

Image Denoising Based on Implementing Threshold Techniques in Multi-Resolution Wavelet Domain and Spatial Domain Filters



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ABSTRACT

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Nowadays, a digital image is often easily corrupted due to different forms of noise and complex processes resulting from the acquisition, compression, encoding, transportation, storage, retrieval, etc. All of these factors cause image quality to be distorted and visual information to be lost; in order to overcome this problem, Image denoising techniques are used widely to eliminate the various forms of noise that exist in the deteriorating image while keeping as many fine details and vital signal features as possible in the digital image. The wavelet denoising method aims to remove unwanted noise from a noisy image while preserving its vital features as a result of its ability to divide the degraded image into four sub-bands (sub-images) and operate at the frequencies of each one separately, where acquiring the original image content is vital to achieving reliable performance. This work introduces and implements a new hybrid system to the image denoising caused by Additive White Gaussian Noise (AWGN). The hybrid system is achieved using a combination of Median and Wiener filters as spatial domain filters with two-dimensional stationary and discrete wavelet transform (2D-SWT, 2D-DWT) as a multi-resolution analysis technique by applying 131 wfilters from the wavelet families (haar, db, sym, coif, bior, rbio, dmey, fk) in image processing at three levels of decomposition based on Hard, SureShrink, Bayesian, and Penalized threshold techniques on both high and low frequencies to distinguish and remove noise from affected pixel units and obtain improved results of the noise reduction process to the noisy image. Then, the multi-level 2D inverse wavelet transform (2D-IWT) eliminates noise and completes the image reconstruction by the hybrid denoising technique. Finally, the performance of the hybrid system has been estimated and measured by the peak signal-to-noise ratio (PSNR) value as an image quality metric. Experimental evaluation findings that the results of the proposed approach improved by about 17.5% by comparing them to the results of the related work, as well as enhancement the essence of image quality in terms of better noise reduction and edge preservation instead of using a multi-resolution WT domain or spatial domain filters separately.

1. INTRODUCTION

The digital revolution has changed people's lives so that all aspects of our lives today have been affected by technology. One aspect of this revolution is the rapid technology development of digital images, which plays a critical role in some of the extensively utilized sectors, including the medical field, astronomy science, color/video processing, remote sensing, pattern recognition, and many other applications. However, the problem is obtaining images from sensors contaminated with what we refer to as "noise." The term "noise" in the digital image refers to damage that happens within the image or unwanted information that damages the quality contents of the original image and thus leads to the loss of its vital properties. It occurs as a result of acquisition and transmission processes or as a result of noise sources near image capturing devices, sensor misfocus, defective memory location, optical aberrations, atmospheric distortions, and motion of objects in the scene result in distorted image quality. A distorted image occurs due to exposure to various types of noise that can affect and damage the digital image, such as salt

and pepper, Gaussian, Speckle, Poisson, and others noise [1, 2].

AWGN noise is one of several types of noise that can occur as a result of poor image acquisition quality, loud surroundings, or transmission mediums. In this research, AWGN noise has been added to all input images. It is essential to eliminate AWGN noise from the noisy image till it approximates the input image. Therefore, noise is one of the major sources of information in several applications and one of the significant constraints in image accuracy [3]. Denoising an image seeks to eliminate or reduce the amount of noise and attempt to retrieve valuable information from degraded and blurred images. As a result, the primary difficulty is to extract as much actual data as possible from the noisy image. Thus, denoising is a critical and necessary approach in digital image analysis and the appropriate first action to actually take before beginning any type of investigation, including image acquisition and understanding, pre-processing, classification, textures analysis, segmentation, features extraction, etc. [4, 5]. Researchers are continuously focusing on it to enhance visual assurance and the success of high-level vision tasks [6].

Additive random noise can be easily eliminated without difficulty when applying threshold techniques. The denoising of digital images affected by AWGN noise with WT techniques is a very effective procedure due to its ability to capture the energy of a signal in a minimal number of energy transform values. Additionally, its ability to access time and frequency information concurrently makes it a powerful tool for signal processing, images denoising, and other applications in both continuous and discrete wavelet transform, which may result in small and less significant coefficients. The WT technique is advantageous for energy compression since it generates small and large coefficients as a result of noise and key image elements. Thresholding the low coefficients is doable without compromising the image's significant features [7].

Wavelet-based analysis techniques such as (DWT and SWT) are among the most frequently used denoising procedures; both focus on signal decomposition and band splitting into a few frequency bands. Thus, the procedure isolates the noisy image into various sub-band images, and then it associates the high-frequency wavelet coefficients as horizontal (H), vertical (V), and diagonal (D). DWT is commonly employed to eliminate noise from degraded images due to the ability to sparse representation for the primary image, which means that it has numerous coefficients close by zero value [8]. In the usual form, each coefficient (H, V, D) indicates the threshold by contrasting it with an estimate, referred to as a threshold value. Numerous threshold strategies have been investigated and used by researchers to ascertain the value of a threshold [9, 10].

In contrast, SWT decomposes the input signal into many levels using high-pass filters and low-pass with wavelet coefficients of equal length at each level. SWT implements a tree-structured algorithm similar to DWT, but it achieves and returns a better approximation result than DWT because the output signal is not decimated (without down-sampling), which DWT cannot achieve. As a result, a multi-layer SWT applies to overcome the limitation of the conventional wavelet transform [7].

This work concludes to introduce and apply a hybrid system designed to denoise the noise from the affected pixel-units that exist in the degraded digital image that has been affected by AWGN noise in terms of noise reduction and improved visual quality of the image (degraded image) in comparison to previous works. To achieve that, the concept use of both Median and Wiener filters as spatial domain filters are combined with a working mechanism based on selecting the threshold technique in both 2D-SWT and 2D-DWT into WT domain as a multi-resolution analysis technique in image processing at three levels of decomposition to achieve improved results of the noise reduction process. The advantage of adopting the hybrid method system is retaining the exact details and preserving the vital features of denoised images concerning AWGN noise elimination as much as possible and structure preservation. Furthermore, it compares hybrid noise removal mechanisms and applies several hard and soft threshold techniques (Hard, Sure shrink, Bayes shrink, and Penalized) for all case scenarios and various percentages of noise levels by calculating the PSNR value.

2. RELATED WORK

Image denoising stands out as a critical practice in the image

processing field. The main goal is to approximate an original image by removing noise from an image with a contaminated version. Image noise can emanate from either extrinsic or intrinsic conditions, which are usually difficult to avoid. Due to this aspect, the image denoising practice is critical because it aids in image restoration and visual tracking. Other outcomes entail image registration, classification, and segmentation. The use of algorithms has proven to be effective in denoising images. However, challenges are still being experienced, thus paving the way for several research programs.

Ferzo and Mustafa (2020) noted that noise can end up corrupting or distorting images. As such, they recommended image denoising by using appropriate techniques. The suggested technique was using the Wiener filter, which was applied before and after employing 2D-DWT based on hard and soft threshold techniques to extract the noise from the image pixels. The use of 2D-DWT with Wiener filter helped to improve image quality by 17.5%, depending on the PNSR value. As a result, the proposed algorithm was more efficient than using the wavelet transform and Wiener filter separately [11].

Fan et al. (2019) acknowledged the critical role of wavelet transformation in denoising the deteriorating digital image affected by GWN noise. The wavelet transform was presented as a technique that assists in analyzing the localization of time-frequencies by using telescopic translation techniques to enhance the signal function on a multi-scale and then adjust it to the time-frequency. The experiment determined that the optimal technique was to threshold wavelet coefficients using either a soft or a hard threshold. The method was compared to the use of Wiener filtering, which was conducted in a wavelet domain. The comparison of both techniques affirmed that Wiener filtering was more powerful in image denoising. Thus, individuals and institutions seek to denoise images [12].

