

# WHAT FACTORS AFFECT CROSS-MODAL SUBSTITUTION? – EVIDENCES FROM THE OSLO AREA

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## ABSTRACT

The vast majority of studies on urban travel demand focus on the effect on the demand of one travel mode given a change in the characteristics of that same transport mode, for example, own-elasticities. Comparatively little is known about cross-elasticities of demand. In particular, there is a need for a better understanding of the underlying mechanisms of modal substitution, that is a better understanding of cross-modal diversion factors (DFs) defined as the proportion of people who leave mode A and switch to mode B. The purpose of this article is to investigate what factors explain variations in DFs across transport modes, submarkets and policy measures. Using a recently developed empirical travel mode choice model for the Oslo area, we simulate over 10,000 different DFs by systematically changing the underlying transport modes, submarkets and policies (size, direction and type of change). With descriptive statistics, we show how the DFs vary on a general level. Most results are immediately intuitive, for example that car drivers mostly substitute to walk for short-distance trips but that those DFs diminish rapidly with increasing distance. Interestingly, we find rather high DFs across different forms of public transportation. With successive regression analyses we show that the number of available alternatives and relative market shares significantly affect DFs.

*Keywords: cross-model substitution, diversion factors, nested logit model, sample enumeration*

## 1 INTRODUCTION

It is safe to say that urban passenger cross-modal substitution is not very well understood. Intermodal interaction was identified by Dodgson [1] back in 1991 as an issue in need of further research. This remains the case. It is widely accepted that it is difficult to generalize results and establish ‘rules of thumb’ because – as opposed to direct effects (own-elasticities) – cross-modal substitution (cross-elasticities) is very context dependent. This is because the availability and quality of travel alternatives differ greatly between study areas. A cross-elasticity towards metro, say, may be very low in city A compared to city B, not just because travellers’ preferences may differ, but because the metro service may be relatively poor in city A. A related factor that adds to the variation across studies with regard to cross-elasticities is the fact that relative market shares (of altered and affected mode) directly affect the absolute value of cross-elasticities (see [2]). Surprisingly, market shares are seldom reported alongside cross-elasticities in the literature [3]. Without controlling for market shares, it is often difficult to explain variation in reported cross-elasticities.

In this article, we take a closer look at the underlying mechanisms of modal substitution by studying cross-modal diversion factors (DF). The notion of DF is straightforward. For example, say 100 persons stop travelling by car as a result of a gas price increase and that 20 of them will walk instead, 10 switch to cycling, 30 to bus, 20 to metro, 10 to train and 10 stay home and do not travel. DFs will then be 10%, 30%, 20%, 10% and 10%, respectively.

As opposed to cross-elasticities, DFs are independent of the relative market shares of the altered mode – at least as a first-order effect – and can therefore be expected to be more stable across studies [4]. Still, differences in availability and quality of alternative travel modes across studies remain a challenge when aiming for generalizable results. Also, the

composition of trips (distribution of trip distance, trip purposes, etc.) in the empirical data is likely to affect overall results. For example, in dense cities, travel distance will be shorter on average and that will, all else being equal, cause DFs towards walking to be higher than in more spread-out cities.

Another element that can affect the comparison of evidence of cross-modal substitution is differences in the methods of data collection and modelling used. Cross-elasticities and DFs can be measured/predicted by different approaches including – among others – before–after studies [5], time series regression models [6–10] and cross-sectional choice modelling based on either stated preference [11–13] or revealed preference (RP) [14–17]. Little is known how the methodological approach may impact on study results.

The literature on DFs is limited. Some key contributions include the following. Acutt and Dodgson [18] asked 25 experts and operators for their opinion on DFs between car and train/metro/bus following fare reductions, that is, the proportion of new train/metro/bus passengers that previously used car. DFs ranged from 1% (London, car to bus) to 25% (intercity, car to train). Storchmann [19] estimated DFs from car to public transport resulting from changes in fuel taxes in Germany for various trip purposes. DFs ranged from zero percent for business, holiday and leisure trips to 100% for education trips. Adler and van Ommeren [20] studied the effects of public transport strikes during 2003–2011 in Rotterdam and found DFs from public transport to car and cycling of 27–29%. Prud'homme et al. [21] did an ex-post survey among 1,000 passengers on a Paris tramline that had been converted from bus, coinciding with a capacity reduction on a parallel road link. Their results suggest that most tram passengers were diverted from other public transport (bus 57% and subway 38%). Only 3% of the tram passengers used car previously. Murphy and Usher [22] surveyed the users of Dublin's inner-city bike sharing scheme and found that its users were diverted from walk (46%), bus (26%), car (20%) and train (9%). The Norwegian empirical evidence is very limited. Fearnley and Nossun [23] evaluated the Norwegian Ministry of Transport's 1990s urban public transport policy packages and found that 42.7% of passengers on new or improved bus services would otherwise have generated a car trip. Fearnley [24] reviewed experiences around the world with free local public transport and concluded that typically, a very low proportion of generated patronage stems from car. New passengers are more likely to generate traffic and divert from walk and cycle.

As seen from this brief literature review, the range of estimated DFs is substantial. It is likely that various factors relating to the context of the study and/or the applied method affect the empirical values.

To tackle the challenge of producing transferable results, our general approach in this article is laid out as follows: First, we control for the general context by keeping the analysis within one study area: the Greater Oslo area. Second, we base our results on one general type of model: the travel mode choice model MPM23 [25]. Third, we simulate DFs for different submarkets and different policy measures with the aim of learning how the (a) availability of modes, (b) quality of modes, (c) trip distance, (d) trip purpose, (e) type of policy, (f) size of policy change affect the simulated DFs from and towards different transport modes (car, train, bus, metro/tram, walk and cycle).

This analytical method can be referred to as *model-internal meta-analysis* as the same model is applied for a large range of policies and submarkets, and subsequent regression analysis is performed on the simulated results in a similar way as in a typical (formal) meta-analysis. Thus, the feature of 'model-internal meta-analysis' (compared to regular meta-analysis) is that the dependent variable in the regression models (DFs) is coming not from different studies found in the literature but from the same geographical context and modelling

approach. Similar methodologic approaches have earlier been applied in analysing the ‘package’ approach to transport policy, whereby strategic or tactical models are run many times and the results are then subject to further analysis (see [26–29]).

## 2 THE DIVERSION FACTOR: SOME MEASURES, THEORY AND PROPERTIES

In several studies, DFs are established based on survey data. They may take the form of direct questions on how respondents would behave if their current mode became unavailable (e.g. [30]), or of transfer time (and cost) questions on intended behaviour of the form ‘How much would your journey cost have to increase before you switch to another mode/don’t make this trip?’ (e.g. [31]). DFs are calculated as the proportion who states that they would switch to each mode (or not travel).

