ENHANCING THE LIFE OF MECHANICAL SYSTEMS SUCH AS REFRIGERATOR IN RAIL TRANSIT BASED ON LIFE-STRESS MODEL AND SAMPLE SIZE FORMULATION

SEONG-WOO WOO[1], DENNIS L. O'NEAL[2], YIMER MOHAMMED HASSEN[1], GEZAE MEBRAHTU[1]
[1]Faculty of Mechanical Engineering, Ethiopian Technical University, Ethiopia.
[2]Department of Mechanical Engineering, School of Engineering and Computer Science, Baylor University, USA.

ABSTRACT

Parametric accelerated life testing (ALT) with the reliability quantitative (RQ) specifications is recommended as reliability methodology to pinpoint design flaws and correct them in transit. It covers (1) cycles of an accumulated failure rate of X% (BX) lifetime with ALT strategy, (2) fatigue design, (3) ALTs with alterations, and (4) discernment if design(s) obtains targeted BX life. The quantum/transport-based (generalized) life-stress failure prototype and sample size formulation for generating RQ specifications were suggested. The equivalent elevated damage potential in parametric ALT was applied, represented by field power spectral density. A case study was used to evaluate a refrigerator fatigued during rail. In first ALT, for RQ specifications – 40 min, refrigerator tubes made of ethylene propylene diene monomer rubber fractured because of mount designs. The failed shape in first ALT was alike to those of the field refrigerator. After mounts and tubes were redesigned, there were no difficulties during second ALT. Refrigerator was assured to fulfill a B1 lifetime for travel distance.

Keywords: design flaws, fatigue, parametric accelerated life testing, quantum/transport-based life-stress model, random vibration, sample size equation.

1 INTRODUCTION

Because of competitive forces in the marketplace, manufacturers need to innovate and enhance the design of their products on a regular basis. As typically driven by consumer demand, such innovation requires new product features to be rapidly implemented into the marketplace. Companies incorporate them into new product's specifications during each design cycle. Without enough reliability testing or expectations of how modifications may be utilized in the field, the introduction of these new attributes may cause unsuccessful products in the field and negatively affect the company's brand image. Prior to release, any product defects might be recognized and adjusted by reliability testing such as parametric accelerated life testing (ALT) [1].

Mechanical fatigue accounts for over 70% of structural failures in machinery parts and load-bearing components, which start as form of cracks from high stress concentrations such as sharp edges, grooves, thin surfaces, etc. and grow it to the end [2]. It affects the reliability of mechanical devices such as running automobiles, airplane wings and main bodies, vessels at sea, nuclear reactors, aircraft engines, and ground-based turbines. With the development of quantum mechanics, designers would recognize that fatigue starts from the lower nanometer size range due to material defects/transport, not explained as traditional continuum design aspects such as strength of materials [3]. That is, field failure stochastically occurs in the areas of locally high stress concentration. For example, the Boeing 737 MAX adopted more efficient Cubic Feet per Minute (CFM) International LEAP-1B engines using the optimized 68-inch fan design; these engines consumed 12% less fuel and were 7% lighter than other engines. If an airplane engine had insufficient stiffness due to decreased engine weight and increased power when subjected to the adjusted (or abruptly increased) random loads from take-offs, cruising, and landings, it could unexpectedly fail due to fatigue in a problematic component – engine.

ISSN: 2046-0546 (paper format), ISSN: 2046-0554 (online), http://www.witpress.com/journals
DOI: 10.2495/CMEM-V10-N4-385-397

The requirements for stiffness, which is the resistance against reversible deformation, rely on the (transit) product application. As large oscillations occur near the natural frequency, engine is designed to protect it from random vibration loads generated in a multi-module product internally (or externally), which cannot precisely predict future dynamic behavior as a deterministic model. To approximate the fatigue damage of a system in field, the equivalent damage method, represented by time-dependent random vibrational spectra transmitted from base supporting structures, needs to be considered for developing an accelerated test [4]. This method allows conversion of these complicated signals into a straightforward Gaussian distribution, represented by the power spectral density (PSD) in frequency field. If based on vibration theory such as force transmissibility, we can get the identical field spectral (or PSD) damage of mechanical system that can be applied to system for accelerated test. However, to do this, it is necessary to have additional numerous fundamental concepts for ALT – fatigue damage spectrum (FDS), life-stress (LS) prototype, and sample size equation.

As an option, the ALT entrenched on reliability block diagrams could be scrutinized. It involves test schemes for the system structure, failure mechanics, elevated testing procedures, sample size formulation, etc. Elsayed [5] categorized physics/statistics, statistical, and physics/experimental-based models for assessment. Hahn and Meeker [6] proposed numerous feasible recommendations to organize an ALT. Modern experimental method would fail to reproduce the field failure(s) due to design imperfections because they evaluate with inadequate samples and testing time for frail components in multi-module products. However, to carry out an ALT, it also requires numerous concepts to be developed such as fracture mechanics, the BX life idea, LS prototype, sample size formulation, etc.

