International Information and Engineering Technology Association
Advancing the World of Information and Engineering

Mathematical Modelling of Engineering Problems
Vol. 9, No. 5, October, 2022, pp. 1243-1250
Journal homepage: http://iieta.org/journals/mmep

The Behavior of the Synchronous and Asynchronous Natural Frequencies for Asymmetric Double Beams

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https://doi.org/10.18280/mmep.090511

Received: 6 May 2022
Accepted: 6 October 2022

Keywords:
double beam, vibration of asymmetric beam, synchronous and asynchronous mode

ABSTRACT

The effect of vibrations on asymmetric double beams is a common engineering problem in various engineering applications. In this paper, the synchronous (lower) and asynchronous (higher) natural frequencies of the asymmetric double beams are calculated using the Bernoulli-Euler method. Where the traditional methods are used to find the frequency equations at different boundary conditions, such as Pinned beam, clamped-Clamped beam, Clamped-Free beam, and Clamped-Pinned beam. The increase in the stiffness of the elastic connected layer leads to an increase in the values of the high frequencies of double beams. The greatest effect of changing the thickness of one of the upper or lower beams is for CF beams and the least effect is for CP beams. The length of the beam affects the higher and lower frequencies in high and close proportions for almost all types of beams, and the least effect is only on the higher frequencies of CF beams. The influence of the modulus elasticity change is relatively small on the lower natural frequencies of all types of beams except for CF beams, and its effect is relatively large on the higher natural frequencies of the most types of beams and comparatively less on the CF beams. The effect of varying the values of mass density is relatively small on the low natural frequencies of all types of beams except for CF beams, and its effect is comparatively large on the higher natural frequencies of all types of beams and relatively less on the CF beams.

1. INTRODUCTION

One of the important industrial applications in aerospace engineering and construction is the double beams because it has distinctive engineering properties such as resistance to stresses and high impacts on external surfaces, with resistance to bending stresses and buckling due to the elastic conduction layer while having a very important property of lightweight. Which made the researchers make their best efforts to analyze them in terms of dynamic loads and resistance to vibrations, especially for asymmetric types. Under arbitrary boundary conditions, Kim et al. [1] examined the free vibration of an elastically linked double-beam structure linked by an elastic layer with a homogeneous elastic stiffness. The vibration of the structure is modeled using Timoshenko theory, which considers the effects of shear deformation as well as rotational inertia. The vibration of linked double-beam with generalized elastic boundary conditions was investigated using the Haar wavelet discretization method. Hao et al. [2] used a modified Fourier–Ritz technique to analyze the vibration of a linked double beam with random boundary conditions and arbitrary fundamental parameters of beams. The displacement components were stated as Fourier cosine series with auxiliary polynomial functions. Hammed et al. [3] examined the dynamical responses of a double Euler-Bernoulli beam system under the influence of a moving distributed force, which is elastically coupled by a two - parameter Pasternak constructional work. The fourth order partial differential equations describing the beam motion were transformed into second order ordinary differential equations using the Finite Fourier sine transformation. Using the differential transformation approach, the dynamic response of the beams was estimated. Yang et al. [4] explored analytically the double-beam system, which consists of two generic beams with an assortment of symmetric boundary conditions and found the double beam mode shapes are similar to those of a single at identical boundary conditions and the amplitude of its for a double-beam system is doubled that of a single beam. He and Feng [5] developed a formula for the dynamic response of an elastically coupled multiple beam system under a moving oscillator using the finite sine-Fourier inverse transform. Stojanovi'c et al. [6] studied a universal approach for determining the buckling loads and natural frequencies for a collection of beam systems subjected to a compressive axial stress. The dynamical behavior of multi-layered microbeam systems in the presence of a moving mass was studied by Khaniki and Hashemi [7]. An analytical solution has been discovered for double- and three-layered microbridge systems utilizing the Laplace transform. A state space technique has also been employed for higher-layered microbridge systems. Abu-Hilal [8] discussed the dynamic behavior of a double-beam system passes by a moving load. The two simply supported beams are parallel, identical to one another, and joined by a viscoelastic layer that runs the length of the beam. Both beams' dynamic deflections are expressed in analytical closed forms. Atiyah and Abdulsahib [9] investigated the effect of four geometric and material characteristics on the vibration of twin beams. The qualities of the intermediate layer are mass density, thickness, and modulus of elasticity of the two beams. The frequencies of the twin beams were computed using the Bernoulli-Euler beam. Mirzabeigy and Madoliat [10] examined the influence of a nonlinear Winkler inner layer on small-amplitude free vibration. The well-known frequency solutions for double-beam systems were used, and it was discovered that the elastic inner layer had the greatest influence on the fundamental frequency, using the first mode of vibration. De Rosa and Lippiello [11] used the differential quadrature method to investigate the vibration of double beams linked by a Winkler-type elastic layer. Vertical translation and rotation elastic restrictions were applied to the ends of the double-beam. Abdulsahib and Atiyah [12] studied the effect of non-linear elasticity on the frequency of sandwich beams under arbitrary boundary conditions. The impact of the inner layer's non-linearity stiffness on those frequencies was calculated using the energy balancing approach. Most of the previous studies focused on the investigation of vibrations of symmetric beams and did not pay much attention to the effect of properties of connecting layers between the beams on the vibration characteristics. The behavior of the higher and lower natural frequencies of the asymmetric doubled beams will be studied under different boundary conditions with the influence of a number of properties, such as the difference in thickness of the two beams, their mass densities, their elasticity modulus, the properties of the connected layer between them, or the length of the two beams. To validate the present results, a comparison is achieved with previous results. The influence of material properties of connecting layer on the vibration asymmetric double beam is examined.