Another way to obtain DFs is to observe the change in demand for mode  $j$  and the proportion that diverts to mode  $i$ . Formally, this would be calculated as

$$DF_{ji} = (Q_{T1i} - Q_{T0i}) / (Q_{T1j} - Q_{T0j}) \quad (1)$$

where  $Q$  is demand (number of passengers);  $T0$  and  $T1$  are time periods or scenarios. In typical scenario analysis (e.g. two model runs),  $j$  is the transport mode that is altered in attributes, while  $i$  remains unchanged.  $DF_{ji}$  is then referred to as DF from mode  $j$  towards mode  $i$  (given a change in mode  $j$ ). This is the standard procedure for deriving DFs from discrete choice models or transport models. A base scenario ( $T0$ ) is compared with an intervention scenario ( $T1$ ) where one (or several) attribute is (are) changed. The resulting  $Q$ s are then plotted in the above formula in order to obtain DFs.

DFs can also be calculated ‘backwards’ from known cross-elasticities, known own-elasticities and known market shares. We have the following relationship, which defines cross-elasticities of demand [2]:

$$\varepsilon_{ij} = |\varepsilon_{jj}| \frac{Q_i}{Q_j} DF_{ji} \quad (2)$$

where  $\varepsilon_{ij}$  is the cross-elasticity of demand for mode  $i$  with respect to an attribute change of mode  $j$ ;  $|\varepsilon_{jj}|$  is the absolute value of mode  $j$ ’s own-elasticity of demand;  $Q_i/Q_j$  is the ratio of market shares or ratio of volumes; and  $DF_{ji}$  is the proportion of those who leave mode  $j$  and switch to mode  $i$ . It follows that

$$DF_{ji} = \frac{\varepsilon_{ji}}{|\varepsilon_{jj}|} \frac{Q_i}{Q_j} \quad (3)$$

When inserting the definition of linear-arc-elasticities in eqn (3), it is straightforward to show that eqn (3) is mathematically equivalent with eqn (1).

Note that the sum of DFs from mode  $j$  to all other transport modes  $i$  adds up to 100% when travel mode choice is the only behaviour dimension in the modelling framework. When trip generation is included (or the choice model includes an option for ‘not travelling’), the sum of DF towards a transport mode which is improved can be smaller than 100% when the improvement creates generated traffic (or a worsening leads to suppressed transport). When transport modes are substitutes (the usual case), DFs are non-negative. For complementary modes, DF can be negative. In this case, for two complementary modes, it can be that DF towards a third mode is above 100%. For example, consider a measure that yields an increase

in train ridership of 100 persons. Assume that metro is – on average – a complement to train and every 10th new train user generates one additional metro trip. Assume this makes 110 fewer bus trips; then the DF from train to bus would be 110%.

DFs are ‘directional’, i.e.  $DF_{ji} \neq DF_{ij}$  in general. It is worth noting that, in the literature, DFs are sometimes defined interchangeably as ‘proportion of travellers that leave mode  $i$  and switch to mode  $j$ ’ on the one side, or as ‘proportion of new travellers on mode  $j$  that switched from mode  $i$ ’. There is no reason at all for these to be the same quantum: To say that 20% of new bus passengers previously used car is in fact essentially different from saying that 20% of motorists who leave the car would switch to bus. The fact that this is often treated as the same phenomenon in the literature may relate to a failure to understand Bayes’s theorem and conditional probabilities.

Note that DF may also be non-symmetric for a given altered mode  $j$ , for example can price increase of mode  $j$  make a higher proportion substitute to mode  $i$ , than a price reduction would attract from mode  $i$ . This is intuitive in real life and an important question relates to which methods would allow to preserve/capture such a non-symmetry. As a point estimate,  $DF_{AB}$  should be the same quantum whether it be ‘the proportion of traffic lost to  $j$  which switches to  $i$  if  $j$  gets worse’ or ‘the proportion of  $j$ ’s new traffic which has come from  $i$  if  $j$  gets better’. For DFs that are calculated as in eqn (1), this may not be the case when the underlying model is nonlinear in attributes. For instance, when quantities are predicted with logit models a certain non-symmetry is expected given the S-shape of the logit model. However, if changes in attributes are small (e.g. 1% and –1% changes) results will tend to be close to symmetrical.

In the introduction section, it was mentioned that DFs are independent of the relative market shares at a first-order effect. That is, eqn (1) does not involve market shares of  $i$  and  $j$ . However, it is likely that market shares represent the competitiveness of travel alternatives and are therefore likely to influence the changes in quantities in eqn (1). For instance,  $(Q_{T1i} - Q_{T0i})$  is likely to be great in absolute terms when mode  $i$  is a highly competitive transport mode and therefore a likely substitute to mode  $j$ .

Furthermore, when quantities in eqn (1) are predicted on the basis of multinomial logit models we can establish a direct relationship (see the appendix for the derivation):

$$DF_{ij} = P_j / (1 - P_i) \quad (4)$$

where  $P_j, P_i$  are (individual) choice probabilities for mode  $j$  and  $i$ , respectively.

Equation (4) holds true on an individual level, in which case  $DF_{ji}$  is interpreted by relative probabilities to switch from mode  $j$  to mode  $i$ . Aggregating over (heterogeneous) individuals (as done in this article by means of sample enumeration), eqn (4) does *not* necessarily hold on a market level, in which case  $P$  represent market shares. Note also that for nested logit models, eqn (4) applies only for modes of the same lowest level nest. The relationship between market shares and DFs is empirically investigated in the later analysis of this article.

### 3 METHODOLOGICAL APPROACH

Recently, Flügel et al. [25] established a travel mode choice model, referred to as MPM23, for short-distance trips within Norway’s capital Oslo and the surrounding county Akershus. MPM23 is a nested logit model that calculates choice probabilities of nine alternatives that are structured into four nests: car (includes choice alternatives: car driver and car passenger), walk, cycle and PT (includes choice alternatives: train, bus, metro/tram, combinations

with train and combination of bus and metro/tram). Model parameters are estimated from travel surveys, where respondents reported trip diaries of the day before the interview was conducted. Respondents do only report their actual behaviours (chosen transport mode, trip purpose, etc.), that is RP data. The model includes the usual level-of-service (LoS) attributes as well as several dummy variables that calibrate the choice probabilities for different submarkets. Trip frequency is not modelled, nor is destination choice or traffic assignment.

The estimated model is implemented in Microsoft Excel with an intuitive user interface for stylized scenario analysis. Users can specify changes in LoS variables in percent of the base values. The model predicts new market shares by sample enumerating choices of 14,947 observations (single trips). The method of sample enumeration has a long tradition (going back to at least Ben-Akiva and Atherton [32]). An attractive feature is that it preserves information at the individual level. This is important in the case of MPM23, among others because the model operates with choice sets defined at the individual level (see later).