An additional engineering perspective for fatigue failures incorporates the strength of material and finite element method. Various engineer conjectures that unexpected structure failures may be judged by (1) strict formulation employing Newtonian or Lagrangian skills, (2) evaluating the system response for (dynamic) loads and identifying its stress, (3) employing the rain-flow counting procedure through von Mises stress, and (4) approximating system damage employing the Palmgren–Miner rule [7]. Nevertheless, making use of an analytical method that may build a closed-form solution frequently necessitate invoking numerous presumptions that do not know multi-module product failures such as small cracks or pre-existing defects.

Random vibration testing in the laboratory can be conducted by using an FDS-measured environmental data. Some of the most typical test guidelines are found in MIL-STD-810. However, there are inherent limitations because actual environmental stresses (alone or in combination) might not be exactly matched in the laboratory. Consequently, engineer does not presume that a successful item during laboratory testing will survive in the field. As an alternative approach, companies will carry out highly accelerated life testing (HALT) to find the product problems in transit. However, because there are no systematic concepts, such as generalized stress model, accelerated factor, sample size, and lifetime target, the tests will just perform without the reliability quantitative (RQ) specification – mission cycle, sample size, etc.

The objective of this paper is to propose parametric ALT as an analytic method, which may develop the RQ specification, such as mission time for finding the design defects of mechanical systems in transit and modifying them. It incorporates (1) a parametric ALT strategy developed on a newly defined product BX life, (2) a PSD load investigation for ALT, (3) an individually modified sample of ALTs with design alternations, and (4) an appraisal of whether the last design(s) of the system fulfills the objective BX life. A case investigation of domestic refrigerators subjected to repeated random vibrations during railway transit is provided.

2 PARAMETRIC ALT FOR MECHANICAL SYSTEMS IN TRANSIT

2.1 Product life – BX life and its targeting

Reliability is interpreted as the product's capacity to execute under expressed circumstances for a specified time or total distance. It might be often explained by the bathtub curve. If a product's reliability pursues the traditional bathtub curve, it can have difficulties in attaining the accomplishment of an aim in the market because of the inherent design flaws. As a mechanical product is upgraded, the normal (slanted) bathtub curving contour might be altered to a line with the shape parameter β (Fig. 1).

Product is required to define a proper lifetime index for design such as BX life (or 'bearing life'). It is stated as the distance (or elapsed time) where *X%* of a finite or infinite collection of items under consideration shall have been unsuccessful.

If a random variable *T* represents the product life, the Weibull Cumulative Distribution Function (CDF), indicated by *F(t)*, might be stated as follows:

$$P\left(T \le t\right) = F\left(t\right), \tag{1}$$

where $F\left(t\right) = 1 - e^{-\left(\frac{t}{\eta}\right)^{\beta}}$ for Weibull distribution.

The failure rate on the bathtub may be expressed as

$$\lambda = \frac{f}{R} = \frac{dF/dt}{R} = \frac{\left(1-R\right)'}{R} = \frac{-R'}{R}, \tag{2}$$

where f is the density function and λ is the failure rate.

If eqn (2) is integrated over a point of time, *X%* accumulative failure, $F(L_B)$, at T = L_B (*BX* life), $L_{B,}$ might be attained:

$$\int \lambda dt = -ln\, R. \tag{3}$$

In other words,

$$A\left(=X\right) = \left\langle \lambda \right\rangle \cdot L_B = \int_0^{T=L_B} \lambda\left(t\right) \cdot dt = -ln\, R\left(L_B\right) = -ln\left(1-F\right) \cong F\left(L_B\right). \tag{4}$$

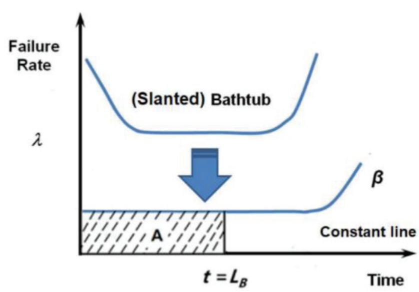


Figure 1: A lifetime index – BX life (L_B) – on the bathtub curve.

We know that the sum of $F(t)$ and $R(t)$ is always 1 or $F(t) + R(t) = 1$ for ≥ 0. Assume that T_1 be a point of time of the earliest failure, we can state reliability function, $R(t)$. In other words,

$$R(t) = P(T_1 > t) = P(\text{no failure} \quad \text{in } (0,t]) = \frac{(m)^0 e^{-m}}{0!} = e^{-m} = e^{-\lambda t}. \tag{5}$$

As product improves, eqn (5) might be obtained from the multiplication of the system life L_B and failure rate λ:

$$R(L_B) = 1 - F(L_B) = e^{-\lambda L_B} \cong 1 - \lambda L_B. \tag{6}$$

Practically, this proportionality in eqn (6) is relevant to less than 20% of accumulative failure rates.

