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Fractional Order Sliding Mode Controller for HBV Epidemic System

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ABSTRACT

The Hepatitis-B (HBV) epidemic's dynamic can be presented as a compartment model. Determining the HBV epidemic control strategy can be considered a nonlinear feedback control problem. The sliding mode controller (SMC) is an effective feedback control method for controlling the dynamical system under disturbances. Recently, the SMC based on fractional order calculus can provide preferable characteristics for a control system such as robustness and convergence rate. In this study, the HBV epidemic system's control policy is proposed using the fractional order sliding mode controller (FOSMC). The control policy with multiple measures including vaccination, isolation, and treatment is formulated to manipulate the susceptible and the infected subpopulations to the desired level. The Lyapunov-based approach is proven for stability analysis. The control policy is applied to the simulation example to verify the feasibility of the proposed FOSMC method. The simulation results are compared with those of the integer order SMC. By the proposed method, the results reveal that the susceptible and infected subpopulations are driven to the desired levels under disturbances with a higher convergence rate compared to that of the integer one. Moreover, the proposed FOSMC method can reduce the chattering occurrence which is the primary drawback of the SMC method.

1. INTRODUCTION

Hepatitis B is one of the most prevalent infectious diseases and concerns public health on a worldwide scale. Recently, the worldwide prevalence of Hepatitis B virus (HBV) is about 292 million people [1]. There are considerable regional differences in the prevalence of infection. In Africa, there is the highest prevalence of HBV infection, followed by Central Asia and the Southeast Asia region [1, 2]. Acute and chronic liver illness can both be caused by HBV infection. The chronic illness might progress to cirrhosis and liver cancer in the infected person [3]. Nowadays, the primary route of HBV transmission is transmitted vertically (mother-to-child) which accounts for roughly 50% of total cases [4, 5]. Therefore, it is essential to develop control strategies to prevent HBV infection and subsequently restrict its transmission. HBV epidemic control can be achieved via different measures. This includes vaccination, treatment, and isolation [6]. Vaccination is the most successful strategy to decrease the prevalence of chronic HBV infection and control mother-to-child transmission [7]. For newborns, HBV vaccines are recommended as routine immunizations in many countries [8-11]. Within 24 hours of birth, infants should receive their first dose of the HBV vaccine for perinatal HBV transmission prevention [12]. For HBV treatment, the primary objective is to reduce the onset of liver fibrosis which might develop into cirrhosis and liver cancer [12, 13]. Additionally, controlling infectious disease epidemics by isolation measures is an effective preventive strategy [14].

A compartmental model is commonly constructed to mathematically represent the dynamic transmission of HBV. Medley et al. [15] developed a SECIR mathematical model to observe the HBV transmission mechanism. This model was also applied to evaluate the epidemic situation in New Zealand [16] and Canada [17]. Pang et al. [18] modified Medley et al.' model to investigate how control strategies affect HBV transmission. Additionally, a four-compartmental model was developed to predict new HBV case numbers in China [19].

In the model, the control policy which consists of multiple measures can be obtained by the optimal control based on Pontryagin's maximum principle [6, 20-23]. However, using this approach to set the control policy requires an accurate model. Moreover, finding the policy in analytical form is complex when the model contained nonlinear terms and the order of the model is high. Based on compartmental model representation, the feedback control approach is simpler to synthesize the control policy in the analytical form [24]. Also, feedback control is suitable for locating the policy for the model with disturbances and uncertainties. As mentioned in [25], the control strategy according to feedback control can handle uncertainties and disturbances which occur in the biological system. Also, it is not complicated to determine a control strategy in an analytical form. These aspects are advantages compared to dynamic optimization. The control strategies for many epidemic diseases including HBV diseases have been synthesized using nonlinear control techniques presented in previous works [24-34].

One of the nonlinear control techniques used in applications for epidemic control is sliding mode control (SMC) [24, 26, 27, 29, 32, 35]. The robustness is an attractive property of the SMC method. However, as noted in the literature [35-37], the fundamental problem of the SMC approach is that it causes chattering, or high frequency in the control input signal. To overcome the disadvantage, fractional order sliding mode control (FOSMC) is proposed as one of the enhanced versions of the SMC method. In the FOSMC, the improvement in terms of chattering reduction [37] and fast response [38-43], is achievable through the concept of fractional order calculus [44]. With these advantages, the FOSMC was utilized in several applications such as aerospace [45-48], missile guidance [49-51], automotive [52-53], robotic [40, 54-57], energy, and power systems [58-61]. Also, the SEIR epidemic model's vaccination control policy was established by the FOSMC [62-63].