Qian, in 2018 upheld the use of an improved wavelet threshold approach in conjunction with the Median filter in denoising from the damaged image. The wavelet decomposing aspects of the image mixed with Gaussian noise were denoised by applying the improved threshold function on wavelet detail coefficients (high-frequency) of each level. The study outcomes indicated that the utilization of an improved threshold brought forth superior outcomes compared to both the hard and soft threshold in effectively eliminating multiple random noises (Gaussian noise and salt & pepper) based on measurement parameters (PSNR). In this regard, it is recommendable for one to rely on combining both wavelet improved threshold and Median filter. The two approaches have a high capacity for denoising images without any abnormalities due to their high adaptability and impact on more effective noise reduction [13].

Golilarz and Demirel (2017) introduced a new approach for denoising in the WT domain to preserve important details and vital features of the image. The method used relied on undecimated wavelet transform (UWT) depending on the soft threshold function to overcome the limitations of DWT. The approach used has effectively achieved positive results in terms of removing noise from the degraded image and so increasing the clarity and quality of the resulting image data. In terms of PSNR value, the findings showed that the UWT methodology outperformed Dong's adaptive method and that the analysis process was enhanced as a result. Accordingly, it should be adopted more in image denoising activities [14].

Ramadhan et al. (2017) recommended a new approach of extracting noise from the degraded image. The subject method was the use of the threshold-based wavelet domain denoising approach (2D-DWT) with the median filter. In the study, numerous types of wavelet filters were utilized together with the median filters. This approach has been undertaken with the core aim of achieving positive results in denoising images. After the study, the 2D-DWT proved to be a powerful image analysis tool, especially after the use of frequencies of sub-bands. The proposed method proved to be effective compared to only using median filter alone or 2D-DWT independently, depending on PSNR value. As a result, the combination of both techniques is necessary for activities that revolve around denoising images [15].

Sultan [16] proposed a reliable concept that can aid in the denoising of images. The subject approach entailed using hybrid denoising algorithms that combine Wiener filter as spatial domain and threshold-based discrete wavelet domain (2D-DWT) and framelet domain (FLT). Three algorithms were proposed, and the first one used a Wiener filter which is a 2-level 2D-DWT. The second one utilized a 2-level FLT, while the third one combined both Wiener filters with a 1-level 2D-DWT, then employed FLT on the low-frequency of the wavelet transform. The results affirmed that the first hybrid denoising algorithm was superior and performed better than the second and third approaches in terms of (PSNR, MSE) values in testing and extracting both Gaussian, salt and pepper noise from images. As a result, the proposed concept is effective and should be highly utilized in image denoising.

Ismael et al. [17] developed a novel denoising method to enhance the overall visual quality of images. The technique aims to apply the hard threshold value to remove additive Gaussian noise from corrupted images, which is performed via a two-stage analysis procedure utilizing 2D-DWT due to the ability that the wavelet transforms to divide an image into four sub-bands and perform independently on each sub-band frequency. The Robust Median Estimator was used to determine the noise ratio for the corrupt image that was affected by noise. As per the observed result, the proposed approach, which employs a variety of wavelet filter families, has improved both (MSE, PSNR) values for the denoised image.

Boyat and Joshi [18] practiced image denoising using the Poisson Gaussian noise model. They also employed the integration of the Wiener filter in a wavelet domain depending on the log energy distribution method to the denoised image corrupted by Poisson Gaussian noise. The two approaches have been explored based on iterative noise variance. The procedure was applied using MATLAB R2021a. The focus was on restoring the original image from a highly contaminated one. In this case, the noisy image was decomposed through the use of a discrete wavelet to create different sub-bands. The wavelet and the wiener filter ended up providing fair and robust noise detection. The method was useful in bringing forth a mathematical model or the combined approach in a high noise environment, and it also gave certain levels of accurate and valid results founded on PSNR value.

Naimi et al. [19] conducted a study that explored an effective approach to extracting noise from the image naturally deteriorated by the noise. The researchers upheld the wavelet transform techniques because they have proved to be useful in image denoising experiments. Precisely, wavelet transform has been utilized to recover infinite-dimensional items such as images, densities, curves, etc. These parameters make it useful,

especially in the healthcare environment where image analysis is a common task—the proposed denoising approach combines dual-tree complex wavelets (DTCWT) based on the hard and soft threshold values with the Wiener filter. The findings showed that images denoised using DTCWT and Wiener filter technique had more accuracy and smoothness based on measurement parameters (SSIM, PSNR, SSIM). Thus, the proposed method is effective and should be highly encouraged since it can enhance image analysis in healthcare facilities.

In this work, we have introduced and implemented a new hybrid system for the image denoising caused by AWGN through several cases are applied to cover the steps and the sequence for using the 2D-SWT and 2D-DWT as a multi-resolution analysis technique via employing 131 filters from wavelet families in image processing at three levels of decomposition based on selecting a threshold techniques (Hard, SureShrink, Bayesian, and Penalized) with the combination of Median and Wiener filters as spatial domain filters on both high and low frequencies to distinguish and remove noise from affected pixel units to eliminate noise and obtain an improved visual quality from the noisy image.

3. DIGITAL IMAGE DENOISING

A denoising technique, also known as reduction of image noise, is a form of image processing that aims to eliminate the various forms of unwanted noise available in degraded digital images. The noise is defined as unwanted information and can contaminate original images. Noise occurs due to various sources, including failure to acquire or error in the data transfer process, storage and retrieval processes, and other causes. Therefore, there is a degradation in the visual quality of the digital image. Thus, the purpose of denoising from images is to eliminate noise value and recover original data from disturbing data while preserving the edges and other detailed features [5].

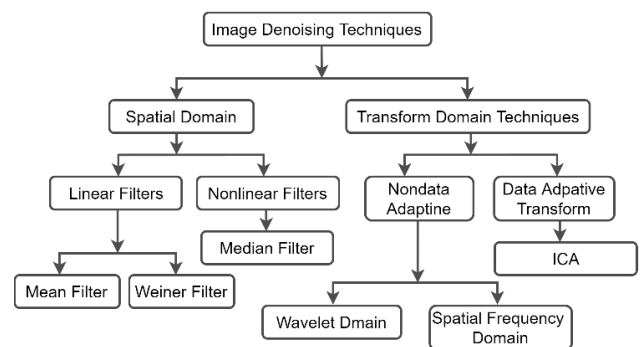


Figure 1. The hierarchical description of various denoising techniques [7, 20]

In image processing, there are several denoising methods. The choice of the denoising technique used in digital images depends on the type of noise present in degraded images and also levels of noise ratios. According to the noise source, it can be roughly divided into two categories: external/internal noise [7]. The fundamental methods of image denoising can be divided into two major approaches: spatial domain filtering and wavelet domain methods, as presented in Figure 1. The methods included in the first approach are at the same level of image and depend on the direct treatment of image elements

(pixels). In contrast, the second approach methods are based on modifying the coefficients of the sub-image to be processed [7, 20].

