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Generalization of Fuzzy SHA-Transform with Medical Application

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ABSTRACT

Fuzzy differential and integral equations and their associated fuzzy boundary (initial) value problems have been solved using a variety of methods., fuzzy integral transforms are significant method for this purpose. In this research, we extend and use the formula of third- order fuzzy derivative of fuzzy SHA-transform (FSHAT) to generalize the nth-order fuzzy derivative of this integral transform. In fact, this process studies under strongly generalized Hukuhara differentiability. In fact, a drug distribution means the amount of drug in the body which is measured by the proportional of this drug to its amount in the plasma or blood, so corresponding to this issue, a drug concentration in an organ equation solves using this proposed fuzzy SHA-transform.

1. INTRODUCTION

To date, various methods have been developed and introduced for the purpose of resolving beginning and boundary value problems involving fuzziness. One popular technique for resolving both differential and integral equations using fuzzy sets is the application of fuzzy transforms. Laplace transform [1, 2] and Sumudu transform [3, 4] are two examples of conventional transforms that have recently been converted to fuzzy transforms for the purpose of solving fuzzy differential equations. A novel about fuzzy complex integral transform called SHA- transform is created by Shaymaa and Alkiffai [5]. In this work, arranged as follows. Basic concepts are given in Section 2. In Section 3, some properties of fuzzy SHA-transform. In Section 4, fuzzy SHA-transform for third order derivative [6].

The general formula of the fuzzy SHA-transform was introduced in Section 5. Finally in order in Section 6, a generalization of fuzzy SHA-transform technique (n^{th}-order fuzzy derivative) is discussed with an illustrative example that related with drug concentration or the amount of drug in any organ in the body which can be measured by its level in the blood (plasma), urine, saliva and other sampled fluids.

2. BASIC CONCEPTS

Definition 2.1 [7]

Parametrically speaking, a fuzzy number is a set of functions that meet the following criteria:

1. $\underline{\omega}(\vartheta)$ defines a left-continuous function in (0,1] continuing to the right and within bounds, and not diminishing 0.
2. $\bar{\omega}(\vartheta)$ defines a left-continuous function in (0,1] right continuous at infinity and bounded non-increasing 0.
3. $\underline{\omega}(\vartheta) \le \bar{\omega}(\vartheta)$, $0\le\vartheta\le1$.

For $\omega = \underline{\omega}(\vartheta), \bar{\omega}(\vartheta)$ and $\nu = \underline{\nu}(\vartheta), \bar{\nu}(\vartheta)$, the sum is defined by us $\omega \oplus \nu$ subtraction $\omega\ominus\nu$ and scalar multiplication by $\varphi>0$ as follows:

(a) Addition: $\omega \oplus \nu = \underline{\omega}(\vartheta) + \underline{\nu}(\vartheta), \bar{\omega}(\vartheta) + \bar{\nu}(\vartheta)$.
(b) Subtraction: $\omega\ominus\nu = \underline{\omega}(\vartheta) - \bar{\nu}(\vartheta), \bar{\omega}(\vartheta) - \underline{\nu}(\vartheta)$.
(c) Scalar multiplication: $\varphi\odot\omega = \begin{cases}(\varphi\underline{\omega}, \varphi\bar{\omega}) & \varphi\ge0\\ (\varphi\bar{\omega}, \varphi\underline{\omega}) & \varphi<0\end{cases}$.

Definition 2.2 [8]

Assume that $\omega,\nu\in E$(E: The set of all fuzzy numbers). If there exists $\rho\in E$such that $\omega+\nu=\rho$ then $\rho$is called the Hukuhara difference of $\omega$and $\nu$and it is identified by $\omega\ominus\nu$.

Definition 2.3 [9]

Let $\varphi(\varpi): (a,b) \to E$; be continuous fuzzy-valued function, φis strongly generalized difference at $\varpi_0 \in (a,b)$. If there was an element $\varphi'(\varpi_0) \in E$ then:

1. For all $\hbar > 0$ small enough $\exists\varphi(\varpi_0+\hbar)\ominus\varphi(\varpi_0), \exists\varphi(\varpi_0)\ominus\varphi(\varpi_0-\hbar)$ and the limit is

$$\varphi'(\varpi_0) = \lim_{\hbar\to0^+}\frac{\varphi(\varpi_0+\hbar)\ominus\varphi(\varpi_0)}{\hbar} = \lim_{\hbar\to0^+}\frac{\varphi(\varpi_0)\ominus\varphi(\varpi_0-\hbar)}{\hbar}.$$

Or

2. For all $\hbar > 0$ small enough $\exists\varphi(\varpi_0)\ominus\varphi(\varpi_0+\hbar), \exists\varphi(\varpi_0-\hbar)\ominus\varphi(\varpi_0)$ and the limit is

$$\varphi'(\varpi_0) = \lim_{\hbar\to0^+}\frac{\varphi(\varpi_0)\ominus\varphi(\varpi_0+\hbar)}{-\hbar} = \lim_{\hbar\to0^+}\frac{\varphi(\varpi_0-\hbar)\ominus\varphi(\varpi_0)}{-\hbar}.$$

Or

3. For all $\hbar > 0$ small enough $\exists\varphi(\varpi_0+\hbar)\ominus\varphi(\varpi_0), \exists\varphi(\varpi_0-\hbar)\ominus\varphi(\varpi_0)$ and the limit is

$$\varphi'(\varpi_0)=\lim_{\hbar\to 0^+}\frac{\varphi(\varpi_0+\hbar)\ominus\varphi(\varpi_0)}{\hbar}=\lim_{\hbar\to 0^+}\frac{\varphi(\varpi_0-\hbar)\ominus\varphi(\varpi_0)}{-\hbar}.$$

Or

For all $\hbar>0$ small enough $\exists\varphi(\varpi_0)\ominus\varphi(\varpi_0+\hbar),\exists\varphi(\varpi_0-\hbar)\ominus\varphi(\varpi_0)$ and the limit is

$$\varphi'(\varpi_0)=\lim_{\hbar\to 0^+}\frac{\varphi(\varpi_0)\ominus\varphi(\varpi_0+\hbar)}{-h}=\lim_{\hbar\to 0^+}\frac{\varphi(\varpi_0-\hbar)\ominus\varphi(\varpi_0)}{\hbar}.$$

Theorem 2.4 [10]

Assume that $\varphi: R\to E$(E: the set of all fuzzy numbers) be a function and indicate $\varphi(\varpi)=\left(\underline{\varphi}(\varpi;\vartheta),\bar{\varphi}(\varpi;\vartheta)\right)$ for each $\vartheta\in[0,1]$. Then:

1. If φ is the 1st form, then $\underline{\varphi}(\varpi;\vartheta)$ and $\bar{\varphi}(\varpi;\vartheta)$ can be differentiated and $\varphi'(\varpi)=\underline{\varphi}(\varpi;\vartheta),\bar{\varphi}(\varpi;\vartheta)$.