To fulfill the target of a system life by ALT, three occasions of the subsystem (or module) in product were necessitated: (1) a changed module, (2) a newly altered module, and (3) a similar module. Module A that will discussed as a case study in section 2.3 may be regarded as an adjusted module because the compressor rubber is redesigned to dampen the transmitted impact in rail transportation. It had yearly failure rates of 0.25% per year and B1.8 life for 7.2 years in the marketplace. After modification, the module was expected to have yearly failure rates of 0.50% per year and B1.8 life for 3.6 years. To achieve the targeted product lifetime, new lifetime of module A was set to B1 life for 10 years with 0.1% per year (Table 1).

2.2 Quantum/transport-based generalized LS prototype and sample size formulation

Fatigue failure starts to appear on the micro-scale structural surface of a component if there is a stress raiser of part such as groove, notch, thin area, etc. that will generate the void and transport it. The quantum/transport-based LS model might be required.

First, consider non-interacting electrons moving (freely) in a constant potential wall. Time-independent Schrodinger's equation in one dimension can be expressed as follows:

Table 1: Entire ALT strategy of mechanical system such as domestic refrigerator.

Modules	Field Data		Expected Reliability				Aimed Reliability	
	Failure Rate Per Year, %/Year	BX Life, Year	Failure Rate Per Year, %/Year			BX Life, Year	Failure Rate Per Year, %/Year	BX Life, Year
A	0.25	7.2	Modified	×2	0.50	3.6	0.10	10(BX = 1.0)
B	0.30	6.0	Given	×1	0.30	6.0	0.10	10(BX = 1.0)
C	0.40	4.5	New	×5	2.00	0.9	0.10	10(BX = 1.0)
D	0.20	9.0	Given	×1	0.20	9.0	0.15	10(BX = 1.5)
E	0.20	9.0	Modified	×2	0.40	4.5	0.10	10(BX = 1.0)
Others	0.50	12.0	Given	×1	0.50	12.0	0.45	10(BX = 4.5)
Product	1.85	4.5	-	-	3.90	0.9	1.00	10(BX = 10)

$$-\frac{h^2}{8\pi^2 m}\frac{d^2\psi_n(x)}{dx^2} = E_n\psi_n, \tag{7}$$

where ψ_n is wave function, E_n is (electron) energy, h is Planck constant, and m is electron mass.

Boundary conditions are as follows: (1) ψ_n finite outside the metal but decaying exponentially, i.e. $\psi_n \to 0$ as $x \to \infty$ and (2) $\psi_n = 0$ at walls. So, we can find the solution as follows:

$$\psi_n(x) = \sqrt{\frac{2}{a}} sin\left(\frac{n\pi}{a}\right)x;\ E_n = \frac{n^2h^2}{8ma^2} n > 0, \tag{8}$$

where $\psi_n(x + \mathrm{a}) = \psi_n(x)$, a is (periodic) distance, and n is principal quantum number.

There is a series of potential barriers in the atoms of the crystal, which hinder the movement of the charged impurities. As electric magneto-motive field, ξ, (driving force) is applied, the barriers of potential junction energy as a function of distance will be reduced and distorted/phase-shifted. The impurities in materials, produced through electronic movement, are straightforwardly migrated to the right because the passage to the left become difficult.
Linear transport processes may be generally stated as follows:

$$J = LX, \tag{9}$$

where J is a flux vector, X is defined as (thermodynamic or driving) force, and L is a phenomenological transport coefficient.

For example, the solid-state diffusion of impurities in silicon materials takes place as follows: [8]

$$\begin{aligned} J &= \left[aC(x-a)\right]\cdot exp\left[-\frac{q}{kT}\left(w-\frac{1}{2}a\xi\right)\right]\cdot v, \\ &= -\left[a^2ve^{-qw/kT}\right]\cdot cosh\frac{qa\xi}{2kT}\frac{\partial C}{\partial x} + \left[2ave^{-qw/kT}\right]C\ sinh\frac{qa\xi}{2kT}, \end{aligned} \tag{10}$$

where C is the concentration, q is the magnitude of electric charge, υ is the frequency, a is the distance between successive potential barriers, ξ is the applied field, k is the Boltzmann's constant, T is the temperature, Q is the energy, $\Phi()$ is a constant, and B is a constant.

Unless the electric field is relatively small, i.e. $\xi \ll \frac{qa}{2kT}$, eqn (10) might be redefined as follows:

$$J = \Phi(x,t,T)\ sinh(a\xi)\ exp\left(-\frac{Q}{kT}\right) = B\ sinh(a\xi)\ exp\left(-\frac{Q}{kT}\right) \tag{11}$$

On the other hand, the reaction process, which relies on speed, could be stated as follows:

$$\begin{aligned} K &= K^+ - K^- = a\frac{kT}{h}e^{-\frac{\Delta E - aS}{kT}} - a\frac{kT}{h}e^{-\frac{\Delta E + aS}{kT}}, \\ &= a\frac{kT}{h}e^{-\frac{\Delta E}{kT}}\ sinh\left(\frac{aS}{kT}\right) = B\ sinh(aS)\ exp\left(-\frac{\Delta E}{kT}\right), \end{aligned} \tag{12}$$

where K is the reaction rate, S is the (chemical) effect, T is the temperature, k is Boltzmann's constant, E is the (activation) energy, B is the constant, and Δ is the difference.