3.1 Noise

Noise is an important factor that interferes with human perception and understanding of information and hindering it. Since the noise itself is unpredictable, it can be considered as a random error that can be specified through probabilities and statistics. Therefore, image noise can be defined as a multidimensional random process that can be described by the probability distribution and the probability density function. According to the relation between the image and noise, image noise can be classified into two types: Additional noise, where the noise is not related to the original image and can be expressed in Eq. (1), and Multiplied noise, where the noise is related to the original image and can be expressed Eq. (2) [21, 22]:

$$f(x, y) = g(x, y) + n(x, y) \quad (1)$$

$$f(x, y) = g(x, y) * n(x, y) \quad (2)$$

where, $f(x,y)$ represents the contaminated image (noise image), $g(x,y)$ represents the original image, and $n(x,y)$ represents the noise value.

3.2 Spatial domain filters

The phrase spatial domain filtering can be termed an effective technique to modify and enhance a digital image that has been degraded due to unwanted noise. Filtering or rather spatial domain operation capitalizes on the proposed value of a current pixel for its ability to handle directly on pixels of an image (x, y) , which is usually affected by with itself pixels or with neighborhood pixels. In this case, filtering stands out as a neighborhood operation. The value of a subject pixel (x, y) in an output image is usually determined by applying algorithms to the values of the pixels in the neighborhood of corresponding input pixels [23, 24]. In a broader sense, a neighborhood pixel can be determined based on locations relative to that pixel. The filter response at this pixel is calculated via the filter mask at each pixel value (x, y) depending on a predetermined relationship (moving the filter mask from the pixel to another pixel value in an image).

The filtering process works by highlighting certain features and eliminating others that are unnecessary ones, with the goal of sharpening, smoothing, and enhancing the edges of an image. The spatial domain filtering is grouped into two types of filtering techniques: linear filter, in which the value of an output pixel is a linear combination of the values of the pixels in the input pixel's neighborhood. While nonlinear filter, its basic operation is to compute the median gray-level value in the neighborhood in which the filter is located. It also has greater performance than linear filtering in noise removal. The utilization of this concept has proven to be critical in image denoising. Thus, its use is recommended when one is interested in enhancing a quality image [25, 26].

3.3 Wavelet transform domain

In image denoising, WT is effective in enhancing the representation of signals that have a high degree of sparsity.

This principle facilitates the non-linear wavelet signal estimation, which is called wavelet denoising. During transmission, images usually become corrupted. Cleaning or enhancing their quality requires the use of appropriate interventions. WT becomes handy because it enhances image purity and clarity. Precisely, WT has wavelet coefficients, which are dominated by noise. To achieve transformation, it is necessary to use an inverse wavelet transform. This approach leads to a reconstruction, which is characterized by minimal noise. The utilization of thresholding techniques becomes crucial at this phase. The reason for this claim is that thresholding is critical in eliminating noise from images [7].

This effective concept works by shrinking coefficients. The shrinking aspect is what aids in the elimination of noise from images. Noise ends up getting reduced regardless of the extent of damage on digital images. Both soft and hard thresholds are utilized in WT to aid in the attainment of positive outcomes. Small thresholds can easily bring forth outcomes that are close to the input value. As such, a significant threshold is required since it triggers results that have little or no noise at all [27].

Wavelet transformation is described by several authors as a mathematical technique that analyzes (or synthesizes) a signal in the time domain using various editions of an enlarged and translated base function named the wavelet prototype or the mother wavelet [28]. Besides, the wavelet transformation is a method that subsumes time and frequency fields and is exactly known as a non-stationary signal's time-frequency representation. In wavelet transform, the signal in the time domain is disintegrated (decomposed) to create two separate parts by carrying it via a high pass filter and low pass filter (low pass (L) and high pass (H) versions) [29]. Wavelet transform can be demonstrated by Eq. (3) as follows [30].

$$X_{a,b} = \int_{-\infty}^{\infty} x(t) \psi_{a,b}(t) dt \quad (3)$$

where, (x) indicates the real signal, (ψ) is an arbitrary mother wavelet, (a) denotes the scale, and (b) is the translation (X is the processed signal).

3.3.1 Hard thresholding

The main goal of image denoising is to achieve results that have no noise. The elimination of noise is necessary since it enhances the digital image's quality. Hard thresholding becomes useful because it facilitates the elimination of noise at extreme levels. In this concept, the framelet coefficients, which are more significant than the preset threshold value gets retained. The remaining ones are then made zero. The aspect of making them zero is what aids in reducing noise, hence making the images better [31]. The hard threshold is effective because it differs from the soft threshold, which is ineffective in reducing coefficient concentration. Image quality enhancement is thus easy when utilizing a hard threshold. The core aspect that explains why a hard threshold is critical in image denoising is turning coefficient to zero. The hard threshold is represented by the following Eq. (4) [32, 33].

$$\widehat{W}_{ji} = \begin{cases} W_{ji} & \text{When } |W_{ji}| \geq T \\ 0 & \text{When } |W_{ji}| < T \end{cases} \quad (4)$$

where, (\widehat{W}_{ji}) means denoised wavelet coefficients, (W_{ji}) denotes noisy wavelet coefficients, (i) is the location of the detail component, (j) is decomposition level, and (T) is the representative of the threshold value.

3.3.2 Sure shrink

Sure Shrink stands out as the other thresholding technique which is effective in image denoising. It can be termed as a combination of both universal and SURE thresholds. It has a core goal of minimizing the Mean Square Error which helps in the identification of a threshold in every sub-band. This outcome is arrived at through the concept called sub-band adaptive thresholding. Sure shrink is also adaptive to smoothness. As such, it is appropriate to state that it supports abrupt changes in an image. It is these changes that lead to the enhancement of image quality after the elimination of noise [34]. It is represented by the following Eq. (5) [29].

$$MSE = \frac{1}{n^2} \sum_{x,y=1}^n (z(x,y) - s(x,y))^2 \quad (5)$$

where, $z(x,y)$ is the estimate of the signal, $s(x,y)$ is the original signal without noise, and (n) is the size of the signal.

Sure Shrink suppresses noise through thresholding the empirical wavelet coefficients [35]. The Sure Shrink threshold t^* is denoted as:

$$t^* = \min(t, \sigma \sqrt{2 \log n}) \quad (6)$$

where, (t) denotes the value that reduces SURE, (σ) is the noise variance computed from Eq. (6), and (n) is the size of the image.

Sure Shrink is following the rule of soft thresholding. The threshold employed is adaptive, i.e., a threshold level is set for each dyadic resolution level through reducing SURE for threshold estimates. It means that if the unknown function contains sudden changes or limits in the image, so does the reconstructed image [36].

3.3.3 Bayesian shrinkage

Bayesian shrinkage has been proposed by Chang et al. (2000) as an adaptive data-driven image denoising threshold by reducing the effects of sampling variation, which is based on the soft threshold estimate based on the hypothesis that the wavelet coefficients are the wavelet coefficients of the natural non-noise image are in the GGD [37, 38]. This method's objective is to predict a threshold estimate that reduces the possibility of Bayesian, which assists in digital image quality enhancement since all the unnecessary components get eliminated. It employs soft thresholding and is sub-band dependent, meaning thresholding is performed in the wavelet decomposition at each resolution band. The Bayes threshold is estimated as shown by Eq. (7) [39].

$$\lambda_{Bayes} = \hat{\sigma}_n^2 / \hat{\sigma}_x \quad (7)$$

where, $(\hat{\sigma}_n^2)$ is calculated by Eq. (8), $(\hat{\sigma}_x)$ is computed through WT coefficients applied in each sub-band. While $(\hat{\sigma}_x)$ value can be derived using Eq. (8) and (9) as shown below:

$$\hat{\sigma}_x = \max \sqrt{\hat{\sigma}_y^2 - \hat{\sigma}_n^2}, 0 \quad (8)$$

$$\sigma_y = \frac{1}{n^2} \sum_{j,k=1}^n w^2 j, k \quad (9)$$

The size of the subband under consideration is denoted by (n) .

