Oscillatory MHD Mass Transfer Channel Flow in a Rotating System with Hall Current

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ABSTRACT

The effects of heat and mass transfer on the flow of an oscillatory convective MHD channelflow of an electrically conducting viscous incompressible fluid in a slip flow regime in the presence of Hall current and thermal radiation are studied. A magnetic field of uniform strength is applied normal to the plates of channel. The two vertical porous plates of the channel are subjected to a constant injection/suction. The entire system rotates about an axis normal to the plates with a uniform angular velocity $\vec{\Omega}$. A closed form analytical solutions to the equations governing the fluid motion are obtained for the velocity, temperature and concentration fields. The numerical calculations for the velocity, temperature and concentration fields, and the coefficient of skin friction, rate of heat and mass transfer at the plates are performed. The results are presented graphically for different values of the physical parameters involved. It is seen that primary fluid flow is retarded due to angular velocity of the system.

Keywords: Convective flow, Hall current, Rotating channel, Slip conditions, Thermal radiation.

1. INTRODUCTION

The phenomenon of heat and mass transfer has been the object of extensive research due to its application in science and technology. Such phenomenon is observed in buoyancy induced motions in the atmosphere, in the bodies of water, quasi-solid bodies such as earth and so on. Oscillatory flows are associated with higher rates of heat and mass transfer. Many studies have been done to understand its characteristics in different systems such as reciprocating engines, pulse combustors and chemical reactors etc. Unsteady oscillatory free convective flows play an importance in aerospace technology, turbo machinery and chemical engineering. Such flows arise due to either unsteady motion of a boundary or boundary temperature. In nature and industrial applications many transport processes exist where the transfer of heat and mass takes place simultaneously as a result of combined buoyancy effects of thermal diffusion and diffusion of chemical species. Cooper et al. [1] has made a detailed study on fluid mechanics of oscillatory and modulated flows and associated applications in heat and mass transfer. Muthucumaraswamy [2] has studied the effect of heat and mass transfer on flow past an oscillatory vertical plate with variable temperature. Ahmed & Kalita [3] discussed the combined heat mass transfer effects on Oscillatory MHD free and forced convection flow through a porous medium in presence of a heat source with variable suction. The hydrodynamic rotating flow of an electrically conducting viscous incompressible fluid has gained considerable

attention because of its numerous applications in physics and engineering. In geophysics, it is applied to measure and study the positions and velocities with respect to a fixed frame of reference on the earth, which rotates with respect to an inertial frame in the presence of its magnetic field. Boundary layer flows in a rotating fluid system are important due to various applications in science and technology. Debnath [4] presented exact solutions of the hydrodynamic and hydro magnetic boundary layer equations in such systems. Oscillatory hydro magnetic Couette flow in a rotating fluid system is investigated by Guria et al. [5]

When the density of an electrically conducting fluid is low and/or applied magnetic field is strong, a current is induced in a direction which is normal to both the electric and magnetic fields. Thus if an electric field is applied at right angle to the magnetic field, the total current will not flow along the electric field. This tendency of the electric current to flow across an electric field in the presence of a magnetic field is called Hall effects and the resulting current is known as Hall current. Hall effects are important when the Hall parameter, which is the ratio between the electron-cyclotron frequency and the electron-atom-collision frequency, is high. This happens when the magnetic field is strong or when the collision frequency is low. Hall currents are of great importance in many astrophysical problems, Hall accelerator and flight MHD as well as flows of plasma in a MHD power generator. Aboeldahab and Elbarbary [6] considered effects of Hall current on MHD free convection flow with heat and mass transfer over a vertical in the presence of a strong

external magnetic field. Sarkar *et al* [7] investigated effects of Hall currents and radiation on MHD free convection flow past an oscillating vertical plate with oscillatory plate temperature in a porous medium.

It is assumed that the boundary condition for a viscous fluid at a solid surface is one of "no - slip" at the macroscopic level. While no – slip boundary condition has been proven experimentally to be accurate for a number of macroscopic flows, it remains an assumption that is based on physical principles. In fact nearly two hundred years ago, Navier's [8] proposes a general a boundary condition that incorporates the possibility of the fluid slip at a solid boundary. Navier's proposed condition assumes that the fluid slip velocity at a solid surface is proportional to the shear stress at the surface. The mathematical form of the Navier's proposed condition on the slip velocity as emphasized by Goldstein [9] is $u = \gamma \left(\dfrac{\partial u}{\partial \eta} \right)$; where γ is the slip strength or slip coefficient, u is the slip velocity and η is the normal coordinate. If $\gamma = 0$ then no-slip boundary condition is obtained. If γ is finite, fluid slip occurs at the wall but its effect depends upon the length scale of the flow. The effect of slip condition on MHD steady flow in a channel with permeable boundaries has been discussed by Makinde and Osalusi [10]. Marques et al. [11] has considered the effect of fluid slippage at the plate for Couette flow. Ahmed and Kalita [12] have studied MHD oscillatory free convective flow past a vertical plate in slip-flow regime with variable suction and periodic plate temperature

Recently Singh and Pathak [13] have studied the effects of Hall current and thermal radiation on MHD convective flow in a vertical rotating porous channel in slip flow regime in presence of uniform magnetic field where the entire system rotates about the axis perpendicular to the planes of the plates of the channel.

