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Initialization of an Iterative Low-Complexity Method for Signal Precoding in MM-Wave Massive MIMO Systems

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ABSTRACT

In the last few years, huge interest has been directed towards research in wireless communications technology, notably at the level of the recently born massive MIMO systems. In such systems, the function of precoding at the base station (BS) plays a central goal in guaranteeing reliable downlink transmission. This paper aims to suggest a new low complexity linear precoding algorithm that can provide enhanced performance for downlink mm-wave massive MIMO systems. For this end, a first iterative solution is briefly computed by the Jacobi (Jac) method and then provided as an initialization for the known iterative symmetric successive over relaxation (SSOR) algorithm. This developed iterative way reduces the complexity by one order of magnitude compared with that of the zero forcing (ZF) near-optimal precoding, which relies on direct calculation of a large inverse matrix. In addition, to prove the performance of the new proposed Jac-SSOR iterative algorithm compared with its origin versions, some benchmarking simulations have been carried out in adequate typical scenario.

1. INTRODUCTION

The last decade has seen enormous research studies aimed in improving the performance of radio mobile communication systems, especially with the increasing requirement of wireless throughput by the growing number of their users. The response to this accelerated user demand is targeted across the implementation of massive MIMO technology, which can play a key role in the actual appearance of fifth-generation (5G) cellular systems. In fact, this new technology employs a large number of antennas with structured signal processing to transmit and receive different signals simultaneously at both communication system extremities [1, 2]. Furthermore, the promised benefits that a massive MIMO (m-MIMO) system can deliver depend upon strong challenges concentrated particularly in channel estimation, pilot contamination, precoding, detection, energy and spectral efficiency.

The preprocessing, or precoding, is an essential signal processing procedure in m-MIMO downlinks that uses the channel state information (CSI) from the prior transmitter. With effective delivery CSI, precoding can provide promising benefits to the m-MIMO system as it can mitigate the negative effects created by path loss and inter-user interference and thus maximize link performance [3]. Using a precoding process allows for the simplification of receiver complexity, enhancement of system spectrum efficiency, and reduction of bit error rate. For all this, precoding has found applications not only in massive MIMO but in several other communication systems such as satellites, power lines, and optics [4].

The precoding techniques can be primarily classified into two categories: non-linear and linear approaches. The implementation of nonlinear precoding techniques is difficult for practical purposes due to their complex signal processing, whereas the less complexity of linear precoding techniques allows for a simple implementation at the base station [5]. In addition, the selection of an adequate precoding technique is based on different parameters, such as the computational resources, the number of users, the number of antennas at the base station, and the particularities of the environment.

Among linear precoding schemes that exist in literature, the zero-forcing (ZF) precoding is considered as benchmark because it can reach the near-optimal performance [6]. Furthermore, for the ZF algorithm, a calculation of matrix inversion of large size is required that causes a high computational complexity which is cubic in regards to the number of users [5]. The ZF precoding predicated on the Neumann series approximation has been considered in the study [7] as one way to lower the complexity of matrix inversion by transforming it into a series of matrix-vector multiplications. On the other hand, various iterative-based precoding techniques derived from linear equations have attracted great interest by turning channel matrix inversion into solving linear equations. Among them, a Jacobi method (JM) based precoding has been proposed in the study [8] to overcome the complex matrix inversion and attain near-optimal performance and capacity approximating the ZF precoding. Additionally, the convergence rate of the JM-based precoding has been quantified, and a faster convergence was revealed with the increasing number of BS antennas. Further, the Gauss-Seidel (GS) and the Successive Over Relaxation (SOR) have also been introduced as a conventional example of iteration based precoding schemes [9], in which a series of low-complexity matrix multiplications and additions are employed instead of complicated matrix inversion.

In order to substantially decrease the complexity of the ZF precoding and to simultaneously accomplish its near-optimal performance, a reduced-complexity linear precoding approach relying on the symmetric successive over relaxation (SSOR) method has recently been proposed [10]. The key idea of the SSOR-based precoding is to exploit the asymptotical orthogonality channel feature in m-MIMO systems to approximate an optimal relaxation parameter. In addition, a proposed least square QR (LSQR) precoding scheme [11] has computed iteratively the expected signal after precoding based on QR decomposition; its goal is to mitigate the ZF complexity by avoiding its undesirable matrix inversion of large size. Also, and as an example, a low-complexity and fast-convergence linear precoding based on modified SOR was more recently proposed [12], in which complicated matrix inversion is directly avoided. This new precoding proposition exploits the diagonal-dominant property of the matrix instead of the original zero-vector solution to get good performance with a small number of iterations.