3.3.4 Penalized threshold

Birge and Massart introduced this threshold technique in

1997. It utilizes level-dependent thresholds obtained from the post-selection rules for WT coefficients. This procedure could arrange the detail coefficients in descending order, and then the succeeding equation threshold value (λ) could be derived in line with this arrangement, as indicated in the following Eq. (10) [40]:

$$\lambda = \text{arg} \min_t \left[- \sum_{k=1}^t d_k^2 + 2\sigma^2 t(\alpha + \ln \frac{n}{t}) \right]; t = 1, \dots, n \quad (10)$$

where, $(\alpha > 1)$ is the sparsity parameter.

4. PROPOSED HYBRID DENOISING SCHEME

This research presents a multi-resolution wavelet domain based on both a hard and soft threshold technique with spatial domain filters adopted as an algorithm of hybrid denoising in the digital image. This combination has achieved a better result for removing noise from the damaged digital image. During the implementation of the algorithm stage, both 2D-SWT and 2D-DWT as a multi-resolution wavelet domain applied by using all filter families with five experimental grayscale images of the same size (256 x 256) pixels will take into consideration as an original image named (Cameraman, Lena, Butterfly, Peppers, and Fruit) as shown in Figure 2. The AWGN noise was inserted in all input images (original images) with zero means ($m=0$) and three levels of noise ratios ($\sigma=10$, $\sigma=15$, and $\sigma=25$) to create the noisy images.

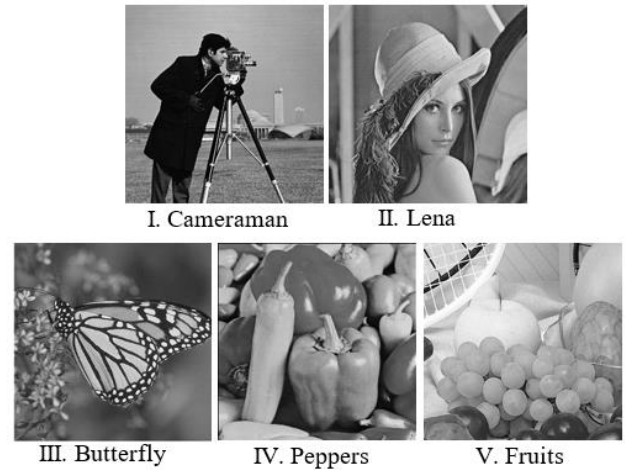


Figure 2. The test original images used in proposed denoising methodology scheme

In order to denoise the deteriorating digital images through the use of multi-resolution wavelet domain, several steps are implemented according to the denoising image algorithm described below in Figure 3.

Step (1): Choose one of five original images to use as an input image in the process.

Step (2): Add three different levels of noise ratios to the input image (10, 15, and 25) to create and keep the noisy form for each case as image files.

Step (3): Perform three levels of wavelet decomposition technique on the noisy image for each case to reduce the noise.

Step (4): Utilize various filter families of the wavelet domain, including haar, db, sym, coif, bior, and so on, to the noisy image (degraded) in order to analysis them.

Step (5): Select one of the denoising methodologies in the proposed system for several cases that will be applied to cover the process for using the threshold-based 2D-WT as per the proposed algorithm diagram in Figure 3.

Step (6): Perform thresholding technique at each wavelet decomposition level of the noisy image for various WT.

Step (7): Perform inverse wavelet transform (IWT) at three levels to obtain a denoised image (output image).

Step (8): Compute and compare the algorithm performance of the noisy image with the denoised image depending on the PSNR value.

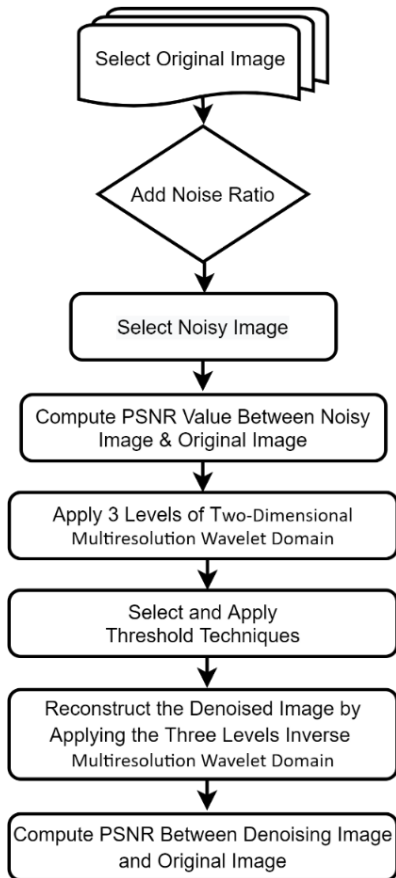


Figure 3. Proposed denoising methodology in the multiresolution wavelet domain steps

The steps listed above are only for image denoising in the multi-resolution WT domain depending on the hard and soft threshold value. One more step is added for hybrid digital image denoising in order to demonstrate the role of spatial domain filters applied before/after the 2D-WT domain, as shown below in Figures 2 and 3.

Both mean-square error (MSE) and peak signal-to-noise ratio (PSNR) criteria are employed to measure the quality of the image compression. MSE denotes the cumulative squared error connecting the compressed and the actual image, while PSNR represents an estimate of the peak error. The error becomes less if the MSE estimate is low. PSNR relies on MSE to calculate the PSNR. The initial block computes the mean-squared error by utilizing Eq. (11) [41, 42].

$$MSE = \frac{1}{M N} \sum_{x=1}^M \sum_{y=1}^N [g(x,y) - f(x,y)]^2 \quad (11)$$

where, (M) denotes the number of rows in the input image, (N) indicates the number of columns in the input image, (g)

represents an input image (noise image), and (f) represents an output image (denoising image).

While the next block computes the PSNR by using Eq. (12) as below:

$$PSNR = 10 \log_{10} \left(\frac{R^2}{MSE} \right) \quad (12)$$

(R) is the greatest fluctuation in the input image data type. The input image contains an 8-bit unsigned integer data type, so R is 255 [43].

In our proposed system, several cases applied to cover the steps and the sequence for using the threshold-based two-dimensional WT (2D-DWT, 2D-SWT) as a transform domain method by selecting a threshold technique (Hard, SureShrink, Bayesian, and Penalized) at each threshold process with spatial filters (Wiener filter, Median filter) apply as a spatial domain filters technique to eliminate noise and obtain an improved visual quality from the noisy image (degraded). Table 1 illustrates the whole process of all cases used in the system.

