International Information and Engineering Technology Association
Advancing the World of Information and Engineering

Mathematical Modelling of Engineering Problems
Vol. 9, No. 6, December, 2022, pp. 1603-1612
Journal homepage: http://iieta.org/journals/mmep

# Some New Properties on Block Matrices Using MATLAB Code

Ahmed A.H. Alkhalidi[1*], Mujtaba Zuhair Ali[1], Mohsen Naeem Yasser[2]

[1] Computer Technical Engineering Department, College of Technical Engineering, The Islamic University, Najaf P. O. Box 9, Iraq
[2] Department of Mathematics, Faculty of Education, University of Kufa, Najaf P. O. Box 9, Iraq

Corresponding Author Email: ahmedalkhalidi8618@iunajaf.edu.iq

https://doi.org/10.18280/mmep.090621

**Received:** 4 August 2022
**Accepted:** 20 November 2022

***Keywords:***
*block matrices, permutation matrices, MATLAB code*

**ABSTRACT**

In this article, we study some new properties of the block matrices. Introduce the methods for finding the block matrices of any matrix. We prove some of these methods by using the matrix analysis methods. Some results of this paper are proved by using the permutations of matrices of the block matrices. In the last of this study, some examples are introduced to explain the results of this work and we give some MATLAB code in the appendix to explain the results of this article.

## 1. INTRODUCTION

In mathematics, a block matrix or a partitioned matrix is a matrix that is interpreted as having been broken into sections called blocks or submatrices [1]. Intuitively, a matrix interpreted as a block matrix can be visualized as the original matrix with a collection of horizontal and vertical lines, which break it up, or partition it, into a collection of smaller matrices [2]. Any matrix may be interpreted as a block matrix in one or more ways, with each interpretation defined by how its rows and columns are partitioned.

Block matrices are widely used in physics and applied mathematics, and they are used naturally to describe systems with several discrete variables (such as quantum spin, quark colour, and flavour, for example) [2-7]. Block matrices are also used in several computational techniques that are common among fluid dynamics researchers [8-10]. For both analytical and numerical applications, it is virtually universally necessary to determine the determinants of these matrices [1, 11-13].

In this section we define the permutation matrices for block matrices in general method and we introduced the definition of permutation block matrices. some new idea are given to calculate the block matrices for any given matrix.

This section contains some results of finding block matrices. These results suggest the new properties of block matrices form the given matrices.

We have two tapes of matrices according the number of rows ($m$) and columns ($n$).

Square matrices when ($m$=$n$) and non-square matrices when ($m$≠$n$).

**SQUARE MATRICES**

It is a matrix in which the number of rows equals the number of columns when ($m$=$n$).

$$A=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}_{n\times n}$$

Write the block matrices from any matrix by using way:

### 1.1 Method equal division

Divided the main matrix in to square sub-matrices and of the same size like in the follows:

$$A=\begin{bmatrix}
\boldsymbol{a}_{11} & \cdots & \boldsymbol{a}_{1(k)} & \boldsymbol{a}_{1(k+1)} & \cdots & \boldsymbol{a}_{1(2k)} & \cdots & \boldsymbol{a}_{1(pk+1)} & \cdots & \boldsymbol{a}_{1(rk=n)} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
\boldsymbol{a}_{(k)1} & \cdots & \boldsymbol{a}_{(k)(k)} & \boldsymbol{a}_{(k)(k+1)} & \cdots & \boldsymbol{a}_{(k)(2k)} & \cdots & \boldsymbol{a}_{(k)(pk+1)} & \cdots & \boldsymbol{a}_{(k)(rk=n)} \\
\boldsymbol{a}_{(k+1)} & \cdots & \boldsymbol{a}_{(k+1)(k)} & \boldsymbol{a}_{(k+1)(k+1)} & \cdots & \boldsymbol{a}_{(k+1)(2k)} & \cdots & \boldsymbol{a}_{(k+1)(pk+1)} & \cdots & \boldsymbol{a}_{(k+1)(rk=n)} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
\boldsymbol{a}_{(2k)1} & \cdots & \boldsymbol{a}_{(2k)(k)} & \boldsymbol{a}_{(2k)(k+1)} & \cdots & \boldsymbol{a}_{(2k)(2k)} & \cdots & \boldsymbol{a}_{(2k)(pk+1)} & \cdots & \boldsymbol{a}_{(2k)(rk=n)} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
\boldsymbol{a}_{(pk+1)1} & \cdots & \boldsymbol{a}_{(pk+1)(k)} & \boldsymbol{a}_{(pk+1)(k+1)} & \cdots & \boldsymbol{a}_{(pk+1)(2k)} & \cdots & \boldsymbol{a}_{(pk+1)(pk+1)} & \cdots & \boldsymbol{a}_{(pk+1)(rk=n)} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
\boldsymbol{a}_{(rk=n)1} & \cdots & \boldsymbol{a}_{(rk=n)(k)} & \boldsymbol{a}_{(rk=n)(k+1)} & \cdots & \boldsymbol{a}_{(rk=n)(2k)} & \cdots & \boldsymbol{a}_{(rk=n)(pk+1)} & \cdots & \boldsymbol{a}_{(rk=n)(rk=n)}
\end{bmatrix}
=\begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1b} \\ A_{12} & A_{22} & \cdots & A_{2b} \\ \vdots & \vdots & \ddots & \vdots \\ A_{b1} & A_{b2} & \cdots & A_{bb} \end{bmatrix}$$

In this case the obtained matrix will be square sup-matrices.

$$A_{11} = \begin{bmatrix} a_{11} & \cdots & a_{1(k)} \\ \vdots & \ddots & \vdots \\ a_{(k)1} & \cdots & a_{(k)(k)} \end{bmatrix}_{k\times k}$$

$$A_{12=}\begin{bmatrix} a_{1(k+1)} & \cdots & a_{1(2k)} \\ \vdots & \ddots & \vdots \\ a_{(k)(k+1)} & \cdots & a_{(k)(2k)} \end{bmatrix}_{k\times k}$$

...

$$A_{bb} = \begin{bmatrix} \boldsymbol{a}_{(\boldsymbol{pk+1})(\boldsymbol{pk+1})} & \cdots & \boldsymbol{a}_{(\boldsymbol{pk+1})(\boldsymbol{rk=n})} \\ \vdots & \ddots & \vdots \\ \boldsymbol{a}_{(\boldsymbol{rk=n})(\boldsymbol{pk+1})} & \cdots & \boldsymbol{a}_{(\boldsymbol{rk=n})(\boldsymbol{rk=n})} \end{bmatrix}_{k\times k}$$

To find the identity block matrices we have two methods:

**Theorem (1-1-1)**

The first method to finding the block matrices by using the main diagonal of the block matrix which ($A_{11}$, $A_{22}$, …, $A_{bb}$) write the identity matrix for these sup-matrices and the rest of the sup-matrices in the same row are zero matrices as follows:

$$A = \begin{bmatrix} A_{11} & \cdots & A_{1b} \\ \vdots & \ddots & \vdots \\ A_{b1} & \cdots & A_{bb} \end{bmatrix}$$

It will be like this

$$I = \begin{bmatrix} I_{11} & 0_{12} & \cdots & 0_{1b} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{b1}2 & \cdots & 0_{b(b-1)} & I_{bb} \end{bmatrix}$$