The reaction rate K from eqns (11) and (12) could make simpler to be understood as follows:

$$K = B\ sinh(aS)\ exp\left(-\frac{E_a}{kT}\right). \tag{13}$$

If eqn (13) takes an inverse expression, the LS prototype could be redefined as follows:

$$TF = A\left[sinh(aS)\right]^{-1} exp\left(\frac{E_a}{kT}\right). \tag{14}$$

The sine hyperbolic term [sinh $(aS)]^{-1}$ in eqn (14) might be expressed as follows: (1) $(S)^{-1}$ in the begin has a linear effect, (2) $(S)^{-n}$ has what is considered as a middle consequence, and (3) $(e^{aS})^{-1}$ at final is rapidly changed (Fig. 2).

A parametric ALT is routinely executed in the middle range. Stress is a matter quantity, which designates the (internal) forces that neighboring particles of matter bring to bear on each other. As the power is stated as the multiplication of effort and flows, stresses can start from effort in a multi-port product. Equation (14) could be restated as follows:

$$TF = A(S)^{-n} \exp\left(\frac{E_a}{kT}\right) = A(e)^{-\lambda} \exp\left(\frac{E_a}{kT}\right) \tag{15}$$

To assess the ride attribute of a system placed on a vehicle, one of the most insightful mathematical representations of a vehicle suspension system is a quarter car model that can be simply analyzed for vibration modes. The quarter car model as two degrees-of-freedom is used to study wheel-hop as well as the bounce mode. It is set up using mutual connections of masses, springs, and dampers. Although it has four state variables, it fulfills the aim of deciding the vehicle movement in transit. In these diagrams, the assumed model of the vehicle includes the sprung mass and the un-sprung mass, individually. The sprung mass, m_s, denotes one-fourth of the body of the vehicle; the un-sprung mass, m_u, denotes one wheel of the vehicle. The principal suspension is formulated as a spring k_s and a damper b_s in parallel, which attaches the un-sprung mass to the sprung mass. The tire (or rail) is modeled as a spring constant of the tire (or rail), k_t, and denotes the movement of the road force to the un-sprung mass (Fig. 3).

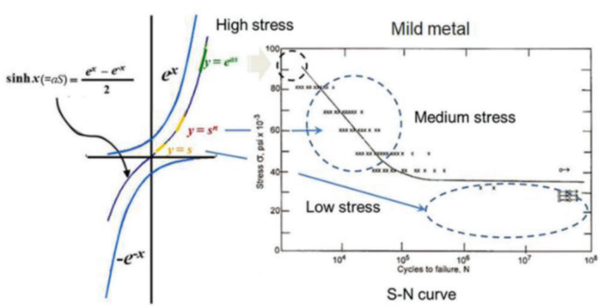


Figure 2: A meaning of hyperbolic sine stress expression in S-N curve or Paris law.

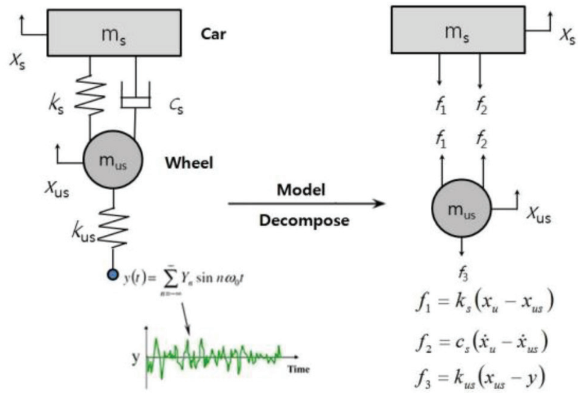


Figure 3: A quarter car model subjected to random loads from the base (or road).