In theory, it would be possible to differentiate the full MPM23 model and extract (individual) DFs directly by eqn (4). However, each individual in each submarket (trip purpose, geography, distance) faces different constraints and different availabilities of transport modes. There is simply not one effect on mode choice that applies to all individuals. As we are interested on results on a market (submarket) level, we must run MPM23, predict individual choice in behaviour but calculate DFs on a market (submarket) level.

For the analysis in this article, nine choice alternatives in MPM23 are merged into six travel modes as described in Table 1.

Reducing from nine to six choice alternatives eases interpretation, streamlines analysis and increases transferability of the results. A disadvantage with this procedure is that the category bus (metro/tram) might include some trips that are actually made by train and metro/tram (bus).

We use MPM23 to predict changes in ridership given policy scenarios and we calculate DFs applying eqn (1). In total, we have calculated 11,560 single DFs. Table 2 lists the variables by which the scenarios differ from each other.

The combination of the latter three categories yields 36 submarkets (2 trip distances \* 3 geographies \* 6 trip purposes). Three of those submarkets had less than 30 observations in the data set and were merged together resulting in 34 submarkets. Table 3 presents sample size and baseline market shares of these submarkets.

Table 1: Travel alternatives in MPM23 and in this article.

<b>Travel modes in MPM23</b>	<b>Shares going into new categorization</b>
Car driver	100% to car
Car passenger	100% to car
Walk	100% to walk
Cycle	100% to cycle
Train (without transfer to other PT)	100% to train
Bus (without transfer to other PT)	100% to bus
Metro/tram (without transfer to other PT)	100% to metro/tram
Combination with train	50% to train, 25% to bus, 25% to metro/tram
Combination with bus and metro (not train)	50% to bus and 50% to metro/tram

Table 2: Underlying variables in scenario simulations.

Travel mode altered	Policy variable	Size of change	Travel mode affected	Trip distance	Geography	Trip purpose
• Car	• In vehicle time	• -30%	• Car	• <5 km	• Urban	• Commuting
• Train	• Out-of-pocket costs*	• -1%	• Walk	• >5 km	• Suburban	• School
• Bus		• +1%	• Cycle		• Urban	• Business
• Metro/ tram	• Access/egress time (not car)	• +30%	• Train		to/from	• Grocery
	• Waiting time (not car)		• Bus		suburban	• Deliver/pick up
	• Number of interchanges (not car)		• Metro/ tram			• Other leisure

\* Includes fuel and road tolls cost for car and single ticket prices for PT (users with season ticket have zero costs in the current version of MPM23).

Table 3: Sample size and baseline market shares of 34 submarkets.

Index	Characteristic of submarket*	N	Baseline market shares (%)					
			Car	Walk	Cycle	Train	Bus	Metro/ tram
1	>5 km; urban; commuting	983	32.7	2.0	8.9	6.9	18.0	31.5
2	>5 km; urban; school	103	8.1	1.1	5.3	5.2	22.3	58.0
3	>5 km; urban; business	73	50.6	1.0	3.5	4.3	15.4	25.1
4	>5 km; urban; grocery	354	56.3	2.4	4.4	3.9	11.9	21.1
5	>5 km; urban; deliver/pick up	135	75.2	2.4	4.2	1.6	6.7	10.0
6	>5 km; urban; other leisure	607	41.3	4.5	6.2	3.4	16.6	28.2
7	>5 km; suburban; commuting	517	86.0	0.5	2.6	4.5	6.0	0.4
8	>5 km; suburban; school	48	23.5	1.5	8.1	17.3	47.4	2.1
9	>5 km; suburban; business	31	91.3	0.4	1.1	3.4	3.6	0.2
10	>5 km; suburban; grocery	456	92.1	0.8	1.7	1.6	3.9	0.1
11	>5 km; suburban; deliver/pick up	193	97.1	0.4	0.8	0.4	1.2	0.0
12	>5 km; suburban; other leisure	514	87.4	1.1	2.0	3.0	6.2	0.3
13	>5 km; u to/from s; commuting	1,423	54.0	0.3	3.6	16.2	18.4	7.5
14	>5 km; u to/from s; school	63	15.6	1.3	4.1	31.2	32.1	15.7
15	>5 km; u to/from s; business	96	67.5	0.1	1.1	9.9	15.9	5.5
16	>5 km; u to/from s; grocery	421	76.4	0.5	1.7	10.5	7.5	3.4
17	>5 km; u to/from s; deliver/pick up	236	92.1	0.5	1.2	1.9	2.6	1.7
18	>5 km; u to/from s; other leisure	771	67.8	1.2	3.0	10.2	12.6	5.3
19	<5 km; urban; commuting	839	18.3	32.9	14.0	0.3	14.0	20.5
20	<5 km; urban; school	96	5.7	27.4	9.9	0.7	24.5	31.9
21	<5 km; urban; business	83	27.6	28.9	6.2	0.3	9.1	27.8
22	<5 km; urban; grocery	1,492	33.0	50.3	5.6	0.0	4.9	6.2
23	<5 km; urban; deliver/pick up	485	49.8	40.6	4.7	0.0	2.8	2.1

(Continued)

Table 3: (Continued)

Index	Characteristic of submarket*	N	Baseline market shares (%)					
			Car	Walk	Cycle	Train	Bus	Metro/ tram
25	<5 km; suburban; commuting	213	60.5	25.6	8.4	0.2	5.3	0.0
26	<5 km; suburban; school	35	21.8	38.5	19.0	1.4	19.2	0.0
27	<5 km; suburban; grocery	773	70.2	24.0	3.5	0.1	2.1	0.0
28	<5 km; suburban; deliver/pick up	370	80.0	17.3	2.1	0.0	0.7	0.0
29	<5 km; suburban; other leisure	751	50.9	41.7	4.7	0.1	2.6	0.0
30	<5 km; u to/from s; commuting	139	56.9	20.7	10.2	1.1	9.3	1.8
31	<5 km; u to/from s; grocery	421	65.6	26.2	3.5	0.2	3.8	0.7
32	<5 km; u to/from s; deliver/pick up	213	79.6	16.0	2.7	0.3	1.3	0.1
33	<5 km; u to/from s; other leisure	393	45.1	44.6	5.1	0.2	4.1	0.9
34	<5 km; remaining	38	64.4	17.2	4.9	0.6	12.9	0.0

\* 'u to/from s' means 'urban areas to/from suburban areas'.

Overall, car has the highest market shares in the Greater Oslo area. This holds true for most submarkets. Exceptions are school trips and most short-distance (<5 km) urban trips.