In accordance with the study [6, 20-23], various measures can be applied for eradicating the HBV epidemic. Consequently, the control policy for the HBV epidemic systems is formulated from various measures. The well-known feedback controls such as synergetic control and time-varying sliding mode control (TVSMC) have been employed to set the control policies containing various measures in an analytical form as presented in studies [32, 34]. However, the robustness of the control epidemic systems has not been investigated in prior research.

This study, according to the feasibility to apply feedback control for setting the epidemic control policy or strategy and the advantages of the FOSMC method. The control policy to eradicate the epidemic of HBV based on the FOSMC method is proposed.

Following is an overview of the study's motivations.

-To the best of the authors' knowledge, the FOSMC method has not been developed for the HBV epidemic systems in any previous studies.

-In the research [63-64], the FOSMC approach was used to set the epidemic control policy with only one control input (vaccination) which is a single input single output (SISO) control system. The focus of these studies [62, 63] has not explored the control strategies with multiple control inputs known as a multiple input multiple output (MIMO) system.

In the research [32, 34], the control system's robustness is not investigated for controlling the HBV epidemic system.

The subsequent sections of the article are structured as follows. In Section II, the mathematical model representing the HBV epidemic, related background about fractional order calculus, FOSMC controller design, and proof of stability are presented. Section III shows the simulation results and discussion corresponding to the control system under the proposed control. This research's conclusion is stated in Section IV.

2. MATHEMATICAL MODELLING AND CONTROLLER DESIGN

2.1 Mathematical model

The HBV control policy can be set using a feedback controller approach based on the HBV compartmental model presented in the study [6], which was used to synthesize the control policy with multiple measures based on the FOSMC method. To demonstrate the dynamic behavior of HBV, this mathematical model is provided with four differential equations as presented in Eq. (1). The nonlinear model includes four epidemiological groups that are susceptible ($S(t)$) individuals who are susceptible to HBV infection, acute infected ($I_1(t)$) individuals with the highly infectious preliminary phase of HBV infection, chronical infected ($I_2(t)$) who can either spread disease or are not infectious, and recovered ($R(t)$) individuals who have recovered from the acute and chronical infected stage. The aim of the control strategy is to minimize the classes of susceptible, acute infected, and chronical infected subpopulations while increasing the number of recovered subpopulations. To achieve the set objectives, the measures for controlling the HBV epidemic of the model are isolation policy ($u_1(t)$), treatment policy ($u_2(t)$), and vaccination policy ($u_3(t)$).

$$\begin{aligned}
\dot{S} &= b-\alpha SI_1(1-u_1)-\mu_0 S-u_3 S \\
\dot{I}_1 &= \alpha SI_2(1-u_1)-(\mu_0+\beta+\gamma_1)I_1-(u_2+u_3)I_1 \\
\dot{I}_2 &= \beta I_1-(\mu_0+\mu_1+\gamma_2)I_2-(u_2+u_3)I_2 \\
\dot{R} &= \gamma_1 I_1+\gamma_2 I_2+u_3 S-\mu_0 R+(u_2+u_3)(I_1+I_2)
\end{aligned} \tag{1}$$

The model's parameters are denoted as follows. Parameters b and μ_0 represent the birth rate and the natural death of susceptible subpopulations respectively. The parameter α defines the rate at which the susceptible subpopulation moves to the acute infected subpopulation. The parameter β represents the moving rate from the acute infected subpopulation to the chronical infected subpopulation. The parameters γ_1 and γ_2 define the recovery rate from the acute and chronical infected subpopulation to the recovered subpopulation. The parameter μ_1 denotes the death rate caused by HBV which involved the growth rate of chronic subpopulation. The constraints of all measures of the control policy are unity constraints and are defined as $0 \le u_1 \le 1$, $0 \le u_2 \le 1$, and $0 \le u_3 \le 1$. Importantly, the nonnegative initial condition of all subpopulations is assumed as $S(0) \ge 0$, $I_1 \ge 0$, $I_2 \ge 0$, and $R(0) \ge 0$.