In a quest to achieve almost the same objective as most of the above mentioned techniques, we consider in this work the mm-wave massive MIMO system and suggest an iterative combined precoding approach with low complexity to reach the performance of the ZF near-optimal precoder. In particular, we initialize the existing SSOR algorithm with an initial solution that is provided by the conventional Jacobi algorithm. Hence, the simple proposed approach is based on matrix channel decomposition to prevent wide matrix inversion calculations before starting the precoding iterative procedure, and this leads to a complexity reduction of about one order of magnitude compared with the ZF near-optimal linear precoding technique.

The balance of this paper is organized in the following manner: In section 2, the problem formulation is made first. Then, some interesting linear pre-coding algorithms including the proposed one, together with the complexity analysis, are presented in section 3. Section 4 offers some numerical results with discussions. Finally, in section 5, the conclusion is given.

2. PROBLEM FORMULATION

In the present letter, we consider a single-cell downlink scenario of a large-scale antenna system where its base station (BS) is leveraged with an array of N antennas to simultaneously transmit data for K users with a single antenna, as simply depicted in Figure 1. In such a system, we often assume that N is much greater than K ($N \gg K$). The $N\times1$ transmitted signal x undergoes a precoding at the BS while the $K\times1$ received signal vector y at the user side can be formulated as:

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_K \end{bmatrix} = \sqrt{\rho_r} \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1N} \\ h_{21} & h_{22} & \cdots & h_{2N} \\ h_{31} & h_{32} & \cdots & h_{3N} \\ \vdots & \vdots & \cdots & \vdots \\ h_{K1} & h_{K2} & \cdots & h_{KN} \end{bmatrix}^T \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_N \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ \vdots \\ n_K \end{bmatrix} \quad (1)$$

Or more succinctly:

$$y = \sqrt{\rho_r} Hx + n \quad (2)$$

where, ρ_r is the average received power, H represents the $K\times N$ flat Rayleigh fading channel matrix whose entries are independent and identically distributed (i.i.d) according the distribution $CN(0, 1)$, and n denotes a $K\times1$ (AWGN) vector whose entries are i.i.d. each one following the distribution $CN(0, \sigma^2)$.

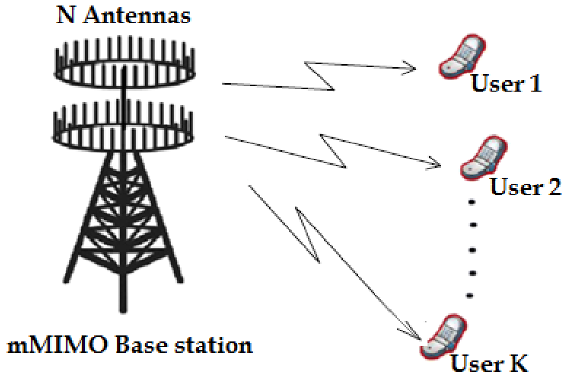

Figure 1. Illustration of a massive MU-MIMO system

When a linear precoding scheme is made in downlink transmission to lower the users' interference, the resultant preceded signal vector x can be expressed as:

$$x = Ws \quad (3)$$

where, W denotes the $N\times K$ precoding matrix, and $s = [s_1, \; s_2, \; \cdots, \; s_K]^T$ accounts for the $K\times1$ vector of original signal for K users to be transmitted. Furthermore, the precoding matrix should be subjected to the following power constraint:

$$tr\left(WW^H\right) = P \quad (4)$$

where, P is the total transmitting power at the BS.

Hence, substituting (3) into (2) yields:

$$y = \sqrt{\rho_r} HWs + n = \sqrt{\rho_r} Qs + n \quad (5)$$

Q=HW is the equivalent channel matrix on which we base to express the signal-to-interference plus noise ratio (SINR) at the reception part. This last parameter can be given for the k^{th} user as:

$$SINR_k = \frac{\frac{\rho_r}{K}\left|q_{k,k}\right|^2}{\frac{\rho_r}{K}\sum_{m\neq k}^{K}\left|q_{m,k}\right|^2 + 1} = \frac{\rho_r}{K}\left|q_{k,k}\right|^2 \quad (6)$$

where, $q_{m,k}$ stands for the element of the matrix Q in the m^{th} row and k^{th} column.