Not surprisingly, the market share for walking varies considerably between short-distance and long-distance trips. Cycling has higher market shares for commuting and school trips. Train is barely used for trips under 5 km and has its highest market shares on longer suburban and 'suburban to/from urban' trips. Also, the market shares for bus vary greatly across submarkets; school trips are particularly often done by bus. Bus – and to an even higher degree metro/tram – has higher market shares for urban than for suburban trips. There are no short-distance metro/tram trips within suburban areas since metro/tram is not available there. The market shares for metro/tram trip departing and/or ending in suburban areas are in reality trips made by combinations of transport modes but are coded as metro/tram with the applied method.

The competitive structure in the 34 submarkets can also be described by the available choice alternatives (Table 4). Whether a travel mode is 'available' or not is defined by MPM23 on a trip level. Car is always available, as the model assumes that you can always be 'car passenger'. For walk and cycle, availability is defined by the trip distance, with limits of availability of 10 and 40 km, respectively. Availability of PT is mainly defined by distance to the nearest station in a similar fashion (see [25] for details).

For submarkets with shorter trips (<5 km), train and, to a lower degree, metro/tram are seldom defined as available due to unreasonably long access/egress times to the nearest station. The availability for walk decreases rapidly for submarkets with longer trip relations.

The overall methodical approach of our analysis is briefly summarized in Fig. 1.

#### 4 DESCRIPTIVE STATISTICS

In this section, we present some descriptive statistics from the results of the model simulations. Regression analyses are presented in Section 5.

As DFs have a close connection to cross-elasticities (see equation (2)), we have also simulated own- and cross-elasticities of demand alongside DFs. All simulated own-elasticities are negative. This is expected given that all analysed policy variables are 'bads', for example an

Table 4: Average distance and availability of modes (averages over trips within submarkets).

Index	Characteristic of submarket*	Average distance (km)	No. of available travel modes	Availability by mode (%)**					
				Car	Walk	Cycle	Train	Bus	Metro/tram
1	>5 km; urban; commuting	9.0	4.9	100	73	100	30	97	90
2	>5 km; urban; school	9.1	4.8	100	71	100	17	93	95
3	>5 km; urban; business	8.6	5.0	100	82	100	27	97	89
4	>5 km; urban; grocery	8.2	4.9	100	83	100	24	97	85
5	>5 km; urban; deliver/pick up	8.8	4.8	100	73	100	22	96	88
6	>5 km; urban; other leisure	8.4	4.8	100	80	100	20	96	86
7	>5 km; suburban; commuting	19.8	4.1	100	25	90	62	96	39
8	>5 km; suburban; school	18.5	4.2	100	38	88	65	100	33
9	>5 km; suburban; business	17.8	4.0	100	32	87	61	87	32
10	>5 km; suburban; grocery	13.0	4.2	100	54	97	53	88	26
11	>5 km; suburban; deliver/p. up	19.3	4.1	100	51	88	50	90	32
12	>5 km; suburban; other leisure	18.9	4.1	100	43	88	55	88	33
13	>5 km; u to/from s; commuting	20.9	4.6	100	21	90	72	97	79
14	>5 km; u to/from s; school	18.7	4.8	100	29	95	81	98	81
15	>5 km; u to/from s; business	21.1	4.5	100	22	91	69	95	75
16	>5 km; u to/from s; grocery	18.9	4.7	100	35	91	76	98	73
17	>5 km; u to/from s; deliver/p. up	18.6	4.5	100	36	88	61	96	68
18	>5 km; u to/from s; other leisure	18.6	4.6	100	32	92	68	96	73
19	<5 km; urban; commuting	2.8	4.4	100	100	100	10	68	58
20	<5 km; urban; school	2.8	4.4	100	100	100	8	70	60
21	<5 km; urban; business	2.5	4.3	100	100	100	13	51	64
22	<5 km; urban; grocery	1.7	3.6	100	100	100	3	30	22
23	<5 km; urban; deliver/pick up	1.8	3.5	100	100	100	1	28	17
24	<5 km; urban; other leisure	2.0	3.8	100	100	100	5	38	34
25	<5 km; suburban; commuting	2.6	3.4	100	100	100	8	32	1
26	<5 km; suburban; school	2.6	3.5	100	100	100	6	40	3

(Continued)



Table 4: (Continued)

Index	Characteristic of submarket*	Average distance (km)	No. of available travel modes	Availability by mode (%)**					
				Car	Walk	Cycle	Train	Bus	Metro/tram
27	<5 km; suburban; grocery	2.2	3.2	100	100	100	2	21	0
28	<5 km; suburban; deliver/pick up	2.3	3.2	100	100	100	1	16	0
29	<5 km; suburban; other leisure	2.2	3.2	100	100	100	3	18	0
30	<5 km; u to/from s; commuting	3.2	4.1	100	100	100	17	70	19
31	<5 km; u to/from s; grocery	2.3	3.6	100	100	100	8	40	12
32	<5 km; u to/from s; deliver/p. up	2.6	3.5	100	100	100	7	37	6
33	<5 km; u to/from s; other leisure	2.1	3.5	100	100	100	7	31	10
34	<5 km; remaining	2.9	3.7	100	100	100	13	50	3

\* 'u to/from s' = 'urban areas to/from suburban areas'.

\*\* As defined on an individual trip level in MPM23.

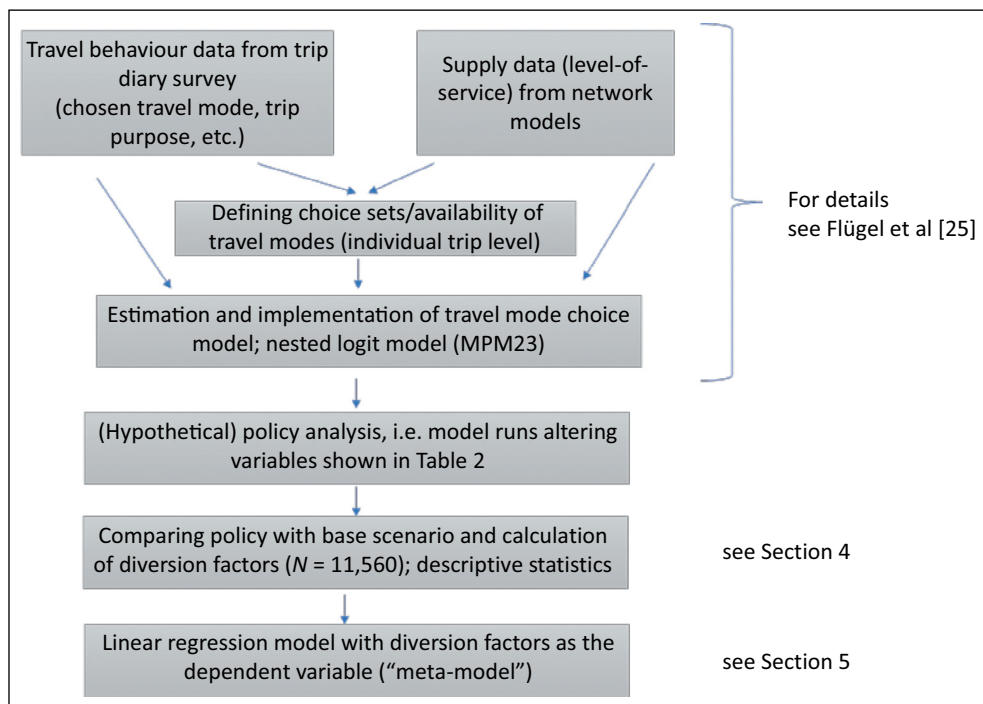


Figure 1: Overall methodological approach.

increase in LoS variable as travel cost, in-vehicle time, waiting time, access–egress times and number of interchanges for mode  $j$  leads to a decrease in ridership of mode  $j$ .