The method of the present work is similar to the work of Singh and Pathak [13] although the flow geometries are not identical. Further in the present work the effect of mass transfer is taken into account which was not considered in the work of Singh and Pathak [13].

The objective of the present work is to investigate the effects of Hall current and thermal radiation on MHD oscillatory convective mass transfer flow through a rectangular channel bounded by two vertical porous plates in a rotating system with slip flow regime.

2. MATHEMATICAL MODELOF THE PROBLEM

The equations governing the unsteady motion of an incompressible, viscous, electrically conducting fluid with a vertical channel system in presence of magnetic field areas follows:

Continuity equation:

$$\vec{\nabla}.\vec{V} = 0 \tag{1}$$

Gauss's law of magnetism:

$$\vec{\nabla}.\vec{B} = 0 \tag{2}$$

Kirchhoff's first law:

$$\vec{\nabla}.\vec{J} = 0 \tag{3}$$

General Ohm's law:

$$\vec{J} + \frac{\omega_e \tau_e}{B} \left(\vec{J} \times \vec{B} \right) = \sigma \left[\vec{E} + \vec{V} \times \vec{B} + \frac{1}{e\eta_e} \nabla p_e \right] \tag{4}$$

Momentum equation:

$$\rho \left[\frac{\partial \vec{V}}{\partial t'} + \vec{\Omega} \times \vec{V} + \left(\vec{V}.\vec{\nabla} \right) \vec{V} \right] = -\vec{\nabla} p + \vec{J} \times \vec{B} + \mu \nabla^2 \vec{V} \\ + g\beta T' + g\bar{\beta}C' \tag{5}$$

Energy equation:

$$\rho C_p \left[\frac{\partial T'}{\partial t'} + \left(\vec{V}.\vec{\nabla} \right) T' \right] = k\nabla^2 T' - \vec{\nabla}.q \tag{6}$$

Species continuity equation:

$$\frac{\partial C'}{\partial t'} + \left(\vec{V}.\vec{\nabla} \right) C' = D\nabla^2 C' \tag{7}$$

We now consider an oscillatory MHD convective flow of an electrically conducting viscous incompressible fluid through a channel bounded by two infinite vertical porous plate separated by distance 'd', taking into account the Hall current. One of the plates of the channel is subjected to slip velocity. The X' axis is taken oriented vertically upward along the centreline of the channel, the Z' axis is taken normal to the planes to the plates which is the axis of rotation and Y' is taken along the width of the either plate.
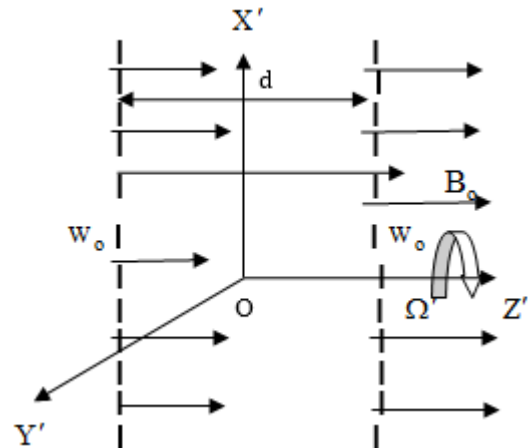

Figure 1. Physical model of the problem

In order to make the mathematical model of the present work idealized, the present investigation is restricted the following assumptions:

i. All the fluid properties are considered constant except the influence of the density with temperature and concentration in the buoyancy force.

ii. The viscous and Ohmic dissipations of energy are negligible.

iii. The magnetic Reynolds number is small that the induced magnetic field can be neglected.

iv. The plate is electrically insulator and the plate temperature as well as the plate concentration varies linearly as the distance along the plate.
v. No external electric field is applied for which the polarization voltage is negligible leading to $\vec{E} = \vec{0}$

Let $\vec{V} = (u', v', w')$ denote the fluid velocity, $\vec{\Omega} = (0, 0, \Omega')$ be the angular velocity, $\vec{B} = (0, 0, B_o)$ be the magnetic flux density and $\vec{J} = (j_{x'}, j_{y'}, j_{z'})$ be the current density at the point $P(x', y', z', t')$ in the fluid. As the plates of the channel are infinite in X' and Y' directions, therefore all the physical quantities are independent of x' and y'.

Equation (1) yields

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} + \frac{\partial w'}{\partial z'} = 0$$

$$\Rightarrow \frac{\partial w'}{\partial z'} = 0$$

which holds for $w' = W_o$ (constant)

The equation (2) is trivially satisfied by the magnetic flux density distribution $\vec{B} = (0, 0, B_o)$

The equation (3) gives, $\dfrac{\partial j_{z'}}{\partial z'} = 0$, which is satisfied with $j_{z'}$ is constant.

Further, as the plates are electrically insulators, so $j_{z'} = 0$ on the walls and, hence $j_{z'} = 0$ for the entire fields.

Again on the basis of the above assumptions, equation (4) takes the form

$$\vec{J} + \frac{\omega_e \tau_e}{B} (\vec{J} \times \vec{B}) = \sigma(\vec{V} \times \vec{B}) \tag{8}$$

Splitting the equation (8), we derive the following equations

$$j_{x'} + m\, j_{y'} - \sigma B_o v' = 0 \tag{i}$$
$$-m\, j_{x'} + j_{y'} + \sigma B_o u' = 0 \tag{ii}$$

Solving (i) and (ii), we obtain

$$j_{x'} = \frac{\sigma B_o (mu' + v')}{1 + m^2} \text{ and } j_{y'} = \frac{\sigma B_o (mv' - u')}{1 + m^2}$$

where $m = \omega_e \tau_e$ is the Hall parameter.