2. THEORETICAL WORK

Figure 1 shows the asymmetric double beam of different properties (ρ, E, b, and h). An elastic layer between them having the elastic stiffness (K_e) connects these beams. The Bernoulli-Euler beam theory for vibrations is utilized to relate the equations of motion [12, 13]:

$$\frac{\partial^2}{\partial x^2}\left(E_1 I_1 \frac{\partial^2 Y_1}{\partial x^2}\right) + K_e(Y_1 - Y_2) + \rho_1 A_1 \frac{\partial^2 Y_1}{\partial t^2} = 0 \tag{1}$$

$$\frac{\partial^2}{\partial x^2}\left(E_2 I_2 \frac{\partial^2 Y_2}{\partial x^2}\right) - K_e(Y_1 - Y_2) + \rho_2 A_2 \frac{\partial^2 Y}{\partial t^2} = 0 \tag{2}$$

where, A_1, A_2, ρ_1, ρ_2, E_1, E_2, I_1, I_2, Y_1 and Y_2 are the cross-sectional area, mass density, modulus of elasticity, moment of area and the deflection for first and second beam, respectively.

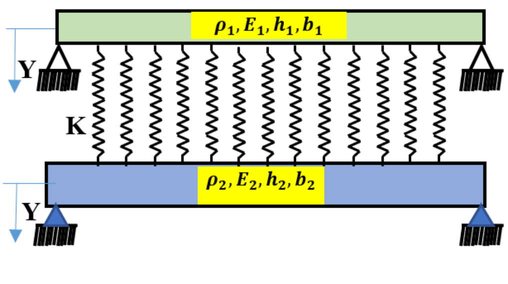

Figure 1. Asymmetric double beam

Assuming the time-harmonic motion as follow [14]:

$$W_i(x,t) = \sum_{n=1}^{\infty} x_n(x) \cdot T_{ni}(t), i = 1,2 \tag{3}$$

where, [15],

$$\zeta = \frac{x}{L} \tag{4}$$

$$y_n(\zeta) = cosh(\Omega_n\zeta) - cos(\Omega_n\zeta) - \sigma_n[sinh(\Omega_n\zeta) - sin(\Omega_n\zeta)], \quad \Omega_n = \frac{\pi(2n+1)}{2}, \quad n = 1,2,3,\ldots, \ \sigma_n \cong 1 \text{ For Clamped beams [15]} \tag{5}$$

$$y_n(\zeta) = sin(\Omega_n\zeta), \quad \Omega_n = n\pi, \quad n = 1,2,3,\ldots \text{ For Pinned beams [15]} \tag{6}$$

$$y_n(\zeta) = cosh(\Omega_n\zeta) + cos(\Omega_n\zeta) - \sigma_n[sinh(\Omega_n\zeta) + sin(\Omega_n\zeta)], \quad \Omega_n = \frac{\pi(2n+1)}{2}, \quad n = 1,2,3,\ldots, \sigma_n \cong 1 \text{ For Free beams [15]} \tag{7}$$

$$y_n(\zeta) = cosh(\Omega_n\zeta) - cos(\Omega_n\zeta) - \sigma_n[sinh(\Omega_n\zeta) - sin(\Omega_n\zeta)], \quad \Omega_n = \frac{\pi(2n-1)}{2}, \quad n = 1,2,3,\ldots, \ \sigma_n \cong 1 \text{ For Cantilever beam [15]} \tag{8}$$

$$y_n(\zeta) = cosh(\Omega_n\zeta) - cos(\Omega_n\zeta) - \sigma_n[sinh(\Omega_n\zeta) - sin(\Omega_n\zeta)], \quad \Omega_n = \frac{\pi(4n+1)}{4}, \quad n = 1,2,3,\ldots, \ \sigma_n \cong 1 \text{ For Clamped-Pinned beams [15]} \tag{9}$$