Now, we can compute the ergodic capacity for the downlink massive MIMO system after precoding using the following expression [13]:

$$C = \sum_{k=1}^{K} \log_2\left(SINR_k + 1\right) \quad (7)$$

The achievable capacity is one of the main factors used to evaluate the performance of precoding methods.

3. PRECODING ALGORITHMS

In this section, we first discuss the three linear precoding schemes such as zero-forcing (ZF), Jacobi, and SSOR. Then, we present in detail our proposed scheme. And finally, we will analyze the computational complexity of each method.

3.1 Zero-forcing precoding algorithm

Zero-forcing (ZF) precoding is a basic algorithm that aims to solve the inter-user interference problem by following the optimization criteria to minimize it. In other words, the ZF precoder attempts to nullify all interferences between users. The ZF algorithm is given as [3]:

$$W_{ZF} = \beta H^H (HH^H)^{-1} = \beta H^H G^{-1} \tag{8}$$

where, $G=HH^H$ forms the Gram matrix, and β is the normalized parameter of the average of the transmit power fluctuations. This parameter is defined as:

$$\beta = \sqrt{\frac{K}{tr(G^{-1})}} \tag{9}$$

The resultant signal to be transmitted after ZF precoding can then be expressed as:

$$x_{ZF} = W_{ZF} s = \beta H^H G^{-1} s = \beta H^H z \tag{10}$$

where, $G^{-1}s=z$ that leads evidently to:

$$Gz = s \tag{11}$$

The corresponding received signal vector after ZF precoding becomes:

$$y_{ZF} = \beta\sqrt{\rho_r} HH^H (HH^H)^{-1} s + n = \beta\sqrt{\rho_r} Es + n \tag{12}$$

where, $E = HH^H(HH^H)^{-1}$ is the ZF equivalent channel matrix.

Then, the corresponding received SINR for any user k can be calculated as:

$$SINR_k = \frac{\frac{\rho_r}{K}\left|e_{k,k}\right|^2}{\frac{\rho_r}{K}\sum_{m\neq k}^{K}\left|e_{m,k}\right|^2 + 1} = \frac{\rho_r}{K}\left|e_{k,k}\right|^2 = \frac{\rho_r}{tr\left(G^{-1}\right)} \tag{13}$$

where, $e_{m,k}$ stands for the element of the matrix E in the m^{th} row and k^{th} column.

According to Eq. (13), we can compute the sum capacity achieved by the ZF precoding for the massive MIMO system using the following expression [14]:

$$C_{ZF} = \sum_{k=1}^{K} \log_2\left(SINR_k + 1\right) = K\log_2\left(\frac{\rho_r}{tr\left(G^{-1}\right)} + 1\right) \tag{14}$$

3.2 Jacobi precoding algorithm

Probably, among all the iterative linear methods, the Jacobi method is the simplest one to avoid the direct calculation of matrix inversion [15]. In fact, the Jacobi method solves a diagonally dominant linear system $At=b$. Its process involves separating the matrix A as follows:

$$A = D + R \tag{15}$$

or more clearly:

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1N} \\ a_{21} & a_{22} & \cdots & a_{2N} \\ \vdots & \vdots & \cdots & \vdots \\ a_{N1} & a_{N2} & \cdots & a_{NN} \end{bmatrix} = \begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & a_{NN} \end{bmatrix} + \begin{bmatrix} 0 & a_{12} & \cdots & a_{1N} \\ a_{21} & 0 & \cdots & a_{2N} \\ \vdots & \vdots & \cdots & \vdots \\ a_{N1} & a_{N2} & \cdots & 0 \end{bmatrix} \tag{16}$$

where, D is the diagonal component matrix and R is the remainder off-diagonal matrix.

When we put $Gz=s$ exactly equivalent to the previous linear equation $At=b$, we can arrive at:

$$\left(D + R\right)z = s \tag{17}$$

Formally, the Jacobi precoding scheme sets:

$$z_{(i+1)} = D^{-1}Rz_{(i)} + D^{-1}s \tag{18}$$

where, i denotes the iteration index.