Under the above assumptions, the governing equations (5), (6) and (7) reduces to

$$\frac{\partial u'}{\partial t'} + w_o \frac{\partial u'}{\partial z'} = -\frac{1}{\rho} \frac{\partial p'}{\partial x'} + \upsilon \frac{\partial^2 u'}{\partial z'^2} + 2\Omega' v' + \frac{\sigma B_o^2 (mv' - u')}{\rho (1 + m^2)} \tag{9}$$
$$+ g\beta(T' - T_o) + g\bar{\beta}(C' - C_o)$$

$$\frac{\partial v'}{\partial t'} + w_o \frac{\partial v'}{\partial z'} = -\frac{1}{\rho} \frac{\partial p'}{\partial y'} + \upsilon \frac{\partial^2 v'}{\partial z'^2} - 2\Omega' u' - \frac{\sigma B_o^2 (mu' + v')}{\rho (1 + m^2)} \tag{10}$$

$$\rho C_p \left(\frac{\partial T'}{\partial t'} + w_o \frac{\partial T'}{\partial z'} \right) = k \frac{\partial^2 T'}{\partial z'^2} - \frac{\partial q}{\partial z'} \tag{11}$$

$$\frac{\partial C'}{\partial t'} + w_o \frac{\partial C'}{\partial z'} = D \frac{\partial^2 C'}{\partial z'^2} \tag{12}$$

The last term in the energy equation (11) stands for radiative heat flux is given by

$$\frac{\partial q}{\partial z'} = 4\alpha^2 (T' - T_o) \tag{13}$$

where α is the mean radiation absorption coefficient

The appropriate boundary and slip flow conditions for the present problem are

$$u' = L\frac{\partial u'}{\partial z'}, \quad v' = L\frac{\partial v'}{\partial z'}, \quad T' = T_o, \quad C' = C_o \quad \text{at } z' = \frac{-d}{2}$$
$$u' = v' = 0, \ T' = T_o(1 + \cos\omega' t'), \ C' = C_o(1 + \cos\omega' t') \text{ at}$$
$$z' = \frac{d}{2} \tag{14}$$

where L is the mean free path which is constant for an incompressible fluid.

The non dimensional quantities are

$$\eta = \frac{z'}{d}, \ x = \frac{x'}{d}, \ y = \frac{y'}{d}, \ u = \frac{u'}{U}, \ v = \frac{v'}{d}, \ T = \frac{T' - T_o}{T_o},$$
$$C = \frac{C' - C_o}{C_o}, \ t = \frac{t'U}{d}, \ \omega = \frac{\omega' d}{U}, \ p = \frac{p'}{\rho U^2} \tag{15}$$

With the help of non dimensional quantities (15), equations (9) to (12) reduces to

$$\frac{\partial u}{\partial t} + \frac{\lambda}{R_e} \frac{\partial u}{\partial \eta} = -\frac{\partial p}{\partial x} + \frac{1}{R_e} \frac{\partial^2 u}{\partial \eta^2} + \frac{2\Omega}{R_e} v + \frac{M^2 (mv - u)}{R_e (1 + m^2)} \tag{16}$$
$$+ \frac{G_r}{R_e} T + \frac{G_m}{R_e} C$$

$$\frac{\partial v}{\partial t} + \frac{\lambda}{R_e} \frac{\partial v}{\partial \eta} = -\frac{\partial p}{\partial y} + \frac{1}{R_e} \frac{\partial^2 v}{\partial \eta^2} - \frac{2\Omega}{R_e} u - \frac{M^2 (mu + v)}{R_e (1 + m^2)} \tag{17}$$

$$\frac{\partial T}{\partial t} + \frac{\lambda}{R_e} \frac{\partial T}{\partial \eta} = \frac{1}{P_r R_e} \frac{\partial^2 T}{\partial \eta^2} - \frac{N^2}{P_r R_e} T \tag{18}$$

$$\frac{\partial C}{\partial t} + \frac{\lambda}{R_e} \frac{\partial C}{\partial \eta} = \frac{1}{S_c R_e} \frac{\partial^2 C}{\partial \eta^2} \tag{19}$$

where,

$$R_e = \frac{Ud}{\upsilon}, \ \Omega = \frac{\Omega' d^2}{\upsilon}, \ \lambda = \frac{w_o d}{\upsilon}, \ G_r = \frac{g\beta d^2 T_o}{\upsilon U},$$

$$G_m = \frac{g\bar{\beta} d^2 C_o}{\upsilon U}, \ P_r = \frac{\mu C_p}{k}, \ S_c = \frac{\upsilon}{D}$$

$$N = \frac{2\alpha d}{\sqrt{k}}, \quad M = B_o d \sqrt{\frac{\sigma}{\rho \upsilon}}$$

All the physical quantities are defined in nomenclature. The boundary conditions in the non-dimensional form becomes

$$\eta = -\frac{1}{2}: \quad u = h\frac{\partial u}{\partial \eta}, \quad v = h\frac{\partial v}{\partial \eta}, \quad T = 0, \quad C = 0$$

$$\eta = \frac{1}{2}: \quad u = v = 0, \quad T = \cos\omega t, \quad C = \cos\omega t \tag{20}$$

where $h = \dfrac{L}{d}$ is the slip flow parameter.