**Proof**

Since all sup-matrix of (k × k) matrices then the identity matrix which in each row and column contains only one element with a value of 1 and the rest of the elements in that row or column are zeros will be in form:

$$I_{ij} = \begin{bmatrix} 1 & 0 & \cdots \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix}_{K\times K}$$

$$AI = \begin{bmatrix} A_{11} & \cdots & A_{1b} \\ \vdots & \ddots & \vdots \\ A_{b1} & \cdots & A_{bb} \end{bmatrix}_{b\times b} \begin{bmatrix} I_{11} & 0_{12} & \cdots & 0_{1b} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{b1} & \cdots & 0_{b(b-1)} & I_{bb} \end{bmatrix}_{b\times b}$$

$$= \begin{bmatrix} A_{11}I_{11} + A_{12}0_{21} + \cdots + A_{1b}0_{b1} & \cdots & A_{11}0_{1b} + \cdots + A_{1(b-1)}0_{b(b-1)} + A_{1b}I_{bb} \\ \vdots & \ddots & \vdots \\ A_{b1}I_{11} + A_{b2}0_{21} + \cdots + A_{bb}0_{b1} & \cdots & A_{b1}0_{1b} + \cdots + A_{b(b-1)}0_{b(b-1)} + A_{bb}I_{bb} \end{bmatrix}$$

$$\therefore A_{ij}I_{ij} = [a_{ij}]_{k\times k}[1]_{k\times k} = [a_{ij}]_{k\times k} = A_{ij}$$

$$\therefore A_{ij}0_{ij} = [a_{ij}]_{k\times k}[0]_{k\times k} = [0]_{k\times k} = 0_{ij}$$

$$AI = \begin{bmatrix} A_{11} & \cdots & A_{1b} \\ \vdots & \ddots & \vdots \\ A_{b1} & \cdots & A_{bb} \end{bmatrix} = A$$

$$I = \begin{bmatrix} I_{11} & 0_{12} & \cdots & 0_{1b} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{b1} & \cdots & 0_{b(b-1)} & I_{bb} \end{bmatrix}$$

The last block matrices called Identity block matrix (I.B.).

**Theorem (1-1-2)**

The second method we use the secondary diagonal for the block sup-matrices which ($A_{1b}$, $A_{2(b-1)}$, …, $A_{b1}$) write the identity matrix for these sup-matrices and the rest of the sub-matrices in the same row are zero matrices as follows:

$$A = \begin{bmatrix} A_{11} & \cdots & A_{1b} \\ \vdots & \ddots & \vdots \\ A_{b1} & \cdots & A_{bb} \end{bmatrix}$$

It will be like this

$$\begin{bmatrix} 0_{11} & \cdots & 0_{1(b-1)} & I_{1b} \\ \vdots & \vdots & \ddots & \vdots \\ I_{b1} & 0_{b2} & \cdots & 0_{bb} \end{bmatrix}$$

**Proof**

In this method get the identity block matrix by using the permutation method of matrices to finding the identity block matrices:

$$I = \begin{bmatrix} I_{b1} & 0_{b2} & \cdots & 0_{bb} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{11} & \cdots & 0_{1(b-1)} & I_{1b} \end{bmatrix}$$

$$AI = \begin{bmatrix} A_{11} & \cdots & A_{1b} \\ \vdots & \ddots & \vdots \\ A_{b1} & \cdots & A_{bb} \end{bmatrix}_{b\times b} \begin{bmatrix} I_{b1} & 0_{b2} & \cdots & 0_{bb} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{11} & \cdots & 0_{1(b-1)} & I_{1b} \end{bmatrix}_{b\times b}$$

$$= \begin{bmatrix} A_{11}I_{b1} + A_{12}0_{(b-1)1} + \cdots + A_{1b}0_{11} & \cdots & A_{11}0_{bb} + \cdots + A_{1(b-1)}0_{2b} + A_{1b}I_{1b} \\ \vdots & \ddots & \vdots \\ A_{b1}I_{b1} + A_{b2}0_{(b-1)1} + \cdots + A_{bb}0_{11} & \cdots & A_{b1}0_{bb} + \cdots + A_{b(b-1)}0_{2b} + A_{bb}I_{1b} \end{bmatrix}$$

$$\therefore A_{ij}I_{ij} = [a_{ij}]_{k\times k}[1]_{k\times k} = [a_{ij}]_{k\times k} = A_{ij}$$

$$\therefore A_{ij}0_{ij} = [a_{ij}]_{k\times k}[0]_{k\times k} = [0]_{k\times k} = 0_{ij}$$

$$AI = \begin{bmatrix} A_{11} & \cdots & A_{1b} \\ \vdots & \ddots & \vdots \\ A_{b1} & \cdots & A_{bb} \end{bmatrix} = A$$

$$I = \begin{bmatrix} I_{b1} & 0_{b2} & \cdots & 0_{bb} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{11} & \cdots & 0_{1(b-1)} & I_{1b} \end{bmatrix}$$

The last block matrix called Identity block matrix (I.B.).

### 1.2 Method of non-equal division

If the matrix is square and the division sup-matrices is not equal, that is mean, all the sup-matrices with different rows and columns from the other,

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}_{n\times n}$$

To find the matrix of blocks for the basic matrix, we use the following methods:

**Theorem (1-2-1)**

Divided the main matrix in to sup-matrices so that the sup-matrices that make up the main diagonal are square matrices like in the follows:

$$A=\begin{bmatrix} a_{11} & \cdots & a_{1(k_1)} & a_{1(k_1+1)} & \cdots & a_{1(k_2)} & \cdots & a_{1(k_p+1)} & \cdots & a_{1(k_r=n)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ a_{(k_1)1} & \cdots & a_{(k_1)(k_1)} & a_{(k_1)(k_1+1)} & \cdots & a_{(k_1)(k_2)} & \cdots & a_{(k_1)(k_p+1)} & \cdots & a_{(k_1)(k_r=n)} \\ a_{(k_1+1)} & \cdots & a_{(k_1+1)(k_1)} & a_{(k_1+1)(k+1)} & \cdots & a_{(k_1+1)(k_2)} & \cdots & a_{(k_1+1)(k_p+1)} & \cdots & a_{(k_1+1)(k_r=n)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ a_{(k_2)1} & \cdots & a_{(k_2)(k_1)} & a_{(k_2)(k+1)} & \cdots & a_{(k_2)(k_2)} & \cdots & a_{(k_2)(k_p+1)} & \cdots & a_{(k_2)(k_r=n)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ a_{(k_p+1)1} & \cdots & a_{(k_p+1)(k_1)} & a_{(k_p+1)(k+1)} & \cdots & a_{(k_p+1)(k_2)} & \cdots & a_{(k_p+1)(k_p+1)} & \cdots & a_{(k_p+1)(k_r=n)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ a_{(k_r=n)1} & \cdots & a_{(k_r=n)(k_1)} & a_{(k_r=n)(k+1)} & \cdots & a_{(k_r=n)(k_2)} & \cdots & a_{(k_r=n)(k_p+1)} & \cdots & a_{(k_r=n)(k_r=n)} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1b} \\ A_{12} & A_{22} & \cdots & A_{2b} \\ \vdots & \vdots & \ddots & \vdots \\ A_{b1} & A_{b2} & \cdots & A_{bb} \end{bmatrix}$$

where, $b=k_r$.

In this case, the matrices representing the diagonal will be about square sup-matrices:

$$A_{11}=\begin{bmatrix} a_{11} & \cdots & a_{1(k_1)} \\ \vdots & \ddots & \vdots \\ a_{(k_1)1} & \cdots & a_{(k_1)(k_1)} \end{bmatrix}_{k_1\times k_1}$$

$$\cdots$$

$$A_{bb}=\begin{bmatrix} a_{(k_p+1)(k_p+1)} & \cdots & a_{(k_p+1)(k_r=n)} \\ \vdots & \ddots & \vdots \\ a_{(k_r=n)(k_p+1)} & \cdots & a_{(k_r=n)(k_r=n)} \end{bmatrix}_{k_r\times k_r}, \forall(i=j)$$

and the rest of the sup-matrices will be non-square:

$$A_{12}=\begin{bmatrix} a_{1(k_1+1)} & \cdots & a_{1(k_2)} \\ \vdots & \ddots & \vdots \\ a_{(k_1)(k_1+1)} & \cdots & a_{(k_1)(k_2)} \end{bmatrix}_{k_1\times k_2}, \cdots, \forall(i\neq j)$$

**Proof**

To find the identity block matrices we have methods.

