DETERMINISTIC-STOCHASTIC BOUNDARY ELEMENT MODELING OF THE BRAIN AND EYE EXPOSED TO HIGH-FREQUENCY RADIATION

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ABSTRACT

The paper reviews the influence of the variability in the morphology and the tissue properties of the human brain and eye, respectively, exposed to high-frequency (HF) radiation. Deterministic-stochastic modeling enables one to estimate the effects of the parameter uncertainties on the maximum induced electric field and Specific Absorption Rate (SAR). Surface Integral Equation (SIE) scheme applied to the brain exposed to HF radiation and hybrid boundary element method (BEM)/finite element method (FEM) scheme used to handle the eye exposure to HF radiation are discussed.

Furthermore, a simple stochastic collocation (SC), through which the relevant parameter uncertainties are taken into account, is presented. The SC approach also provides the assessment of corresponding confidence intervals in the set of obtained numerical results. The expansion of statistical output in terms of the mean and variance over a polynomial basis (via SC approach) is shown to be robust and efficient method providing a satisfactory convergence rate. Some illustrative numerical results for the maximum induced field and SAR in the brain and eye, respectively, are given in the paper, as well.

Keywords: boundary integral equations, deterministic modeling, human exposure to electromagnetic fields, stochastic modeling.

1 INTRODUCTION

Human exposure to artificial electromagnetic fields has raised many questions regarding potential adverse effects [1], particularly for the brain and eye exposure to high-frequency (HF) radiation. The assessment of HF exposure is based on the evaluation of specific absorption rate (SAR) distribution and related temperature rise in a tissue. As a measurement of fields induced in the body is not possible, human exposure assessment is carried out via sophisticated computational models [2–5]. Contrary to the simple canonical models used in the 60s and 70s (plane slab, cylinders, homogeneous and layered spheres and prolate spheroids [6]) modern realistic, anatomically based computational models comprising cubical cells are mostly related to the use of the Finite Difference Time Domain (FDTD) methods [7]. The Finite Element Method (FEM) and Boundary Element Method (BEM) are generally used to a somewhat lesser extent [3, 8].

One of the significant difficulties arising in the area of computational bioelectromagnetics is the appreciable variation of the input parameter set, i.e. possible differences in individual size and age (morphology), or the general variability of permittivity and conductivity, due to difference in age or sex. The uncertainty of the input parameters set eventually leads to the

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uncertainty of dosimetric model outputs such as induced electric field and SAR. Thus, one of the key challenges that numerical dosimetry faces today is the quantification and the treatment of these uncertainties.

A novel approach to tackle this problem is to use so called stochastic dosimetry [9, 10], combining deterministic electromagnetic techniques with certain statistical methods.

The present paper deals with the influence of the variability in the morphology and the tissue properties of the brain and eye, respectively, to the related SAR due to the exposure to HF electromagnetic fields. Stochastic-deterministic modeling provides a satisfactory theoretical basis for estimating the effects of the corresponding uncertainties on the maximum induced local and average SAR, respectively. An efficient BEM scheme to treat the brain exposure and hybrid BEM/finite element method (FEM) used to handle the eye exposure are considered. Having completed the deterministic modeling a simple stochastic collocation (SC) formalism is applied to accurately account for uncertainties and to assess confidence intervals in the set of obtained numerical results.

2 FORMULATION: DETERMINISTIC AND STOCHASTIC APPROACH

The main task of HF dosimetry is to quantify thermal effects, i.e. to assess the level and distribution of the electromagnetic energy absorbed by the body. The main dosimetric quantity for quantifying the effects of HF radiation is the SAR and related temperature increase.

2.1 Specific absorption rate

The *SAR*, a fundamental quantity in HF dosimetry, is defined in terms of the rate of energy *W* absorbed by, or dissipated in the unit body mass *m*:

$$SAR = \frac{dp}{dm} = \frac{\sigma}{2\rho}\left|E^{ind}\right|^2 \cdot \tag{1}$$

where *P* is the dissipated power, E^{ind} is the peak value of the electric field induced inside a tissue, ρ is the tissue density and s is the tissue conductivity.