The time functions are assumed as follow [14]:

$$T_{ni} = D_i e^{jw_n t}, i = 1,2 \tag{10}$$

The double beams have the following Boundary conditions:

Cantilever:

$$Y_i(0,t) = Y_i'(0,t) = Y_i''(1,t) = Y_i'''(1,t) = 0, \ i = 1,2 \tag{11}$$

Clamped:

$$Y_i(0,t) = Y_i'(0,t) = Y_i(1,t) = Y_i'(1,t) = 0, \ i = 1,2 \tag{12}$$

Simply Supported:

$$Y_i(0,t) = Y_i''(0,t) = Y_i(1,t) = Y_i''(1,t) = 0, \ i = 1,2 \tag{13}$$

Free

$$Y_i''(0,t) = Y'''(0,t) = Y_i''(1,t) = Y_i'''(1,t) = 0, \ i = 1,2 \tag{14}$$

Substituting the above expressions in to Eqns. (1) and (2) will get:

$$(E_1 I_1 \Omega_n^4 + K_e - \rho_1 A_1 \omega_n^2) D_1 - K_e D_2 = 0 \tag{15}$$

$$(E_2 I_2 \Omega_n^4 + K_e - \rho_2 A_2 \omega_n^2) D_2 - K_e D_1 = 0 \tag{16}$$

For simplifying the solution of Eqns. (15) and (16), the following parameters are assumed:

$$\Omega_{n1} = K_e + \Omega_n^4 E_1 I_1 \quad (17)$$

$$\Omega_{n2} = K_e + \Omega_n^4 E_2 I_2 \quad (18)$$

$$A = (\rho_2 A_2) \cdot \Omega_{n1} + (\rho_1 A_1) \cdot \Omega_{n2} \quad (19)$$

$$B = (\rho_2 A_2)^2 \cdot \Omega_{n1}^2 + (\rho_1 A_1)^2 \cdot \Omega_{n2}^2 \quad (20)$$

$$C = 2(\rho_1 A_1) \cdot (\rho_2 A_2) \cdot \left[2{K_e}^2 - \Omega_{n1} \cdot \Omega_{n2}\right] \quad (21)$$

The lower and higher (synchronous and asynchronous) natural frequencies for asymmetric double beams at arbitrary boundary conditions can get as follow:

$$\omega_{1n} = \sqrt{\frac{A - \sqrt{B + C}}{2(\rho_1 A_1) \cdot (\rho_2 A_2)}} \quad (22)$$

$$\omega_{2n} = \sqrt{\frac{A + \sqrt{B + C}}{2(\rho_1 A_1) \cdot (\rho_2 A_2)}} \quad (23)$$

3. RESULTS AND DISCUSSION

In order to validate the accuracy of Eqns. (22) and (23), comparison tests were made between the results of the present work and other references. Those comparisons are shown in Tables 1 and 2. An excellent identification between the present work and the results of the references [2, 3] can be observed.

Table 1. The natural frequencies of double beam at PP and CP boundary conditions

$$\rho_1 A_1 = \frac{1}{2}\rho_2 A_2 = 300\frac{kg}{m}, E_1 I_1 = \frac{1}{2}E_2 I_2 = 6 \times 10^6, L = 8\ m, K_e = 2.5 \times 10^5 \frac{N}{m^2}$$

No. of Mode	Pinned-Pinned		Clamped-Pinned	
	Present	Ref. [2]	Present	Ref. [2]
1	21.8090	21.8090	34.0697	34.0697
2	41.5407	41.5407	49.0993	49.0994
3	87.2358	87.2358	110.4076	110.4080
4	94.1280	94.1281	115.9303	115.9300
5	196.2806	196.2810	230.3563	230.3560
6	199.4394	199.4390	233.0537	233.0540
7	348.9432	348.9430	393.9221	393.9220
8	350.7298	350.7300	395.5055	395.5060

Table 2. The natural frequencies of double beam at P-P, C-C and C-F boundary conditions

$$\rho_1 A_1 = \frac{1}{2}\rho_2 A_2 = 100\frac{kg}{m}, E_1 I_1 = \frac{1}{2}E_2 I_2 = 4 \times 10^6, L = 10\ m, K_e = 1 \times 10^5 \frac{N}{m^2}$$

No. of mode	Pinned-Pinned		Clamped-Clamped		Clamped-Free	
	Present	Ref. [3]	Present	Ref. [3]	Present	Ref. [3]
1	19.7392	19.7392	44.7466	44.7451	7.0320	7.0320
2	43.4699	43.4699	59.1799	59.1790	39.3630	39.3630
3	78.9568	78.9564	123.3457	123.3403	44.0690	44.0690
4	87.9442	87.9439	129.2832	129.2791	58.6692	58.6688
5	177.6529	177.6508	241.8068	241.7925	123.3943	123.3918
6	181.8256	181.8239	244.8888	244.8888	129.3296	129.3297