After the quarter car model is decomposed, the government equation can be represented as follows:

$$m_s\ddot{x}_s = -k_s\left(x_s - x_{us}\right) - c_s\left(\dot{x}_s - \dot{x}_{us}\right). \tag{16}$$

$$m_{us}\ddot{x}_{us} = k_s\left(x_s - x_{us}\right) + c_s\left(\dot{x}_s - \dot{x}_{us}\right) - k_{us}\left(x_{us} - y\right). \tag{17}$$

So, eqns (16) and (17) might be simply defined as follows:

$$\begin{bmatrix} m_s & 0 \\ 0 & m_{us} \end{bmatrix}\begin{bmatrix} \ddot{x}_s \\ \ddot{x}_{us} \end{bmatrix} + \begin{bmatrix} c_s & -c_s \\ -c_s & c_s \end{bmatrix}\begin{bmatrix} \dot{x}_s \\ \dot{x}_{us} \end{bmatrix} + \begin{bmatrix} k_s & -k_s \\ -k_s & k_{us}+k_s \end{bmatrix}\begin{bmatrix} x_s \\ x_{us} \end{bmatrix} = \begin{bmatrix} 0 \\ k_{us}y \end{bmatrix}. \tag{18}$$

Consequently, eqn (18) might be stated in a matrix form:

$$\left[M\right]\ddot{X} + \left[C\right]\dot{X} + \left[K\right]X = F_T e^{i\omega t}. \tag{19}$$

For eqn (19), the (random) system response of an approximated single degree of freedom is described. That is,

$$m_e\ddot{X}\left(t\right) + c_e\dot{X}\left(t\right) + k_e X\left(t\right) = c_e\dot{Y}\left(t\right) + kY\left(t\right). \tag{20}$$

As the variables in the time domain are changed into the frequency domain, we can obtain a frequency domain function called FDS. So, eqn (20) can be stated as follows:

$$\ddot{X}(f) + 2j\varsigma\omega_n\dot{X}(f) + \omega_n^2 X(f) = \left[-2j\varsigma\frac{fn}{f} - \left(\frac{fn}{f}\right)^2\right]\ddot{Y}\left(f\right), \tag{21}$$

where ζ is the damping ratio, ω_n ($=2\pi f_n$) is the natural frequency, and ω($=2\pi f$) is the excitation frequency.

The acceleration $\ddot{X}(f)$ can be expressed in the enforced acceleration $\ddot{Y}(f)$. That is,

$$\ddot{X}(f)=\left[\frac{2j\varsigma\left(\frac{f_n}{f}\right)+1}{1-\left(\frac{f_n}{f}\right)^2+2j\varsigma\left(\frac{f_n}{f}\right)}\right]\ddot{Y}(f)=H(jf)\ddot{Y}(f), \tag{22}$$

where *H(jf)* is the frequency transfer function.

From eqn (22), the PSD as the strength of the variations (energy) in the frequency domain can be attained. PSD is used extensively to analyze the mechanical system characteristics for random loads such as forces and moments. The PSD of the acceleration $W_{\ddot{X}}$ can be stated in the PSD of the transmitted acceleration $W_{\ddot{Y}}$. That is,

$$W_{\ddot{X}}(f)=\left|H(f)\right|^2 W_{\ddot{Y}}(f), \tag{23}$$

where $$\left|H(f)\right|=\left[\frac{\left(2\varsigma\frac{fn}{f}\right)^2+1}{\left(1-\left(\frac{fn}{f}\right)^2\right)^2+\left(2\varsigma\frac{fn}{f}\right)^2}\right]^{1/2}. \tag{24}$$

The root mean square (rms) magnitude of the acceleration $\ddot{X}$ can be computed as follows:

$$a=\ddot{X}_{rms}=\int_0^\infty W_{\ddot{X}}(f)df. \tag{25}$$

$$\ddot{X}_{rms}(f_n)=\sqrt{\sum_{k=1}^{N}\left|H(f_n,f_k)\right|^2 W_{\ddot{X}}(f_k)\Delta f_k}. \tag{26}$$

In other words, from eqns (18) and (19), the stresses come from the transmitted vibration loads (F_T) that can be expressed as the PSD level of acceleration for a certain frequency band. Equation (15) can be defined as follows:

$$TF=A(S)^{-n}exp\left(\frac{E_a}{kT}\right)=A(e)^{-\lambda}exp\left(\frac{E_a}{kT}\right)=B(F_T)^{-\lambda}exp\left(\frac{E_a}{kT}\right), \tag{27}$$

where A and B are constants.

The force transmissibility, Q, due to base load, F_T, from eqn (24) can be modeled as follows:

$$Q=\frac{F_T}{kY}=r^2\left[\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}\right]^{1/2}=r^2\left|H(f)\right|, \tag{28}$$

where $r = \omega_n/\omega = f_n/f$, ζ is damping ratio ($c/c_c = c/2\omega_n$), k is spring constant, Y is range of sinusoidal base excitations.

We also know that the PSD level for the given frequency band, a, is calculated from eqns (25) and (26). Thus, the acceleration factor (AF) is expressed as the proportion between the typical and the elevated circumstance. It may be defined as the multiplication of the amplitude proportion of gravitational acceleration R and force transmissibility Q. In other words,

$$\mathrm{AF} = \left(\frac{S_1}{S_0}\right)^n \left[\frac{E_a}{k}\left(\frac{1}{T_0} - \frac{1}{T_1}\right)\right] = \left(\frac{a_1}{a_0}\frac{F_T}{kY}\right)^\lambda \left[\frac{E_a}{k}\left(\frac{1}{T_0} - \frac{1}{T_1}\right)\right] = (R \times Q)^\lambda \left[\frac{E_a}{k}\left(\frac{1}{T_0} - \frac{1}{T_1}\right)\right], \quad (29)$$

where a_1 is the accelerated PSD level for the determined frequency band, a_0 is the normal PSD level for the determined frequency band, T_o is the normal temperature, and T_1 is the accelerated temperature.