In addition to incident field parameters, the absorbing and reflecting effects of the environment, as well as the properties of the exposed body, also significantly influence the *SAR* distribution. Maximal values of *SAR* in the body are induced when the electric field is oriented parallel to the longitudinal body axis.

2.2 Deterministic modeling

The lossy dielectric model of the brain exposed to HF electromagnetic fields is based on the surface integral equation (SIE) formulation which could be derived from the equivalence theorem and by forcing the corresponding interface conditions for the electric and/or magnetic field, [11, 12] as indicated in Fig. 1.

The lossy homogeneous object representing the brain is excited by the incident electromagnetic field characterized by $\vec{E}^{inc}$ and $\vec{H}^{inc}$.

Performing some mathematical manipulations, the following set of coupled SIEs is obtained [11, 12]:

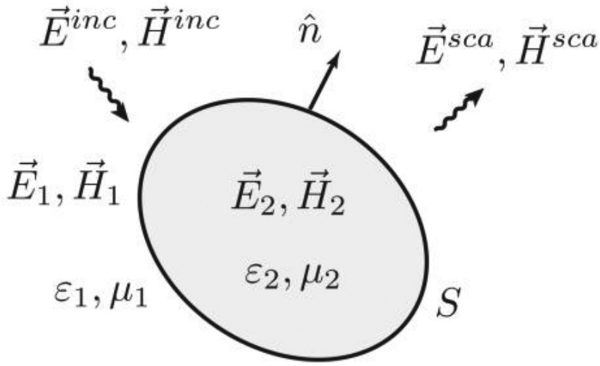

Figure 1: The lossy homogeneous dielectric brain model.

$$
\begin{aligned}
& j\omega\mu_n \iint_S \vec{J}\left(\vec{r}'\right) G_n\left(\vec{r},\vec{r}'\right) \mathrm{d}S' - \\
& -\frac{j}{\omega\varepsilon_n} \iint_S \nabla'_S \cdot \vec{J}\left(\vec{r}'\right) \nabla G_n\left(\vec{r},\vec{r}'\right) \mathrm{d}S' + \\
& + \iint_S \vec{M}\left(\vec{r}'\right) \times \nabla' G_n\left(\vec{r},\vec{r}'\right) \mathrm{d}S' = \begin{cases} \vec{E}^{inc}, & n=1 \\ 0, & n=2 \end{cases} \cdot
\end{aligned} \tag{2}
$$

where E^{inc} stands for the incident electric field illuminating the biological tissue, $\vec{J}$ and $\vec{M}$ are equivalent electric and magnetic current density, respectively, G_n is the interior/exterior Green function given by [11]:

$$
G_n\left(\vec{r},\vec{r}'\right) = \frac{e^{-jk_n R}}{4\pi R}; \quad R = \left|\vec{r} - \vec{r}'\right| \cdot \tag{3}
$$

and R is the distance from the source to observation point, respectively, while k_n denotes the wave number of a medium n.

The set of integral equations (2) is solved by means of an efficient BEM scheme reported in [11, 12].

Plane wave incidence on the corneal part of the eye representing an unbounded scattering problem can be formulated via the Stratton-Chu formulation, i.e. the time-harmonic electric field in the exterior domain is expressed by the following boundary integral equation [3]:

$$
\alpha \vec{E} = \vec{E}_i + \oint_{\partial V'} \vec{n}' \times (\nabla' \times \vec{E}) G dS' + \oint_{\partial V'} \left[\left(\vec{n}' \times \vec{E}\right) \times \nabla' G + \left(\vec{n}' \cdot \vec{E}\right) G \right] dS' \tag{4}
$$

where E_i stands for the incident electric field, $\vec{n}$ is an outer normal to surface $\partial V'$. bounding the volume V representing the eye and α is the solid angle subtended at an observation point, while G represents the fundamental solution of the corresponding Helmholtz equation [9].

Mathematical details for hybrid BEM/FEM are available in [3, 13].