The change of the natural frequencies with the difference in the stiffness values of the connected layer between the two beams can be noted in Figure 2 and Table 3. When the stiffness values of the elastic layer are increased from 100 kN/m^2 to 2,100 kN/m^2, an increase in the values of high frequencies (asynchronous) is observed up to 125% for PP beams, 40% for CC beams, 280% for CF beams, and 170% for CP beams. The different hardness values of the connected layer have no effect on the values of the lower natural frequencies of double beams, as these frequencies maintain their values despite the increase and decrease in the stiffness values of the connected layer. From the above, it can be concluded that the greatest effect of the stiffness layer is on the values of the higher natural frequencies of CF beams and the least effect is on the PP beams. Generally, the increase in the stiffness of the elastic connected layer leads to an increase in the values of the high frequencies of those beams and does not affect the values of the lower natural frequencies of them.

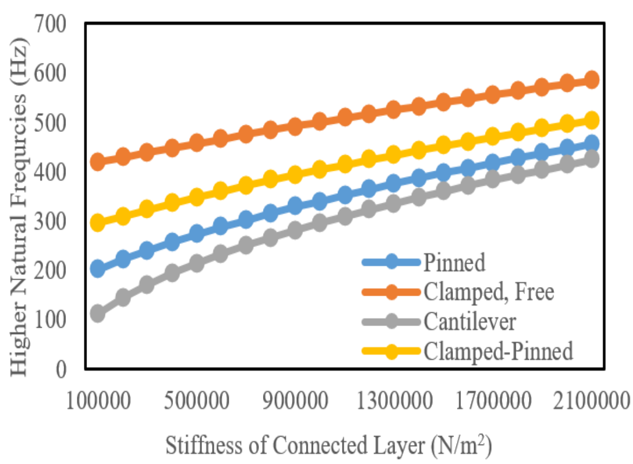

Figure 2. Higher natural frequencies vs. stiffness of connected layer

Table 3. Higher natural frequencies (Hz) vs. stiffness of connected layer (N/m²)

$$E_1 = E_2 = 10\ Gpa., h_1 = h_2 = 20\ mm, b_1 = b_2 = 40\ cm,$$
$$L = 10\ m, \rho_1 = \rho_2 = 3000\ kg/m^3$$

K_e	PP	CC, FF	CF	CP
100000	201.998	418.555	111.598	295.928
200000	221.667	428.394	144.179	309.688
300000	239.728	438.012	170.648	322.863
400000	256.521	447.424	193.531	335.520
500000	272.280	456.641	213.980	347.717
600000	287.175	465.677	232.639	359.500
700000	301.335	474.540	249.908	370.909
800000	314.859	483.240	266.059	381.978
900000	327.826	491.787	281.284	392.734
1000000	340.298	500.188	295.726	403.204
1100000	352.330	508.450	309.495	413.409
1200000	363.964	516.580	322.678	423.368
1300000	375.237	524.584	335.342	433.098
1400000	386.182	532.467	347.545	442.614
1500000	396.825	540.236	359.334	451.929
1600000	407.189	547.894	370.748	461.057
1700000	417.297	555.447	381.821	470.007
1800000	427.165	562.898	392.582	478.790
1900000	436.810	570.253	403.056	487.415
2000000	446.247	577.513	413.264	495.890
2100000	455.488	584.683	423.227	504.222

Table 4. Higher natural frequencies (Hz) vs. thickness of upper beam (m)

$$E_1 = E_2 = 10\ Gpa., h_2 = 20\ mm, b_1 = b_2 = 40\ cm,$$
$$L = 10\ m, \rho_1 = \rho_2 = 3000\frac{kg}{m^3}, K_e = 100\frac{kN}{m^2}$$

h_1	PP	CC	CF	CP
0.010	212.061	423.503	128.922	302.887
0.015	205.407	420.211	117.656	298.266
0.020	201.998	418.555	111.598	295.928
0.025	199.924	417.558	107.800	294.517
0.030	198.530	416.892	105.191	293.572
0.035	197.528	416.416	103.288	292.896
0.040	196.773	416.058	101.837	292.387
0.045	196.184	415.780	100.694	291.991
0.050	195.712	415.557	99.770	291.674
0.055	195.324	415.375	99.008	291.414
0.060	195.001	415.223	98.368	291.197
0.065	194.727	415.094	97.824	291.014
0.070	194.491	414.984	97.355	290.856
0.075	194.287	414.888	96.946	290.720
0.080	194.108	414.805	96.587	290.600
0.085	193.950	414.731	96.270	290.495
0.090	193.810	414.665	95.986	290.401
0.095	193.684	414.606	95.732	290.317
0.100	193.571	414.554	95.503	290.242
0.105	193.468	414.506	95.295	290.173
0.110	193.375	414.462	95.105	290.111