2. If φ is the 2nd form, then $\underline{\varphi}(\varpi;\vartheta)$ and $\bar{\varphi}(\varpi;\vartheta)$ can be differentiated and $\varphi'(\varpi)=\bar{\varphi}(\varpi;\vartheta),\underline{\varphi}(\varpi;\vartheta)$.

Theorem 2.5 [9]

Let $\varphi(\varpi;\vartheta): R\to E$(E: the set of all fuzzy numbers) and it is represented by $[\underline{\varphi}(\varpi;\vartheta),\bar{\varphi}(\varpi;\vartheta)]$. For any fixed $\vartheta\in(0,1]$ assume that $\underline{\varphi}(\varpi;\vartheta)$ and $\bar{\varphi}(\varpi;\vartheta)$ are Riemann-integrable functions on [a, b] for every $b\geq a$, there are two positive functions $\underline{M}_\vartheta$ and $\overline{M}_\vartheta$ such that $\int_a^b\left|\underline{f}(\varpi;\vartheta)\right|d\varpi\leq\underline{M}_\vartheta$ and $\int_a^b\left|\bar{\varphi}(\varpi;\vartheta)\right|d\varpi\leq\overline{M_\vartheta}$. Then, $f(\varpi)$ is an improper fuzzy Riemann-integrable function on [a,∞). Furthermore, we have:

$$\int_a^\infty\varphi(\varpi)\,d\varpi=\left[\int_a^\infty\underline{\varphi}(\varpi;\vartheta)\,d\varpi,\int_a^\infty\bar{\varphi}(\varpi;\vartheta)\,d\varpi\right]$$

Definition 2.6 [5]

Let $\varphi(\sigma)$ exist as a continuous fuzzy-valued function, let's say that $\varepsilon\int_0^\infty e^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\sigma}\varphi(\sigma)d\sigma$ is an improper fuzzy Riemann-integrable on [0, ∞), then $\varepsilon\int_0^\infty e^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\sigma}\varphi(\sigma)d\sigma$ is called $\widehat{SHA}$ -transform and is denoted as: $SHA[\varphi(\sigma)]=SHA(\varepsilon)=\varepsilon\int_0^\infty e^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\sigma}\varphi(\sigma)d\sigma\quad \mathrm{n}\geq 1$.

From Theorem 2:

$$\varepsilon\int_0^\infty e^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\sigma}\varphi(\sigma)\,d\sigma=\varepsilon\int_0^\infty e^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\sigma}\underline{\varphi}(\sigma;\phi)\,d\sigma,$$
$$\varepsilon\int_0^\infty e^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\sigma}\overline{\varphi}(\sigma;\phi)\,d\sigma$$

Also by the definition of classic SHA-transform:

$$SHA\left[\underline{\varphi}(\sigma;\phi)\right]=\varepsilon\int_0^\infty e^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\sigma}\underline{\varphi}(\sigma;\phi)\,d\sigma$$
$$,SHA\left[\overline{\varphi}(\sigma;\phi)\right]=\varepsilon\int_0^\infty e^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\sigma}\overline{\varphi}(\sigma;\phi)\,d\sigma$$

So:

$$SHA[\varphi(\sigma;\phi)]=SHA\left[\underline{\varphi}(\sigma;\phi)\right],\mathrm{SHA}\left[\overline{\varphi}(\sigma;\phi)\right].$$

Theorem 2.7 [8]

Duality Between Fuzzy Laplace –$\widehat{SHA}$transforms. If F(p) is fuzzy Laplace transform of $\varphi(\sigma)$ and $SHA(\varepsilon)$ is $\widehat{SHA}$-transform of $\varphi(\sigma)$ then $SHA(\varepsilon)=\varepsilon F\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)$.

Theorem 2.8

Let $\Im(\delta)$ by fuzzy function $\delta\geq 0$, $\eta(\varepsilon)=\varepsilon$, $\varepsilon\neq 0$ be positive real function and $\beta(\varepsilon)=i\sqrt[2n]{\varepsilon}+\varepsilon,\varepsilon\neq 0$ be positive complex function then the derivatives of $\Im(\delta)$ for nth - order will be as following:

1. $SHA\{\delta\,\Im(\delta)\}=-\dfrac{\varepsilon}{i\sqrt[2n]{\varepsilon}+\varepsilon}\left(\dfrac{SHA(\Im(\delta),\varepsilon)}{\varepsilon}\right)'$

2. $SHA\{\delta^2\Im(\delta)\}=(-1)^2\dfrac{\varepsilon}{i\sqrt[2n]{\varepsilon}+\varepsilon}\left(\dfrac{1}{i\sqrt[2n]{\varepsilon}+\varepsilon}\left(\dfrac{SHA(\Im(\delta),\varepsilon)}{\varepsilon}\right)'\right)'$

$SHA\{\delta^n\Im(\delta)\}=(-1)^n\dfrac{\varepsilon}{i\sqrt[2n]{\varepsilon}+\varepsilon}$

3. $$\left(\frac{1}{i\sqrt[2n]{\varepsilon}+\varepsilon}\left(\frac{1}{i\sqrt[2n]{\varepsilon}+\varepsilon}\left(\cdots\left(\frac{1}{i\sqrt[2n]{\varepsilon}+\varepsilon}\left(\frac{SHA(\Im(\delta),\varepsilon)}{\varepsilon}\right)'\right)'\right)'\right)\cdots\right)'$$

Proof:

