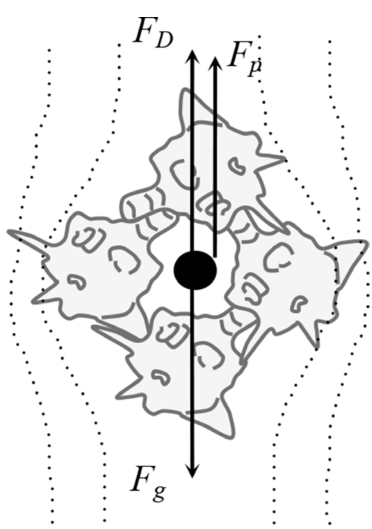

Figure 7: Forces acting on the agglomerate.

The drag force that arises as a result of the flow of liquid through channels formed by macropores can be determined on the basis of the Darcy–Weisbach equation, on the basis of which the relationship for a porous bed was derived, called the Leva equation [16]:

$$\Delta p = \lambda \frac{h}{d_{eII}} \frac{\rho_l v_o^2}{2} \frac{(1-\varepsilon)^{3-n}}{\varepsilon^3} \psi^{3-n}. \tag{14}$$

Falling agglomerates are treated as separate porous beds, therefore the height of such bed is treated as the height of a cylinder with a base diameter equal to the diameter of the equivalent agglomerate and a volume equal to the volume of a sphere with a diameter equal to the diameter of the equivalent agglomerate.

$$h = \frac{2}{3} d_e. \tag{15}$$

The shape factor ψ determines how many times the surface of all particles is greater than the surface of the spheres in the number equal to the number of particles. For agglomerates formed in various physicochemical processes, this factor should be determined experimentally.

$$\psi = \frac{6A_f}{n\pi d_e^2}. \tag{16}$$

For particles for which the calculated Reynolds number is lower than 10, the value of the drag coefficient λ is calculated from the formula (17) and the Leva exponent $n = 1$:

$$\lambda = \frac{400}{Re}. \tag{17}$$

The pressure drop that occurs during the flow of fluid through the porous bed is the result of the drag force. According to the definition of pressure, this force can be expressed by the equation:

$$F_p = \Delta p \frac{\pi d_e^2}{4}. \tag{18}$$

Taking the above relationships into account, the formula for the drag force for fluid flow through the agglomerates takes the form:

$$F_p = \frac{\pi}{8} \lambda \frac{d_e^3}{d_{eII}} \rho_l v_o^2 \frac{(1-\varepsilon)^2}{\varepsilon^3} \psi^2, \tag{19}$$

where *deII* is the average equivalent diameter of the particles from which the agglomerate was formed.

Since the sedimentation velocity of the particles is very low, the movement of the liquid between the pores of the agglomerate will be laminar. Thus, eqn (19) is correct for agglomerates with high porosity. Following the example of Kozena's solution [17], which stated that the free flow of liquid between the pores in a porous bed occurs for a porosity greater than 0.5, it can be assumed that the falling velocity can be calculated from the equation for the balance of forces acting on the particle for particles with porosity $\varepsilon > 0.5$. In the case of particles with lower porosity, eqn (10) should be used treating the particle as a compact volume in which there is no flow between the pores. Based on the balance of forces described by eqs (12), (13), and (19), the formula for the agglomerate falling velocity takes the form:

$$v_o = \frac{(\rho_s - \rho_l) g (1-\varepsilon_s) d_e}{\frac{18\mu_L}{d_e} + \frac{300}{d_{eII}} \frac{(1-\varepsilon)^2}{\varepsilon^3} \psi^2}. \tag{20}$$

4 COMPARISON OF MODEL AND EXPERIMENTAL RESULTS

In order to verify the adopted models of calculating the free fall of particles on a laboratory stand, the sedimentation velocity was measured for three types of particles of the post-coagulation mixture. The tests were carried out for water of density ρc = 1006.18 kg/m^3 i dynamic viscosity μL= 0.000804 Pas. The density of the material from which the solid particles are made was ρs = 2127 kg/m^3. The average diameter of the spherical particles *deI* is 0.0000047 m, and their porosity εs is 0.28. For non-spherical particles, the average diameter *deII* is 0.000057 m, and coefficient ($\sqrt{C_D^2 + C_M^2}$) is 4882. Average equivalent diameter of the agglomerate *deIII* is 0.0002 m, porosity ε is 0.62, and the experimentally determined shape factor ψ is 1.1.