Figure 3 and Table 4 manifest the relationship between the increase in the upper beam thickness and the change in the values of higher natural frequencies. When the thickness of the beam is increased by about 20 times, a decrease in higher frequencies is noted about 9% for PP beams, 9% for CC beams, 9% for FF beams, 27% for CF beams, and about 4% for CP beams. The values of the higher natural frequencies vary in the same proportions when the ratio of the thickness of the lower layer of the beam changes for all types of beams. It was also found that the values of lower natural frequencies are not affected by the change in the thickness of the upper or lower layer of the beams. As a result, it can be concluded that the greatest effect of changing the thickness of one of the upper or lower beams, or the thickness of one of them to the other is for CF beams and the least effect is for CP beams. In general, the influence of thickness is small on the frequencies compared to the rest of the factors studied in this research.

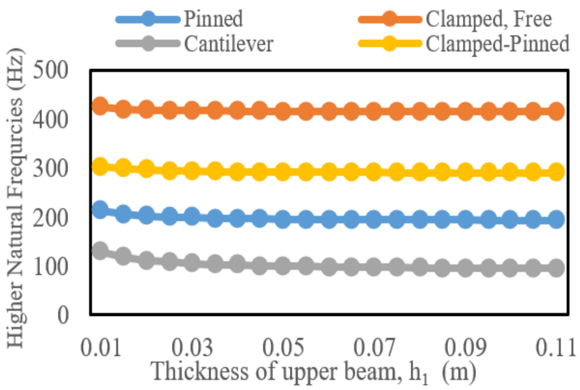

Figure 3. Higher natural frequencies vs. thickness of upper beam

Figure 4 and Table 5 represent the behavior of low frequencies with the change in beam length. When the length increases about 50%, the lower frequencies will decrease approximately 55% for all types of beams. While Figure 5 evinces the relationship of higher frequencies with the variation in length of beam. In this figure, it is observed that when the length of the beam is increased by 50%, the higher frequencies decrease 48% for PP beams, 44% for CC beams, 25% for CF beams, and about 52% for CP beams. As a result, the length of the beam affects the higher and lower frequencies in high and close proportions for almost all types of beams, and the least effect is only on the higher frequencies of CF beams.

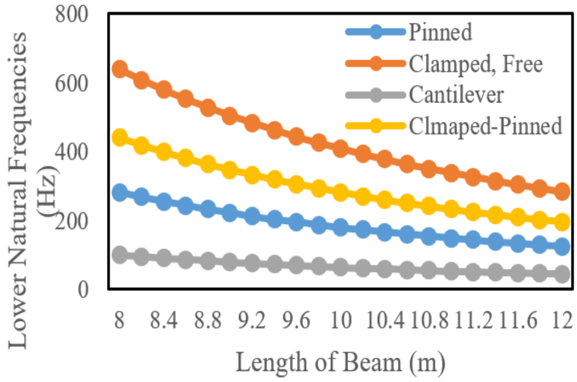

Figure 4. Lower natural frequencies vs. length of beam

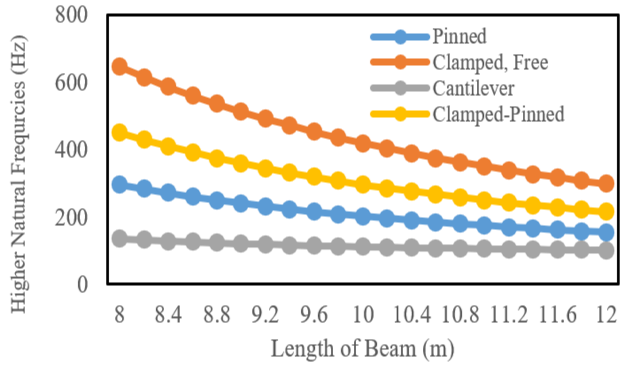

Figure 5. Higher natural frequencies vs. length of beam

Table 5. Natural frequencies (Hz) vs. length of beam (m)

$$E_1 = E_2 = 10\ Gpa., h_1 = h_2 = 20\ mm, b_1 = b_2 = 40\ cm,$$
$$\rho_1 = \rho_2 = 3000\frac{kg}{m^3}, K_e = 100\frac{kN}{m^2}$$