The initial solution of the Jacobi algorithm is given by:

$$z_{(0)} = D^{-1}s \tag{19}$$

The precoded signal vector resulting from the Jacobi precoding algorithm is:

$$x_{Jac} = \beta H^H z \tag{20}$$

3.3 SSOR precoding algorithm

The symmetric successive over relaxation (SSOR) method is considered as a modified variant of its original SOR method. In the SSOR method, each iteration is formed with two half iterations: a forward iteration, which is the SOR method, followed by a backward iteration, which is actually the SOR method with equations in reverse order. The goal of the SSOR based precoding is to overcome the complicated matrix inversion problem in an iterative manner. Since the precoding matrix W is Hermitian positive definite, as demonstrated in [10], we can use the SSOR method for linear precoding according to these three steps:

Step1: we decompose the precoding matrix W, or rather the Gram matrix G, into three parts as below:

$$G = D + L + U = D + L + L^H \tag{21}$$

where, in order, D, L, and $U=L^H$ stand for the diagonal, strictly lower triangular, and strictly upper triangular constituents of G.

Step 2: basing on the SOR method, we compute directly the forward iteration of the precoded signal to be transmitted using the following mathematical expression:

$$z_{(i+\frac{1}{2})} = \left(D+\omega L\right)^{-1}\left(\left(1-\omega\right)Dz_{(i)} - \omega L^{H} z_{(i)} + \omega s\right) \tag{22}$$

Step 3: expressing the SOR method in reverse order, we compute the backward iteration of the precoded signal to be transmitted as follows:

$$z_{(i+1)} = \left(D+\omega L^{H}\right)^{-1}\left(\left(1-\omega\right)Dz_{(i+\frac{1}{2})} - \omega L z_{(i+\frac{1}{2})} + \omega s\right) \tag{23}$$

here, the "relaxation factor" ω is a real number selected in the interval $0<\omega<2$. Moreover, an optimal value of this parameter has been selected through a detailed mathematical analysis [10], which is given as:

$$\omega_{opt} = \frac{2}{1+\sqrt{2\left(1-\rho\left[B\right]\right)}} \tag{24}$$

where, $\rho[\cdot]$ stands for the spectral radius of a matrix. Here B is the iteration matrix of the Jacobi method, which is presented as [16]:

$$B = D^{-1}(L+L^{H}) = D^{-1}R \tag{25}$$

Finally, the SSOR precoded signal can be given as:

$$x_{SSOR} = \beta H^{H} z \tag{26}$$

It is recalled that β is the normalized parameter of the average of the transmit power, which is expressed in (9).

3.4 Proposed Jac-SSOR precoding algorithm

As we have previously mentioned, the Jac-SSOR linear precoding scheme is simply a SSOR algorithm initialized by the Jacobi method. So, this new proposed scheme consists of two main parts, which are the initialization and the iteration. We summarize below the complete steps of our proposed precoding approach in algorithm I.

Algorithm I: Proposed Jac-SSOR precoding scheme

Input:
s: original signal
H: channel matrix
N: number of BS antennas
K: number of users
I: number of iterations
Initialization step:
$G=HH^{H}$: Gram matrix
$\beta = \sqrt{\frac{K}{tr(G^{-1})}}$: normalized parameter of the average of the transmit power
$D = diag\left(G\right),\ L = tril\left(G\right), R = L + L^{H}, B = D^{-1}R$
$\omega_{opt} = \frac{2}{1+\sqrt{2\left(1-\rho\left[B\right]\right)}}$: optimal relaxation factor
$z_{(0)} = D^{-1}s$: first Jacobi solution
$z_{(1)} = (D-L)^{-1}(L^{H} z_{(0)} s)$: initialization with Jacobi method when (i=1)
Iteration step:
for $i = 1 : I-1$
$z_{(i+\frac{1}{2})} = \left(D+\omega L\right)^{-1}\left(\left(1-\omega\right)Dz_{(i)} - \omega L^{H} z_{(i)} + \omega s\right)$
$z_{(i+1)} = \left(D+\omega L^{H}\right)^{-1}\left(\left(1-\omega\right)Dz_{(i+\frac{1}{2})} - \omega L z_{(i+\frac{1}{2})} + \omega s\right)$
end
Output:
$x_{Jac-SSOR} = \beta H^{H} z$: Jac-SSOR precoded signal

3.5 Computational complexity analysis

In the ensuing, we analyse first the computational load of the proposed Jac-SSOR precoder, and then we compare it with the basic ZF and the other iterative schemes studied in this work (see Table 1). As it is known, the computational complexity can be expressed generally in terms of the two fundamental arithmetical operations, which are addition and multiplication. However, the last operation dominates the complexity of numerous precoding schemes.