The simulated cross-elasticities are typically positive but some cross-elasticities that involve metro/tram are negative (but rather low in size). These are typically cases for suburban areas where metro/tram is only used in combination with bus or train. In these cases, metro/tram is a complement rather than a substitute to bus and train. This would be the case for commuters who take a train or bus into central Oslo and from there take metro or tram to their final destination within Oslo.

It is important to note that cross-elasticities are highly dependent on the relative market shares between the affected and altered transport modes. If the relative market share is high (the affected mode has a much higher market share than the altered mode), cross-elasticities are typically very close to zero (Fig. 2). This underlies the point made in the introduction section about cross-elasticities, which is context dependent and difficult to interpret without considering the underlying market shares.

DFs are less affected by market shares, as shown in Fig. 3. Even for high relative market shares we find a widely spread DFs. However, there appears to be a positive relationship between relative market shares and DFs, which may relate to the theoretical properties of the underlying logit models (see eqn 4). A positive correlation between market shares and DFs does generally make sense because the market share of mode  $i$  is likely to be proxy for the competitiveness (or ‘LoS’) of mode  $i$ .

The following figures show how DFs vary by transport mode combination and for group of submarkets (aggregates of the 34 submarkets used for simulation). Figure 4 shows average

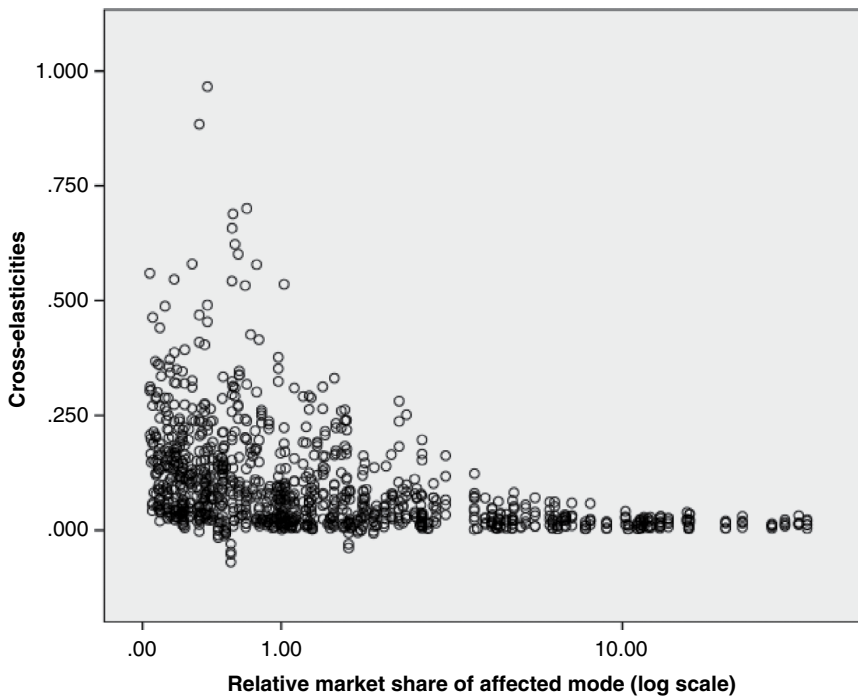


Figure 2: Simulated cross-elasticities and relative market shares.

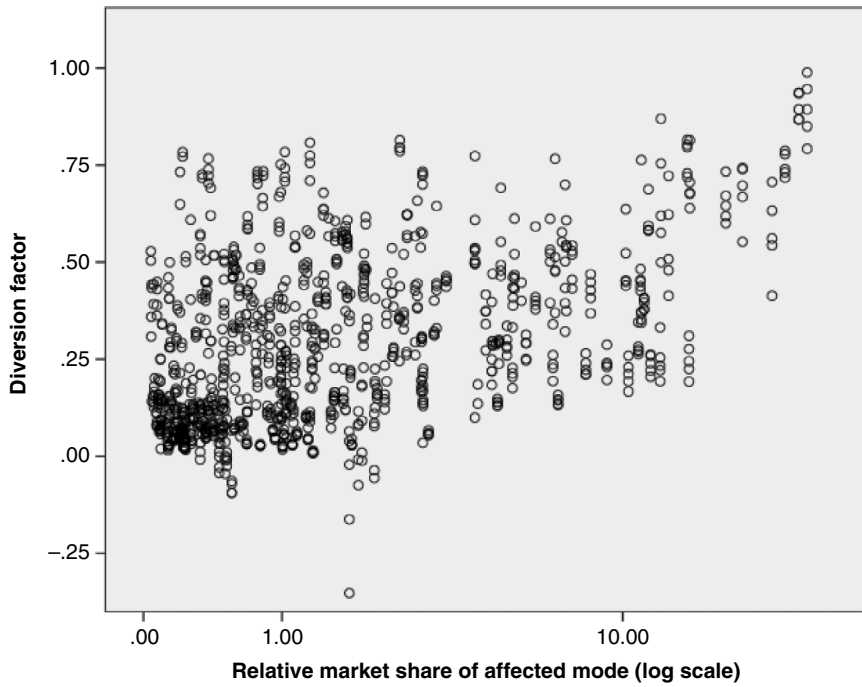


Figure 3: Simulated diversion factors and relative market shares.