The compartment model can be expressed in the nonlinear affine state-space system. The state and the control vectors are defined as $x(t)=[x_1\ x_2\ x_3\ x_4]^T=[SI_1\ I_2\ R]^T$ and $u(t)=[u_1\ u_2\ u_3]^T$, respectively. The model can be represented using matrices and vectors:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \\ f_4(x) \end{bmatrix} + \begin{bmatrix} g_{11}(x) & g_{12}(x) & g_{13}(x) \\ g_{21}(x) & g_{23}(x) & g_{33}(x) \\ g_{31}(x) & g_{32}(x) & g_{33}(x) \\ g_{41}(x) & g_{42}(x) & g_{43}(x) \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \tag{2}$$

where,

$$f_1(x) = b-\alpha x_1 x_3-\mu_0 x_1$$

$$f_2(x) = \alpha x_1 x_3 - (\mu_0 + \beta + \gamma_1)x_2$$
$$f_3(x) = \beta x_2 - (\mu_0 + \mu_1 + \gamma_2)x_3$$
$$f_4(x) = \gamma_1 x_2 + \gamma_2 x_3 + \mu_0 x_4$$
$$g_{11}(x) = \alpha x_1 x_3, g_{12}(x) = 0, g_{13}(x) = -x_1$$
$$g_{21}(x) = -\alpha x_1 x_3, g_{22}(x) = -x_2, g_{23}(x) = -x_2$$
$$g_{31}(x) = 0, g_{32}(x) = -x_3, g_{33}(x) = -x_3$$
$$g_{41}(x) = 0, g_{42}(x) = (x_2 + x_3),\ g_{43}(x) = (x_1 + x_2 + x_3)$$

2.2 Fractional order calculus

As the design process of FOSMC design involved fractional order derivative, important definitions and related basic knowledge about fractional derivative operators are summarized as follows.

The definition of the integro-differential equation operator is presented in the study [64-71]:

$$ {}_aD_t^{\lambda} f(t) = \begin{cases} \dfrac{d^{\lambda}}{dt^{\lambda}} f(t), \mathrm{Re}(\lambda) > 0 \\ f(t), \mathrm{Re}(\lambda) = 0 \\ \int_a^t f(\tau)(d\tau)^{-\lambda}, \mathrm{Re}(\lambda) < 0 \end{cases} \tag{3}$$

The order λ could be real or complex number. The upper and lower limit of operation are denoted by a and t. Three common definitions of fractional order derivative are presented as follows [64-71]:

Riemann Louisville's definition

$$ {}_aD_t^{\lambda} f(t) = \frac{1}{\Gamma(n-\lambda)} \frac{d^n}{dt^n} \int_a^t \frac{f(\tau)}{(t-\tau)^{\lambda-n+1}} d\tau \tag{4}$$

where, D_t^{λ} is the differentiation operator and p represents the fractional order of the derivative and define as n-1<λ<n. The variable Γ is a gamma function.

Caputo's definition

$$ {}_aD_t^{\lambda} f(t) = \frac{1}{\Gamma(n-\lambda)} \int_a^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\lambda-n+1}} d\tau \tag{5}$$

The term n in Eq. (4) and Eq. (5) denotes the order of derivative.

Grunwald-Letnikov's definition

$$ {}_aD_t^{\lambda} f(t) = \lim_{h\to 0} h^{-\lambda} \sum_{i=0}^{[(t-a)/h]} (-1)^{-i} \binom{\lambda}{i} f(t-ih) \tag{6}$$

where, $[(t\text{-}a)/h]$ refers to the integer part of $(t\text{-}a)/h$.

In this study, the definition of Caputo's fractional order derivative was utilized in the FOSMC controller design procedure.

2.3 Control objectives

As presented in the study [6], the aim of the policy is to minimize the classes of susceptible subpopulation, acute infected subpopulation, and chronical infected subpopulation. Thus, the control goal of designing feedback controls is to minimize the error between each of these subpopulations and the corresponding desired level. The errors can be expressed as

$$e_i = x_i - x_{ir} \tag{7}$$

where, x_{ri}=0 for i=1,2,3 are desired levels of susceptible, acute infected, and chronical infected subpopulations.