1. Since

$$SHA\{\Im(\delta),\varepsilon\}=\varepsilon\int_0^\infty\underline{\Im}(\delta;\varepsilon)e^{-\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)\delta}d\delta,\varepsilon\int_0^\infty\overline{\Im}(\delta;\varepsilon)e^{-\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)\delta}d\delta$$
$$\Rightarrow\frac{SHA\{\Im(\delta),\varepsilon\}}{\varepsilon}=\int_0^\infty\underline{\Im}(\delta;\varepsilon)e^{-\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)\delta}d\delta,\varepsilon\int_0^\infty\overline{\Im}(\delta;\varepsilon)e^{-\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)\delta}d\delta$$

Derivative above equation with respect ε, to get:

$$\left(\frac{SHA\{\Im(\delta),\varepsilon\}}{\varepsilon}\right)'=\frac{d}{d\varepsilon}\begin{bmatrix}\int_0^\infty\underline{\Im}(\delta;\varepsilon)e^{-\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)\delta}d\delta,\\ \int_0^\infty\overline{\Im}(\delta;\varepsilon)e^{-\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)\delta}d\delta\end{bmatrix}$$

$$\left(\frac{SHA\{\Im(\delta),\varepsilon\}}{\varepsilon}\right)'=-\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)\int_0^\infty\underline{\Im}(\delta;\varepsilon)e^{-\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)\delta}d\delta,$$
$$-\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)\int_0^\infty\overline{\Im}(\delta;\varepsilon)e^{-\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)\delta}d\delta\tag{1}$$

From Eq. (1), to get:

$$\left(\frac{SHA\left\{\Im(\delta),\varepsilon\right\}}{\varepsilon}\right)' = -\left(i\,\sqrt[2n]{\varepsilon}+\varepsilon\right)\frac{SHA\left\{\underline{\Im}(\delta;\varepsilon),\varepsilon\right\}}{\varepsilon},$$

$$-\left(i\,\sqrt[2n]{\varepsilon}+\varepsilon\right)\frac{SHA\left\{\overline{\Im}(\delta;\varepsilon),\varepsilon\right\}}{\varepsilon}$$

$$\left(\frac{SHA\left(\Im(\delta),\varepsilon\right)}{\varepsilon}\right)' = -\left(i\,\sqrt[2n]{\varepsilon}+\varepsilon\right)\frac{SHA\left\{\delta\ \Im(\delta),\varepsilon\right\}}{\varepsilon}$$

Then, to get:

$$SHA\left\{\delta\ \Im(\delta)\right\} = -\frac{\varepsilon}{i\,\sqrt[2n]{\varepsilon}+\varepsilon}\left(\frac{SHA\left(\Im(\delta),\varepsilon\right)}{\varepsilon}\right)'$$

2. Since from the first part, we have:

$$SHA\left\{\delta\ \Im(\delta),\varepsilon\right\} = -\frac{\varepsilon}{i\,\sqrt[2n]{\varepsilon}+\varepsilon}\left(\frac{SHA\left(\Im(\delta),\varepsilon\right)}{\varepsilon}\right)'$$

Taking the derivative for both sides of the above equation:

$$-\left(i\,\sqrt[2n]{\varepsilon}+\varepsilon\right)\frac{1}{\varepsilon}\int_0^\infty \delta^2\,\underline{\Im}(\delta;\varepsilon)e^{-\left(i\,\sqrt[2n]{\varepsilon}+\varepsilon\right)\delta}d\delta,$$

$$-\left(i\,\sqrt[2n]{\varepsilon}+\varepsilon\right)\int_0^\infty \delta^2\,\overline{\Im}(\delta;\varepsilon)e^{-\left(i\,\sqrt[2n]{\varepsilon}+\varepsilon\right)\delta}d\delta = \left(-\frac{\varepsilon}{\left(i\,\sqrt[2n]{\varepsilon}+\varepsilon\right)}\left(\frac{SHA\left\{\Im(\delta),\varepsilon\right\}}{\varepsilon}\right)'\right)'$$

$$-\frac{\left(i\,\sqrt[2n]{\varepsilon}+\varepsilon\right)}{\varepsilon}SHA\left\{\delta^2\,\underline{\Im}(\delta;\vartheta)\right\},$$

$$\frac{\left(i\,\sqrt[2n]{\varepsilon}+\varepsilon\right)}{\varepsilon}SHA\left\{\delta^2\,\overline{\Im}(\delta;\vartheta)\right\} = \left(-\frac{\varepsilon}{\left(i\,\sqrt[2n]{\varepsilon}+\varepsilon\right)}\left(\frac{SHA\left\{\Im(\delta),\varepsilon\right\}}{\varepsilon}\right)'\right)'$$

Thus:

$$SHA\left\{\delta^2\Im(\delta)\right\} = (-1)^2\frac{\varepsilon}{i\,\sqrt[2n]{\varepsilon}+\varepsilon}\left(-\frac{1}{i\,\sqrt[2n]{\varepsilon}+\varepsilon}\left(\frac{SHA\left(\Im(\delta),\varepsilon\right)}{\varepsilon}\right)'\right)'.$$

3. In similar way, we can prove the third part

$$SHA\left\{\delta^2\Im(\delta)\right\} = (-1)^2\frac{\varepsilon}{i\,\sqrt[2n]{\varepsilon}+\varepsilon}\left(-\frac{1}{i\,\sqrt[2n]{\varepsilon}+\varepsilon}\left(\frac{SHA\left(\Im(\delta),\varepsilon\right)}{\varepsilon}\right)'\right)'$$

derivative both side of above equation (n-2)-Times, we get:

$$SHA\left\{\delta^n\Im(\delta)\right\}=(-1)^n\frac{\varepsilon}{i\,\sqrt[2n]{\varepsilon}+\varepsilon}$$

$$\left(\frac{1}{i\,\sqrt[2n]{\varepsilon}+\varepsilon}\left(\frac{1}{i\,\sqrt[2n]{\varepsilon}+\varepsilon}\left(\cdots\left(\frac{1}{i\,\sqrt[2n]{\varepsilon}+\varepsilon}\left(\frac{SHA\left(\Im(\delta),\varepsilon\right)}{\varepsilon}\right)'\right)'\right)'\right)'\cdots\right)'.$$