2.3 Stochastic analysis

Once the deterministic modeling via BEM and BEM/FEM, respectively is carried out a stochastic post-processing of the obtained numerical results arising from the deterministic model can be performed via stochastic Collocation (SC) technique by simply choosing one or more random variables depending on the problem of interest. It is worth noting that stochastic analysis is found to be rather useful in many cases with various uncertainties in the input data set, such as wire structures in complex environment, grounding systems in a rocky terrain, ground penetrating radar (GPR) or particularly in bioelectromagnetics. The need for stochastic analysis is quite obvious in the case of bioelectromagnetics as one hardly could determine the conductivity and/or dielectric constant of the brain or eye precisely.

The fundamental principle of SC technique is to use the polynomial approximation of the output *E* of interest for N given random parameters. The random parameter Z is defined as [9, 10]:

$$Z = Z^0 + \hat{u} \tag{5}$$

where Z^0 is the initial (mean) value, while û is the random variable (RV) with the assigned statistical distribution.

The function $t \to E\left(Z^0, t\right)$ is expanded over a stochastic space using the Lagrangian basis functions set [9, 10]:

$$E\left(Z^0, t\right) = \sum_{i=1}^{n} E_i\left(Z^0\right) L_i\left(t\right) \tag{6}$$

where $E\left(Z^0, t\right)$ is the output of interest (electric field in this work), while Lagrange polynomial L_i(t) are defined, as follows:

$$L_i\left(t\right) = \prod_{j=1, j\neq i}^{n} \frac{\left(t - t_j\right)}{\left(t_j - t_i\right)} \tag{7}$$

Exploiting the property of the Lagrangian basis, yields:

$$E\left(Z^0, t_i\right) = E_i\left(Z^0\right) \tag{8}$$

According to the statistical definition for the mean μ [7],

$$\mu = \left\langle E\left(Z^0, t\right)\right\rangle = \int_D E\left(Z^0, u\right) p\left(u\right) du \tag{9}$$

where *p(u)* denotes the probability density function of RV û from (6) and *D* denotes the random variable domain, the expected value < > of the considered output *E* taking into account (6) and (8) can be written, as follows:

$$\left\langle E\left(Z^o, t\right)\right\rangle = \sum_{i=1}^{n} E_i\left(Z^0\right) \int_D L_i\left(u\right) p\left(u\right) du \tag{10}$$

Expression (10) can be written in the form:

$$\left\langle E\left(Z^0,t\right)\right\rangle = \sum_{i=1}^{n} E_i\left(Z^0\right) w_i \tag{11}$$

where w_i is given by integral:

$$w_i = \int_D L_i\left(u\right) p\left(u\right) du \tag{12}$$

Furthermore, according to statistical definition the variance σ_{var}^2 is defined by following expression:

$$\sigma_{\text{var}}^2 = \int_D \left[E\left(Z^0,u\right) - \left\langle E\left(Z^0,t\right)\right\rangle \right]^2 p\left(u\right) du \tag{13}$$

which, after performing some mathematical manipulations, can be written, as follows:

$$\sigma_{\text{var}}^2 = \sum_{i=0}^{n} w_i E_i^2\left(Z^0\right) - \left(\sum_{i=0}^{n} w_i E_i\left(Z^0\right) \right)^2 \tag{14}$$

The order (n-1) of approximation, i.e. the convergence, depends on the number of chosen collocation points. The computation of integral (12) is based on Gaussian quadrature.

The stochastic analysis is presented for the case of single RV but it can be easily extended to the case of N RVs [9, 10]. Also, in addition to the mean, other higher statistical moments can be readily determined for the case of one-dimensional and multidimensional RV case [10].

Further mathematical details for SC procedures can be found elsewhere, e.g. in [9, 10].

3 COMPUTATIONAL EXAMPLES

The first set of Figures is related to the results obtained from deterministic brain modeling.