Table 1. All cases applied in the proposed system

No	Case	Description (Domain)
1	Wiener Filter (WF)	Spatial domain filtering
2	Median Filter (MF)	Spatial domain filtering
3	2D-DWT	2D wavelet transform domain
4	2D-SWT	2D stationary transform domain
5	WF_2D-DWT	Wiener Filter before 2D-DWT
6	WF_2D-SWT	Wiener Filter before 2D-SWT
7	2D-DWT_WF	2D-DWT before Wiener Filter
8	2D-SWT_WF	2D-SWT before Wiener Filter
9	MF_2D-DWT	Median Filter before 2D-DWT
10	MF_2D-SWT	Median Filter before 2D-SWT
11	2D-DWT_MF	2D-DWT before Median Filter
12	2D-SWT_MF	2D-SWT before Median Filter

To estimate the performance of the proposed image denoising algorithms, the PSNR value of denoising images was estimated and compared to the PSNR value of the noisy images for all filter families in the wavelet domain were applied and stored. Matlab programming language version R2020a and Microsoft Excel were used with the Visio graphics application to show the final results and charts. The cases can be explained as follow:

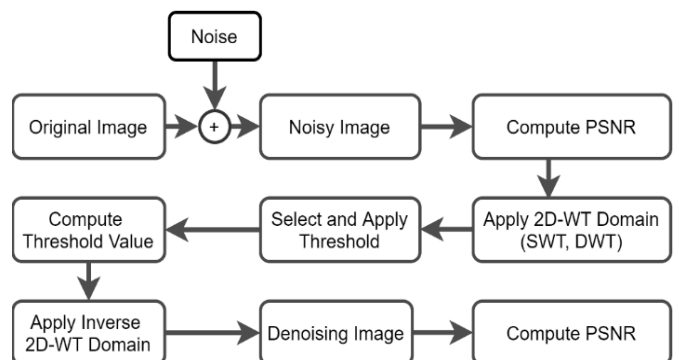


Figure 4. Block diagram of 2D-WT domain to denoise the image

Case 1: Both 2D-DWT and 2D-SWT are used separately as a multi-resolution 2D-WT domain depending on selecting the hard or soft threshold techniques to eliminate noise from the noisy image (degraded) and improve it. Then re-construct the

denoised image (output) through 2D-IWT from the approximation and detail coefficients. Figure 4 illustrates the process of applying the 2D-WT domain to denoise the image only.

Case 2: Both Wiener and Median filters are used separately as spatial domain filtering to eliminate noise and enhance the distorted image quality (degraded). Figure 5 illustrates the process of applying the spatial filters technique (WF, MF) to denoise the image.

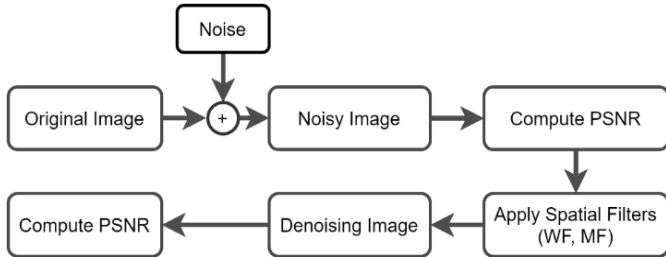


Figure 5. Block diagram of spatial domain filtering (WF, MF) to denoise the image

Case 3: Apply spatial domain filters (WF, MF) before using 2D-WT based on threshold value to implement this case of the proposed method. In the first step of the procedure, both Wiener and Median filters apply separately as a spatial filters technique on the noisy image. As a second step, 2D-DWT and 2D-SWT separately depend on selecting threshold techniques at each threshold process to eliminate noise and obtain an improved visual quality from the noisy image (degraded). Then, the denoised image (output) is re-constructed by 2D-IWT from the approximation and detail coefficients. Figure 6 illustrates the process of using the spatial domain filtering (WF, MF) before the 2D-WT domain to denoise the image (hybrid method).

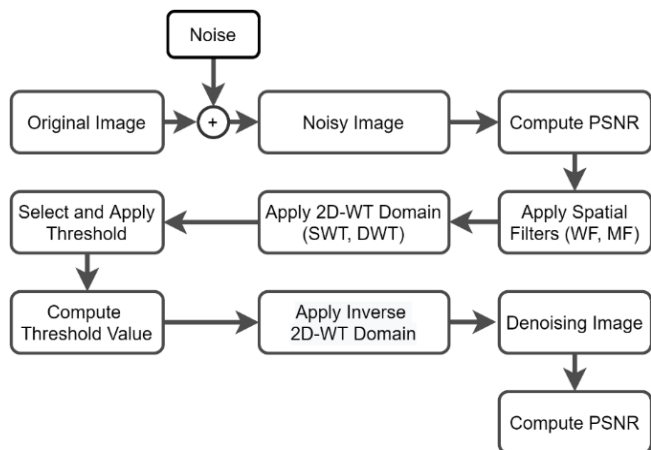


Figure 6. Block diagram of using the spatial domain filtering (WF, MF) before the 2D-WT domain to denoise the image

Case 4: Apply spatial domain filters (WF, MF) after using 2D-WT based on threshold technique to obtain an improved visual quality from the noisy images (degraded). In this method, the noisy image is handled in a couple of phases. As a first process, both 2D-DWT and 2D-SWT are applied separately as a pre-processing 2D-WT domain method based on selecting threshold technique at each threshold process to remove noise from the noisy image (degraded). Then, the denoised image (output) is re-constructed by 2D-IWT from the

approximation and detail coefficients. In the second and final process, the Wiener and Median filters are both applied separately as a spatial filters technique. Figure 7 illustrates the process of using the spatial domain filtering (WF, MF) after the 2D-WT domain to denoise the image (hybrid method).

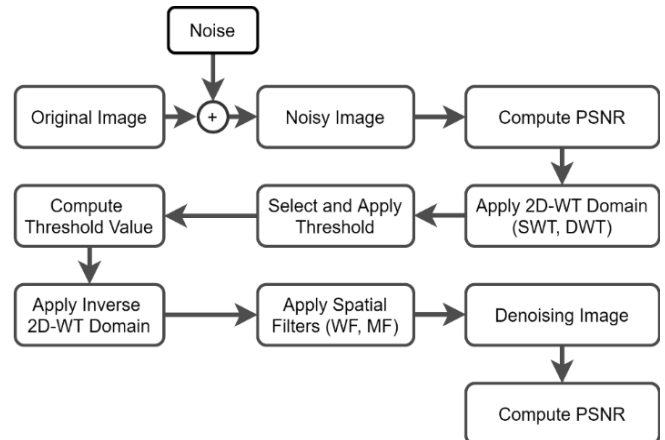


Figure 7. Block diagram of using the spatial domain filtering (WF, MF) after the 2D-WT domain to denoise the image

5. EXPERIMENTAL RESULTS

Following the implementation of the proposed cases to five grayscale images of the same size will take into consideration as original images named (Cameraman, Lena, Butterfly, Peppers, and Fruits) in all process steps depending on the best PSNR values. Table 2 represents this evaluation focusing on the different five grayscale images with three levels of noise ratios ($\sigma=10$, $\sigma=15$, and $\sigma=25$). Furthermore, Table 2 shows that the best value of PSNR result and best-proposed method of denoising images are repeated frequently when applying 2D-SWT based on the Sure Shrink threshold technique with selected Haar, Biorthogonal, and Reverse biorthogonal Wavelet Filters Families (db1, bior1.1, and rbio1.1), which have a better PSNR value compared to one of the other cases (process steps) according to the process steps at each noise ratios ($\sigma=10$, $\sigma=15$ and $\sigma=25$), which mean that the PSNR value of denoising image, in these cases, are better than in other cases. While the best PSNR value at all over-tested cases for all images with all noise ratios equal is (32.5460). As a result, it can have concluded that it is better to use both of Sure Shrink threshold technique and the Reverse biorthogonal filter together to achieve more suitable results.

Based on Table 2, results were selected as the best result of denoising images in the proposed process based on two important properties to obtain this better outcome: the wavelet filters based on thresholding techniques with spatial domain filters which had an effective role in obtaining these results and conclusions.

The chart in Figure 8 shows the best results in all cases among PSNR values after evaluating the proposed methods of denoising images at noise ratio ($\sigma=10$, $\sigma=15$, and $\sigma=25$).