Since the matrices which are the main diagonal of the block matrix and which ($A_{11}, A_{22}, \ldots, A_{bb}$) are squares, write each row and column contains only one sup array with the value I and the rest of the sup-matrices in that row or column are zeros sup-matrices the identity matrix of these sup-matrices as follows:

$$A\begin{bmatrix} A_{11} & \cdots & A_{1b} \\ \vdots & \ddots & \vdots \\ A_{b1} & \cdots & A_{bb} \end{bmatrix}$$

It will be like this

$$I=\begin{bmatrix} I_{11} & 0_{12} & \cdots & 0_{1b} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{b1} & \cdots & 0_{b(b-1)} & I_{bb} \end{bmatrix}$$

Since all sub-matrix of ($k_i\times k_i$) matrices then the identity matrix which in each row and column contains only one element with a value of 1 and the rest of the elements in that row or column are zeros will be in form:

$$I_{ij}=\begin{bmatrix} 1 & 0 & \cdots \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix}_{K_i\times K_i}, i=1,2,\cdots,p,r$$

$$AI=\begin{bmatrix} A_{11} & \cdots & A_{1b} \\ \vdots & \ddots & \vdots \\ A_{b1} & \cdots & A_{bb} \end{bmatrix}_{b\times b}\begin{bmatrix} I_{11} & 0_{12} & \cdots & 0_{1b} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{b1} & \cdots & 0_{b(b-1)} & I_{bb} \end{bmatrix}_{b\times b}$$

$$=\begin{bmatrix} A_{11}I_{11}+A_{12}0_{21}+\cdots+A_{1b}0_{b1} & \cdots & A_{11}0_{1b}+\cdots+A_{1(b-1)}0_{b(b-1)}+A_{1b}I_{bb} \\ \vdots & \ddots & \vdots \\ A_{b1}I_{11}+A_{b2}0_{21}+\cdots+A_{bb}0_{b1} & \cdots & A_{b1}0_{1b}+\cdots+A_{b(b-1)}0_{b(b-1)}+A_{bb}I_{bb} \end{bmatrix}$$

$$\therefore A_{ij}I_{ij}=[a_{ij}]_{k_i\times k_i}[1]_{k_i\times k_i}=[a_{ij}]_{k_i\times k_i}$$
$$=A_{ij},(i,j)=1,2,\cdots,p,r$$
$$\therefore A_{ij}I_{ij}=[a_{ij}]_{k_i\times k_j}[1]_{k_j\times k_j}=[a_{ij}]_{k_i\times k_j}$$
$$=A_{ij},(i,j)=1,2,\cdots,p,r$$
$$\therefore A_{ij}0_{ij}=[a_{ij}]_{k_i\times k_j}[0]_{k_j\times k_t}=[0]_{k_i\times k_t}$$
$$=0_{it},(i,j,t)=1,2,\cdots,p,r$$
$$AI=\begin{bmatrix} A_{11} & \cdots & A_{1b} \\ \vdots & \ddots & \vdots \\ A_{b1} & \cdots & A_{bb} \end{bmatrix}=A$$

$$I=\begin{bmatrix} I_{11} & 0_{12} & \cdots & 0_{1b} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{b1} & \cdots & 0_{b(b-1)} & I_{bb} \end{bmatrix}$$

The last block matrices called Identity block matrix (I.B.).

**Theorem (1-2-2)**

Divided the secondary matrix in to sup-matrices so that the sup-matrices that make up the secondary diagonal are square matrices like in the follows:

$$
A = \begin{bmatrix}
\boldsymbol{a_{11}} & \cdots & \boldsymbol{a_{1(k_1)}} & \boldsymbol{a_{1(k_1+1)}} & \cdots & \boldsymbol{a_{1(k_2)}} & \cdots & \boldsymbol{a_{1(k_p+1)}} & \cdots & \boldsymbol{a_{1(k_r=n)}} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
\boldsymbol{a_{(k_r)1}} & \cdots & \boldsymbol{a_{(k_r)(k_1)}} & \boldsymbol{a_{(k_r)(k_1+1)}} & \cdots & \boldsymbol{a_{(k_r)(k_2)}} & \cdots & \boldsymbol{a_{(k_r)(k_p+1)}} & \cdots & \boldsymbol{a_{(k_r)(k_r=n)}} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
\boldsymbol{a_{(k_3+1)}} & \cdots & \boldsymbol{a_{(k_3+1)(k_1)}} & \boldsymbol{a_{(k_3+1)(k+1)}} & \cdots & \boldsymbol{a_{(k_3+1)(k_2)}} & \cdots & \boldsymbol{a_{(k_3+1)(k_p+1)}} & \cdots & \boldsymbol{a_{(k_3+1)(k_r=n)}} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
\boldsymbol{a_{(k_2)1}} & \cdots & \boldsymbol{a_{(k_2)(k_1)}} & \boldsymbol{a_{(k_2)(k+1)}} & \cdots & \boldsymbol{a_{(k_2)(k_2)}} & \cdots & \boldsymbol{a_{(k_2)(k_p+1)}} & \cdots & \boldsymbol{a_{(k_2)(k_r=n)}} \\
\boldsymbol{a_{(k_2+1)1}} & \cdots & \boldsymbol{a_{(k_2+1)(k_1)}} & \boldsymbol{a_{(k_2+1)(k+1)}} & \cdots & \boldsymbol{a_{(k_2+1)(k_2)}} & \cdots & \boldsymbol{a_{(k_2+1)(k_p+1)}} & \cdots & \boldsymbol{a_{(k_2+1)(k_r=n)}} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
\boldsymbol{a_{(k_1=m)1}} & \cdots & \boldsymbol{a_{(k_1=n)(k_1)}} & \boldsymbol{a_{(k_1=n)(k+1)}} & \cdots & \boldsymbol{a_{(k_1=n)(k_2)}} & \cdots & \boldsymbol{a_{(k_1=n)(k_p+1)}} & \cdots & \boldsymbol{a_{(k_1=n)(k_r=n)}}
\end{bmatrix}
= \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1b} \\ A_{12} & A_{22} & \cdots & A_{2b} \\ \vdots & \vdots & \ddots & \vdots \\ A_{b1} & A_{b2} & \cdots & A_{bb} \end{bmatrix}
$$

In this case, the sup matrices representing the secondary diameter of the block matrix will be square matrices of degree ($k_i \times k_i$).

$$
A_{1b} = \begin{bmatrix} \boldsymbol{a_{1(k_p+1)}} & \cdots & \boldsymbol{a_{1(k_r=n)}} \\ \vdots & \ddots & \vdots \\ \boldsymbol{a_{(k_r)(k_p+1)}} & \cdots & \boldsymbol{a_{(k_r)(k_r=n)}} \end{bmatrix}_{k_r \times k_r}
$$

$$\cdots$$

$$
A_{b1} = \begin{bmatrix} \boldsymbol{a_{(k_2+1)1}} & \cdots & \boldsymbol{a_{(k_2+1)(k_1)}} \\ \vdots & \ddots & \vdots \\ \boldsymbol{a_{(k_1=m)1}} & \cdots & \boldsymbol{a_{(k_1=m)(k_1)}} \end{bmatrix}_{k_1 \times k_1}
$$

and the rest of the sup-matrices will be non-square:

$$
A_{12} = \begin{bmatrix} \boldsymbol{a_{1(k_1+1)}} & \cdots & \boldsymbol{a_{1(k_2)}} \\ \vdots & \ddots & \vdots \\ \boldsymbol{a_{(k_1)(k_1+1)}} & \cdots & \boldsymbol{a_{(k_1)(k_2)}} \end{bmatrix}_{k_1 \times k_2}
$$

**Proof**

To find the identity block matrices we have methods.