Figures 2 and 3 show the *SAR* distribution in the brain at f = 900 MHz and f = 1,800 MHz, respectively, for the case of vertical polarization. The power density of the incident plane wave is P = 5 mW/cm^2. The brain parameters of interest are presented in Table 1.

The solution is carried out via the BEM by discretizing the brain to 696 triangular elements and 1,044 edge-elements.

The obtained peak and average *SAR* values for vertical polarization at f = 900 MHz and f = 1,800 MHz are given in Table 2.

The obtained peak *SAR* values in the brain do not exceed the ICNIRP limits [14] for localized *SAR* in the head averaged per 10 g of tissue (10 W/kg for the occupational exposure).

However, the exposure limit for the general public exposure limit (2 W/kg localized in the head and trunk) has been exceeded at f = 1,800 MHz.

Fig. 4 shows the SC convergence with 3, 5 and 7 points for the SAR value calculations and different random variables (RV_k; k=1,. . . ,5).

The dimensions of the average adult human brain are, as follows: width 131.8 mm, length 161.1 mm, height 139 mm, while the frequency dependent parameters of the human brain are discussed in [12]. Furthermore, the value for the relative permittivity and the electrical conductivity of the brain are: ε_r = 45.805 and σ = 0.766 S/m, respectively. Each RVk

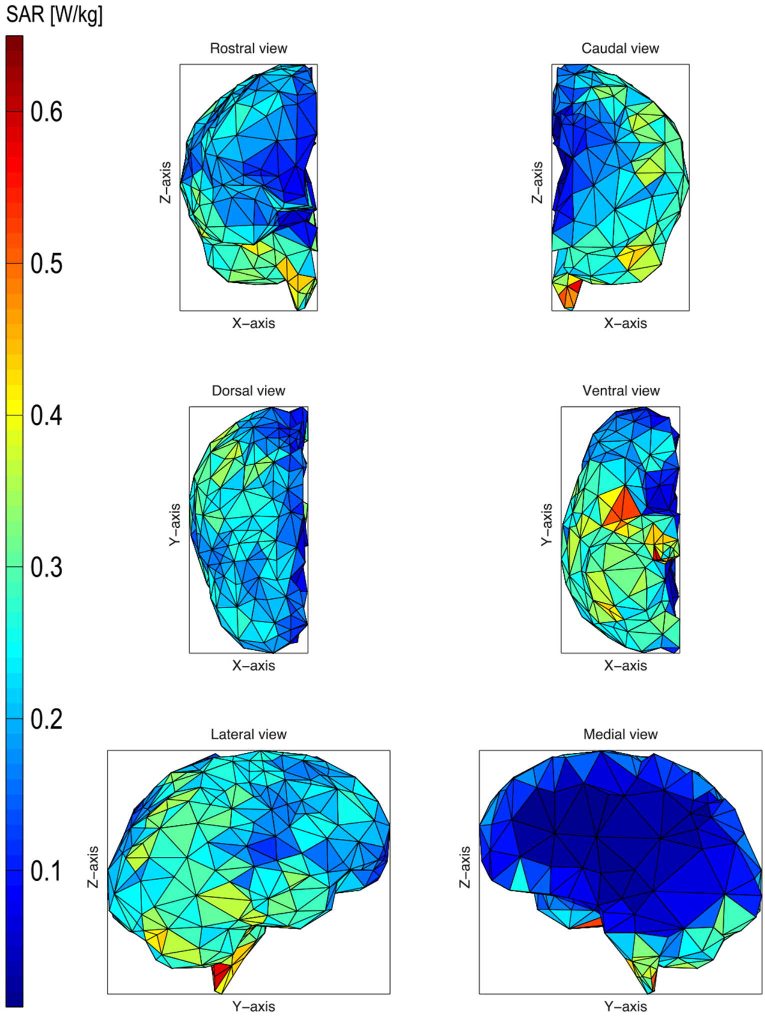

Figure 2: *SAR* distribution at *f* = 900 MHz.