Furthermore, the comparison results of denoising technique between the proposed methods of the hybrid system in this work and the results of the related works respectively referred to as Related_1, Related_2, and Related_3, represented by the references [1, 15, 17] are shown in Tables 3, 4, 5, 6 and 7 with all noise ratios ($\sigma=10$, $\sigma=15$, and $\sigma=25$).

Table 2. The evaluation of the proposed method depending on the best PSNR values

Image	Noise Ratio	PSNR	Case	Threshold Technique	Wavelet Filter
Cameraman	$\sigma=10$	31.6320	2D-SWT	Sure Shrink	db1, bior1.1, rbio1.1
	$\sigma=15$	28.6777	2D-SWT	Sure Shrink	db1, bior1.1, rbio1.1
	$\sigma=25$	25.7784	2D-SWT	Sure Shrink	db1, bior1.1, rbio1.1
Lena	$\sigma=10$	31.8136	2D-SWT	Sure Shrink	coif1
	$\sigma=15$	29.2087	2D-SWT	Sure Shrink	db1, bior1.1, rbio1.1
	$\sigma=25$	26.6966	2D-SWT_WF	Bayes Shrink	rbio6.8
Butterfly	$\sigma=10$	31.1761	2D-SWT	Sure Shrink	bior2.2
	$\sigma=15$	28.2307	2D-SWT	Sure Shrink	bior1.1
	$\sigma=25$	25.1847	2D-SWT	Sure Shrink	db1, bior1.1, rbio1.1
Peppers	$\sigma=10$	32.5460	2D-SWT	Sure Shrink	rbio2.4
	$\sigma=15$	29.9717	2D-SWT	Sure Shrink	fk4
	$\sigma=25$	27.6841	2D-SWT_WF	Bayes Shrink	bior1.5
Fruits	$\sigma=10$	31.9359	2D-SWT	Sure Shrink	db1, bior1.1, rbio1.1
	$\sigma=15$	29.2377	2D-SWT	Sure Shrink	db1, bior1.1, rbio1.1
	$\sigma=25$	27.0317	2D-SWT_WF	Bayes Shrink	bior1.5

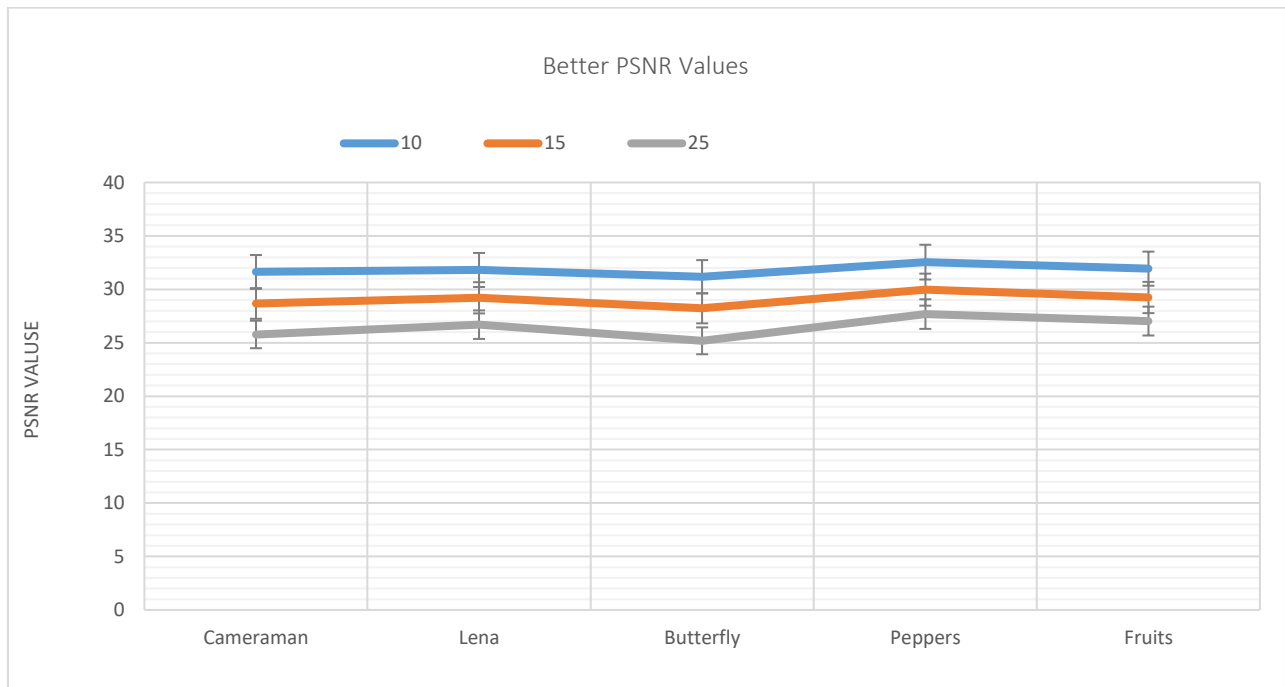


Figure 8. Best results in all cases among PSNR values after evaluating the proposed methods of denoising images at noise ratio ($\sigma=10$, $\sigma=15$, and $\sigma=25$)

Table 3. PSNR values of comparing the proposed case with related works for the Cameraman at noise ratios ($\sigma=10$, $\sigma=15$, and $\sigma=25$)

Image	Noise Ratio	Author	PSNR	Case	Threshold Technique	Wavelet Filter
Cameraman	$\sigma=10$	Related_1	----	----	----	----
		Related_2	----	----	----	----
		Related_3	29.7910	2D-DWT	Hard	rbio3.7
		Proposed	31.6320	2D-SWT	Sure Shrink	db1, bior1.1, rbio1.1
Cameraman	$\sigma=15$	Related_1	27.1889	WF_2D-DWT	Penalized	bior3.9
		Related_2	25.1232	MF_2D-DWT	Hard	rbio
		Related_3	27.1640	2D-DWT	Hard	sym5
		Proposed	28.6777	2D-SWT	Sure Shrink	db1, bior1.1, rbio1.1
Cameraman	$\sigma=25$	Related_1	25.4744	2D-DWT_WF	Bayes Shrink	bior1.3
		Related_2	23.6200	2D-DWT_MF	Hard	db
		Related_3	24.8630	2D-DWT	Hard	sym5
		Proposed	25.7784	2D-SWT	Sure Shrink	db1, bior1.1, rbio1.1

According to comparison results, the PSNR values for five grayscale images of the same size will take into consideration as an original image named (Cameraman, Lena, Butterfly, Peppers, and Fruits) in all process steps depending on the best PSNR values for three levels of noise ratios ($\sigma=10$, $\sigma=15$, and

$\sigma=25$) between both the proposed cases and the results of the related works; the proposed cases have better PSNR values compared to related works indicated by the references [11, 15, 17].

The charts in Figures 9, 10, and 11 show how effective the hybrid system proposed as a solution is in removing the noise

in images when compared to related work at each noise ratio ($\sigma = 10, \sigma = 15, \sigma = 25$).