2.4 Controller design procedure

According to references [45, 59, 73, 74], the FOSMC design procedure for setting the HBV control policy is summarized as follows:

First, select the sliding surface. From the structure of the compartmental model in Eq. (2), the sliding surface is defined according to the fractional order sliding surface presented in the study [55] as

$$s_i = D^{1-\lambda}(e_i(t)) + k_{Ii} D^{-\lambda}(e_i(t)) \tag{8}$$

Second, synthesize control inputs based on the reaching law [39]:

$$\dot{s}_i = \theta_i \tag{9}$$

where, θ_i=-[k_{swi} sign(s_i)+$k_{spi}s_i$] for i=1,2,3. From the properties of fractional order derivative's definition, Eq. (9) can be obtained as

$$D^{1-\lambda} D^{\lambda} [D^{1-\lambda} e_i(t) + k_{Ii} D^{1-\lambda} e_i(t)] = \theta_i \tag{10}$$

Substituting the dynamic from Eq. (2) into Eq. (10) yields

$$\begin{gathered} D^{1-\lambda}[f_i(x) - \dot{x}_{ir} + e_i] + D^{1-\lambda} \sum_{j=1}^{3} g_{ij} u_j = -\theta_i \\ D^{1-\lambda} \sum_{j=1}^{3} g_{ij} u_j = -D^{1-\lambda}[f_i(x) - \dot{x}_{ir} + e_i] - \theta_i \\ \sum_{j=1}^{3} g_{ij} u_j = v_i \end{gathered} \tag{11}$$

where, $v_i = -[f_i(x) - \dot{x}_{ir} + e_i] - D^{\lambda-1}\theta_i$ for i=1,2,3. Eq. (11) can be expressed in a matrix form and solved as

$$\begin{gathered} \begin{bmatrix} g_{11}(x) & g_{12}(x) & g_{13}(x) \\ g_{21}(x) & g_{22}(x) & g_{23}(x) \\ g_{31}(x) & g_{32}(x) & g_{33}(x) \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \\ \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} g_{11}(x) & g_{12}(x) & g_{13}(x) \\ g_{21}(x) & g_{22}(x) & g_{23}(x) \\ g_{31}(x) & g_{32}(x) & g_{33}(x) \end{bmatrix}^{-1} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \end{gathered} \tag{12}$$

2.5 Proof of stability

According to the control input presented in Eq. (12), the Lyapunov stability theorem is used to demonstrate the stability of the HBV epidemic control system. The Lyapunov function is selected as

$$V = s^T s = 0.5\sum_{i=1}^{3} s_i^2 \tag{13}$$

The derivative of Eq. (13) is calculated as

$$\begin{aligned} V &= s^T \dot{s} = s^T (D^{1-\lambda} D^{\lambda} s) \\ &= [s_1\ s_2\ s_3] \left\{ \begin{bmatrix} D^{1-\lambda}[f_1(x) + \dot{x}_{1r} - e_1] \\ D^{1-\lambda}[f_2(x) + \dot{x}_{2r} - e_2] \\ D^{1-\lambda}[f_3(x) + \dot{x}_{3r} - e_3] \end{bmatrix} + D^{1-\lambda} \left(\begin{bmatrix} g_{11}(x) & g_{12}(x) & g_{13}(x) \\ g_{21}(x) & g_{22}(x) & g_{23}(x) \\ g_{31}(x) & g_{32}(x) & g_{33}(x) \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \right) \right\} \end{aligned} \tag{14}$$

If matched disturbances, $d_1(t)$, $d_2(t)$ and $d_3(t)$ occur in the susceptible and infected individuals in the HBV epidemic system, and substituting Eq. (12) into Eq. (14) yield