Theorem 2.9

Let $\eta(\varepsilon)=\varepsilon$ and $\beta(\varepsilon)=i\,\sqrt[2n]{\varepsilon}+\varepsilon$ are differentiable functions such that $\Im(\delta)$ be fuzzy function, then:

$$SHA\left\{\delta\Im^{(n)}(\delta)\right\} = -\frac{\varepsilon}{i\,\sqrt[2n]{\varepsilon}+\varepsilon}\frac{d}{d\varepsilon}\left(\frac{SHA\left(\Im^{(n)}(\delta)\right)}{\varepsilon}\right)$$

Proof:
Since

$$SHA\left\{\Im^{(n)}(\delta),\varepsilon\right\} = \varepsilon\int_0^\infty \underline{\Im}^{(n)}(\delta;\vartheta)e^{-\left(i\,\sqrt[2n]{\varepsilon}+\varepsilon\right)\delta}d\delta,\ \varepsilon\int_0^\infty \overline{\Im}^{(n)}(\delta;\vartheta)e^{-\left(i\,\sqrt[2n]{\varepsilon}+\varepsilon\right)\delta}d\delta$$

$$\frac{SHA\left\{\Im^{(n)}(\delta),\varepsilon\right\}}{\varepsilon} = \int_0^\infty \underline{\Im}^{(n)}(\delta;\vartheta)e^{-\left(i\,\sqrt[2n]{\varepsilon}+\varepsilon\right)\delta}d\delta,\ \int_0^\infty \overline{\Im}^{(n)}(\delta;\vartheta)e^{-\left(i\,\sqrt[2n]{\varepsilon}+\varepsilon\right)\delta}d\delta$$

By derivative above equation respect to ε, then:

$$\frac{d}{d\varepsilon}\left[\frac{SHA\left\{\Im^{(n)}(\delta),\varepsilon\right\}}{\varepsilon}\right] = \left[\int_0^\infty \underline{\Im}^{(n)}(\delta;\vartheta)e^{-\left(i\,\sqrt[2n]{\varepsilon}+\varepsilon\right)\delta}d\delta,\ \int_0^\infty \overline{\Im}^{(n)}(\delta;\vartheta)e^{-\left(i\,\sqrt[2n]{\varepsilon}+\varepsilon\right)\delta}d\delta\right]$$

$$\frac{d}{d\varepsilon}\left[\frac{SHA\left\{\Im^{(n)}(\delta),\varepsilon\right\}}{\varepsilon}\right] = -(i\,\sqrt[2n]{\varepsilon}+\varepsilon)\int_0^\infty \underline{\Im}^{(n)}(\delta;\vartheta)e^{-\left(i\,\sqrt[2n]{\varepsilon}+\varepsilon\right)\delta}d\delta,$$

$$-(i\,\sqrt[2n]{\varepsilon}+\varepsilon)\int_0^\infty \overline{\Im}^{(n)}(\delta;\vartheta)e^{-\left(i\,\sqrt[2n]{\varepsilon}+\varepsilon\right)\delta}d\delta \tag{2}$$

From Eq. (2):

$$\frac{d}{d\varepsilon}\left[\frac{SHA\left\{\Im^{(n)}(\delta),\varepsilon\right\}}{\varepsilon}\right] = -(i\,\sqrt[2n]{\varepsilon}+\varepsilon)\frac{SHA\left\{\delta\underline{\Im}^{(n)}(\delta;\vartheta),\varepsilon\right\}}{\varepsilon},$$

$$-(i\,\sqrt[2n]{\varepsilon}+\varepsilon)\frac{SHA\left\{\delta\overline{\Im}^{(n)}(\delta;\vartheta),\varepsilon\right\}}{\varepsilon}$$

$$\frac{d}{d\varepsilon}\left[\frac{SHA\left\{\Im^{(n)}(\delta),\varepsilon\right\}}{\varepsilon}\right] = -(i\,\sqrt[2n]{\varepsilon}+\varepsilon)\frac{SHA\left\{\delta\Im^{(n)}(\delta;\vartheta),\varepsilon\right\}}{\varepsilon}$$

Then**:** $SHA\left\{\delta\Im^{(n)}(\delta)\right\} = -\dfrac{\varepsilon}{i\,\sqrt[2n]{\varepsilon}+\varepsilon}\dfrac{d}{d\varepsilon}\left(\dfrac{SHA\left(\Im^{(n)}(\delta)\right)}{\varepsilon}\right)$

Theorem 2.10

Assume that $\varphi^{\backslash}(\sigma)$ be continuous fuzzy-valued function and $\varphi(\sigma)$ the primitive of $\varphi^{\backslash}(\sigma)$ on $[0,\infty)$, we have:

1. $SHA\left[\varphi^{\backslash}(\sigma)\right] = (i\sqrt[2n]{\varepsilon}+\varepsilon)SHA\left[\varphi(\sigma)\right]\ominus\varepsilon\varphi(0)$, where, φ is the first form differentiable
2. $SHA\left[\varphi^{\backslash\backslash}(\sigma)\right] = -\varepsilon\varphi(0)\ominus\left(-i\sqrt[2n]{\varepsilon}+\varepsilon\right)SHA\left[\varphi(\sigma)\right]$, where, φ is the second form differentiable.