The initialization part of the proposed Jac-SSOR precoding scheme requires $i(4K^2\text{-}2K)$, which is equivalent to the number of multiplications in the conventional Jacobi method, whereas the same number of multiplications as in the SSOR scheme is required for its iteration part. Thus, the total required number of complex multiplications of the Jac-SSOR precoding scheme is $6iK^2$.

Table 1. Complexity of the investigated precoding schemes

Precoding schemes	Number of multiplications
ZF	$N+NK+K^3$
Jacobi	$i\left(4K^2\text{-}2K\right)$
SSOR	$i\left(2K^2\text{+}2K\right)$
Proposed Jac-SSOR	$6iK^2$

In addition, the proposed scheme is able to keep the same order of the required complexity as their original algorithms while at the same time lowering it by one order relative to the ZF precoder complexity.

4. NUMERICAL SIMULATION RESULTS

In this section, we take the BER and sum-rate as the principal performances of the proposed Jac-SSOR precoding in order to compare it with the existing Jacobi and SSOR techniques on the one hand, and we consider the basic ZF as a reference for comparison on the other hand. For this end, we assume that the typical massive MIMO configuration involves 128 transmitting antennas at the base station and 16 single antenna users. The base station takes 64 QAM as the modulation scheme, while Rayleigh fading is selected as the system channel model.

Figure 2, Figure 3, and Figure 4 depict a comparison of BER performance between proposed Jac-SSOR, classical Jacobi, and classical SSOR for different iterations for the Rayleigh fading channel of a massive MIMO system. In addition, the BER curve of ZF precoding accompanies the other curves as a benchmark for comparison.

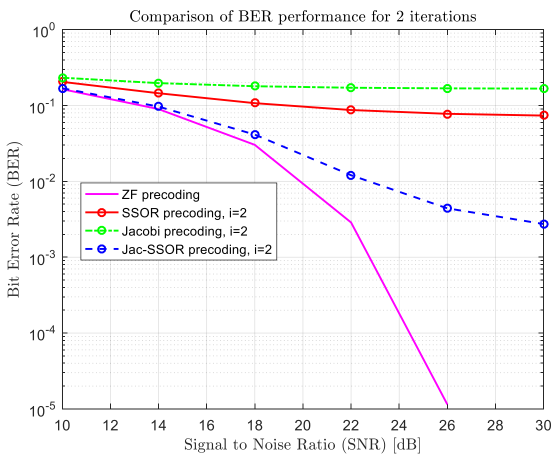

Figure 2. Performance comparison of precoding iterative techniques (i=2 iterations) in terms of BER for $N \times K$=128×16 massive MIMO configuration in Rayleigh fading channels

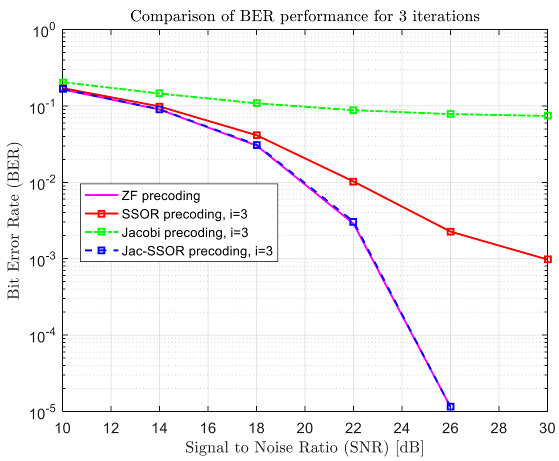

Figure 3. Performance comparison of precoding iterative techniques (i=3 iterations) in terms of BER for $N \times K$=128×16 massive MIMO configuration in Rayleigh fading channels