$$\begin{aligned} V &= [s_1\ s_2\ s_3] \left\{ \begin{bmatrix} D^{1-\lambda}[f_1(x) + d_1(t) + \dot{x}_{1r} - e_1] \\ D^{1-\lambda}[f_2(x) + d_2(t) + \dot{x}_{2r} - e_2] \\ D^{1-\lambda}[f_3(x) + d_3(t) + \dot{x}_{3r} - e_3] \end{bmatrix} + D^{1-\lambda} \left(\begin{bmatrix} -[f_1(x) - \dot{x}_{1r} + e_1] - D^{\lambda-1}\theta_1 \\ -[f_2(x) - \dot{x}_{2r} + e_2] - D^{\lambda-1}\theta_2 \\ -[f_3(x) - \dot{x}_{3r} + e_3] - D^{\lambda-1}\theta_3 \end{bmatrix} \right) \right\} \\ \dot{V} &= [s_1\ s_2\ s_3] \begin{bmatrix} D^{1-\lambda} d_1(t) - k_{sw1} sign(s_1) - k_{sp1} s_1 \\ D^{1-\lambda} d_2(t) - k_{sw2} sign(s_2) - k_{sp2} s_2 \\ D^{1-\lambda} d_3(t) - k_{sw3} sign(s_3) - k_{sp3} s_3 \end{bmatrix} \\ \dot{V} &= -k_{sp1} s_1^2 - k_{sp2} s_2^2 - k_{sp3} s_3^2 \\ &\quad + (D^{1-\lambda} d_1(t) - k_{sw1} sign(s_1)) \\ &\quad + (D^{1-\lambda} d_2(t) - k_{sw2} sign(s_2)) \\ &\quad + (D^{1-\lambda} d_3(t) - k_{sw3} sign(s_3)) \end{aligned} \tag{15}$$

Then,

$$\begin{aligned} \dot{V} \le &-k_{p1} s_1^2 - k_{p2} s_2^2 - k_{p3} s_3^2 + \left(M_{d1} s_1 - k_{sw1} sign(s_1) s_1 \right) \\ &+ \left(M_{d2} s_2 - k_{sw2} sign(s_2) s_2 \right) + \left(M_{d3} s_3 - k_{sw3} sign(s_3) s_3 \right) \end{aligned} \tag{16}$$

Since $|D^{1-\lambda} d_i(t)| < M_{di}$, $0 \le M_{di} < \infty$ and the values of k_{swi} is chosen such that $k_{swi} > M_{di}$ for i=1,2,3, it can be obtained as follows:

$$\begin{aligned} \dot{V} < &-k_{p1} s_1^2 - k_{p2} s_2^2 - k_{p3} s_3^2 + \left(M_{d1} - k_{sw1} \right) |s_1| \\ &+ \left(M_{d2} - k_{sw2} \right) |s_2| + \left(M_{d3} - k_{sw3} \right) |s_3| < 0 \end{aligned} \tag{17}$$

According to Eq. (17), clearly, the specified Lyapunov function's derivative is negative definite [72-73]. Thus, the control HBV epidemic system is stable and robust under the control input.

3. RESULTS AND DISCUSSION

The simulation is used as a tool to assess the FOSMC method's capabilities to prevent the spread of HBV. This section is divided into two parts: simulation example; and simulation results.

3.1 Simulation example

In this study, the mathematical model with system parameters and initial conditions used in a simulation example is in accordance with the study [6]. The numerical values of these parameters are presented as follows: α=0.8, β=0.025, γ_1=0.05, γ_2=0.5, μ_0=0.0121, μ_1=0.02 and b=0.0121. Initial conditions are assumed as $x_1(0)$=100, $x_2(0)$=20, $x_3(0)$=20 and $x_4(0)$=12. First, the controlled epidemic HBV system without disturbances under the FOSMC was simulated. The simulation results were compared with those under the integer order SMC. Second, in order to investigate the control policy's robustness, it is assumed that there are bounded disturbances $d_2(t)$ and $d_3(t)$ which occur and affect the growth rates of acute and chronical infected individuals, while the disturbance which affected the susceptible individual is zero as $d_1(t)$=0. The bounded disturbances are defined as

$$d_i(t) = \begin{cases} 0, 0 \le t < t_s \\ A_{di} \sin(5t) + d_{ci}, t_s \le t < t_e \\ 0, t_e < t < \infty \end{cases} \tag{18}$$

where, T_s=15 days, T_e=20 days, A_{di}=0.25, d_{ci}=0.5 for i=2,3. Then, the FOSMC policy with λ=0.65 was applied to the HBV system affected by the disturbances. The rest of the controller parameters were selected as follows: k_{Ii}=0.001, k_{spi}=0.001, and k_{swi}=50.

MATLAB Simulink was implemented to simulate the control HBV system from the initial time of t=0 day to 30 days with 0.001 day of incremental time. The Oustaloup filter was utilized to implement the fractional order derivative as presented in the study [39, 65, 74-76].