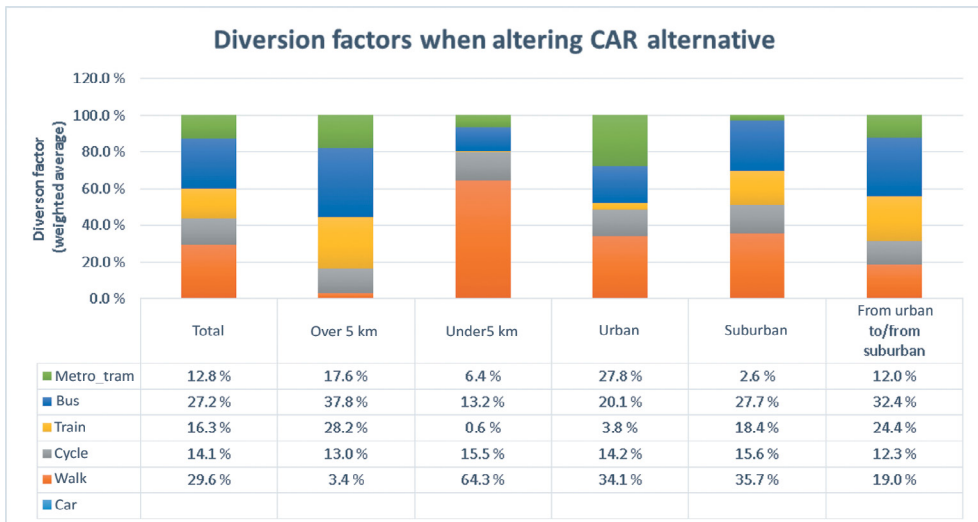


Figure 4: Diversion factors, total and by main categories, when car is altered.

values of DFs when the car alternative is altered. Observations are weighted by the size of each submarket and the market share of car in these submarkets.

Note that both  $-30\%$ ,  $-1\%$ ,  $1\%$  and  $30\%$  changes in attributes are included; that is potential asymmetry is not taken into account here (see later discussion).

Overall, bus and walk have the highest DFs than car travel. The DF of 27.2% for bus and 29.6% for walk can be interpreted with a hypothetical scenario that leads to 1,000 fewer (more) car trips. About 272 would come from (go to) bus, while 296 would come from (go to) walk.

DFs vary with submarkets. Walk dominates for trips under 5 km. Bus, together with train, is the best alternative to car for longer trips. For urban trips, metro/tram has a relatively high DF. This is directly related to availability (see Table 4). For suburban trips its DF is low. The opposite pattern is observed for train.

Figure 5 shows the corresponding picture when bus attributes are altered. The highest DFs are found for car on longer trips and trips within suburban areas, and for metro/tram on shorter and urban trips. For suburban travel, metro/tram and bus seem rather to be complementary, as indicated by the slightly negative DF. In total, close to 50% (33.1% + 16.2%) of bus users divert to other PT options. This finding is discussed in Section 6.

Figure 6 gives the results of simulations where train attributes are altered. Not surprisingly, the DF for walk (and cycle) is very small. Bus and car are main competitors for train as judged from the simulated DFs although a substantial diversion to metro/tram can be seen on short and urban trips.

Some interesting patterns are shown in Fig. 7, where DFs are calculated for attribute changes in metro/tram. For trips within suburban areas (where metro/tram has rather low market shares, and most of the ridership stems from trips where metro/tram is used in combination with PT modes), we find negative diversion to both bus and train. Fewer metro/tram passengers will also reduce bus and train patronage. In this market, there is therefore complementarity between metro/tram and bus and train. Apart from suburban trips, bus has high DFs when metro/tram is altered. Diversion to car is also significant in all submarkets, with the exception of short trips where diversion to walk is prominent.

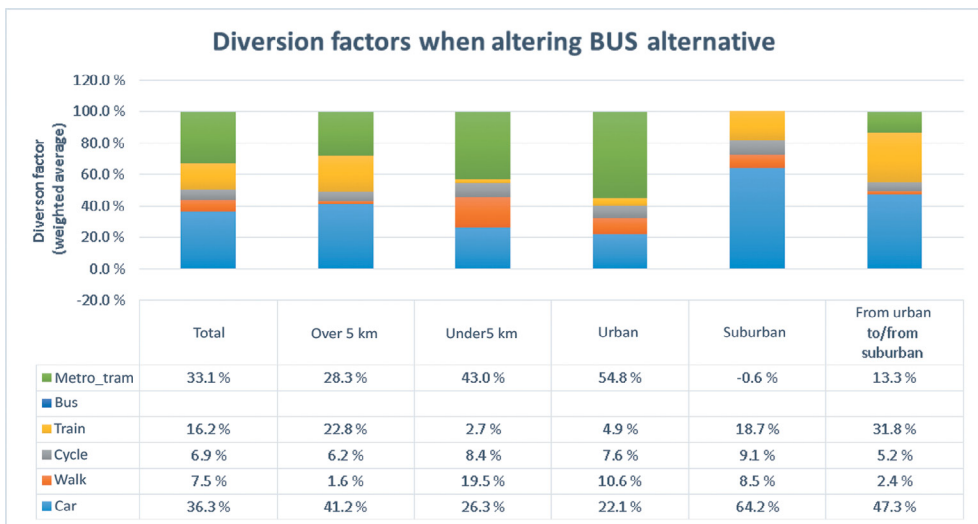


Figure 5: Diversión factors, total and by main categories, when bus is altered.

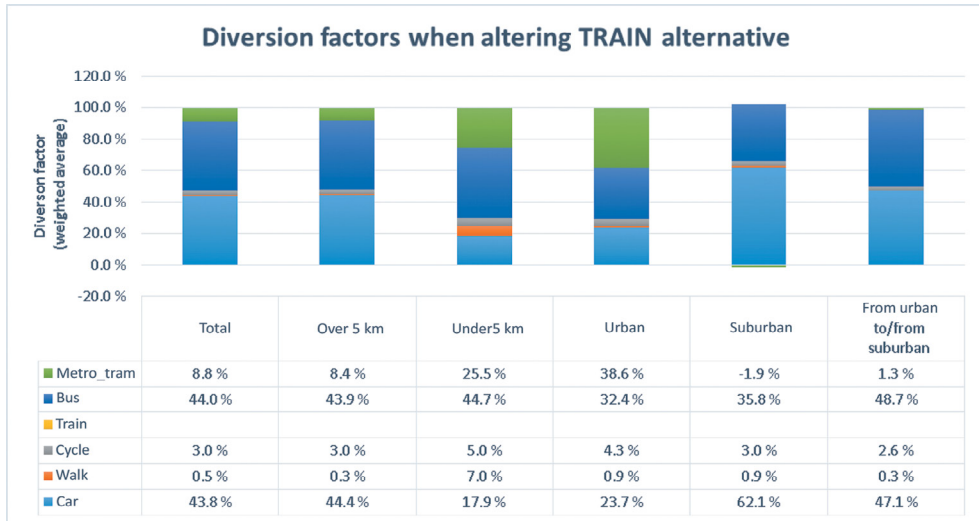


Figure 6: Diversion factors, total and by main categories, when train is altered.