To secure the mission cycle – RQ specification – of ALTs from the aimed BX life on the test strategy, the sample size formulation united with the AF in eqn (19) could be obtained as follows: [1]

$$n \geq (r+1) \times \frac{1}{x} \times \left(\frac{L_B^*}{AF \times h_a}\right)^\beta + r. \quad (30)$$

When the target of a product life, such as the household compressor, is put to be a B1 life for 10 years, the assigned cycles could be obtained for an allotted set of samples. In ALTs, the design imperfections of new system might be found to help achieve the life objective.

2.3 Case study: improving fatigue of refrigerator subjected to random vibrations during rail transit

Rail transportation of products is a leading way of moving products from cargo ships to the product's eventual destination – which could be a distributor, warehouse, or end user. According to market data, after both shipping, the refrigerator compressor rubber in the mechanical sections were torn out and the joining tubes to the compressor had fractured under unknown stress conditions during the railway transport. For Europe, the distance at which failure first occurred during rail transportation was roughly 2,400 km in Nis, Serbia. Over 2 days, when the refrigerators travelled the 2,520-km distance from London to Skopje, 10% of the products failed. On the other hand, in the United States, the distance at which failure first occurred during rail transportation was roughly 2,500 km over 2 days. In Chicago, 27% of the transported products failed. Over 7 days, when the refrigerators travelled the 7,200-km distance from Los Angeles to Boston, 67% of the products failed (Fig. 4).

Data from the field indicated that the unsuccessful refrigerators had design flaws. After identifying the problematic refrigerator designs in laboratory tests, manufacturer could modify the problematic designs.

To attain the PSD obtained along rail routes in Fig. 4(b) that incorporated mainline, sideline, and industrial line in the United States, it was measured in vertical and horizontal directions. That is, a SAVER 3X90 shock and vibration field data recorder (Lansmont Corp., CA, USA) was utilized to gather the spectral data for whole travel. The SAVER was installed immediately to the floor situated to the middle of the storing section where the frame hole in a doorway is. After analyzing them, we could obtain the vibration environment spectra, which

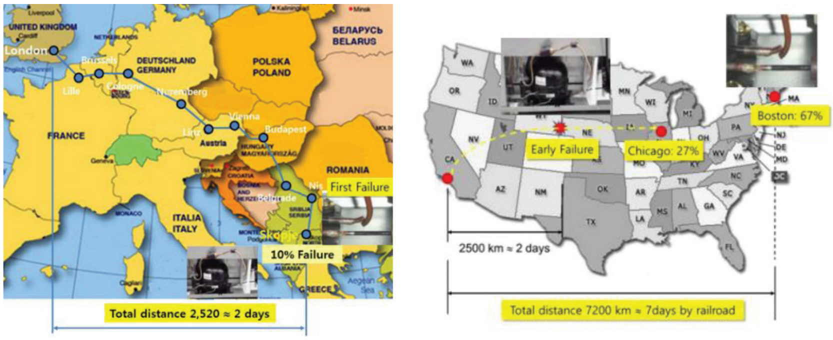
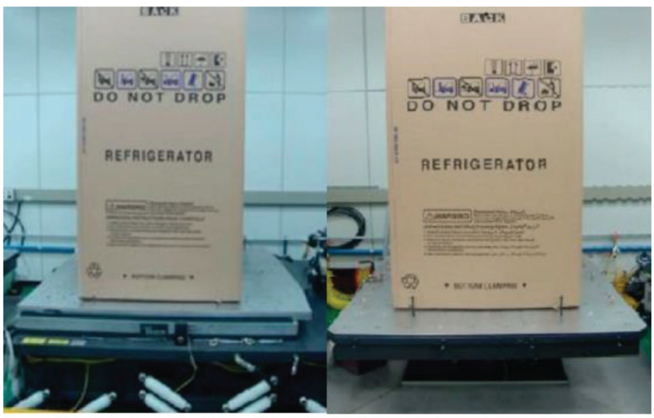


Figure 4: Failed places from the marketplace.



(a) Horizontal vibration (Left ↔ Right) (b) Vertical vibration (Up ↔ Down)

Figure 5: Vibration test for the parametric ALT.

showed a graphic chart of the PSD levels versus frequency. In this investigation, the PSD spectra were represented from 0.3 to 400 Hz to assess the design of refrigerator.