Theorem 2.11 [6]

Assume that, $\varphi(\sigma)$, $\varphi^{\backslash}(\sigma)$ are continuous fuzzy-valued function on $[0,\infty)$, fuzzy derivative of FSHA-Transform about second order will be as following:

1. If $\varphi,\varphi^{\backslash}$ are first form then:

$$SHA\left[\varphi^{\backslash\backslash}(\sigma)\right] = \left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)^2 SHA\left[\varphi(\sigma)\right]\ominus\varepsilon\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)\varphi(0)\ominus\varepsilon\varphi^{\backslash}(0)$$

2. If φ first form and $\varphi^{\backslash}$ then the second form:

$$SHA\left[\varphi^{\backslash\backslash}(\sigma)\right] = -\varepsilon\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)\varphi(0)\ominus\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)^2 SHA\left[\varphi(\sigma)\right]\ominus\varepsilon\varphi^{\backslash}(0)$$

3. If φ consists of a second form and $\varphi^{\backslash}$ initial form followed by:

$$SHA\left[\varphi^{\backslash\backslash}(\sigma)\right] = -\varepsilon\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)\varphi(0)\ominus\left(-i\sqrt[2n]{\varepsilon}+\varepsilon\right)^2 SHA\left[\varphi(\sigma)\right]-\varepsilon\varphi^{\backslash}(0)$$

4. If $\varphi,\varphi^{\backslash}$ are second form then:

$$SHA\left[\varphi^{\backslash\backslash}(\sigma)\right] = \left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)^2 SHA\left[\varphi(\sigma)\right]\ominus\varepsilon\left(-i\sqrt[2n]{\varepsilon}+\varepsilon\right)\varphi(0)-\varepsilon\varphi^{\backslash}(0)$$

Theorem 2.12 [8]

Assume that $\Im(\delta),\Im^{\backslash}(\delta),\dots,\Im^{n-1}(\delta)$ be from continuous fuzzy-valued functions on $[0,\infty)$. Let $\Im^{(i_1)}(\delta),\Im^{(i_2)}(\delta),\dots,\Im^{(i_m)}(\delta)$ are differentiable functions 2^{st} from functions for $0\le i_1\le i_2\ldots\le i_m\le n-1$ and $\Im^{(p)}$ is the 1^{st} from differentiable function for p≠i_j, j=1,2,...,m, then:

1. If m is an even number, we have

$$L\left(\Im^{(n)}(\delta)\right) = p^n L\left(\Im(\delta)\right)\ominus p^{n-1}\Im(0)\otimes\sum_{k=1}^{n-1}p^{n-(k+1)}\Im^{(k)}(0)$$

such that

$$\otimes=\begin{cases}\ominus, & \text{In case the second order repetition count is equal to the first order repetition count 2 among } i_1,\dots,i_k \text{ ,is an even number}\\ -, & \text{If the frequency of occurrence of the number 2 among } i_1,\dots,i_k \text{ is an odd number}\end{cases}$$

2. If m is an odd number, we have

$$L\left(\Im^{(n)}(\delta)\right) = -p^{n-1}\Im(0)\ominus\left(-p^n\right)L\left(\Im(\delta)\right)\otimes\sum_{k=1}^{n-1}p^{n-(k+1)}\Im^{(k)}(0)$$

such that

$$\otimes=\begin{cases}\ominus, & \text{If the frequency of occurrence of the number 2 among } i_1,\dots,i_k \text{ is an odd number}\\ -, & \text{If the frequency of occurrence of the number 2 among } i_1,\dots,i_k \text{ is an even number}\end{cases}$$

Theorem 2.13

Suppose that, $\Im(\delta)$, $\Im^{\backslash}(\delta)$ and $\Im^{\backslash\backslash}(\delta)$ are continuous fuzzy-valued function on $[0,\infty)$, fuzzy derivative of fuzzy SHA-transform about third order it will be:

1. If $\Im,\Im^{\backslash},\Im^{\backslash\backslash}$ are first form then:

$$SHA\left[\Im^{\backslash\backslash\backslash}(\delta)\right] = \left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^3 SHA\left[\Im(\delta)\right] \ominus\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^2\Im(0)\ominus\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)\Im^{\backslash}(0)\ominus\varepsilon\Im^{\backslash\backslash}(0)$$

2. If $\Im^{\backslash},\Im^{\backslash\backslash}$ are first form and $\Im$ second form then:

$$SHA\left[\Im^{\backslash\backslash\backslash}(\delta)\right] = \varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^2\Im(0)\ominus\left(-\sqrt[2n]{\varepsilon}+\varepsilon\right)^3 SHA\left[\Im(\delta)\right] -\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)\Im^{\backslash}(0)-\varepsilon\Im^{\backslash\backslash}(0)$$

3. If $\Im,\Im^{\backslash\backslash}$ are first form and $\Im^{\backslash}$ second form then:

$$SHA\left[\Im^{\backslash\backslash\backslash}(\delta)\right] = -\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^2\Im(0)\ominus\left(-\sqrt[2n]{\varepsilon}+\varepsilon\right)^3 SHA\left[\Im(\delta)\right]\ominus\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)\Im^{\backslash}(0)\ominus\varepsilon\Im^{\backslash\backslash}(0)$$

4. If $\Im,\Im^{\backslash}$ are first form and $\Im^{\backslash\backslash}$ second form then:

$$SHA\left[\Im^{\backslash\backslash\backslash}(\delta)\right] = -\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^2\Im(0)\ominus\left(-\sqrt[2n]{\varepsilon}+\varepsilon\right)^3 SHA\left[\Im(\delta)\right] -\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)\Im^{\backslash}(0)\ominus\varepsilon\Im^{\backslash\backslash}(0)$$

5. If $\Im,\Im^{\backslash}$ are second form and $\Im^{\backslash\backslash}$ first form then:

$$SHA\left[\Im^{\backslash\backslash\backslash}(\delta)\right] = \left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^3 SHA\left[\Im(\delta)\right]\ominus\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^2\Im(0) -\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)\Im^{\backslash}(0)-\varepsilon\Im^{\backslash\backslash}(0)$$

6. If $\Im,\Im^{\backslash\backslash}$ are second form and $\Im^{\backslash}$ first form then:

$$SHA\left[\Im^{\backslash\backslash\backslash}(\delta)\right] = \left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^3 SHA\left[\Im(\delta)\right]\ominus\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^2 \Im(0)\ominus\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)\Im^{\backslash}(0)-\varepsilon\Im^{\backslash\backslash}(0)$$

7. If $\Im', \Im''$ are second form and $\Im$ first form then:

$$\begin{aligned}&SHA\left[\Im'''(\delta)\right]=\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^3 SHA\left[\Im(\delta)\right]\ominus\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^2\\&\Im(0)-\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)\Im'(0)\ominus\varepsilon\Im''(0)\end{aligned}$$