Since the matrices which are the secondary diagonal of the block matrix and which ($A_{1b}, A_{2(b-1)}, \ldots, A_{b1}$) are squares, write each row and column contains only one sup array with the value $I$ and the rest of the sup-matrices in that row or column are zeros sub-matrices the identity matrix of these sup-matrices as follows:

$$
A = \begin{bmatrix} A_{11} & \cdots & A_{1b} \\ \vdots & \ddots & \vdots \\ A_{b1} & \cdots & A_{bb} \end{bmatrix}
$$

It will be like this:

$$
\begin{bmatrix} 0_{11} & \ldots & 0_{1(b-1)} & I_{1b} \\ \vdots & \vdots & \ddots & \vdots \\ I_{b1} & 0_{b2} & \cdots & 0_{bb} \end{bmatrix}
$$

In this method get the identity block matrix by using the permutation method of matrices to finding the identity block matrices:

$$
I = \begin{bmatrix} I_{n1} & 0_{nb} & \cdots & 0_{bb} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{11} & \cdots & 0_{1(b-1)} & I_{1b} \end{bmatrix}
$$

$$
AI = \begin{bmatrix} A_{11} & \cdots & A_{1n} \\ \vdots & \ddots & \vdots \\ A_{b1} & \cdots & A_{bb} \end{bmatrix}_{b \times b} \begin{bmatrix} I_{b1} & 0_{b2} & \cdots & 0_{bb} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{11} & \cdots & 0_{1(b-1)} & I_{1b} \end{bmatrix}_{b \times b}
$$

$$
= \begin{bmatrix} A_{11}I_{b1} + A_{12}0_{(b-1)1} + \cdots + A_{1b}0_{11} & \cdots & A_{11}0_{bb} + \cdots + A_{1(b-1)}0_{2b} + A_{1b}I_{1b} \\ \vdots & \ddots & \vdots \\ A_{b1}I_{b1} + A_{b2}0_{(b-1)1} + \cdots + A_{bb}0_{11} & \cdots & A_{b1}0_{bb} + \cdots + A_{b(b-1)}0_{2b} + A_{bb}I_{1b} \end{bmatrix}
$$

$$
\therefore A_{ij}I_{ij} = [a_{ij}]_{k_i \times k_i}[1]_{k_i \times k_i} = [a_{ij}]_{k_i \times k_i} = A_{ij}, (i,j) = 1,2,\cdots,p,r
$$

$$
\therefore A_{ij}I_{ij} = [a_{ij}]_{k_i \times k_j}[1]_{k_j \times k_j} = [a_{ij}]_{k_i \times k_j} = A_{ij}, (i,j) = 1,2,\cdots,p,r
$$

$$
\therefore A_{ij}0_{ij} = [a_{ij}]_{k_i \times k_j}[0]_{k_j \times k_t} = [0]_{k_i \times k_t} = 0_{it}, (i,j,t) = 1,2,\cdots,p,r
$$

$$
AI = \begin{bmatrix} A_{11} & \cdots & A_{1b} \\ \vdots & \ddots & \vdots \\ A_{b1} & \cdots & A_{bb} \end{bmatrix} = A
$$

$$
I = \begin{bmatrix} I_{b1} & 0_{b2} & \cdots & 0_{nn} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{11} & \cdots & 0_{1(n-1)} & I_{1n} \end{bmatrix}
$$

The last block matrix called Identity block matrix (I.B.).

**Remark (1-2-1)**

Avoid some divisions in which all sup matrices are not square for which the Identity block matrix (I.B.) cannot be found. As in the following example:

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}_{4\times 4} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

where,

$$A_{11}=[a_{11} \quad a_{12}], A_{12}=[a_{13} \quad a_{14}],$$
$$A_{21}=\begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{bmatrix}, A_{22}=\begin{bmatrix} a_{23} & a_{24} \\ a_{33} & a_{34} \\ a_{43} & a_{44} \end{bmatrix}$$

We also note that there are no square matrices, so we cannot be found Identity block matrix (I.B.).

**NON-SQUARE MATRICES**

A matrix whose number of rows is not equal to the number of columns (*m≠n*).

$$A=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}_{m\times n}$$

To find the matrix of blocks for the base matrix, we use the following method:

$$A=\begin{bmatrix} \boldsymbol{a_{11}} & \dots & \boldsymbol{a_{1(k_1)}} & \boldsymbol{a_{1(k_1+1)}} & \cdots & \boldsymbol{a_{1(k_2)}} & \cdots & \boldsymbol{a_{1(k_p+1)}} & \cdots & \boldsymbol{a_{1(k_r=n)}} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \boldsymbol{a_{(k_1)1}} & \cdots & \boldsymbol{a_{(k_1)(k_1)}} & \boldsymbol{a_{(k_1)(k_1+1)}} & \cdots & \boldsymbol{a_{(k_1)(k_2)}} & \cdots & \boldsymbol{a_{(k_1)(k_p+1)}} & \cdots & \boldsymbol{a_{(k_1)(k_r=n)}} \\ \boldsymbol{a_{(k_1+1)}} & \cdots & \boldsymbol{a_{(k_1+1)(k_1)}} & \boldsymbol{a_{(k_1+1)(k+1)}} & \cdots & \boldsymbol{a_{(k_1+1)(k_2)}} & \cdots & \boldsymbol{a_{(k_1+1)(k_p+1)}} & \cdots & \boldsymbol{a_{(k_1+1)(k_r=n)}} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \boldsymbol{a_{(k_2)1}} & \cdots & \boldsymbol{a_{(k_2)(k_1)}} & \boldsymbol{a_{(k_2)(k+1)}} & \cdots & \boldsymbol{a_{(k_2)(k_2)}} & \cdots & \boldsymbol{a_{(k_2)(k_p+1)}} & \cdots & \boldsymbol{a_{(k_2)(k_r=n)}} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \boldsymbol{a_{(k_p+1)1}} & \cdots & \boldsymbol{a_{(k_p+1)(k_1)}} & \boldsymbol{a_{(k_p+1)(k+1)}} & \cdots & \boldsymbol{a_{(k_p+1)(k_2)}} & \cdots & \boldsymbol{a_{(k_p+1)(k_p+1)}} & \cdots & \boldsymbol{a_{(k_p+1)(k_r=n)}} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \boldsymbol{a_{(k_r=m)1}} & \cdots & \boldsymbol{a_{(k_r=m)(k_1)}} & \boldsymbol{a_{(k_r=m)(k+1)}} & \cdots & \boldsymbol{a_{(k_r=m)(k_2)}} & \cdots & \boldsymbol{a_{(k_r=m)(k_p+1)}} & \cdots & \boldsymbol{a_{(k_r=m)(k_r=n)}} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1b} \\ A_{12} & A_{22} & \cdots & A_{2b} \\ \vdots & \vdots & \ddots & \vdots \\ A_{b1} & A_{b2} & \cdots & A_{bb} \end{bmatrix}$$

In this case, the mass matrix identity cannot be found because there are non-square partial matrices of the major or minor diagonals for which the Identity matrix is unknown. And as in the following:

In this case, the matrices representing the diagonal will be about square sup-matrices:

$$A_{11}=\begin{bmatrix} a_{11} & \cdots & a_{1(k_1)} \\ \vdots & \ddots & \vdots \\ a_{(k_1)1} & \cdots & a_{(k_1)(k_1)} \end{bmatrix}_{\boldsymbol{k_1\times k_1}}$$
$$\cdots$$
$$A_{bb}=\begin{bmatrix} \boldsymbol{a_{(k_p+1)(k_p+1)}} & \cdots & \boldsymbol{a_{(k_p+1)(k_r=n)}} \\ \vdots & \ddots & \vdots \\ \boldsymbol{a_{(k_r=m)(k_p+1)}} & \cdots & \boldsymbol{a_{(k_r=m)(k_r=n)}} \end{bmatrix}, \forall (i=j)$$

We note that the matrix $A_{cc}$ is a non-square matrix because the number of its rows is not equal to the number of its columns, so the identity matrix for it is not known. Therefore, the identity block matrices cannot be found.

It is divided into two parts.

## 2. PERMUTATION BLOCK MATRICES

**Definition (2-1)**

The permutation block matrices is a regular matrix, which in each row and column contains only one block matrix with the value I and the rest of the sup matrices in that row or column are zero matrices The number of permutations is equal to the factorial of the length of the block matrix *(n!)*. Permutational matrix P permutation of matrix A permutational matrices Applications in linear algebra, combinations and cryptography.