3.2 Simulation results

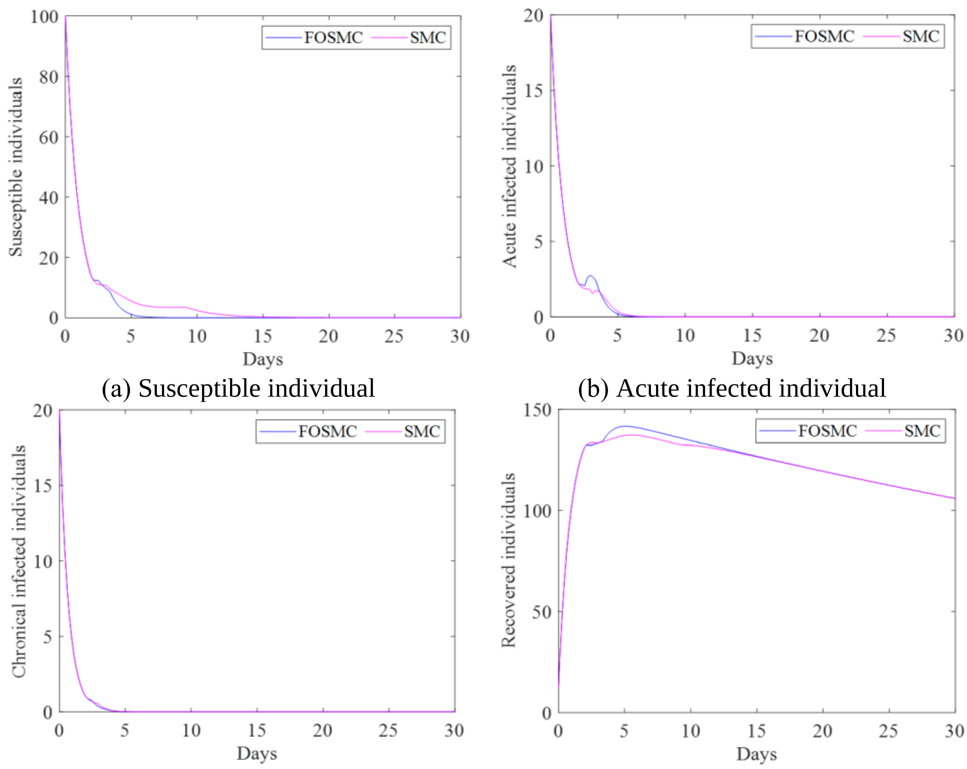
The enhancement of the convergence property of our FOSMC policy was illustrated in the simulation results. The results of the control HBV system manipulated by the policy are based on FOSMC with fractional order of 0.65 compared with those corresponding to the integer order SMC (λ=1).

Without the effect of disturbances, the time responses of the susceptible individual, acute infected individual, chronical infected individual, and recovered individual of the control HBV epidemic system are presented in Figure 1. All target individuals corresponding to the FOSMC policy converge faster to the desired level than those corresponding to the integer order SMC scheme. The proposed FOSMC policy can minimize the susceptible, acute infected, and chronical infected subpopulations as the major goal of the control policy. In terms of convergence performance, the improvement of time response of the FOSMC approach over the SMC approach concurs with the studies carried out by the study [38-43]. The control input of our proposed FOSMC and SMC corresponding to the measures are presented in Figure 2. Based on this figure, the isolation and vaccination policies are the key strategies that require their maximum attempt to prevent HBV infection and its transmission for most of the duration. However, the treatment policy is necessary for a short period of time to limit the deleterious effects of antiviral medications.

To investigate the robustness of the FOSMC policy, the response of the control system affected by the disturbances are presented in Figure 3. When disturbances occurred, both approaches (FOSMC and SMC) still influence all target individuals the corresponding desired levels. In addition, as demonstrated in Figure 3, all target individuals corresponding to the FOSMC policy are driven and converge faster to the desired level than those corresponding to the integer order-based SMC scheme. The control inputs under the effect of disturbance of the proposed FOSMC and SMC are shown in Figure 4. Thus, the control objective is satisfied as that defined in the previous study [6] where the control policy is synthesized based on a dynamic optimization approach in numerical form.