L (m)	Lower Frequency				Higher Frequency			
	PP	CC	CF	CP	PP	CC	CF	CP
8.0	281.552	638.248	100.302	439.838	295.982	644.743	135.624	449.212
8.2	267.986	607.493	95.469	418.644	283.107	614.314	132.090	428.482
8.4	255.376	578.909	90.977	398.946	271.202	586.063	128.880	409.257
8.6	243.636	552.296	86.795	380.606	260.177	559.790	125.963	391.401
8.8	232.688	527.477	82.894	363.503	249.954	535.318	123.308	374.790
9.0	222.461	504.294	79.251	347.526	240.463	512.490	120.889	359.316
9.2	212.894	482.607	75.843	332.581	231.640	491.165	118.682	344.882
9.4	203.931	462.289	72.650	318.579	223.431	471.216	116.668	331.400
9.6	195.523	443.228	69.654	305.443	215.783	452.531	114.826	318.793
9.8	187.623	425.321	66.840	293.103	208.653	435.007	113.141	306.990
10.0	180.194	408.478	64.193	281.496	201.998	418.555	111.598	295.928
10.2	173.196	392.617	61.701	270.566	195.781	403.090	110.183	285.550
10.4	166.599	377.661	59.350	260.259	189.970	388.538	108.884	275.805
10.6	160.372	363.544	57.132	250.531	184.533	374.830	107.691	266.644
10.8	154.487	350.204	55.035	241.338	179.442	361.907	106.594	258.026
11.0	148.920	337.586	53.052	232.642	174.673	349.710	105.584	249.911
11.2	143.649	325.637	51.175	224.407	170.201	338.190	104.653	242.264
11.4	138.653	314.311	49.395	216.602	166.006	327.299	103.794	235.053
11.6	133.913	303.566	47.706	209.198	162.068	316.995	103.001	228.248
11.8	129.412	293.363	46.103	202.166	158.369	307.238	102.268	221.821
12.0	125.134	283.666	44.579	195.484	154.893	297.992	101.590	215.748

The effect of varying the modulus of elasticity of the upper beam on the values of the lower frequencies of double beams is depicted in Figure 6 and Table 6. When the values of the elastic modulus are increased from 10 GPa. to 30 GPa., the lower frequencies increase about 6% for PP beams, 1% for CC beams, 26% for CF beams, and about 1% For CP beams. Also, from Table 6 and Figure 7, when the modulus of elasticity increased from 10 GPa. to 30 GPa., the higher natural frequencies increase about 58% for PP beams, 70% for CC beams, 21% for CF beams, and about 66% for CP beams. As a result, the influence of the change of the modulus elasticity is relatively small on the lower natural frequencies of all types of beams except for CF beams, and its influence is relatively large on higher natural frequencies of the most types of beams and comparatively less on the CF beams. The same effect was seen for the variation of the modulus of elasticity of the lower or upper beam on the natural frequencies of the beams in the same ratios.