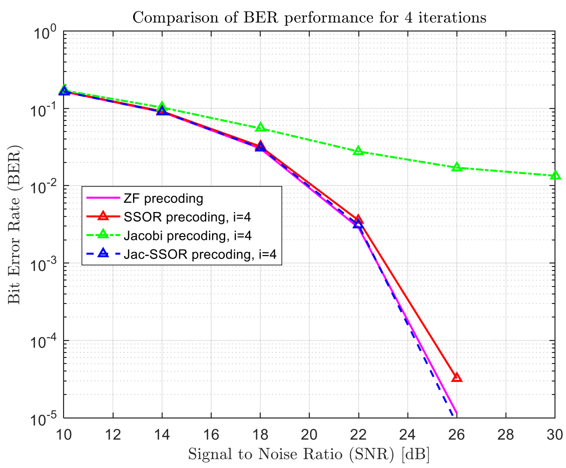

Figure 4. Performance comparison of precoding iterative techniques (i=4 iterations) in terms of BER for $N \times K$=128×16 massive MIMO configuration in Rayleigh fading channels

Figure 2 shows that when the number of iterations is smaller, the Jacobi and SSOR conventional methods suffer from low convergence and poor BER performance. Nevertheless, a significant difference in BER performance is observed between the proposed method and the other two conventional methods, especially with the increasing SNR.

It is also clear from Figures 3 and 4 that the BER performance of the two basic precoding methods can be improved proportionally with the increase in iterations. Furthermore, the SSOR method has always had a faster convergence rate compared with Jacobi method. We further note that the near-optimal BER performance of the ZF precoding technique can be achieved by the Jac-SSOR in only 3 iterations.

Basing on the results obtained from the previous figures, we conclude that the newly devised approach is more satisfying compared to its two original constructive methods in a comprehensive view. The convergence rate of the new approach is fast, and its performance in terms of BER is very close to that of the reference ZF precoding method.

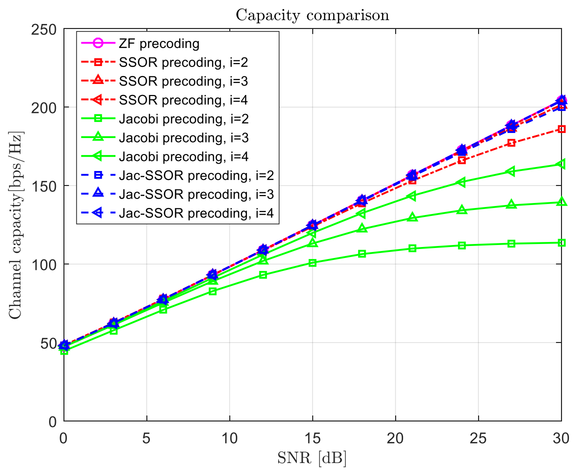

Figure 5. Comparison of the channel capacity of the studied precoding iterative techniques for an $N \times K$=128×16 massive MIMO configuration

Figure 5 compares the channel capacities of Jacobi, SSOR, Jac-SSOR, and ZF precoding methods against the Signal to Noise Ratio (SNR). It is clear from this figure that there is a significant improvement in the achievable channel capacity of the Jacobi-based precoding with increasing iteration value. However, a number of iterations equal to 4 remains insufficient for the Jacobi method to be comparable to the optimal ZF precoding. Besides, we can observe that SSOR performs better than Jacobi, whereas a little difference in its performance compared to ZF can be noticed even with a lower number of iterations. Also, Jac-SSOR iterative precoding can get a lot closer to the benchmark ZF with matrix inversion from the point of view of channel capacity since its second iteration, and with only 3 iterations, its achieved capacity becomes identical to that of the ZF precoding. This good result confirms further the validity and accuracy of our proposed scheme for precoding in a downlink massive MIMO system.

5. CONCLUSIONS

In this paper, an efficient hybrid linear-based precoder for mm-wave massive MIMO systems is devised by combining the individual good aspects of both Jacobi and symmetric successive over relaxation (SSOR) classical algorithms. The low complexity Jac-SSOR approach uses the conventional Jacobi to compute a first iterative solution that will be used as an initialization for the SSOR algorithm in order to follow the iteration procedure. Basing on all our comparison results, we can clearly notice the effectiveness of the proposed Jac-SSOR precoder compared with its origin versions in terms of bit error rate and achievable rate at the same time. In other words, our developed Jac-SSOR-based precoding approach allows achieving the near optimal properties of low computational complexity, fast convergence, and significant channel capacity. Its superiority in massive MIMO linear precoding performance has been confirmed in comparison with the Jacobi and SSOR classical algorithms.

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