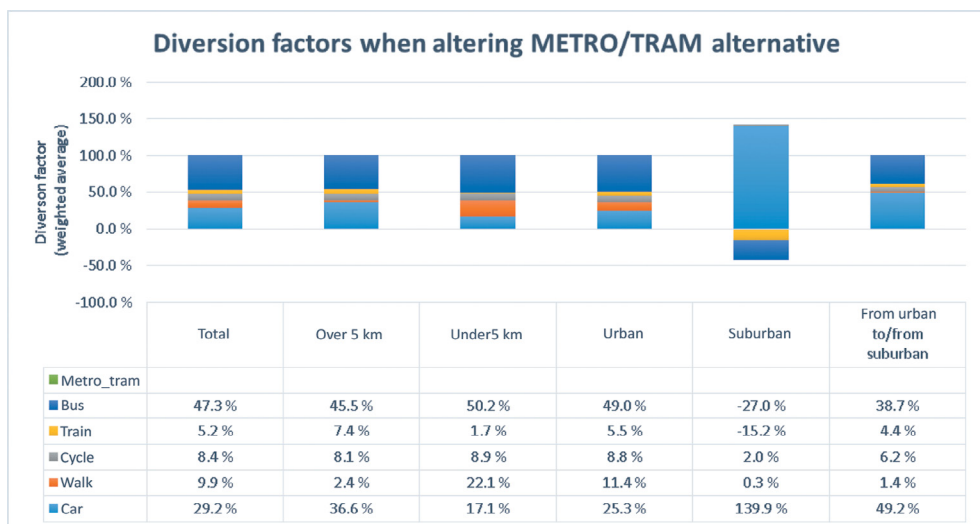


Figure 7: Diversion factors, total and by main categories, when metro/tram is altered.

### 5 REGRESSION MODELS

In this section, we present regression analysis on the simulated data. The purpose is to obtain information about which explanatory variables have a significant effect on DFs after controlling for other explanatory variables. It is convenient to run linear regression models even though those types of models do not guarantee that DFs (over a given altered mode) do add up to 100%.

It is important to note that simulated DFs for a given transport mode pair (altered and affected mode) are very similar within a given submarket. That is, the variation by type, direction and size of the policy change is very low. As a consequence, tests showed that the explanatory variables related to the direction and size (intensity) of change (if it is a  $-30\%$ ,  $-1\%$ ,  $1\%$  or  $30\%$  change) and the policy variable (price, travel time, etc.) are highly insignificant. In the following regression analysis, we look therefore only at  $1\%$  price increases. This implies a substantial reduction in the size of the data set. Note that without this adjustment,  $t$ -values for the other variables would be inflated. We apply weights to the likelihood function given by the market share of the affected transport mode in the given submarket.

We present two model versions (M1 and M2). In the former we include a generic coefficient for the number of available alternatives in the submarket. Diagnostic tests indicated a multicollinearity issue related to this variable (seemingly because of substantial correlation with some of the constant terms). After removing this variable (model M2), multicollinearity issue appears resolved. However, as models M1 give reasonable coefficient estimates, we opt to present results of M1 as well.

Table 5 shows estimation results for models with simulated DF as explanatory variables. The goodness-of-fit indicators of the estimation models are high, which is not surprising since many of the included explanatory variables were used to create the variation in simulated DFs in the first place.

Table 5: Estimated models.

Model index		M1*		M2	
$N$		622		622	
Adjusted $R^2$		0.925		0.925	
Variable	Type of variable	Value	$t$ -Stat	Value	$t$ -Stat
<b>Generic coefficients</b>					
No. of available alternatives	Continuous (count)	-0.0289	-2.32		
Relative market share of affected mode	Cont. log-transformed	0.0064	2.49	0.0054	2.10
<b>Coefficients for diversion factor towards car</b>					
Distance (car)	Continuous (km)	0.0160	4.99	0.0146	4.62
Urban (car)	Dummy	-0.1184	-2.48	-0.1451	-3.12
Work-related (car)	Dummy	-0.0067	-0.26	-0.0088	-0.34
<b>Coefficients for diversion factor towards train</b>					
Distance (train)	Continuous (km)	0.0168	13.01	0.0162	12.76
Urban (train)	Dummy	0.0144	0.72	-0.0057	-0.31
Work-related (train)	Dummy	-0.0188	-1.17	-0.0195	-1.21
<b>Coefficients when diversion factor towards bus</b>					
Distance (bus)	Continuous (km)	0.0151	12.68	0.0145	12.43
Urban (bus)	Dummy	0.0563	2.80	0.0431	2.23
Work-related (bus)	Dummy	-0.0074	-0.44	-0.0078	-0.46
<b>Coefficients when diversion factor towards metro/tram</b>					
Distance (metro/tram)	Continuous (km)	0.0068	4.93	0.0063	4.62
Urban (metro/tram)	Dummy	0.2812	11.11	0.2791	10.99
Work-related (m/t)	Dummy	0.0004	0.02	0.0011	0.07

(Continued)

Table 5: (Continued)

Model index		M1*		M2	
<b><i>Coefficients when diversion factor towards walk</i></b>					
Distance (walk)	Continuous (km)	-0.0338	-17.89	-0.0370	-28.72
Urban (walk)	Dummy	-0.2584	-12.78	-0.2806	-15.71
Work-related (walk)	Dummy	-0.0056	-0.36	-0.0089	-0.57
<b><i>Coefficients when diversion factor towards cycle</i></b>					
Distance (cycle)	Continuous (km)	-0.0008	-0.55	-0.0028	-2.51
Urban (cycle)	Dummy	0.0013	0.06	-0.0203	-1.13
Work-related (cycle)	Dummy	0.0285	1.83	0.0273	1.75
<b><i>Constant terms for mode pair (altered mode → affected mode)</i></b>					
Car → train	Dummy	0.1012	2.57	0.0297	1.21
Car → bus	Dummy	0.1804	5.59	0.1195	6.37
Car → metro/tram	Dummy	0.0881	2.00	0.0122	0.41
Car → walk	Dummy	0.8055	38.51	0.7743	48.19
Car → cycle	Dummy	0.2155	9.23	0.1793	10.31
Train → car	Dummy	0.3908	5.64	0.3467	5.18
Train → bus	Dummy	0.1322	2.75	0.0586	1.62
Train → metro/tram	Dummy	-0.0316	-0.61	-0.1076	-2.68
Train → walk	Dummy	0.7590	19.57	0.7240	20.20
Train → cycle	Dummy	0.1268	3.23	0.0874	2.46
Bus → car	Dummy	0.4444	7.17	0.3884	6.78
Bus → train	Dummy	0.0291	0.65	-0.0491	-1.67
Bus → metro/tram	Dummy	0.1176	2.27	0.0248	0.75
Bus → walk	Dummy	0.7084	20.69	0.6579	24.82
Bus → cycle	Dummy	0.1742	4.92	0.1189	4.53
Metro/tram → car	Dummy	0.4763	7.30	0.4307	6.90
Metro/tram → train	Dummy	-0.0037	-0.09	-0.0714	-2.27
Metro/tram → bus	Dummy	0.2665	5.62	0.1801	6.14
Metro/tram → walk	Dummy	0.7068	21.11	0.6643	23.64
Metro/tram → cycle	Dummy	0.1811	5.21	0.1338	4.74

\*To facilitate understanding of the model, consider a situation where petrol prices increase and the task is to provide an estimate of the DF from car to bus. Assume there are four alternative modes so that the relative market share of bus/car is 0.666, travel distances are 10 km on average and we look at urban non-work trips. Using the model M3, we estimate  $DF_{car \rightarrow bus} = 4*(-0.0289) + 0.0064*LN(0.6666) + 10*0.0151 + 0.0563 + 0 + 0.1804 = 0.2695$  or 26.95%.