We assume that the fluid flow under the influence of a non-dimensional oscillatory pressure gradient varying periodically with the time in the direction of x-axis of the following form

$$-\frac{\partial p}{\partial x} = P\cos\omega t$$

3. SOLUTION OF THE PROBLEM

We introduce a complex velocity function of the form

$$F(\eta, t) = u(\eta, t) + iv(\eta, t) \tag{21}$$

Equations (16) and (17) can be combined to give

$$\frac{\partial F}{\partial t} + \frac{\lambda}{R_e}\frac{\partial F}{\partial \eta} = P\cos\omega t + \frac{1}{R_e}\frac{\partial^2 F}{\partial \eta^2} - \frac{2i\Omega}{R_e}F - \frac{M^2(1+im)}{R_e(1+m^2)}F \tag{22}$$
$$+ \frac{G_r}{R_e}T + \frac{G_m}{R_e}C$$

The boundary conditions (20) in complex form become

$$\eta = -\frac{1}{2}: \quad F = h\frac{\partial F}{\partial \eta}, \quad T = 0, \quad C = 0$$

$$\eta = \frac{1}{2}: \quad F = 0, \quad T = \cos\omega t, \quad C = \cos\omega t \tag{23}$$

In order to solve equations (18), (19) and (22) under the boundary condition (23), we assume in complex form the solution of the form

$$F(\eta, t) = F_o(\eta)e^{i\omega t}, \quad T(\eta, t) = \theta_o(\eta)e^{i\omega t}, \quad C(\eta, t) = \xi_o(\eta)e^{i\omega t} \tag{24}$$

and $-\dfrac{\partial p}{\partial x} = Pe^{i\omega t}$

The boundary condition (23) reduces to

$$\eta = -\frac{1}{2}: \quad F_o = h\frac{\partial F_o}{\partial \eta}, \quad \theta_o = 0, \quad \xi_o = 0$$

$$\eta = \frac{1}{2}: \quad F_o = 0, \quad \theta_0 = 1, \quad \xi_o = 1 \tag{25}$$

Substituting equation (24) in equations (18), (19) and (22), we get

$$\frac{d^2 F_0}{d\eta^2} - \lambda\frac{dF_o}{d\eta} - c^2 F_o = -PR_e - G_r\theta_o - G_m\xi_o \tag{26}$$

$$\frac{d^2\theta_0}{d\eta^2} - \lambda P_r\frac{d\theta_o}{d\eta} - a^2\theta_0 = 0 \tag{27}$$

$$\frac{d^2\xi_0}{d\eta^2} - \lambda S_c\frac{d\xi_o}{d\eta} - b^2\xi_0 = 0 \tag{28}$$

where,

$$c^2 = i\omega R_e + 2\Omega i + \frac{M^2(1+im)}{(1+m^2)}, \quad a^2 = N^2 + i\omega P_r R_e,$$

$$b^2 = i\omega S_c R_e$$

The ordinary differential equations (26), (27) and (28) are solved under the boundary condition (25) and we get the following expression for the velocity, temperature and concentration fields

$$F(\eta, t) = \begin{bmatrix} A_3 e^{c_1\eta} + A_4 e^{c_2\eta} + \dfrac{PR_e}{c^2} \\ -A_1\left\{(a_2^2 - \lambda a_2 - c^2)e^{\left(a_1\eta - \frac{a_2}{2}\right)} - (a_1^2 - \lambda a_1 - c^2)e^{\left(a_2\eta - \frac{a_1}{2}\right)}\right\} \\ -A_2\left\{(b_2^2 - \lambda b_2 - c^2)e^{\left(b_1\eta - \frac{b_2}{2}\right)} - (b_1^2 - \lambda b_1 - c^2)e^{\left(b_2\eta - \frac{b_1}{2}\right)}\right\} \end{bmatrix} e^{i\omega t} \tag{29}$$

$$T(\eta, t) = \left[\frac{e^{\left(a_1\eta - \frac{a_2}{2}\right)} - e^{\left(a_2\eta - \frac{a_1}{2}\right)}}{2\sinh\left(\dfrac{a_1 - a_2}{2}\right)}\right] e^{i\omega t} \tag{30}$$

$$C(\eta, t) = \left[\frac{e^{\left(b_1\eta - \frac{b_2}{2}\right)} - e^{\left(b_2\eta - \frac{b_1}{2}\right)}}{2\sinh\left(\dfrac{b_1 - b_2}{2}\right)}\right] e^{i\omega t} \tag{31}$$