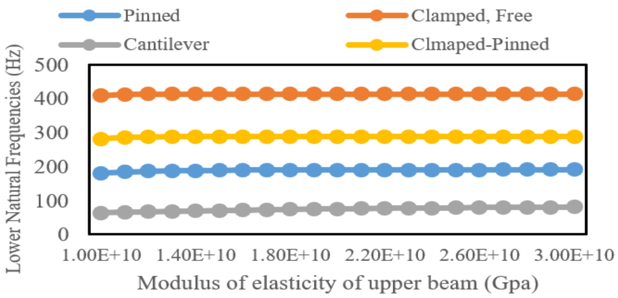

Figure 6. Lower natural frequencies vs. modulus of elasticity of upper beam

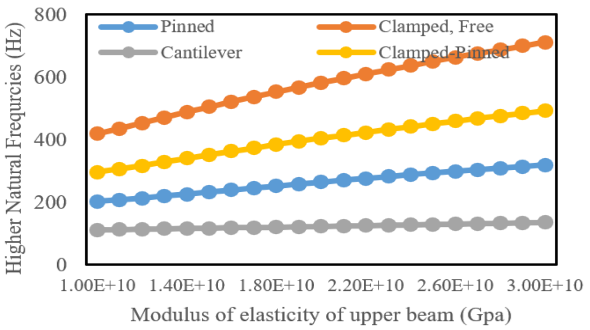

Figure 7. Higher natural frequencies vs. modulus of elasticity of upper beam

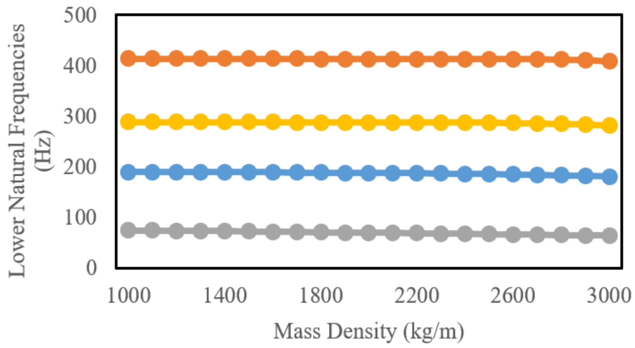

Figure 8. Lower natural frequencies vs. mass density of upper beam

Figure 8 and Table 7 elucidate the relationship between the changes in the mass density of the upper layer of the beam with the lower natural frequencies of the double beams. When the mass density increases from 1,000 kg/m^3 to 3,000 kg/m^3, the lower frequencies decrease about 5% for PP beams, 1% for CC beams, 14%, and about 2% for CP beams. The behavior of higher natural frequencies is displayed in Figure 9 and Table 7. In addition, while the mass density increases from 1,000 kg/m^3 to 3,000 kg/m^3, the higher frequencies decrease about 40% for PP beams, 42% for CC beams, 33% for CF beams, and about 40% for CP beams. Therefore, the influence of changing the values of mass density is comparatively small on the low natural frequencies of all types of beams except for CF beams, and its comparatively is relatively large on the higher natural frequencies of all types of beams and relatively less on the CF beams. Furthermore, there is no difference between the changes in the mass density of the upper or lower layer; both have the same effect and percentage change of frequencies.

Table 6. Natural frequencies (Hz) vs. modulus elasticity of beam (m)

$$E_2 = 10\ Gpa., h_1 = h_2 = 20\ mm, b_1 = b_2 = 40\ cm,$$
$$L = 10\ m, \qquad \rho_1 = \rho_2 = 3000\frac{kg}{m^3}, K_e = 100\frac{kN}{m^2}$$

E$_1$ (Gpa.)	**Lower Frequency**				**Higher Frequency**			
	PP	**CC**	**CF**	**CP**	**PP**	**CC**	**CF**	**CP**
1.0E+10	180.194	408.478	64.193	281.496	201.998	418.555	111.598	295.928
1.1E+10	183.815	412.358	65.740	285.691	206.716	434.384	112.540	305.153
1.2E+10	186.013	412.927	67.175	287.016	212.522	452.664	113.519	316.676
1.3E+10	187.343	413.131	68.508	287.572	218.898	470.555	114.533	328.463
1.4E+10	188.188	413.234	69.744	287.869	225.491	487.875	115.581	340.061
1.5E+10	188.758	413.296	70.891	288.051	232.117	504.635	116.661	351.369
1.6E+10	189.163	413.338	71.955	288.174	238.688	520.872	117.770	362.372
1.7E+10	189.464	413.368	72.942	288.263	245.163	536.627	118.907	373.077
1.8E+10	189.696	413.390	73.857	288.330	251.524	551.938	120.069	383.499
1.9E+10	189.879	413.408	74.707	288.382	257.763	566.839	121.254	393.657
2.0E+10	190.027	413.422	75.496	288.424	263.879	581.360	122.459	403.566
2.1E+10	190.149	413.433	76.228	288.458	269.875	595.530	123.681	413.243
2.2E+10	190.252	413.442	76.908	288.487	275.755	609.371	124.920	422.703
2.3E+10	190.339	413.451	77.541	288.511	281.522	622.906	126.172	431.958
2.4E+10	190.414	413.457	78.130	288.532	287.181	636.154	127.435	441.021
2.5E+10	190.480	413.463	78.678	288.550	292.737	649.132	128.708	449.904
2.6E+10	190.537	413.469	79.190	288.566	298.195	661.856	129.989	458.615
2.7E+10	190.588	413.473	79.667	288.580	303.558	674.341	131.276	467.166
2.8E+10	190.633	413.477	80.113	288.592	308.832	686.599	132.568	475.564
2.9E+10	190.673	413.481	80.530	288.603	314.021	698.642	133.863	483.817
3.0E+10	190.710	413.484	80.920	288.613	319.127	710.481	135.161	491.932

Table 7. Natural frequencies (Hz) vs. mass density of beam (kg/m^3)

$$E_1 = E_2 = 10\ Gpa., h_1 = h_2 = 20\ mm, b_1 = b_2 = 40\ cm,$$
$$L = 10\ m, \qquad \rho_2 = 3000\frac{kg}{m^3}, K_e = 100\frac{kN}{m^2}$$