### 2.1 Row permutation block matrices

In this method use the permutation of rows number of permutation equal half to the factorial of the length of the block matrix ($\frac{n!}{2}$) in the block matrices.

$$A=\begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{12} & A_{22} & \cdots & A_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nn} \end{bmatrix}$$
$$I=\begin{bmatrix} I_{11} & 0_{12} & \cdots & 0_{1n} \\ 0_{21} & I_{22} & 0_{23}\cdots & 0_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{n1} & \cdots & 0_{n(n-1)} & I_{nn} \end{bmatrix}$$
$$AI=\begin{bmatrix} I_{11} & 0_{12} & \cdots & 0_{1n} \\ 0_{21} & I_{22} & 0_{23} & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ 0_{n1} & \cdots & 0_{n(n-1)} & I_{nn} \end{bmatrix}$$
$$P_1A=\begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{12} & A_{22} & \cdots & A_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nn} \end{bmatrix}$$
$$P_1=\begin{bmatrix} 0_{21} & I_{22} & 0_{23}\cdots & 0_{2n} \\ I_{11} & 0_{12} & \cdots & 0_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{n1} & \cdots & 0_{n(n-1)} & I_{nn} \end{bmatrix}$$
$$P_2A=\begin{bmatrix} A_{12} & A_{22} & \cdots & A_{2n} \\ A_{11} & A_{12} & \cdots & A_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nn} \end{bmatrix}$$

$$\vdots$$

$$P_K = \begin{bmatrix} I_{11} & 0_{12} & \cdots & \cdots & \cdots & 0_{1n} \\ 0_{21} & I_{22} & 0_{23} & \cdots & \cdots & 0_{2n} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0_{k1} & \cdots & 0_{k(k-1)} & I_{kk} & 0_{k(k+1)} & \cdots 0_{kn} \\ 0_{(k+1)1} & \cdots & \cdots & 0_{(k+1)k} & I_{(k+1)(k+1)} & \cdots 0_{(k+1)n} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0_{n1} & \cdots & \cdots & \cdots & 0_{n(n-1)} & I_{nn} \end{bmatrix}$$

$$P_kA = \begin{bmatrix} A_{11} & A_{12} & \cdots & \cdots & \cdots & \cdots & A_{1n} \\ A_{12} & A_{22} & \cdots & \cdots & \cdots & \cdots & A_{2n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ A_{(k+1)1} & A_{(k+1)2} & \cdots & A_{(k+1)k} & A_{(k+1)(k+1)} & \cdots & A_{(k+1)n} \\ A_{(k)1} & A_{(k)2} & \cdots & A_{(k)(k)} & A_{(k)(k+1)} & \cdots & A_{(k)n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ A_{n1} & A_{n2} & \cdots & \cdots & \cdots & \cdots & A_{nn} \end{bmatrix}$$

$$\vdots$$

$$P_{\frac{n!}{2}} = \begin{bmatrix} 0_{n1} & \cdots & 0_{n(n-1)} & I_{nn} \\ 0_{21} & I_{22} & 0_{23} & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ I_{11} & 0_{12} & \cdots & 0_{1n} \end{bmatrix}$$

$$P_{\frac{n!}{2}}A = \begin{bmatrix} A_{n1} & A_{n2} & \cdots & A_{nn} \\ A_{12} & A_{22} & \cdots & A_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ A_{11} & A_{12} & \cdots & A_{1n} \end{bmatrix}$$

### 2.2 Column permutations block matrices

In this method we use the column permutation of column block matrices the number permutation equal half to the factorial of the length of the block matrix ($\frac{n!}{2}$) of block matrices.

$$A = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{12} & A_{22} & \cdots & A_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nn} \end{bmatrix}$$

$$I = \begin{bmatrix} I_{11} & 0_{12} & \cdots & 0_{1n} \\ 0_{21} & I_{22} & 0_{23} \cdots & 0_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{n1} & \cdots & 0_{n(n-1)} & I_{nn} \end{bmatrix}$$

$$\vdots$$

$$AI = \begin{bmatrix} I_{11} & 0_{12} & \cdots & 0_{1n} \\ 0_{21} & I_{22} & 0_{23} & \cdots 0_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{n1} & \cdots & 0_{n(n-1)} & I_{nn} \end{bmatrix}$$

$$AP_1 = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{12} & A_{22} & \cdots & A_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nn} \end{bmatrix}$$

$$P_1 = \begin{bmatrix} 0_{12} & I_{11} & \cdots & 0_{1n} \\ I_{22} & 0_{21} & 0_{23} & \cdots 0_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \cdots & 0_{n1} & 0_{n(n-1)} & I_{nn} \end{bmatrix}$$

$$AP_2 = \begin{bmatrix} A_{12} & A_{11} & \cdots & A_{1n} \\ A_{22} & A_{12} & \cdots & A_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n2} & A_{n1} & \cdots & A_{nn} \end{bmatrix}$$

$$\vdots$$

$$P_k = \begin{bmatrix} I_{11} & 0_{12} & \cdots & \cdots & \cdots & \cdots & \cdots & 0_{1n} \\ 0_{21} & I_{22} & 0_{23} & \cdots & \cdots & \cdots & \cdots & \cdots 0_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0_{(k)1} & 0_{(k)2} & \cdots & 0_{(k)(k-1)} & 0_{(k)(k+1)} & I_{(k)(k)} & \cdots & 0_{(k)n} \\ 0_{(k+1)2} & 0_{(k+1)2} & \cdots & 0_{(k+1)(k-1)} & I_{(k+1)(k+1)} & 0_{(k+1)(k)} & \cdots & 0_{(k+1)n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0_{n1} & \cdots & \cdots & \cdots & \cdots & \cdots & 0_{n(n-1)} & I_{nn} \end{bmatrix}$$

$$AP_k = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1(k+1)} & A_{1(k)} & \cdots & A_{1n} \\ A_{12} & A_{22} & \cdots & A_{2(k+1)} & A_{2(k)} & \cdots & A_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ A_{n1} & A_{n2} & \cdots & \cdots & \cdots & \cdots & A_{nn} \end{bmatrix}$$

$$P_{\frac{n!}{2}} = \begin{bmatrix} 0_{1n} & 0_{12} & \cdots & I_{11} \\ 0_{2n} & I_{22} & 0_{23} & 0_{21} \\ \vdots & \vdots & \ddots & \vdots \\ I_{nn} & \cdots & 0_{n(n-1)} & 0_{n1} \end{bmatrix}$$

$$P_{\frac{n!}{2}}A = \begin{bmatrix} A_{1n} & A_{12} & \cdots & A_{11} \\ A_{2n} & A_{22} & \cdots & A_{12} \\ \vdots & \vdots & \ddots & \vdots \\ A_{nn} & A_{n2} & \cdots & A_{n1} \end{bmatrix}$$

## 3. EXAMPLES FOR EXPLAIN THE MAIN RESULT

**Example (3-1):** Find Identity block matrix (I.B.) of the following:

$$A = \begin{bmatrix} 2 & 4 & 6 & 8 \\ 21 & 5 & 8 & 9 \\ 9 & 10 & 3 & 4 \\ 12 & 5 & 2 & 16 \end{bmatrix}$$