As mentioned above, chattering is the SMC method's primary weakness which it generates a high frequency in the input signal. The control inputs' results are presented in Figure 2 (without disturbances) and Figure 4 (with disturbances). By comparing with and without disturbances, the control inputs perform similar dynamic efforts to control the target individuals. However, for SMC policy, the effect of disturbance brings about notable differences in control inputs during the bound disturbance period. The chattering of each control input provided by the FOSMC policy occurs less than each control input corresponding to the integer order one. The reduction of chattering occurrence from the FOSMC method agrees with the results in the literature [37]. In practice, chattering free control action is rational to apply to real-world patient management.

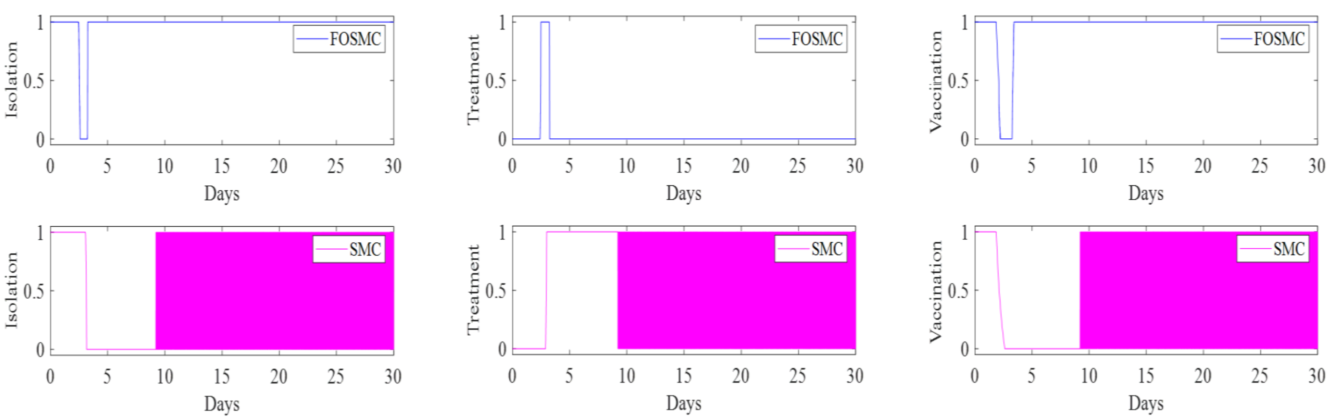
As a result, the simulation outcomes demonstrated that the proposed control policy with multiple measures provides desirable characteristics for the control HBV epidemic system: robustness, convergence rate improvement, and chattering reduction.



(a) Susceptible individual (b) Acute infected individual

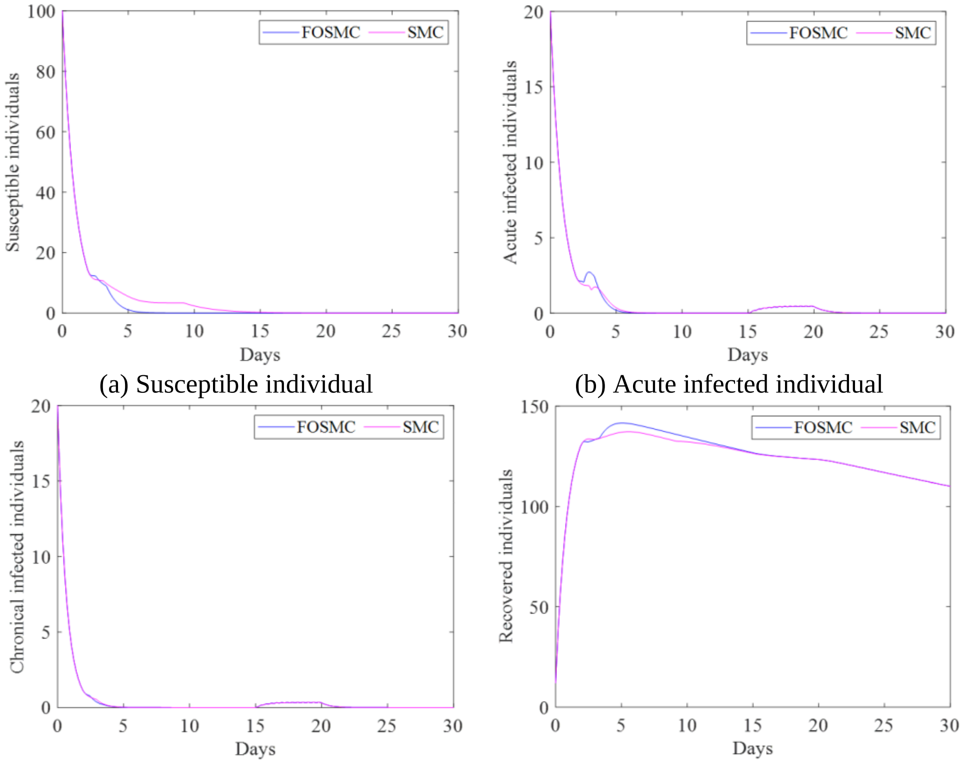
(c) Chronical infected individual (d) Recovered individual