8. If $\Im, \Im', \Im''$ are second form then:

$$\begin{aligned}&SHA\left[\Im'''(\delta)\right]=-\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^2\Im(0)\ominus\left(-\sqrt[2n]{\varepsilon}+\varepsilon\right)^3\\&SHA\left[\Im(\delta)\right]\ominus\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)\Im'(0)-\varepsilon\Im''(0)\end{aligned}$$

Proof: We will prove four cases:

Case 1

1. $\Im, \Im', \Im''$ are first form and for any arbitral $\phi \in [0, 1]$, then:

$$SHA\left[\Im'''(\delta)\right]=SHA\left[\underline{\Im}'''(\delta,\phi)\right],\mathrm{SHA}\left[\overline{\Im}'''(\delta,\phi)\right]$$

By definition of SHA derivative of second order we get:

$$\begin{aligned}&SHA\left[\underline{\Im}'''(\delta)\right]=\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^3 SHA\left[\underline{\Im}(\delta,\phi)\right]-\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^2\\&\underline{\Im}(0,\phi)-\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)\underline{\Im'(0,\phi)}-\varepsilon\underline{\Im''(0,\phi)}\end{aligned}\tag{3}$$

$$\begin{aligned}&SHA\left[\overline{\Im}'''(\delta)\right]=\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^3 SHA\left[\overline{\Im}(\delta,\phi)\right]-\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^2\\&\overline{\Im}(0,\phi)-\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)\overline{\Im'(0,\phi)}-\varepsilon\overline{\Im''(0,\phi)}\end{aligned}\tag{4}$$

If we substitute (3) in (4) and since $\Im'(\delta)$, $\Im''(\delta)$ are first form we get:

$$\begin{aligned}&SHA\left[\underline{\Im}'''(\delta)\right]=\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^3 SHA\left[\underline{\Im}(\delta,\phi)\right]-\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^2\\&\underline{\Im}(0,\phi)-\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)\underline{\Im'(0,\phi)}-\varepsilon\underline{\Im''(0,\phi)}\\&,\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^3 SHA\left[\overline{\Im}(\delta,\phi)\right]-\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^2\\&\overline{\Im}(0,\phi)-\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)\overline{\Im'(0,\phi)}-\varepsilon\overline{\Im''(0,\phi)}\end{aligned}$$

by Theorem (2) we get:

$$\begin{aligned}&SHA\left[\Im'''(\delta)\right]=\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^3 SHA\left[\Im(\delta)\right]\ominus\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^2\\&\Im(0)\ominus\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)\Im'(0)\ominus\varepsilon\Im''(0)\end{aligned}$$

2. If $\Im', \Im''$ are first form and *f* second form then:

$$SHA\left[\Im''(\delta)\right]=SHA\left[\underline{\Im}'''(\delta,\phi)\right],\mathrm{SHA}\left[\overline{\Im}'''(\delta,\phi)\right]$$

By definition of SHA derivative of second order we get:

$$\begin{aligned}&SHA\left[\underline{\Im}'''(\delta)\right]=\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^3 SHA\left[\underline{\Im}(\delta,\phi)\right]-\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^2\\&\underline{\Im}(0,\phi)-\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)\underline{\Im'(0,\phi)}-\varepsilon\underline{\Im''(0,\phi)}\end{aligned}\tag{5}$$

$$\begin{aligned}&SHA\left[\overline{\Im}'''(\delta)\right]=\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^3 SHA\left[\overline{\Im}(\delta,\phi)\right]-\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^2\\&\overline{\Im}(0,\phi)-\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)\overline{\Im'(0,\phi)}-\varepsilon\overline{\Im''(0,\phi)}\end{aligned}\tag{6}$$

If we substitute (5) in (6) and since $\Im'(\delta)$, $\Im''(\delta)$ are first form we get:

$$\begin{aligned}&SHA\left[\underline{\Im}'''(\delta)\right]=\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^3 SHA\left[\overline{\Im}(\delta,\phi)\right]-\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^2\\&\overline{\Im}(0,\phi)-\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)\overline{\Im'(0,\phi)}-\varepsilon\overline{\Im''(0,\phi)}\\&,\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^3 SHA\left[\underline{\Im}(\delta,\phi)\right]-\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^2\\&\underline{\Im}(0,\phi)-\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)\underline{\Im'(0,\phi)}-\varepsilon\underline{\Im''(0,\phi)}\end{aligned}$$

by Theorem (2) we get:

$$\begin{aligned}&SHA\left[\Im'''(\delta)\right]=\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^2\Im(0)\ominus\left(-\sqrt[2n]{\varepsilon}+\varepsilon\right)^3\\&SHA\left[\Im(\delta)\right]-\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)\Im'(0)-\varepsilon\Im''(0)\end{aligned}$$

3. If $\Im, \Im'$ are second form and $\Im''$ first form then:

$$SHA\left[\Im''(\delta)\right]=SHA\left[\underline{\Im}'''(\delta,\phi)\right],\mathrm{SHA}\left[\overline{\Im}'''(\delta,\phi)\right]$$

By definition of SHA derivative of second order we get:

$$\begin{aligned}&SHA\left[\underline{\Im}'''(\delta)\right]=\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^3 SHA\left[\underline{\Im}(\delta,\phi)\right]-\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^2\\&\underline{\Im}(0,\phi)-\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)\underline{\Im'(0,\phi)}-\varepsilon\underline{\Im''(0,\phi)}\\&SHA\left[\overline{\Im}'''(\delta)\right]=\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^3 SHA\left[\overline{\Im}(\delta,\phi)\right]-\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^2\\&\overline{\Im}(0,\phi)-\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)\overline{\Im'(0,\phi)}-\varepsilon\overline{\Im''(0,\phi)}\end{aligned}$$

If we substitute (4) in (3) and since $\Im'(\delta)$ second and $\Im''(\delta)$ first form we get:

$$\begin{aligned}&SHA\left[\underline{\Im}'''(\delta)\right]=\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^3 SHA\left[\underline{\Im}(\delta,\phi)\right]-\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^2\\&\underline{\Im}(0,\phi)-\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)\underline{\Im'(0,\phi)}-\varepsilon\underline{\Im''(0,\phi)}\\&,\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^3 SHA\left[\overline{\Im}(\delta,\phi)\right]-\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^2\\&\overline{\Im}(0,\phi)-\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)\overline{\Im'(0,\phi)}-\varepsilon\overline{\Im''(0,\phi)}\end{aligned}$$

by Theorem (2) we get:

$$\begin{aligned}&SHA\left[\Im'''(\delta)\right]=\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^3 SHA\left[\Im(\delta)\right]\ominus\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^2\\&\Im(0)-\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)\Im'(0)-\varepsilon\Im''(0)\end{aligned}$$