The variable ‘Number of available alternatives’ has a negative and significant impact on DFs. This is intuitive, as the DF towards a given mode should decrease – *ceteris paribus* – when more alternatives are available.

The variable ‘Relative market share of affected mode’ (i.e. relative to altered mode) is positive, meaning that a transport mode with a relatively high market share within a submarket attracts relative more travellers from the affected mode. This is expected given that the relative market share may capture the competitiveness of the affected mode in a given submarket and as such be an indicator of quality (that is, a proxy for the underlying LoS of the affected mode).

The coefficient estimates for distance, urban and work-related trip purpose resemble the general pattern that we already saw in Section 4. Trip distance plays the most prominent role in explaining differences in DFs across affected modes. Clearly, the DF towards walk reduces with increased trip distance. The dummy for urban trips is, as expected, significantly negative for DFs towards car and significantly positive for DFs towards metro/tram. The dummy for urban trips is also significantly negative for walk trips. This may be surprising at first glance but it must be noted that this result is after controlling for trip distance. The results for work-related trips are not significant. We observe a tendency towards cycling having higher DFs for work-related trips. This is likely to relate to the fact that cycling is impractical/inconvenient for some other trip purposes such as grocery shopping and escorting children.

The constant terms for the transport mode pair (altered → affected mode) resemble DFs given trip distance of zero and apply for the normalized segment (no-work suburban trips). The constant terms towards walk are naturally high, as walking is an attractive mode for very short-distance trips.

## 6 CONCLUSIONS AND DISCUSSION

Using our ‘model internal meta-analysis’ method, we have obtained the following results that conform with prior expectation:

1. DFs to walk are in general high but decrease rapidly with increasing distance
2. DFs to cycling tend to be higher for work-related trips
3. DFs to car and train increase with distance
4. The public transport internal DFs (i.e. between public transport modes) are rather high (typically around 50%)
5. DFs are in general lower, the higher the number of available transport modes
6. DFs are in general higher to transport modes with a relative high market share

While results 5 and 6 are more of theoretical interest, results 1–4 may have interesting policy implications. Oslo has a political goal that all future passenger transport growth is facilitated by walk, cycle or public transport. This implies a strong need for cross-modal substitution from car to other modes, since Oslo is experiencing high population growth and the underlying trend is for continued growth in car use. Taking a closer look at results 1–4, we may suggest the following implications for policy:

1. There is greatest potential to get car drivers to substitute to walk for short-distance travel. The DF from car to walk is found to be 64%. Policies that discourage short-distance car use (e.g. parking fees) may therefore be effective.
2. A rather high share of car trips seems to be substitutable with cycle. This appears especially true for work-related trips. In addition to restricting workplace parking availability and pricing, facilitating changing rooms, showers and safe bicycle parking at workplaces may be effective ways to encourage a shift away from car.
3. For longer distance travel in the Greater Oslo area, train is clearly the best substitute to car. Improving train options would therefore result in a relatively high share of long-distance car trips to be transferred to PT.
4. To avoid a strong ‘cannibalization’ between PT modes, it appears important to improve all public transport options. If only one PT option is improved, a relatively large share of new users will come from other PT alternatives.



Our methodological approach was motivated by learning more about variations in DFs (which vary greatly across studies) by holding the general context (study area) and the data and modelling methods fixed.

Important questions relate to the degree to which our results are method/model-driven, and to which results may be specific for the Oslo area and therefore not generalizable.

Result 1 is likely to be universally true. However, the degree is likely to vary between contexts. We regard our estimated values to be transferable as a proxy value for other cities.

On the other hand, result and implication point number 4 ('cannibalizing') may be influenced by the nesting structure of the underlying choice models that have a rather high nest parameter for the PT nest [25]. The nesting structure is defined by the researcher and the value of the estimated nest parameter is (indirectly) conditioned on the specification of utility functions such that high DFs may partly be a consequence of model building. However, the applied nesting structure was the one that fitted the survey data best and should – at least to some extent – represent the 'true' substitution pattern. Note that the high degree of substitutions between PT modes may also be specific to the Oslo area and may only apply to similar cities with an advanced, frequent and widespread public transportation network where several PT options 'overlap'.

Our methodological approach has a few weaknesses that need to be kept in mind. The underlying choice model does not calculate generated (or suppressed) transport which may impact on the absolute size of DFs towards other transport modes. Another important limitation is that we simulate symmetrical DFs for both *direction* and *size* of change, that is, our obtained DFs were close to identical for attribute changes of -30%, -1%, 1% and 30%, respectively. Furthermore, DFs with our method are widely unaffected by the type of attribute that is subject to change (price changes, travel time changes). This may or may not be the case in the real world. Despite these important caveats, we believe that this article has thrown new and to some degree transferable and generalizable light on the under-researched area of DFs and modal substitution.

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#### APPENDIX: DIVERSION OF EQN (4)

In multinomial logit models, we have  $P_i = \frac{\exp(V_i)}{\sum_j \exp(V_j)}$  where the  $V$ s are generalized costs

of the form  $V = b_0 + b_1 X_1 + b_2 X_2 + \dots$  and where the  $X$ s are specific to the alternatives  $j$ , and the  $b$ s may be generic or specific. For simplicity, assume that  $V = a + bX$  without loss of generality. Recall that logit cross-price elasticities (denoted  $\varepsilon$ ) for mode  $i$  are defined as  $\varepsilon_{ii} = b \cdot X_i (1 - P_i)$  and  $\varepsilon_{ij} = -b X_j P_j$  for a linear additive utility function (and similarly for  $j$ ). Using the DF relationship between own- and cross-elasticities, we have  $\varepsilon_{ij} = -\varepsilon_{ji} (P_i / P_j) DF_{ij}$  and substituting in the formula for  $\varepsilon_{jj}$  and  $\varepsilon_{ij}$  we have  $-b \cdot X_j \cdot P_j = -b \cdot X_j \cdot (1 - P_j) \cdot (P_j / P_i) DF_{ji}$  so that  $DF_{ji} = P_i / (1 - P_j)$ .