3.1 Skin friction

The non dimensional skin friction at the left plate is given by

$$\tau_L = \left(\frac{\partial F}{\partial \eta}\right)_{\eta = -\frac{1}{2}}$$

$$= \begin{bmatrix} A_3 c_1 e^{-\frac{c_1}{2}} + A_4 c_2 e^{-\frac{c_2}{2}} - A_1(a_2 - a_1)(a_1 a_2 + c^2)e^{-\left(\frac{a_1 + a_2}{2}\right)} \\ -A_2(b_2 - b_1)(b_1 b_2 + c^2)e^{-\left(\frac{b_1 + b_2}{2}\right)} \end{bmatrix} e^{i\omega t}$$

$$\tag{32}$$

3.2 Heat transfer coefficient

The dimensionless expressions for the rate of heat transfer at the left plate is given by

$$N_u = \left(\frac{\partial T}{\partial \eta}\right)_{\eta=-\frac{1}{2}} = \left[\frac{(a_1 - a_2)e^{-\left(\frac{a_1+a_2}{2}\right)}}{2\sinh\left(\frac{a_1-a_2}{2}\right)}\right]e^{i\omega t} \quad (33)$$

3.3 Mass transfer coefficient

In non-dimensional form, the rate of mass transfer at the left plate is given by

$$S_h = \left(\frac{\partial C}{\partial \eta}\right)_{\eta=-\frac{1}{2}} = \left[\frac{(b_1 - b_2)e^{-\left(\frac{b_1+b_2}{2}\right)}}{2\sinh\left(\frac{b_1-b_2}{2}\right)}\right]e^{i\omega t} \quad (34)$$

where,

$$a_1 = \frac{\lambda P_r + \sqrt{\lambda^2 P_r^2 + 4a^2}}{2}, \quad a_2 = \frac{\lambda P_r - \sqrt{\lambda^2 P_r^2 + 4a^2}}{2},$$

$$b_1 = \frac{\lambda S_c + \sqrt{\lambda^2 S_c^2 + 4b^2}}{2}, \quad b_2 = \frac{\lambda S_c - \sqrt{\lambda^2 S_c^2 + 4b^2}}{2},$$

$$c_1 = \frac{\lambda + \sqrt{\lambda^2 + 4c^2}}{2}, \quad c_2 = \frac{\lambda - \sqrt{\lambda^2 + 4c^2}}{2},$$

$$A_1 = \frac{G_r}{2\sinh\left(\frac{a_1-a_2}{2}\right)(a_1^2 - \lambda a_1 - c^2)(a_2^2 - \lambda a_2 - c^2)},$$

$$A_2 = \frac{G_m}{2\sinh\left(\frac{b_1-b_2}{2}\right)(b_1^2 - \lambda b_1 - c^2)(b_2^2 - \lambda b_2 - c^2)},$$

$$A_3 = -B_1\left[B_2 + A_1(B_4 - B_8) + A_2(B_6 - B_{10})\right],$$

$$A_4 = B_1\left[B_3 + A_1(B_5 - B_9) + A_2(B_7 - B_{11})\right]$$

$$B_1 = \frac{1}{(1-c_2 h)e^{\left(\frac{c_1-c_2}{2}\right)} - (1-c_1 h)e^{-\left(\frac{c_1-c_2}{2}\right)}},$$

$$B_2 = \frac{PR_e}{c^2}\left[(1-c_2 h)e^{-\frac{c_2}{2}} - e^{\frac{c_2}{2}}\right],$$

$$B_3 = \frac{PR_e}{c^2}\left[(1-c_1 h)e^{-\frac{c_1}{2}} - e^{\frac{c_1}{2}}\right],$$

$$B_4 = e^{-\left(\frac{a_1+a_2-c_2}{2}\right)}(a_2 - a_1)(a_1 + a_2 - \lambda - ha_1 a_2 - c^2 h),$$

$$B_5 = e^{-\left(\frac{a_1+a_2-c_1}{2}\right)}(a_2 - a_1)(a_1 + a_2 - \lambda - ha_1 a_2 - c^2 h),$$

$$B_6 = e^{-\left(\frac{b_1+b_2-c_2}{2}\right)}(b_2 - b_1)(b_1 + b_2 - \lambda - hb_1 b_2 - c^2 h),$$

$$B_7 = e^{-\left(\frac{b_1+b_2-c_1}{2}\right)}(b_2 - b_1)(b_1 + b_2 - \lambda - hb_1 b_2 - c^2 h),$$

$$B_8 = (1-c_2 h)e^{-\frac{c_2}{2}}\left\{(a_2^2 - \lambda a_2 - c^2)e^{\left(\frac{a_1-a_2}{2}\right)} - (a_1^2 - \lambda a_1 - c^2)e^{-\left(\frac{a_1-a_2}{2}\right)}\right\},$$

$$B_9 = (1-c_1 h)e^{-\frac{c_1}{2}}\left\{(a_2^2 - \lambda a_2 - c^2)e^{\left(\frac{a_1-a_2}{2}\right)} - (a_1^2 - \lambda a_1 - c^2)e^{-\left(\frac{a_1-a_2}{2}\right)}\right\},$$

$$B_{10} = (1-c_2 h)e^{-\frac{c_2}{2}}\left\{(b_2^2 - \lambda b_2 - c^2)e^{\left(\frac{b_1-b_2}{2}\right)} - (b_1^2 - \lambda b_1 - c^2)e^{-\left(\frac{b_1-b_2}{2}\right)}\right\},$$

$$B_{11} = (1-c_1 h)e^{-\frac{c_1}{2}}\left\{(b_2^2 - \lambda b_2 - c^2)e^{\left(\frac{b_1-b_2}{2}\right)} - (b_1^2 - \lambda b_1 - c^2)e^{-\left(\frac{b_1-b_2}{2}\right)}\right\}.$$