**Solution**

By using Theorem (1-1-1) we find the main diagonal of block matrix

$$A = \begin{bmatrix} 2 & 4 & 6 & 8 \\ 21 & 5 & 8 & 9 \\ 9 & 10 & 3 & 4 \\ 12 & 5 & 2 & 16 \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 2 & 4 \\ 21 & 5 \end{bmatrix} & \begin{bmatrix} 6 & 8 \\ 8 & 9 \end{bmatrix} \\ \begin{bmatrix} 9 & 10 \\ 12 & 5 \end{bmatrix} & \begin{bmatrix} 3 & 4 \\ 2 & 16 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

where,

$$A_{11} = \begin{bmatrix} 2 & 4 \\ 21 & 5 \end{bmatrix}, A_{12} = \begin{bmatrix} 6 & 8 \\ 8 & 9 \end{bmatrix}$$

$$A_{21} = \begin{bmatrix} 9 & 10 \\ 12 & 5 \end{bmatrix}, A_{22} = \begin{bmatrix} 3 & 4 \\ 2 & 16 \end{bmatrix}$$

$$I = \begin{bmatrix} I_{11} & 0_{12} \\ 0_{21} & I_{22} \end{bmatrix}$$

$$AI = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} I_{11} & 0_{12} \\ 0_{21} & I_{22} \end{bmatrix} = \begin{bmatrix} A_{11}I_{11} + A_{12}0_{21} & A_{11}0_{12} + A_{12}I_{22} \\ A_{21}I_{11} + A_{22}0_{21} & A_{21}0_{12} + A_{22}I_{22} \end{bmatrix}$$

$$=\begin{bmatrix}\begin{bmatrix}2&4\\21&5\end{bmatrix}\begin{bmatrix}1&0\\0&1\end{bmatrix}+\begin{bmatrix}6&8\\8&9\end{bmatrix}\begin{bmatrix}0&0\\0&0\end{bmatrix} & \begin{bmatrix}2&4\\21&5\end{bmatrix}\begin{bmatrix}0&0\\0&0\end{bmatrix}+\begin{bmatrix}6&8\\8&9\end{bmatrix}\begin{bmatrix}1&0\\0&1\end{bmatrix}\\ \begin{bmatrix}9&10\\12&5\end{bmatrix}\begin{bmatrix}1&0\\0&1\end{bmatrix}+\begin{bmatrix}3&4\\2&16\end{bmatrix}\begin{bmatrix}0&0\\0&0\end{bmatrix} & \begin{bmatrix}9&10\\12&5\end{bmatrix}\begin{bmatrix}0&0\\0&0\end{bmatrix}+\begin{bmatrix}3&4\\2&16\end{bmatrix}\begin{bmatrix}1&0\\0&1\end{bmatrix}\end{bmatrix}=\begin{bmatrix}\begin{bmatrix}2&4\\21&5\end{bmatrix}&\begin{bmatrix}6&8\\8&9\end{bmatrix}\\ \begin{bmatrix}9&10\\12&5\end{bmatrix}&\begin{bmatrix}3&4\\2&16\end{bmatrix}\end{bmatrix}=\begin{bmatrix}A_{11}&A_{12}\\A_{21}&A_{22}\end{bmatrix}=A$$

$\therefore I=\begin{bmatrix}I_{11}&0_{12}\\0_{21}&I_{22}\end{bmatrix}$ is identity block matrix (I.B.).

**Example (3-2):** Find Identity block matrix (I.B.) of the following:

$$A=\begin{bmatrix}2&4&6&8\\21&5&8&9\\9&10&3&4\\12&5&2&16\end{bmatrix}$$

**Solution**

By using Theorem (1-1-2) we find the secondary diagonal of block matrix:

$$A=\begin{bmatrix}2&4&6&8\\21&5&8&9\\9&10&3&4\\12&5&2&16\end{bmatrix}=\begin{bmatrix}\begin{bmatrix}2&4\\21&5\end{bmatrix}&\begin{bmatrix}6&8\\8&9\end{bmatrix}\\ \begin{bmatrix}9&10\\12&5\end{bmatrix}&\begin{bmatrix}3&4\\2&16\end{bmatrix}\end{bmatrix}=\begin{bmatrix}A_{11}&A_{12}\\A_{21}&A_{22}\end{bmatrix}$$

where,

$$A_{11}=\begin{bmatrix}2&4\\21&5\end{bmatrix},A_{12}=\begin{bmatrix}6&8\\8&9\end{bmatrix}$$

$$A_{21}=\begin{bmatrix}9&10\\12&5\end{bmatrix},A_{22}=\begin{bmatrix}3&4\\2&16\end{bmatrix}$$

$\begin{bmatrix}0_{11}&I_{12}\\I_{21}&0_{22}\end{bmatrix}$ and in order $I=\begin{bmatrix}I_{21}&0_{22}\\0_{11}&I_{12}\end{bmatrix}$

$$AI=\begin{bmatrix}A_{11}&A_{12}\\A_{21}&A_{22}\end{bmatrix}\begin{bmatrix}I_{21}&0_{22}\\0_{11}&I_{12}\end{bmatrix}=\begin{bmatrix}A_{11}I_{21}+A_{12}0_{11}&A_{11}0_{22}+A_{12}I_{12}\\A_{21}I_{21}+A_{22}0_{11}&A_{21}0_{22}+A_{22}I_{12}\end{bmatrix}$$

$$=\begin{bmatrix}\begin{bmatrix}2&4\\21&5\end{bmatrix}\begin{bmatrix}1&0\\0&1\end{bmatrix}+\begin{bmatrix}6&8\\8&9\end{bmatrix}\begin{bmatrix}0&0\\0&0\end{bmatrix} & \begin{bmatrix}2&4\\21&5\end{bmatrix}\begin{bmatrix}0&0\\0&0\end{bmatrix}+\begin{bmatrix}6&8\\8&9\end{bmatrix}\begin{bmatrix}1&0\\0&1\end{bmatrix}\\ \begin{bmatrix}9&10\\12&5\end{bmatrix}\begin{bmatrix}1&0\\0&1\end{bmatrix}+\begin{bmatrix}3&4\\2&16\end{bmatrix}\begin{bmatrix}0&0\\0&0\end{bmatrix} & \begin{bmatrix}9&10\\12&5\end{bmatrix}\begin{bmatrix}0&0\\0&0\end{bmatrix}+\begin{bmatrix}3&4\\2&16\end{bmatrix}\begin{bmatrix}1&0\\0&1\end{bmatrix}\end{bmatrix}=\begin{bmatrix}\begin{bmatrix}2&4\\21&5\end{bmatrix}&\begin{bmatrix}6&8\\8&9\end{bmatrix}\\ \begin{bmatrix}9&10\\12&5\end{bmatrix}&\begin{bmatrix}3&4\\2&16\end{bmatrix}\end{bmatrix}=\begin{bmatrix}A_{11}&A_{12}\\A_{21}&A_{22}\end{bmatrix}=A$$

$\therefore I=\begin{bmatrix}I_{21}&0_{22}\\0_{11}&I_{12}\end{bmatrix}$ is identity block matrix (I.B.).

**Example (3-3):** Find Identity block matrix (I.B.) of the following:

$$A=\begin{bmatrix}2&4&6&8\\21&5&8&9\\9&10&3&4\\12&5&2&16\end{bmatrix}$$

**Solution**

By using Theorem (1-2-1) we find the sup-matrices that make up the main diagonal are square matrices:

$$A=\begin{bmatrix}2&4&6&8\\21&5&8&9\\9&10&3&4\\12&5&2&16\end{bmatrix}$$

$$=\begin{bmatrix}[2]&[4\quad 6\quad 8]\\ \begin{bmatrix}21\\9\\12\end{bmatrix}&\begin{bmatrix}5&8&9\\10&3&4\\5&2&16\end{bmatrix}\end{bmatrix}=\begin{bmatrix}A_{11}&A_{12}\\A_{21}&A_{22}\end{bmatrix}$$

where,

$$A_{11}=[2],A_{12}=[4\quad 6\quad 8]$$

$$A_{21}=\begin{bmatrix}21\\9\\12\end{bmatrix},A_{22}=\begin{bmatrix}5&8&9\\10&3&4\\5&2&16\end{bmatrix}$$

$$I=\begin{bmatrix}I_{11}&0_{12}\\0_{21}&I_{22}\end{bmatrix}$$

$$AI=\begin{bmatrix}A_{11}&A_{12}\\A_{21}&A_{22}\end{bmatrix}\begin{bmatrix}I_{11}&0_{12}\\0_{21}&I_{22}\end{bmatrix}=\begin{bmatrix}A_{11}I_{11}+A_{12}0_{21}&A_{11}0_{12}+A_{12}I_{22}\\A_{21}I_{11}+A_{22}0_{21}&A_{21}0_{12}+A_{22}I_{22}\end{bmatrix}$$

$$=\begin{bmatrix}[2][1]+[4\quad 6\quad 8]\begin{bmatrix}0\\0\\0\end{bmatrix} & [2][0\quad 0\quad 0]+[4\quad 6\quad 8]\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\\ \begin{bmatrix}21\\9\\12\end{bmatrix}[1]+\begin{bmatrix}5&8&9\\10&3&4\\5&2&16\end{bmatrix}\begin{bmatrix}0\\0\\0\end{bmatrix} & \begin{bmatrix}21\\9\\12\end{bmatrix}[0\quad 0\quad 0]+\begin{bmatrix}5&8&9\\10&3&4\\5&2&16\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\end{bmatrix}$$

$$=\begin{bmatrix}[2]&[4\quad 6\quad 8]\\ \begin{bmatrix}21\\9\\12\end{bmatrix}&\begin{bmatrix}5&8&9\\10&3&4\\5&2&16\end{bmatrix}\end{bmatrix}=\begin{bmatrix}A_{11}&A_{12}\\A_{21}&A_{22}\end{bmatrix}=A$$