An exemplary post-coagulation suspension sedimentation process was carried out on a laboratory stand. In order to ensure the repeatability of the tests, coagulation was performed for model water. Model water was prepared by dissolving a specific salt content i.e. 8 g/dm^3 of sodium chloride NaCl, 4 g/dm^3 of sodium sulfate Na_2SO_4, and 0.05 g/dm^3 of iron (II) $FeSO_4$ sulfate in distilled water. The pH was adjusted with sodium hydroxide NaOH to the range 7.9–8.1. In order to precipitate iron, a highly basic, pre-hydrolyzed aluminum coagulant called Flokor 1,2A with the general formula $Al_m(OH)_{3m-1}Cl \cdot H_2O$ was used. It is highly effective and does not require high doses. It contains aluminum ions from 9% to 13% in the form of monomers and polymers [1, 3]. A coagulant dose of 60 g/m^3 was used. The sedimentation velocity was determined based on the observation of the fall of the zone boundary (Fig. 8). As zone falls should be performed for a monodisperse mixture, only pH correction was used for fraction I. This solution causes the fraction to precipitate and, however, its flocculation is marginal. As a result, the sedimentation process concerns a mono-dispersion mixture consisting of particles of the first fraction. In order to prepare a mixture mainly con-

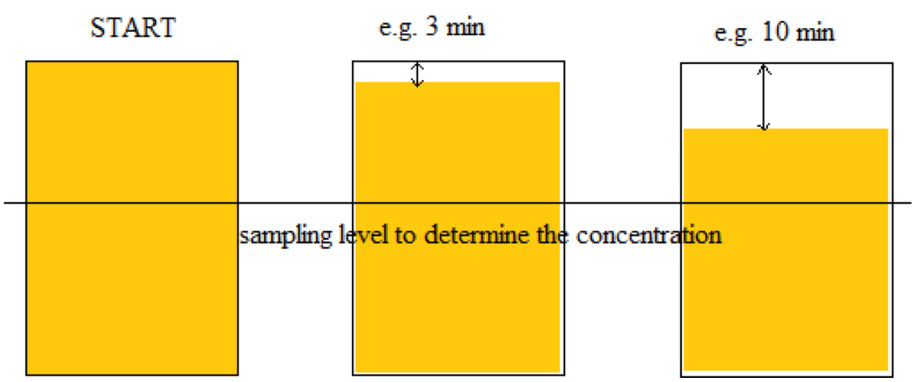

Figure 8: Determination of the sedimentation velocity of monodisperse suspensions in an experimental way.

Table 1: Comparison of the results of the calculations and the experiment.

		Experiment		Calculations	
Fraction	***de* [m]**	***vs* [m/s]**	***vo* [m/s]**	***vs* [m/s]**	***vo* [m/s]**
I	0.0000048	0.0000089	0.000013	0.0000083	0.000012
II	0.000057	0.00025	0.00034	0.00025	0.00035
III	0.0002	0.00015	0.00019	0.00016	0.00019

sisting of fraction II, a coagulant was introduced into the water, which was mixed quickly, and then the liquid movement was reduced by introducing a mechanical plate damper into the vessel. During rapid mixing, mainly fraction II is formed, which, due to the high velocity of the liquid, does not flocculate into fraction III. Quick stop of mixing causes mainly fraction II to remain in the vessel, which only sediments with marginal flocculation. In order to prepare a mono-dispersion mixture consisting of fraction III particles, a full de-ironing process was performed by adjusting the pH, adding the coagulant followed by rapid mixing followed by slow mixing. After stratification occurred, it was possible to separate fraction III, which was reintroduced into the control vessel where the sedimentation velocity was measured.

In addition, at the time intervals in which the height by which the suspension fell in the vessel at the sampling level was measured, a sample was taken to determine the fraction volume fraction by the microscopic method. The equivalent diameters and the particle porosity measurement resulted from the measured approximately one hundred particle diameters of each fraction. Often, more than one particle was present in the microscopic photos, and the diameter was calculated for each of them. In the case of determining the experimental velocities of free fall and sedimentation, 10 repetitions for each velocities were analyzed.

The results of tests and calculations are summarized in Table 1. The value of the particle free-fall velocity *vo* was calculated on the basis of the formula (2) after its transformation to the form in which the volume fraction and the measured sedimentation velocity are known. The values of the n' coefficient were adjusted to the type of fraction: n' = 2.1 for fraction I, n' = 1.5 for fraction II, and n' = 1 for fraction III free falling of the particles of selected fractions. On this basis, theoretical sedimentation velocities from eqn (2) were calculated.

When comparing the obtained results (Table 1), a satisfactory convergence of the experiment was found with the experiment. This proves the correct development of models of particle fall and the sedimentation process.

5 CONCLUSIONS

The presented method of determining the sedimentation velocity of post-coagulation particles has been experimentally verified. The results are satisfactory. The differences between the theoretical and experimental results amount to about 10%, which in this type of processes is sufficient for the process needs. The presented formulas for calculating the sedimentation rate of monodisperse suspensions can be applied to other suspensions with particles of various shapes. The presented work presents averaged research results. As the particles formed in the coagulation process differ to some extent from each other, it may be interesting to study the distribution of the particle population. The results of such research will be the subject of further studies.

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