4. RESULTS AND DISCUSSION

In order to have a physical insight into the problem, the numerical calculations for the velocity, temperature and the concentration fields, and the coefficient of skin friction, rate of heat transfer and rate of mass transfer at the plates are performed by assigning by some specific values to the physical parameter involved in the problem namely rotation parameter Ω, Reynolds number R_e, injection/suction parameter λ, slip-flow parameter h, Hartman number M, Hall parameter m, thermal Grashof number G_r, solutal Grashof number G_m, Prandtl number P_r, Schmidt number S_c, radiation parameter N, pressure gradient P, the frequency of oscillation ω and time t. The values of Schmidt number S_c are chosen to be 0.3, 0.6 and 0.92 to represent Helium, Water Vapour and Carbon dioxide respectively. The numerical results are demonstrated through different graphs and results are interpreted.

Figures 2 to 4 demonstrate the behaviour of the primary fluid velocity distribution against the normal coordinate η under the influence of rotation parameter Ω, Schmidt number S_c and mass Grashof number G_m. It is observed from Figure 2 that there is a steady fall in primary fluid velocity for increasing values of the rotation parameter Ω. Figure 3 indicates the fact that an increase in Schmidt number S_c decelerates the fluid flow marginally. In other words high mass diffusivity causes the velocity to increase.
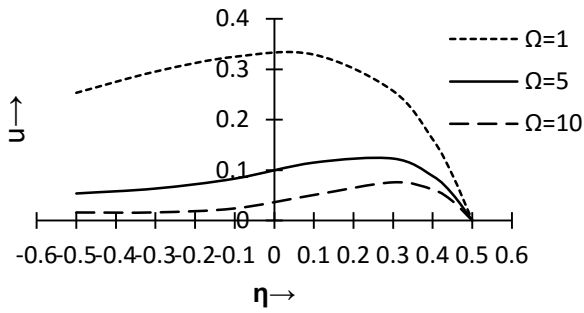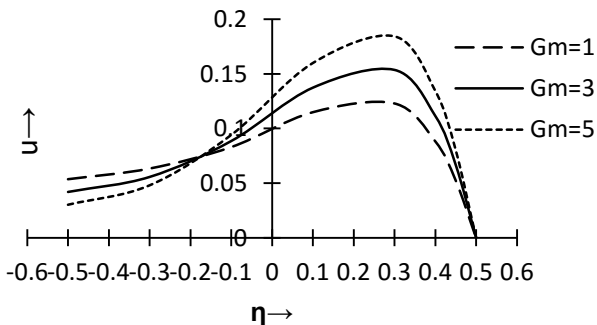
Figure 2. Primary velocity distribution versus η for $P_r=0.71$, $S_c=0.3$, $R_e=1$, $N=1$, $M=1$, $P=5$, $h=1$, $m=1$, $G_r=1$, $G_m=1$, $\lambda=1$, $\omega=5$, $t=0$
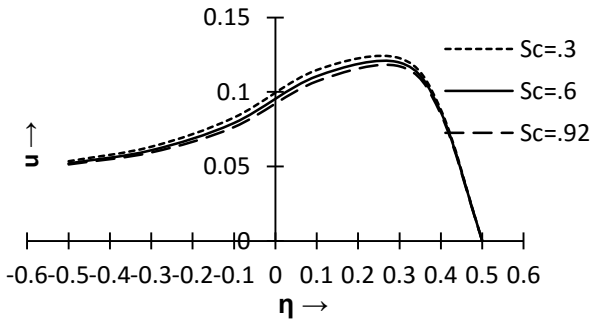
Figure 3. Primary velocity distribution versus η for $Pr=0.71$, $R_e=1$, $N=1$, $M=1$, $P=5$, $h=1$, $m=1$, $G_r=1$, $G_m=1$, $\lambda=1$, $\omega=5$, $\Omega=5$, $t=0$

An interesting behaviour of fluid velocity under solutal Grashof number G_m is observed in Figure 4. It is observed in this Figure that the flow is accelerated due to solutal buyonacy force near the right wall and opposite trend of behaviour is marked near the left wall.

Figure 4. Primary velocity distribution versus η for $_{PR}=0.71$, $S_c=0.3$ $R_e=1$, $N=1$, $M=1$, $P=5$, $h=1$, $m=1$, $G_r=1$, $\lambda=1$, $\Omega=5$, $t=0$

The variations in the secondary velocity v versus Ω, G_m and η are presented in Figures 5 and 6. It is inferred from Figure 5 that there is a substantiall fall in the magnitude of the secondary fluid velocity under the effect of rotation

parameter Ω where as Figure 6 shows that an increase in solutal Grashof number G_m results in steady increase in the secondary velocity.
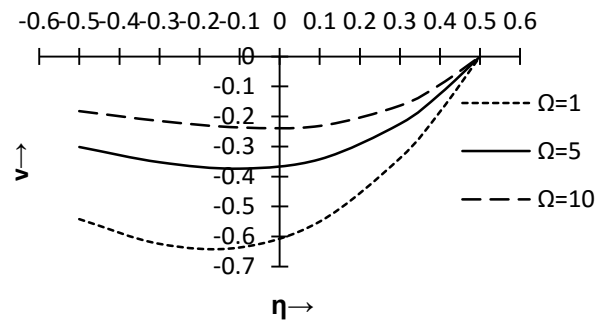