$\therefore I=\begin{bmatrix}I_{11}&0_{12}\\0_{21}&I_{22}\end{bmatrix}$ is identity block matrix (I.B.).

**Example (3-4):** Find Identity block matrix (I.B.) of the following:

$$A=\begin{bmatrix}2&4&6&8\\21&5&8&9\\9&10&3&4\\12&5&2&16\end{bmatrix}$$

**Solution**

By using Theorem (1-2-2) we find the sup-matrices that make up the secondary diagonal are square matrices:

$$A=\begin{bmatrix}2&4&6&8\\21&5&8&9\\9&10&3&4\\12&5&2&16\end{bmatrix}=\begin{bmatrix}A_{11}&A_{12}\\A_{21}&A_{22}\end{bmatrix}$$

$\begin{bmatrix} 0_{11} & I_{12} \\ I_{21} & 0_{22} \end{bmatrix}$ and in order $I = \begin{bmatrix} I_{21} & 0_{22} \\ 0_{11} & I_{12} \end{bmatrix}$

where,

$$A_{11} = [2 \quad 4 \quad 6], A_{12} = [8]$$

$$A_{21} = \begin{bmatrix} 21 & 5 & 8 \\ 9 & 10 & 3 \\ 12 & 5 & 2 \end{bmatrix}, A_{22} = \begin{bmatrix} 9 \\ 4 \\ 16 \end{bmatrix}$$

$$AI = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} I_{21} & 0_{22} \\ 0_{11} & I_{12} \end{bmatrix} = \begin{bmatrix} A_{11}I_{21} + A_{12}0_{11} & A_{11}0_{22} + A_{12}I_{12} \\ A_{21}I_{21} + A_{22}0_{11} & A_{21}0_{22} + A_{22}I_{12} \end{bmatrix}$$

$$= \begin{bmatrix} [2 \quad 4 \quad 6]\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + [8][0 \quad 0 \quad 0] & [2 \quad 4 \quad 6]\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + [8][1] \\ \begin{bmatrix} 21 & 5 & 8 \\ 9 & 10 & 3 \\ 12 & 5 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 9 \\ 4 \\ 16 \end{bmatrix}[0 \quad 0 \quad 0] & \begin{bmatrix} 21 & 5 & 8 \\ 9 & 10 & 3 \\ 12 & 5 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 9 \\ 4 \\ 16 \end{bmatrix}[1] \end{bmatrix}$$

$$= \begin{bmatrix} [2 \quad 4 \quad 6] & [8] \\ \begin{bmatrix} 21 & 5 & 8 \\ 9 & 10 & 3 \\ 12 & 5 & 2 \end{bmatrix} & \begin{bmatrix} 9 \\ 4 \\ 16 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = A$$

$\therefore I = \begin{bmatrix} I_{21} & 0_{22} \\ 0_{11} & I_{12} \end{bmatrix}$ is identity block matrix (I.B.).

**Example (3-5):** Find Identity block matrix (I.B.) of the following:

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 \\ 16 & 17 & 18 & 19 & 20 \end{bmatrix}$$

**Solution**

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 \\ 16 & 17 & 18 & 19 & 20 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

where,

$$A_{11} = \begin{bmatrix} 1 & 2 \\ 6 & 7 \end{bmatrix}, A_{12} = \begin{bmatrix} 3 & 4 & 5 \\ 8 & 9 & 10 \end{bmatrix}$$

$$A_{21} = \begin{bmatrix} 11 & 12 \\ 16 & 17 \end{bmatrix}, A_{22} = \begin{bmatrix} 13 & 14 & 15 \\ 18 & 19 & 20 \end{bmatrix}$$

$A_{12}$, $A_{22}$ is non-square matrices since 3×2 so the identity matrix is unknown therefore has not Identity block matrix (I.B.).

**Example (3-6):** Find permutations of rows and columns of a block matrix:

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$$

**Solution**

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

where,

$$A_{11} = [1], A_{12} = [2 \quad 3], A_{13} = [4],$$

$$A_{21} = \begin{bmatrix} 5 \\ 9 \end{bmatrix}, A_{22} = \begin{bmatrix} 6 & 7 \\ 10 & 11 \end{bmatrix}, A_{23}\begin{bmatrix} 8 \\ 12 \end{bmatrix},$$

$$A_{31}[13], A_{32} = [14 \quad 15], A_{33} = [16]$$

$$I = \begin{bmatrix} I_{11} & 0_{12} & 0_{13} \\ 0_{21} & I_{22} & 0_{23} \\ 0_{31} & 0_{32} & I_{33} \end{bmatrix}$$

$$AI = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} I_{11} & 0_{12} & 0_{13} \\ 0_{21} & I_{22} & 0_{23} \\ 0_{31} & 0_{32} & I_{33} \end{bmatrix}$$

$$= \begin{bmatrix} A_{11}I_{11} + A_{12}0_{21} + A_{13}0_{31} & A_{11}0_{12} + A_{12}I_{22} + A_{31}0_{32} & A_{11}0_{13} + A_{12}0_{23} + A_{13}I_{33} \\ A_{21}I_{11} + A_{22}0_{21} + A_{23}0_{31} & A_{21}0_{12} + A_{22}I_{22} + A_{23}0_{32} & A_{21}0_{13} + A_{22}0_{23} + A_{23}I_{33} \\ A_{31}I_{11} + A_{32}0_{21} + A_{33}0_{31} & A_{31}0_{12} + A_{32}I_{22} + A_{33}0_{32} & A_{31}0_{31} + A_{32}0_{32} + A_{33}I_{33} \end{bmatrix}$$

$$= \begin{bmatrix} [1][1] + [2 \quad 3]\begin{bmatrix} 0 \\ 0 \end{bmatrix} + [4][0] & [1][0 \quad 0] + [2 \quad 3]\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + [4][0 \quad 0] & [1][0] + [[2 \quad 3]]\begin{bmatrix} 0 \\ 0 \end{bmatrix} + [4][1] \\ \begin{bmatrix} 5 \\ 9 \end{bmatrix}[1] + \begin{bmatrix} 6 & 7 \\ 10 & 11 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 8 \\ 12 \end{bmatrix}[0] & \begin{bmatrix} 5 \\ 9 \end{bmatrix}[0 \quad 0] + \begin{bmatrix} 6 & 7 \\ 10 & 11 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 8 \\ 12 \end{bmatrix}[0 \quad 0] & \begin{bmatrix} 5 \\ 9 \end{bmatrix}[0] + \begin{bmatrix} 6 & 7 \\ 10 & 11 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 8 \\ 12 \end{bmatrix}[1] \\ [12][1] + [14 \quad 15]\begin{bmatrix} 0 \\ 0 \end{bmatrix} + [16][01] & [13][0 \quad 0] + [14 \quad 15]\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + [16][0 \quad 0] & [13][0] + [14 \quad 15]\begin{bmatrix} 0 \\ 0 \end{bmatrix} + [16][1] \end{bmatrix}$$

$$= \begin{bmatrix} [1] & [2 \quad 3] & [4] \\ \begin{bmatrix} 5 \\ 9 \end{bmatrix} & \begin{bmatrix} 6 & 7 \\ 10 & 11 \end{bmatrix} & \begin{bmatrix} 8 \\ 12 \end{bmatrix} \\ [13] & [14 \quad 15] & [16] \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