Table 1: The brain electrical parameters.

	f = 900 MHz	*f* = 1,800 MHz
ε_r	46	84
σ[S/m]	0.8	1.2

Table 2: The peak and average *SAR* values for different exposure scenarios.

	f = 900 MHz	*f* = 1,800 MHz
SAR_{max}[W/kg]	0.866	2.678
SAR_{avg} [W/kg]	0.158	0.348

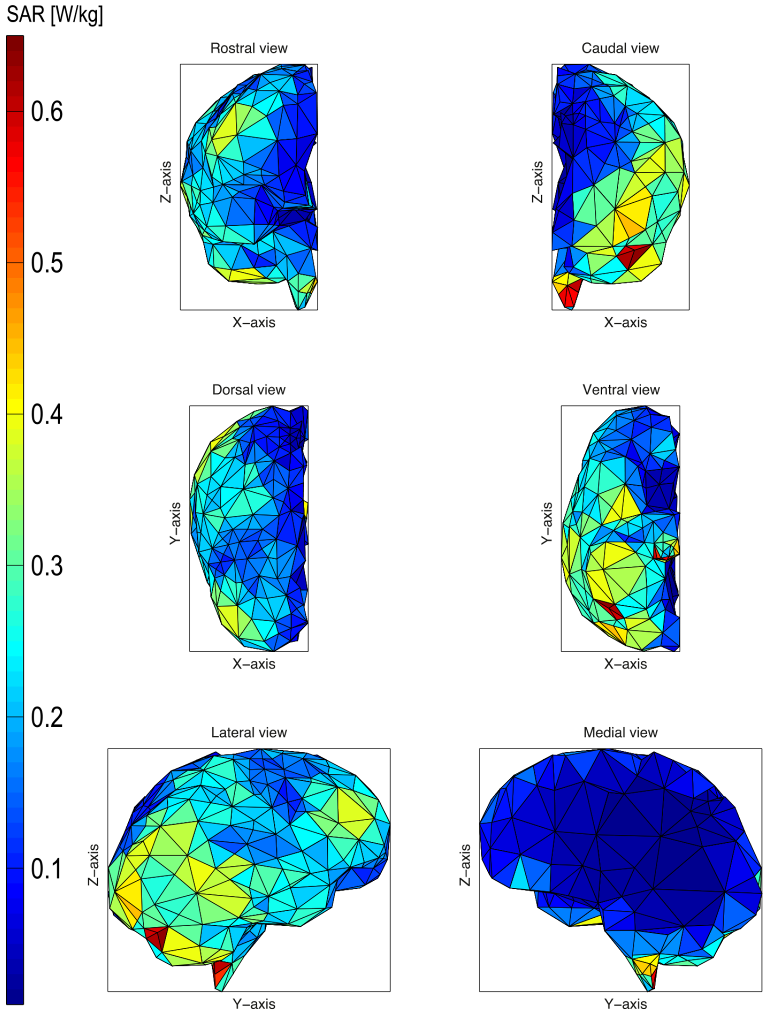

Figure 3: *SAR* distribution at *f* = 1,800 MHz.

(k = 1,. . . , 5) is assumed to be uniformly distributed around deterministic values with common coefficient of variation equal to 5.77% [12].

Although the output (maximum SAR) is highly non linear, SC provides an assessment of the 1st statistical moment with 5 simulations only. Note that several univariate calculations, i.e. calculations for a single random variable, are carried out.

The following set of Figures is related to the results obtained from deterministic and stochastic eye modeling.

Figure 5 shows the *SAR* distribution in the eye due to the exposure to plane wave with power density of 10 W/m^2 obtained by the use of hybrid BEM/FEM. The eye parameters are available elsewhere, e.g. in [13]. Figure 6 gives information of the first-order sensitivity of the model to a corresponding random variable for the SAR assessment in the eye exposed to plane wave, i.e. the SAR variance.