Figure 5. Secondary velocity distribution versus η for $P_r=0.71$, $S_c=0.3$, $R_e=1$, $N=1$, $M=1$, $P=5$, $h=1$, $m=1$, $G_r=1$, $G_m=1$, $\lambda=1$, $\omega=5$, $t=0$
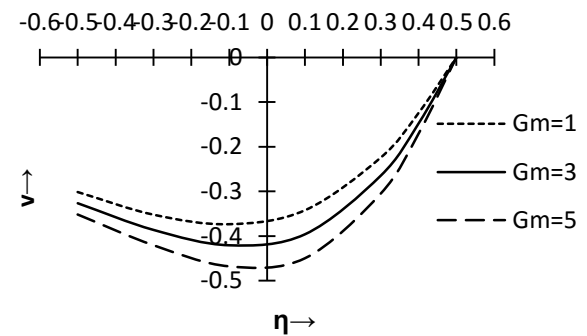
Figure 6. Secondary velocity distribution versus η for $Pr=0.71$, $S_c=0.3$, $R_e=1$, $N=1$, $M=1$, $P=5$, $h=1$, $m=1$, $G_r=1$, $\lambda=1$, $\omega=5$, $\Omega=5$, $t=0$

Figures 7 to 9 demonstrate the behaviour of concentration field under the effects λ, S_c and R_e. From these figures we see that there is steady fall in concentration for the increasing values of λ, S_c and R_e. In other words concentration boundary layer rises for enhancement of mass diffusivity and viscosity.
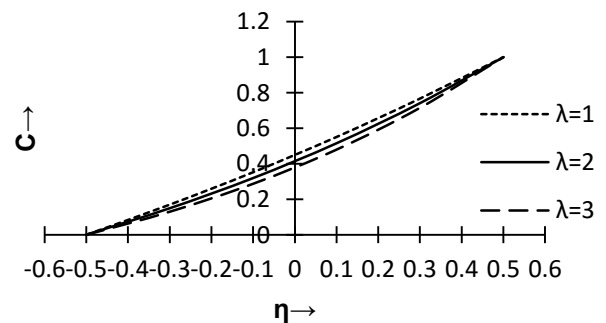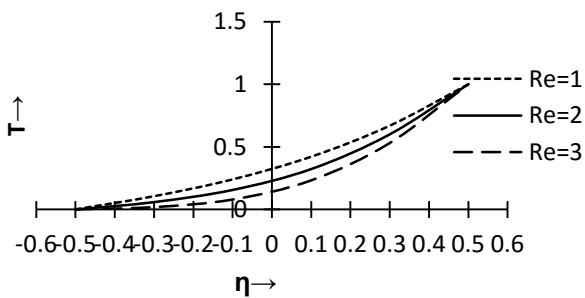

Figure 7. Concentration distribution versus η for $S_c=0.3$, $R_e=1$, $\omega=5$, $t=0$
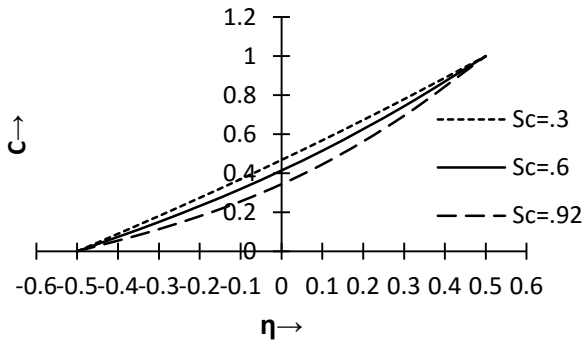
Figure 8. Concentration distribution versus η for Re=1, λ=1, ω=5, t=0
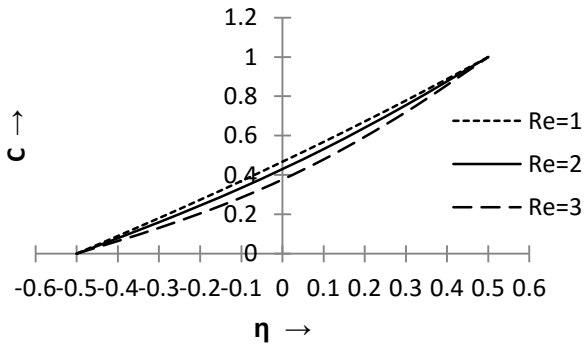
Figure 9. Concentration distribution versus η for S_c=0.3, λ=1, ω=5, t=0

**Figure 10.** Temperature distribution versus η for P_r=0.71, N=1, λ=1, ω=5, t=0

The effect of Reynolds number R_e on temperature field against η is displayed in figure 10. It is marked from this figure that the temperature boundary layer decline with the increase of R_e.