After analyzing the measured vibration spectra, we calculated the AF that multiplies the amplitude proportion of acceleration R and force transmissibility Q from eqn (28) if we know the damping ratio, natural frequency, the normal PSD level, and the accelerated PSD level. A programmed shaker table was used to apply the elevated external random vibrations, expressed by PSD to each refrigerator sample. A table was placed with refrigerator test samples that could be driven by actuators. As random waves and periodic waves such as sine, rectangular, and triangular may be selected as the input motion, random shaking using observed strong PSD motions could particularly simulate the robustness of the product and the design weaknesses for RQ specification. The amplitude of the powerful random vibrations, expressed as G_{rms}, could be adjusted according to the capacity of the specimen for accelerated loads.

In applying the harmonic waves with various frequencies on the refrigerator, the natural frequency of the vibration (left ↔ right) parallel to the plane of the horizon was found to be 5 Hz. The natural frequency of the vibration (up ↔ down) parallel to the plane of the vertical was 9 Hz in the vibration test. To attain the AF for horizontal or vertical directions, based on random vibrations observed in field, amplified PSD loads for each orientation were applied to the refrigerator on a shaking table (Fig. 5).

Because the damping ratio ($\zeta = 0.096 \approx 0.1$) with a settling time of 2 sec and roughly 5% overshoot due to direct contact between the train/refrigerator and frequency ratio was expected, r ($=\omega/\omega_n$) = 1 at the natural frequency ω_n was applied to the refrigerator by rail, the force transmissibility, Q, had a quantity of roughly 5.3 from eqn (28). The AF due to gravitational acceleration for 1 G_{rms} was 4.0 because the refrigerator reached an acceleration of 1 G_{rms} on a shaker table, compared to that of worst-case 0.25 G_{rms}. Using an accumulative damage coefficient, λ, of 2.0, the entire AF in eqn (29) was determined to be 450.0 (Table 3).

Based on the calculated AF, the needed testing time for a given sample size was acquired if the lifetime target was assigned. That is, suppose that the shape parameter in the Weibull plot was 2.0, lifetime objective was put to be B1 life for 7 days. The test time acquired from eqn (30) was roughly 130 min for three sample pieces. If the refrigerator fails less than once in 130 min, the refrigerator design was suitable for a whole travel distance of 7,200 km (7 days) to endure the fatigue damage by random vibration and have a B1 lifetime with roughly a 60% level of confidence.

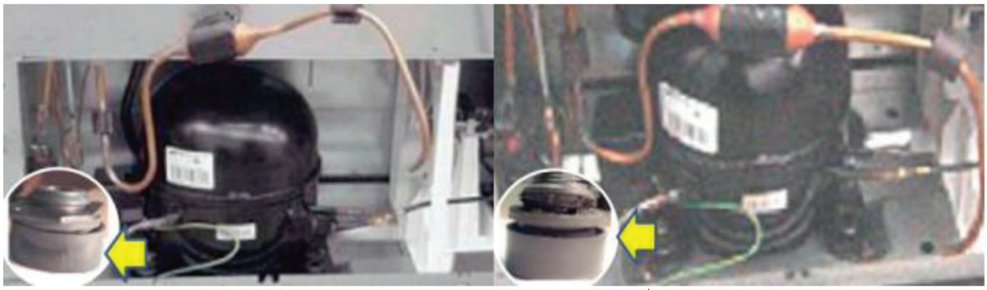
3 RESULTS AND DISCUSSION

As 1.00 G_{rms} at the natural frequency ($r = 1.0$, $\zeta \approx 0.1$) was applied, three refrigerators in the first parametric ALT fractured as follows: one sample at 20 min and two samples at 40 min from the horizontal vibration (left ↔ right). As the rubber mounts tore, the connecting refrigerant tubing in the machine compartment broke, and the refrigerant gas leaked out of the system, the three samples no longer worked. Figure 6 manifests the fractured products from the market and the unsuccessful samples from the parametric ALT, individually. The forms and places of the failures in the ALT were alike to those shown in the marketplace.

Under the close repeated stresses near resonance, we knew that the failure patterns showed in the first ALT and market were close to the failure patterns in the refrigerator from the field. So, the shape parameter, β, was affirmed to be 6.13. Based on both test results, the ALT was

Table 3: Conditions for parametric ALT of the refrigerator.

System Conditions	Worst Case	ALT	AF
Transmissibility, Q ($r = 1.0$, $\zeta = 0.096 \approx 0.1$)	-	5.30 (From Eq. 17)	28.1①
Amplitude ratio of acceleration, R (a_1/a_0)	0.25 G_{rms}	1 G_{rms}	16.0 ②
Total AF (= ① × ②)			450



(a) Field (b) 1^{st} ALT Results

Figure 6: Unsuccessful refrigerator tubes and rubber from the marketplace and first ALT.