ρ_1 (kg/m^3)	Lower Frequency				Higher Frequency			
	PP	CC	CF	CP	PP	CC	CF	CP
1000.0	189.558	413.363	74.737	288.263	332.586	716.391	166.025	500.532
1100.0	189.464	413.354	74.271	288.234	317.265	683.068	159.290	477.285
1200.0	189.361	413.343	73.795	288.203	303.924	654.005	153.494	457.016
1300.0	189.246	413.331	73.307	288.168	292.177	628.366	148.453	439.140
1400.0	189.119	413.317	72.809	288.129	281.739	605.528	144.032	423.223
1500.0	188.976	413.302	72.302	288.084	272.392	585.017	140.124	408.935
1600.0	188.815	413.285	71.786	288.034	263.967	566.465	136.650	396.019
1700.0	188.633	413.264	71.262	287.976	256.332	549.578	133.544	384.273
1800.0	188.426	413.241	70.731	287.909	249.385	534.124	130.755	373.533
1900.0	188.187	413.213	70.195	287.829	243.041	519.913	128.240	363.671
2000.0	187.911	413.180	69.654	287.735	237.235	506.789	125.964	354.578
2100.0	187.590	413.140	69.109	287.621	231.914	494.624	123.898	346.170
2200.0	187.213	413.090	68.561	287.481	227.039	483.311	122.016	338.376
2300.0	186.766	413.025	68.012	287.305	222.579	472.761	120.298	331.141
2400.0	186.235	412.940	67.462	287.078	218.514	462.902	118.725	324.426
2500.0	185.600	412.823	66.912	286.775	214.832	453.679	117.282	318.207
2600.0	184.839	412.651	66.363	286.356	211.527	445.054	115.956	312.483
2700.0	183.932	412.378	65.816	285.755	208.596	437.024	114.734	307.287
2800.0	182.860	411.888	65.272	284.858	206.038	429.658	113.607	302.700
2900.0	181.613	410.862	64.730	283.496	203.845	423.241	112.564	298.865
3000.0	180.194	408.478	64.193	281.496	201.998	418.555	111.598	295.928

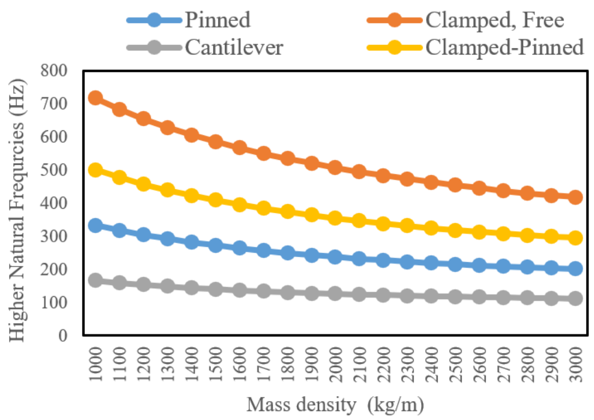

Figure 9. Higher natural frequencies vs. mass density of upper beam

4. CONCLUSIONS

An excellent agreement was found between the numerical results obtained from the current proposed mathematical model and the results of a number of previous literatures. The increase in the stiffness of the elastic connected layer leads to an increase in the values of the high frequencies of those beams and does not affect the values of their lower natural frequencies. The greatest influence of changing the thickness of one of the upper or lower beams, or the thickness of one of them to the other, is for CF beams and the least effect is for CP beams. In general, the effect of thickness is small on the frequencies compared to the rest of the factors studied in this research. The length of the beam affects the higher and lower frequencies in high and close proportions for almost all types of beams, and the least influence is only on the higher frequencies of CF beams. The effect of the change of the modulus elasticity is relatively small on the lower natural frequencies of all types of beams except for CF beams, and its effect is comparatively large on the higher natural frequencies of the most types of beams and relatively less on the CF beams. The same influence was observed for the variation of the modulus of elasticity of the lower or upper beam on the natural frequencies of the beams in the same ratios.

The effect of changing the values of mass density is comparatively small on the low natural frequencies of all types of beams except for CF beams, and its influence is relatively large on the higher natural frequencies of all types of beams and comparatively less on the CF beams. In addition, there is no difference between the changes in the mass density of the upper or lower layer; both have the same effect and percentage change of frequencies.

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NOMENCLATURE

A_1	Cross-sectional area of upper beam
A_2	Cross-sectional area of lower beam
E_1	Modulus of elasticity of upper beam
E_2	Modulus of elasticity of lower beam
I_1	Second moment of area of upper beam
I_2	Second moment of area of upper beam
K	Modulus of elasticity of elastic layer
L	Length of the beams
$W1$	Transverse deflection of upper beam
$W1$	Transverse deflection of lower beam

Greek symbols

ρ_1	Mass density of upper beam
ρ_2	Mass density of lower beam
ω_{1n}	Lower (synchronous) natural frequency for i^{th} mode of the two beams
ω_{2n}	Higher (asynchronous) natural frequency for i^{th} mode of the two beams
$\Omega 1$	Dimensionless natural frequency for i^{th} mode