4. If $\Im, \Im', \Im''$ are second form then:

$$SHA\left[\Im''(\delta)\right]=SHA\left[\underline{\Im}'''(\delta,\phi)\right],\mathrm{SHA}\left[\overline{\Im}'''(\delta,\phi)\right]$$

By definition of SHA derivative of second order we get:

$$SHA\left[\underline{\Im}^{\backslash\backslash\backslash}(\delta)\right]=\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^3 SHA\left[\underline{\Im}(\delta,\phi)\right]-\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^2$$
$$\underline{\Im}(0,\phi)-\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)\underline{\Im^{\backslash}(0,\phi)}-\varepsilon\underline{\Im^{\backslash\backslash}(0,\phi)}$$
$$SHA\left[\overline{\Im}^{\backslash\backslash\backslash}(\delta)\right]=\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^3 SHA\left[\overline{\Im}(\delta,\phi)\right]-\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^2$$
$$\overline{\Im}(0,\phi)-\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)\overline{\Im^{\backslash}(0,\phi)}-\varepsilon\overline{\Im^{\backslash\backslash}(0,\phi)}$$

If we substitute (6) in (5) and since $\Im'(\delta),\Im''(\delta)$ are first form we get:

$$SHA\left[\underline{\Im}^{\backslash\backslash\backslash}(\delta)\right]=\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^3 SHA\left[\overline{\Im}(\delta,\phi)\right]-\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^2$$
$$\overline{\Im}(0,\phi)-\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)\overline{\Im^{\backslash}(0,\phi)}-\varepsilon\overline{\Im^{\backslash\backslash}(0,\phi)}$$
$$,\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^3 SHA\left[\underline{\Im}(\delta,\phi)\right]-\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^2$$
$$\underline{\Im}(0,\phi)-\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)\underline{\Im^{\backslash}(0,\phi)}-\varepsilon\underline{\Im^{\backslash\backslash}(0,\phi)}$$

by Theorem (2) we get:

$$SHA\left[\Im^{\backslash\backslash\backslash}(\delta)\right]=-\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^2\Im(0)\ominus\left(-\sqrt[2n]{\varepsilon}+\varepsilon\right)^3$$
$$SHA\left[\Im(\delta)\right]\ominus\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)\Im^{\backslash}(0)-\varepsilon\Im^{\backslash\backslash}(0)$$

5. Fuzzy SHA-Transform for Fuzzy nth- Order Derivative

Theorem 2.14

Assume that $\Im(\delta),\Im^{\backslash}(\delta),...,\Im^{n-1}(\delta),\Im^{(n)}(\delta)$ are continuous fuzzy-valued functions on $[0,\infty)$ and of exponential order and that $\Im^{(n)}(\delta)$ be pricewise continuous fuzzy-valued on $[0,\infty)$. Let $\Im^{(i_1)}(\delta),\Im^{(i_2)}(\delta),...,\Im^{(i_\varphi)}(\delta)$ are the second form differentiable functions for $0\le i_1\le i_2\ldots\le i_\varphi\le n-1$ and $\Im^{(p)}$ be first form differentiable function for $p\neq i_j$, j=1, 2,..., φ, then:

(1) If φ is an even number, we have

$$M\left(\Im^{(n)}(\delta)\right)=\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^n M\left(\Im(\delta)\right)\ominus\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)$$
$$\Im(0)®\sum_{\kappa=1}^{n-1}\varepsilon^{n-\kappa+1}\Im^{(\kappa)}(0)$$

such that

$$®=\begin{cases}\ominus, & \text{If there is a detectable difference between the occurrences of the second form and the first, then } i_1,...,i_\kappa \text{ is an even number}\\ -, & \text{If there is a detectable difference between the occurrences of the second form and the first, then } i_1,...,i_\kappa \text{ is an odd number}\end{cases}$$

(2) If φ is an odd number, we have

$$M\left(\Im^{(n)}(\delta)\right)=-\varepsilon^{n+1}\Im(0)\ominus\left(-\varepsilon^n\right)M\left(\Im(\delta)\right)®\sum_{\kappa=1}^{n-1}\varepsilon^{n-\kappa+1}\Im^{(\kappa)}(0),$$

such that

$$®=\begin{cases}\ominus, & \text{iIf there is a detectable difference between the occurrences of the second form and the first, then} i_1,...,i_\kappa \text{ is an odd number}\\ -, & \text{If there is a detectable difference between the occurrences of the second form and the first, then } i_1,...,i_\kappa \text{ is an even number}\end{cases}$$

Proof: The proof depends on the duality between fuzzy Laplace –SHA-Transforms from as follows:

$$SHA(\varepsilon)=SHA\left[\Im(\varepsilon)\right]\ F(p)=L\left[\Im(\aleph)\right]$$
$$SHA_n(\varepsilon)=SHA\left[\Im^{(n)}(\delta)\right]\text{and}\,F_n(p)=L\left[\Im^{(n)}(\aleph)\right]$$

From duality relation (1), we have

$$SHA_n(\delta)=SHA\left[\Im^{(n)}(\delta)\right]=\varepsilon F_n\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)$$

Let φ is an even number. Then from Theorem 4 when φ is an even number, Eq. (2) becomes:

$$SHA_n(\delta)=\varepsilon\begin{bmatrix}\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^n F\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)\ominus\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^{(n-1)}\Im(0)\\ ®\sum_{\kappa=1}^{n-1}\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^{n-(\kappa+1)}\Im^{(\kappa)}(0)\end{bmatrix}$$
$$=\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^n\left[\varepsilon F\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)\right]\ominus\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^{(n-1)}$$
$$\Im(0)®\varepsilon\sum_{\kappa=1}^{n-1}\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^{n-(\kappa+1)}\Im^{(\kappa)}(0)$$
$$=\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^n SHA\left[\Im(\delta)\right]\ominus\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^{(n-1)}$$
$$\Im(0)®\varepsilon\sum_{\kappa=1}^{n-1}\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^{n-(\kappa+1)}\Im^{(\kappa)}(0)$$