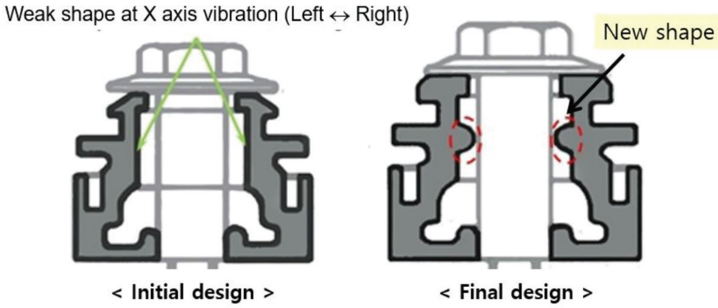


Figure 7: Design modifications.

well-founded because it recognized the design fragilities that were liable for the failures from the field.

The rubber tearing in the first ALT happened because there was no support for the rubber mount to withstand the horizontal vibration (left ↔ right). Due to the design imperfections – such as no rubber support in the high-stress regions – the repetitive random loading may have produced the rubber tearing and broken connection pipes. As action plans, the refrigerator was redesigned: (1) a reshaped rubber mount, C1 (Fig. 7a), and (2) a reshaped joining tube, C2 (Fig. 7b). Second ALTs were then conducted using these modified designs.

The magnitudes of AF and β in Table 3 and Fig. 7 were affirmed to be 452.0 and 6.41, respectively. Based on the test data, because the lifetime objectives of the new samples were less than a B1 lifetime for the whole travel distance (7 days), the recomputed test time in eqn (30) for the three sample refrigerators was 40 min, which would be the specification of the parametric ALT. During the second ALT, the altered designs were successful in protecting the refrigerator from the random-induced vibrations. As a result, the refrigerators were not fractured until 60 min. When a refrigerator reached an acceleration of 1 G_{rms} on a shaking table, we found that the natural frequency of the vibration (left ↔ right) parallel to the plane of the horizon moved from 5 to 8 Hz due to the increase of damping in the system.

Through two rounds of ALTs, as enduring the fatigue damage by random vibration, the new refrigerator was convinced to have B1 lifetime for the whole travel distance of 7,200 km (7 days).

4 CONCLUSIONS

To extend the life of a mechanical system in transit, parametric ALT with RQ specification was proposed. We suggested the quantum/transport based (generalized) time-to-failure equation and sample size formulation. As a case inspection, we investigated the lifetime design of a refrigerator fatigued by random vibrations during rail transportation.

- By inspecting problematic products returned from the field, we found the fracturing of compressor mounts made of ethylene propylene diene monomer rubber and connecting refrigerant tubes in domestic refrigerators. As result, refrigerant gas was leaked and refrigerator did not work.
- To reproduce the problematic product from the marketplace, amplified PSD loads – 1 G_{rms} at resonance in first parametric ALT were performed for RQ specification – 40 min. Like that of field failure, we found the fractured rubber and pipe at 20 and 40 min. After the rubber mounts and the jointing tubes were designed in a different way, in the second ALT

there were no failures for 40 min. Thus, the refrigerator was assured to endure the fatigue damage by random vibration and fulfill the product lifetime needs – B1 lifetime for the whole transit interval of 7,200 km (7 days).

- Examination of the unsuccessful system, load examination, and two rounds of parameter ALTs for RQ specification revealed that the lifetime of the redesigned refrigerator in transit was dramatically enhanced. This structured reliability way may be applicable to other engineered mechanical goods such as airplanes, appliance, automobiles, etc. in transit.

REFERENCES

[1] Woo, S., Pecht, M. & O'Neal, D., Reliability design and case study of the domestic compressor subjected to repetitive internal stresses. *Reliability Engineering & System Safety*, **193**, p. 106604, 2020. https://doi.org/10.1016/j.ress.2019.106604

[2] Duga, J. J., Fisher, W. H., Buxaum, R. W., Rosenfield, A. R., Buhr, A. R., Honton, E. J. & McMillan, S. C., *The Economic Effects of Fracture in the United States*, Final Report, Battelle Laboratories, Columbus, OH. Available as NBS Special Publication, September 30, 1982.

[3] Weingart, R. G. & Stephen, P., Timoshenko: Father of Engineering Mechanics in the U.S. Structure Magazine, 1 August 2007.

[4] Lalanne, C., Mechanical *Vibration and Shock Analysis, Specification Development*. Volume 5. 3rd ed. Hoboken (NJ): John Wiley & Sons, Inc., 2014.

[5] Elsayed, E. A., *Reliability Engineering*. 3rd ed. Hoboken (NJ): John Wiley & Sons, Inc.; 2021.

[6] Hahn, G. J. & Meeker, W. Q., *How to Plan an Accelerated Life Test* (E-Book). Milwaukee: ASQ Quality Press, 2004.

[7] Miner, M. A., Cumulative damage in fatigue. *Journal of Applied Mechanics*, **12**, pp. 149–164, 1945. https://doi.org/10.1115/1.4009458

[8] Grove, A., *Physics and Technology of Semiconductor Device*. 1st Ed. Wiley International Edition, 1967.