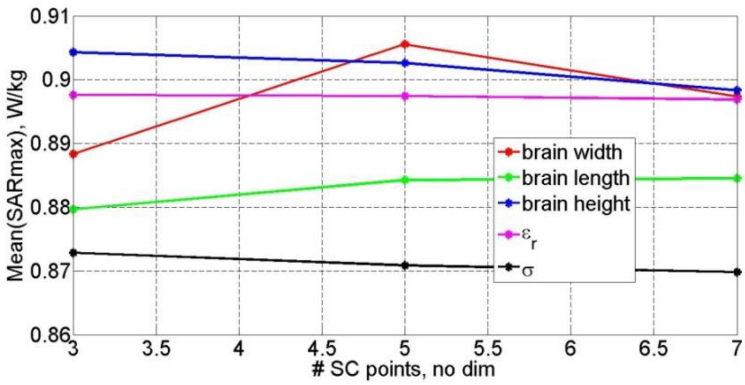

Figure 4: Mean of maximum SAR.

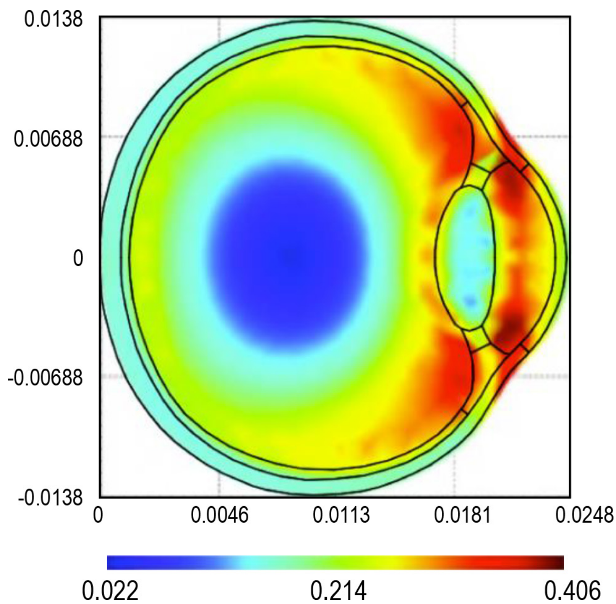

Figure 5: SAR in the eye due to plane wave exposure 10 W/m^2 at f = 1 GHz.

The conductivity of the vitreous body is $\sigma_v^0 = 0.70112$ S/m at f = 6 GHz and is assumed to be uniformly distributed around interval $\sigma_v = \sigma_v^0 \pm 0.6$ S/m.

As depicted in Fig. 6, higher levels of σ_{SAR} are concentrated inside the vitreous body.

4 CONCLUDING REMARKS

The paper presents deterministic-stochastic analysis of the exposure of the brain and eyes to the HF radiation, thus taking into account the uncertainty variations of the several input parameters. The deterministic model is based on the corresponding boundary integral equa-

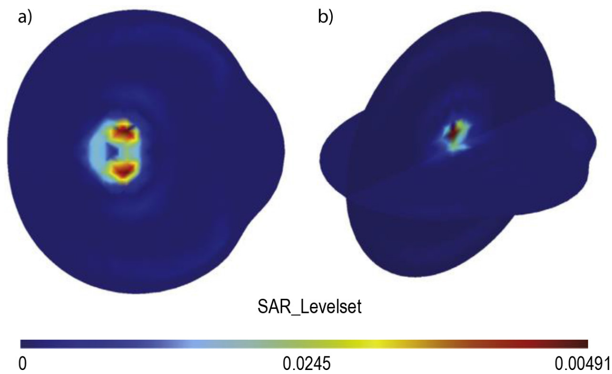

Figure 6: σ_{SAR} inside the eye (stochastic variations around vitreous body conductivity) induced by plane wave with power density 10 W/m^2 at f = 6 GHz and SC approximation order n = 2: (a) z-plane; (b) x-plane and y-plane.

tion formulation and related solution methods based on the BEM scheme (the brain exposure), and hybrid BEM/finite element method (BEM/FEM) scheme (the eye exposure). The expansion of the statistical output in terms of the mean and variance over a polynomial basis via stochastic collocation (SC) is shown to be robust and efficient technique providing a satisfactory convergence rate of the SC technique.

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