Table 4. PSNR values of comparing the proposed case with related works for the Lena image at noise ratios ($\sigma=10, \sigma=15,$ and $\sigma=25$)

Image	Noise Ratio	Author	PSNR	Case	Threshold Technique	Wavelet Filter
Lena	$\sigma=10$	Related_1	----	----	----	----
		Related_2	----	----	----	----
		Related_3	29.9320	2D-DWT	Hard	bior5.5
		Proposed	31.8136	2D-SWT	Sure Shrink	coif1
Lena	$\sigma=15$	Related_1	----	----	----	----
		Related_2	26.7990	2D-DWT_MF	Hard	db
		Related_3	28.2200	2D-DWT	Hard	sym4
		Proposed	29.2087	2D-SWT	Sure Shrink	db1, bior1.1, rbio1.1
Lena	$\sigma=25$	Related_1	----	----	----	----
		Related_2	25.2960	2D-DWT_MF	Hard	rbio
		Related_3	25.1730	2D-DWT	Hard	sym5
		Proposed	26.6965	2D-SWT_WF	Bayes Shrink	rbio6.8

Table 5. PSNR values of comparing the proposed case with related works for the Butterfly image at noise ratios ($\sigma=10, \sigma=15,$ and $\sigma=25$)

Image	Noise Ratio	Author	PSNR	Case	Threshold Technique	Wavelet Filter
Butterfly	$\sigma=10$	Related_1	----	----	----	----
		Related_2	----	----	----	----
		Related_3	----	----	----	----
		Proposed	31.1761	2D-SWT	Sure Shrink	rbio2.2
Butterfly	$\sigma=15$	Related_1	26.4665	2D-DWT	Bayes Shrink	rbio2.4
		Related_2	24.7068	MF_2D-DWT	Hard	coif
		Related_3	----	----	----	----
		Proposed	28.2307	2D-SWT_WF	Bayes Shrink	bior1.3
Butterfly	$\sigma=25$	Related_1	24.6412	WF_2D-DWT	Penalized	bior3.9
		Related_2	23.6200	MF_2D-DWT	Hard	sym
		Related_3	----	----	----	----
		Proposed	25.1847	2D-SWT	Sure Shrink	db1, bior1.1, rbio1.1

Table 6. PSNR values of comparing the proposed case with related works for the Peppers image at noise ratios ($\sigma=10, \sigma=15,$ and $\sigma=25$)

Image	Noise Ratio	Author	PSNR	Case	Threshold Technique	Wavelet Filter
Peppers	$\sigma=10$	Related_1	----	----	----	----
		Related_2	----	----	----	----
		Related_3	30.1010	2D-DWT	Hard	rbio3.7
		Proposed	32.5459	2D-SWT	Sure Shrink	rbio2.4
Peppers	$\sigma=15$	Related_1	29.1246	WF_2D-DWT	Penalized	bior2.8
		Related_2	----	----	----	----
		Related_3	27.6360	2D-DWT	Hard	db4
		Proposed	29.9716	2D-SWT	Sure Shrink	fk4
Peppers	$\sigma=25$	Related_1	27.1144	WF_2D-DWT	Penalized	db10
		Related_2	----	----	----	----
		Related_3	25.0020	2D-DWT	Hard	db4
		Proposed	27.6840	2D-SWT_WF	Bayes Shrink	bior1.5

Table 7. PSNR values of comparing the proposed case with related works for the Fruits image at noise ratios ($\sigma=10, \sigma=15,$ and $\sigma=25$)

Image	Noise Ratio	Author	PSNR	Case	Threshold Technique	Wavelet Filter
Fruits	$\sigma=10$	Related_1	----	----	----	----
		Related_2	----	----	----	----
		Related_3	----	----	----	----
		Proposed	31.9358	2D-SWT	Sure Shrink	db1, bior1.1, rbio1.1
Fruits	$\sigma=15$	Related_1	28.2649	WF_2D-DWT	Penalized	bior3.9
		Related_2	25.9091	MF_2D-DWT	Hard	bior
		Related_3	----	----	----	----
		Proposed	29.2376	2D-SWT	Sure Shrink	db1, bior1.1, rbio1.1
Fruits	$\sigma=25$	Related_1	26.6877	WF_2D-DWT	Penalized	bior1.5
		Related_2	24.5582	MF_2D-DWT	Hard	rbio
		Related_3	----	----	----	----
		Proposed	27.0316	2D-SWT_WF	Bayes Shrink	bior1.5

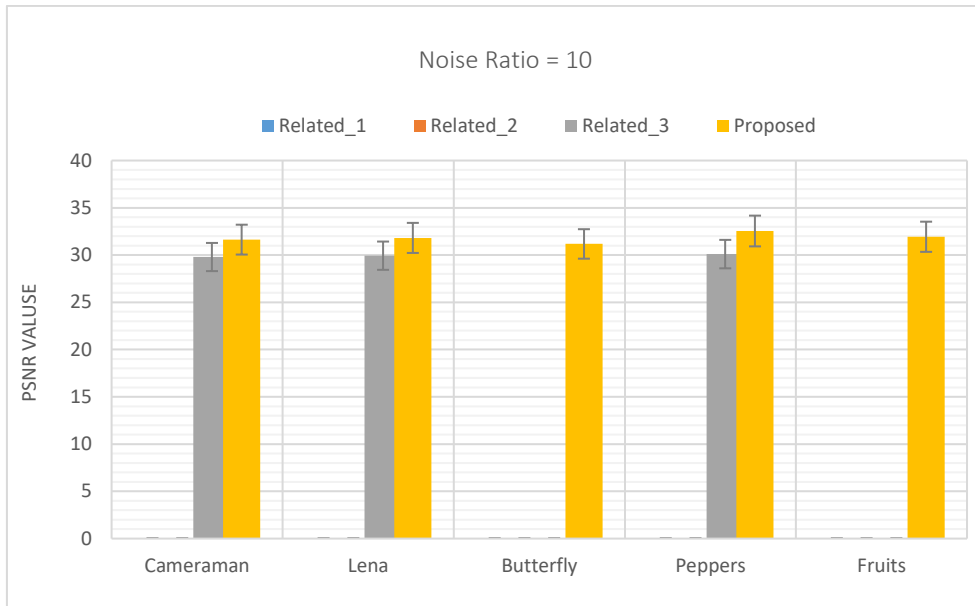


Figure 9. PSNR values of comparing the proposed case with related works for all images at noise ratios $\sigma=10$

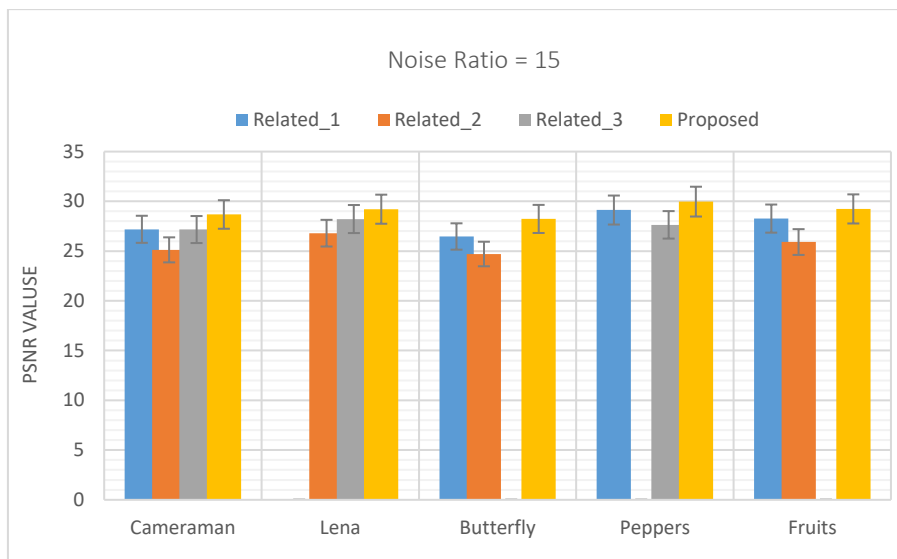


Figure 10. PSNR values of comparing the proposed case with related works for all images at noise ratios $\sigma=15$