Let φ is an odd number. Then from Theorem 4, Eq. (3) becomes:

$$SHA_n(\varepsilon)=\begin{bmatrix}-\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^{(n-1)}\Im(0)\ominus\left(-\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^n\right)F\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)\\ ®\sum_{\kappa=1}^{n-1}\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^{n-(\kappa+1)}\Im^{(\kappa)}(0)\end{bmatrix}$$

$$=-\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^{(n-1)}\Im(0)\ominus\left(-\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^n\right)\left[\varepsilon F\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)\right]$$
$$®\varepsilon\sum_{\kappa=1}^{n-1}\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^{n-(\kappa+1)}\Im^{(\kappa)}(0)$$

$$=-\varepsilon\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^{(n-1)}\Im(0)\ominus\left(-\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^n\right)SHA\left[\Im(\delta)\right]$$
$$®\varepsilon\sum_{\kappa=1}^{n-1}\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)^{n-(\kappa+1)}\Im^{(\kappa)}(0)$$

Example 2.15

The concentration of a drug in an organ. If blood transports a specific drug into an organ at a variable rate of rI(t) cm^3/s and out of the organ at a variable rate of rO(t) cm^3/s, and the organ has an initial blood volume of V cm^3, the following equation can be used: For c(t)g/cm^3 of blood entering the organ, compute an ODE for the amount of drug present in the organ at time t following the partnership.

$$\Im^{\backslash}(t) = \Psi\Upsilon - \hbar\Im(t), \quad \Im(0) = \left(\Im(0,r), \Im(r,0)\right)$$
$$\text{such that } \Psi = 1, \Upsilon = c(t) = 1, \hbar = v(t) = 1$$

To solve the equation, we have two cases:

Case 1

By using FSHA-Transform for both sides of the original equation we get:

$$\left(\sqrt[2n]{\varepsilon} + \varepsilon\right) SHA\left[\Im(\delta)\right] \ominus \varepsilon\Im(0) = -SHA\left[\Im(\delta)\right]$$

Eq. (4) becomes:

$$\left(\sqrt[2n]{\varepsilon} + \varepsilon\right) SHA\left[\underline{\Im}(\delta)\right] - \varepsilon\underline{\Im}(0) = -SHA\left[\underline{\Im}(\delta)\right],$$
$$\left(\sqrt[2n]{\varepsilon} + \varepsilon\right) SHA\left[\overline{\Im}(\delta)\right] - \varepsilon\overline{\Im}(0) = -SHA\left[\overline{\Im}(\delta)\right]$$

By solve Eq. (5) we get:

$$SHA\left[\underline{\Im}(\delta;\vartheta)\right] = \frac{\varepsilon}{\left(\sqrt[2n]{\varepsilon} + \varepsilon\right) + 1}\underline{\Im}(0;\delta),$$
$$SHA\left[\overline{\Im}(\delta;\vartheta)\right] = \frac{\varepsilon}{\left(\sqrt[2n]{\varepsilon} + \varepsilon\right) + 1}\overline{\Im}(0;\delta)$$

Now we use inverse fuzzy SHA-Transform for above equation:

$$\underline{\Im}(\delta;\vartheta) = e^{-\delta}\underline{\Im}(0;\delta), \overline{\Im}(\delta;\vartheta) = e^{-\delta}\overline{\Im}(0;\delta)$$

Case 2

By using fuzzy SHA-Transform for both sides of the original equation we get:

$$-\varepsilon\Im(0) \ominus \left(-\left(\sqrt[2n]{\varepsilon} + \varepsilon\right)\right) SHA\left[\Im(\delta)\right] = -SHA\left[\Im(\delta)\right] \quad (5)$$

Eq. (4) becomes:

$$\left(\sqrt[2n]{\varepsilon} + \varepsilon\right) SHA\left[\overline{\Im}(\delta)\right] - \varepsilon\overline{\Im}(0) = -SHA\left[\underline{\Im}(\delta)\right],$$
$$\left(\sqrt[2n]{\varepsilon} + \varepsilon\right) SHA\left[\underline{\Im}(\delta)\right] - \varepsilon\underline{\Im}(0) = -SHA\left[\overline{\Im}(\delta)\right]$$

By solve Eq. (5) we get:

$$SHA\left[\underline{\Im}(\delta;\vartheta)\right] = \frac{\varepsilon\left(\sqrt[2n]{\varepsilon} + \varepsilon\right)}{\left(\sqrt[2n]{\varepsilon} + \varepsilon\right)^2 - 1}\overline{\Im}(0;\delta) - \frac{\varepsilon}{\left(\sqrt[2n]{\varepsilon} + \varepsilon\right)^2 - 1}\underline{\Im}(0;\delta)$$
$$SHA\left[\overline{\Im}(\delta;\vartheta)\right] = \frac{\varepsilon\left(\sqrt[2n]{\varepsilon} + \varepsilon\right)}{\left(\sqrt[2n]{\varepsilon} + \varepsilon\right)^2 - 1}\underline{\Im}(0;\delta) - \frac{\varepsilon}{\left(\sqrt[2n]{\varepsilon} + \varepsilon\right)^2 - 1}\overline{\Im}(0;\delta)$$

Now we use inverse fuzzy SHA-transform for above equation:

$$\underline{\Im}(\delta;\vartheta) = \cosh\varpi\,\overline{\Im}(0;\delta) - \sinh\varpi\,\underline{\Im}(0;\delta),$$
$$\overline{\Im}(\delta;\vartheta) = \cosh\varpi\,\underline{\Im}(0;\delta) - \sinh\varpi\,\overline{\Im}(0;\delta)$$

3. CONCLUSIONS

In this research we prove some properties of fuzzy SHA-transform and found fuzzy derivatives of fuzzy SHA-transform for third and generalized order and use these formulas to solve system equation that related with drug concentration or the amount of drug in any organ in the body which can be measured by its level in the blood (plasma), urine, saliva and other sampled fluids.

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