**Column permutations**

$$P_1 = \begin{bmatrix} 0_{21} & I_{11} & 0_{13} \\ I_{22} & 0_{21} & 0_{23} \\ 0_{32} & 0_{31} & I_{33} \end{bmatrix}, AP_2 = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} 0_{21} & I_{11} & 0_{13} \\ I_{22} & 0_{21} & 0_{23} \\ 0_{32} & 0_{31} & I_{33} \end{bmatrix} = \begin{bmatrix} A_{12} & A_{11} & A_{13} \\ A_{22} & A_{21} & A_{23} \\ A_{32} & A_{31} & A_{33} \end{bmatrix}$$

$$P_2 = \begin{bmatrix} I_{11} & 0_{13} & 0_{21} \\ 0_{21} & 0_{23} & I_{22} \\ 0_{31} & I_{33} & 0_{32} \end{bmatrix}, AP_2 = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} 0_{21} & I_{11} & 0_{13} \\ I_{22} & 0_{21} & 0_{23} \\ 0_{32} & 0_{31} & I_{33} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{13} & A_{12} \\ A_{21} & A_{23} & A_{22} \\ A_{31} & A_{33} & A_{32} \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 0_{13} & 0_{21} & I_{11} \\ 0_{23} & I_{22} & 0_{21} \\ I_{33} & 0_{32} & 0_{31} \end{bmatrix}, AP_2 = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} 0_{21} & I_{11} & 0_{13} \\ I_{22} & 0_{21} & 0_{23} \\ 0_{32} & 0_{31} & I_{33} \end{bmatrix} = \begin{bmatrix} A_{12} & A_{11} & A_{13} \\ A_{22} & A_{21} & A_{23} \\ A_{32} & A_{31} & A_{33} \end{bmatrix}$$

**Row Permutation**

$$P_4 = \begin{bmatrix} 0_{21} & I_{22} & 0_{23} \\ I_{11} & 0_{12} & 0_{13} \\ 0_{31} & 0_{32} & I_{33} \end{bmatrix}, AP_4 = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} 0_{21} & I_{22} & 0_{23} \\ I_{11} & 0_{12} & 0_{13} \\ 0_{31} & 0_{32} & I_{33} \end{bmatrix} = \begin{bmatrix} A_{21} & A_{22} & A_{23} \\ A_{11} & A_{12} & A_{13} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$$P_5 = \begin{bmatrix} I_{11} & 0_{12} & 0_{13} \\ 0_{31} & 0_{32} & I_{33} \\ 0_{21} & I_{22} & 0_{23} \end{bmatrix}, AP_4 = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} I_{11} & 0_{12} & 0_{13} \\ 0_{31} & 0_{32} & I_{33} \\ 0_{21} & I_{22} & 0_{23} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{31} & A_{32} & A_{33} \\ A_{21} & A_{22} & A_{23} \end{bmatrix}$$

$$P_6 = \begin{bmatrix} 0_{31} & 0_{32} & I_{33} \\ 0_{21} & I_{22} & 0_{23} \\ I_{11} & 0_{12} & 0_{13} \end{bmatrix}, AP_6 = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} 0_{31} & 0_{32} & I_{33} \\ 0_{21} & I_{22} & 0_{23} \\ I_{11} & 0_{12} & 0_{13} \end{bmatrix} = \begin{bmatrix} A_{31} & A_{32} & A_{33} \\ A_{21} & A_{22} & A_{23} \\ A_{11} & A_{12} & A_{13} \end{bmatrix}$$

## 4. CONCLUSION

This study it has been shown the methods for finding the block matrices. Some important examples were given to explain the main results of this paper. A new result was discussed to find the permutations matrices for a block matrices in two different methods. Using MATLAB code to prove the results of the methods that used for proof the theorems in this study.


## ACKNOWLEDGMENT

This work was supported by The Islamic University research grant PRG-IUNAJAF-2019.


## REFERENCES


[1] Zhang, F. (2011). Matrix theory: Basic results and techniques. New York: Springer.

[2] Golub, G.H., Van Loan, C.F. (2013). Matrix computations. JHU Press.

[3] Alkhalidi, A.H., Alrikabi, H.M.B., Ali M.Z. (2022). An existence of three different non-trivial solutions for discrete anisotropic equations with two real parameters. Nonlinear Functional Analysis and Applications, 27(4): 855-867. https://doi.org/10.22771/nfaa.2022.27.04.12

[4] Alkhalidi, A.A.H., Afrouzi, G.A., Khademloo, S. (2019). Variational approaches for lagrangian discrete nonlinear systems. Mathematics, 7(3): 276. https://doi.org/10.3390/math7030276

[5] Axelsson, O., Lukáš, D. (2019). Preconditioning methods for eddy-current optimally controlled time-harmonic electromagnetic problems. Journal of Numerical Mathematics, 27(1): 1-21. https://doi.org/10.1515/jnma-2017-0064

[6] Gharaibeh, M.A. (2022). Numerical solution for the mechanical bending of two elastically coupled plates problem. Mathematical Modelling of Engineering Problems, 9(5): 1432-1439. https://doi.org/10.18280/mmep.090535

[7] Menni Y., Chamkha A.J., Zidani C., Benyoucef B. (2019). Numerical analysis of heat and nanofluid mass transfer in a channel with detached and attached baffle plates, Mathematical Modelling of Engineering Problems, 6(1): 52-60. https://doi.org/10.18280/mmep.060107

[8] Horn, R.A., Johnson, C.R. (2012). Matrix analysis. Cambridge University Press.

[9] Lay, D.C., Lay, S.R., McDonald, J.J. (2016). Linear algebra and its applications. Pearson.

[10] Okunev, P., Johnson, C.R. (2005). Necessary and sufficient conditions for existence of the LU factorization of an arbitrary matrix. arXiv preprint math/0506382

[11] Meyer, C.D. (2000). Matrix analysis and applied linear algebra. Siam.

[12] Stroud, K.A., Booth, D.J. (2020). Advanced engineering mathematics. Bloomsbury Publishing.

[13] Ford, W. (2014). Numerical linear algebra with applications: Using MATLAB. Academic Press.


## APPENDIX

In this section we wrote some codes by using MATLAB program to explain the results in chapter two and we introduced the general code to find the blocks matrices for any square matrix

**<u>Cod (1): Find block matrices</u>**

```
clc
clear
A = input('A=')
Wanted = nan(length(A));
m=length(A)
k=fix(2\m)
i=1
j=1
D1 = A(i:i+(k-1),i:i+(k-1))
D2 = A(i:i+(k-1),j+k:j+(m-1))
D3 = A(j+k:j+(m-1),i:i+(k-1))
D4 = A(i+k:i+(m-1),j+k:j+(m-1))
F=[D1 D2;D3 D4]
```

**<u>Cod (2): Multiplying block matrices in MATLAB</u>**

```
clc
clear
A = input('A=')
B = input('B=')
Wanted = nan(length(A));
Wanted = nan(length(B));
```

```
m=length(A)
k=fix(2\m)
i=1
j=1
D1 = A(i:i+(k-1),i:i+(k-1))*B(i:i+(k-1),i:i+(k-1))
D2 = A(i:i+(k-1),j+k:j+(m-1))*B(i:i+(k-1),j+k:j+(m-1))
D3 = A(j+k:j+(m-1),i:i+(k-1))*B(j+k:j+(m-1),i:i+(k-1))
D4 = A(i+k:i+(m-1),j+k:j+(m-1))*B(i+k:i+(m-1),j+k:j+(m-1))
F=[D1 D2;D3 D4]
```