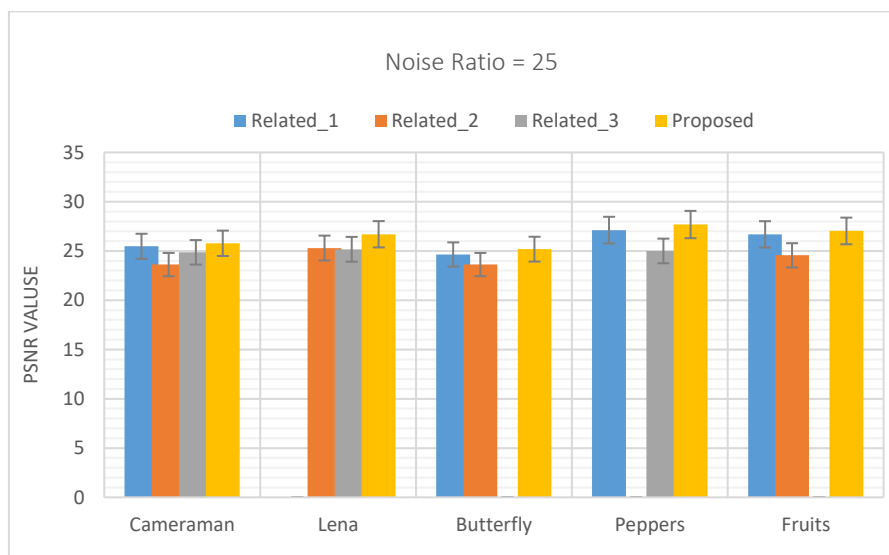


Figure 11. PSNR values of comparing the proposed case with related works for all images at noise ratios $\sigma=25$

Based on Tables 3, 4, 5, 6 and 7 comparing with Related_1 and Related_2 has a mechanism of applying hybrid denoising method by merging the 2D-DWT with median filter in the process of images denoising, it can be concluded that the Wiener filter is more appropriate to be used with 2D-DWT in the hybrid denoising process than the median filter with 2D-DWT for the type AWGN noise. Compared to our proposed, by merging the 2D-SWT with spatial domain, it is observed that the proposed method is better in removing noise and increasing the quality of images compared to other related works and methods. While Related_3, which depends only on removing noise in the wavelet domain, it is observed that the proposed method is a better intern of increasing the quality of images by removing the noise in both domains (spatial and wavelet). The experimental assessment showed that the results of proposed cases of process steps gave an improvement in the denoising operation when compared to the results of related works.

6. CONCLUSIONS

Through this work, the main aim was to remove noise from digital images that have been affected by Additive White Gaussian Noise (AWGN) with three ratios of the noise sigma as 10, 15, and 25. A new approach is designed and implemented as a noise removal system based on evaluating the effect of implementing the proposed methods, which are designed to be a hybrid system by merging the work of the spatial domain filters (Median, Wiener) with the work of the multi-resolution wavelet domain (2D-SWT, 2D-DWT) by applying 131 wfilters from the wavelet families (haar, db, sym, coif, bior, rbio, dmey, fk) in image processing at three levels of decomposition depending on the four threshold techniques (Hard, SureShrink, Bayesian, and Penalized). Then, the multi-level 2D inverse wavelet transform (2D-IWT) eliminates noise and completes the image reconstruction by the hybrid denoising technique.

The hybrid system has been built by using the Matlab programming environment with several kinds of the wavelet transform, and threshold techniques have been tested on five grayscale images of the same size (Cameraman, Lena, Butterfly, Peppers, and Fruit) to achieve improved results of the image noise reduction process by distinguishing and removing the noise from the affected pixel units on both high and low frequencies to obtain improved results of the noise reduction process to the noisy images.

Finally, the performance metrics peak signal-to-noise ratio (PSNR) as an image quality metric is calculated to estimate the evaluation of the suggested method (hybred). The experimental evaluation outcomes of the proposed method reveal a better improvement of image quality concerning minimizing noise and edge preservation compared to the results of the related works respectively referred to as Related_1, Related_2, and Related_3, represented by the references [11, 15, 17] instead of using a threshold-based WT domain or spatial domain filters separately.

All of the findings were recorded directly from Matlab on Excel files, and by analyzing the results, then the following conclusions can be reached:

- The use of the WT analysis of three levels gave supplementary amendment to this effectiveness in eliminating noise as it decreases a greater quantity of noise during this analysis, and this was exhibited by the

results obtained for the criteria used in the performance evaluation of PSNR.

- By increasing the level of the analysis phase in 2D-DWT, the increase of the image enhancement has been noticed. This enhancement will be limit through the limited number of analysis levels because there will be no improvement due to the saturation of the denoising operation. The saturation of the denoising operation means that the number of points in the data is minimal and will not affect the result at all.
- During applying the cases of this work, the wavelet transform has been used in two forms; 2D-DWT and 2D-SWT. And, through the total results, the 2D-SWT provided the best improvement to the PSNR of the work cases.
- The results improved by using the 2D-SWT because in this form of the wavelet transformation, all the point used and there are no down-sampling for the points. Without a down-sample will let the algorithm deals with all the points which contain the information and also remove as much as possible of the noise.
- The 2D-SWT increased the improvement ratio of the PSNR results, but the cost was the processing time, which increased because there is no down-sampling in 2d-SWT, and all the points were used.
- The method of applying the threshold technique in the wavelength domain has shown effectiveness and efficiency in removing the noise of AWGN type by dealing adaptively with each frequency band and the corresponding threshold value.
- In this work and for all cases that belong to the 2D-DWT and 2D-SWT of the wavelet transform, four methods of the threshold have been used which are (Hard, Sure Shrink, Bayes Shrink, and Penalized). The results showed that for all images and cases with 2D-SWT, the improved PSNR values were by 80% with the Soft Threshold Sure Shrink and 20% with the Soft Threshold Bayes Shrink only. While for all images and cases with 2D-DWT, the improved PSNR values were by 20% with the Hard Threshold, 26.66% with the Soft Threshold Sure Shrink, 40% with the Soft Threshold Bayes Shrink, and 13.34% with the Soft Threshold Penalized.
- Through the results of all the cases applied in this work, the largest percentage of the best results (about 87%) was obtained when using filters belonging to the families (Daubechies, Biorthogonal, and Reverse biorthogonal) of the wavelet transform filter families. This indicates the effectiveness of these filter families to be applied in noise reduction applications in digital images. The best results were by using the following filters with the cases of this work: (db1, bior1.1, bior1.3, bior1.5, rbio1.1, rbio2.4, rbio6.8, coif1, and fk4).
- The findings showed good indicators that the hybrid system process of denoising technique by merging between the work of spatial domain filters with multi-resolution wavelet domain based on threshold value leads to improved results than utilizing either technique separately.
- The comparison findings of denoising technique between the proposed methods of the hybrid system in this work and the results of the related works respectively referred to as Related_1 Related_2, and Related_3, represented by the references [11, 15, 17] indicate that the hybrid system is achieved better PSNR values compared to the related

works of all the noise values 10,15 and 25. The improvement in PSNR results was about 17.5%.

As a recommendation for future work in this sector, it is preferable to design and implement the proposed system in this research using high-performance Field Programmable Gate Array (FPGA) circuits. Also, additional performance measures can be determined in order to investigate and analyze the behavior of hybrid system methods that achieve a successful balance of noise removal in various noise reduction fields. As well as, apply the proposed denoising system with several other different types of noise, such as (salt & pepper, Poisson, speckle noise, and etc.) to remove several noise sources in various digital images.

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