Figure 1. Plot of state variables versus time of FOSMC (λ=0.65) and SMC (λ=1) without disturbances



(a) Isolation control (b) Treatment control (c) Vaccination control

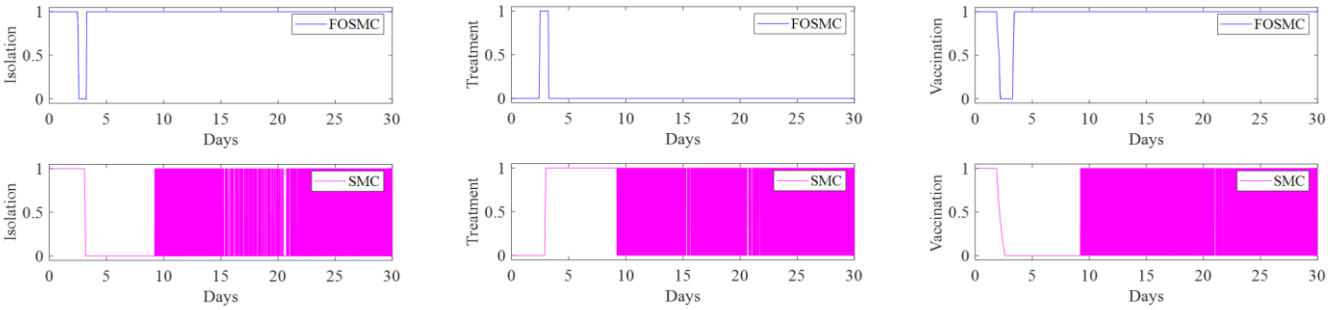
Figure 2. Plot of control inputs versus time of FOSMC (λ=0.65) and SMC (λ=1) without disturbances



(a) Susceptible individual (b) Acute infected individual

(c) Chronical infected individual (d) Recovered individual

Figure 3. Plot of state variables versus time of FOSMC (λ=0.65) and SMC (λ=1) under disturbances



(a) Isolation control (b) Treatment control (c) Vaccination control

Figure 4. Plot of control inputs versus time of FOSMC (λ=0.65) and SMC (λ=1) under disturbances

4. CONCLUSION

This paper proposes the FOSMC method to establish the control policy for the HBV epidemic system. The proposed control law representing the HBV epidemic control policy is presented in an analytical form to minimize susceptible, acute infected, and chronical infected subpopulations. Based on the results of this study, the main contributions of the study are summarized as follows.

-It is more logical and efficient to control HBV transmission with the FOSMC method using a control policy with multiple measures including isolation, vaccination, and treatment than with a control policy with a single control measure. Setting a multiple-measure control policy using the FOSMC approach is regarded as a multiple-input, multiple-output (MIMO) nonlinear feedback control problem. This is more challenging than the case of a single measure control which is considered a single input and single output (SISO) nonlinear feedback control problem.

-The impact of bounded disturbances is applied to investigate the robustness of the FOSMC control system. Under the proposed control policy, robustness and an improved rate of convergence are achieved.

-The chattering reduction in the measures of our proposed FOSMC policy is satisfactory, compared to the integer order-based SMC policy.

The present study has limitations due to some state variables corresponding to subpopulations in the model cannot be observed or being unavailable for a full state feedback controller. Setting the control policy based on an observer-based FOSMC controller can enhance the ability of the HBV control policy. This is a plausible future research direction.

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