Figure 11 presents the variation of skin friction ζ_x due to primary velocity versus frequency parameter ω under the influence on R_e. The figure 11 shows wavy character of the skin friction ζ_x under ω. It is also observed in figure 11 that the magnitude of ζ_x rises as R_e increases. We recall that R_e increases means viscosity drag falls gradually as fluid friction increases. Further it is noticed that for large values ω the effect of R_e on ζ_x is not so pronounced.
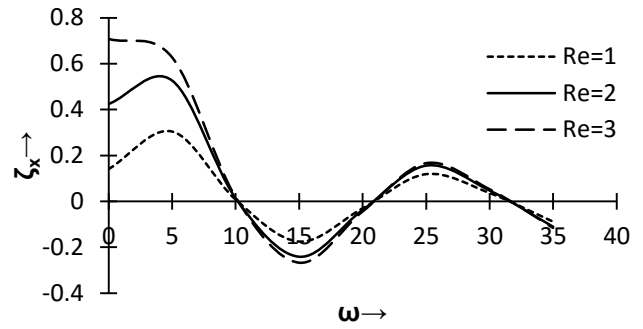

Figure 11. Primary skin friction versus ω for P_r=0.71, S_c=0.3, N=1, M=1, P=5, h=1, m=1, G_r=1, G_m=1, λ=1, Ω=5, t=0.3

The effect of R_e on ζ_y, the skin friction due to secondary velocity is demonstrated in figures 12. This figure shows that the change of behaviour of ζ_y under R_e and ω are analogous to the corresponding behaviour of ζ_x under the same parameters. That is magnitude of ζ_y increases as R_e increases. Further, figure 12 registers the fact that ζ_y is not influenced by R_e for large ω.
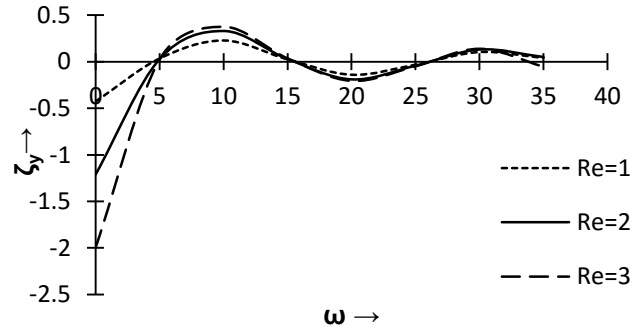
Figure 12. Secondary skin friction versus ω for $_{PR}$=0.71, S_c=0.3, N=1, M=1, P=5, h=1, m=1, G_r=1, G_m=1, λ=1, Ω=5, t=0.3
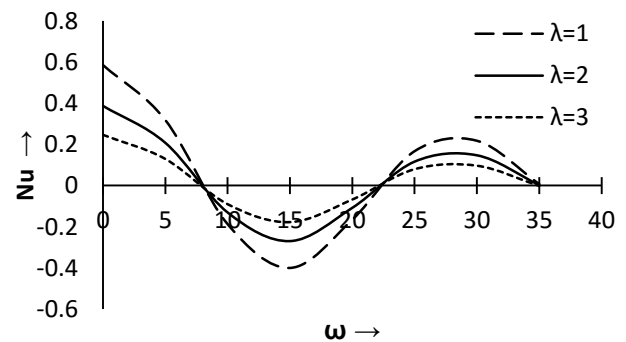
Figure 13. Nusselt number versus ω for P_r=0.71, R_e=1, N=1, t=0.3

The wavy character of behaviour of the Nusselt number Nu like ζ_x and ζ_y under ω are reflected in the figure 13. This figure depicts that the magnitude of Nusselt number rises as injection / suction parameter λ decreases. In other words injection / suction causes of heat transfer to decrease substantially.

Figures 14 and 15 illustrate the variation in Sherwood number under the effect of S_c and R_e. It is marked from this figures that curves are wavy as like figures 15 to 20 and for increasing values of S_c or R_e, magnitude of Shewood number Sh increases. In other words rate of mass transfer decreases for enhancement of mass diffusivity and viscosity. Further it is noticed that for small values of ω, the effect of S_c and R_e is almost unpronounced.

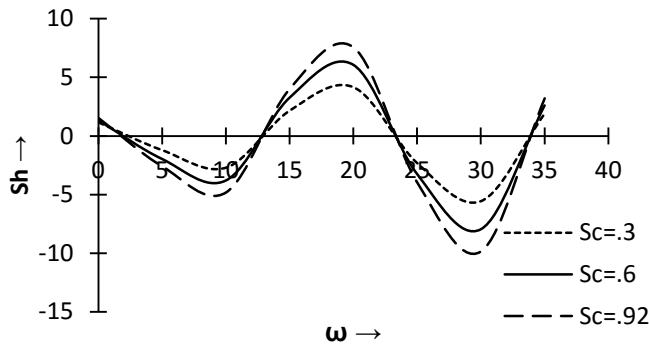**Figure14.** Sherwood number versus ω for R_e=1, λ=1, t=0.3

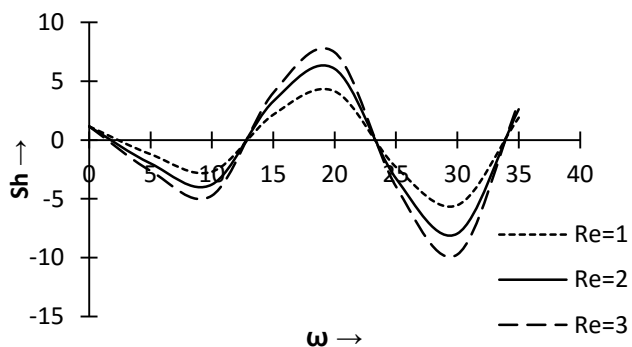**Figure 15.** Sherwood number versus ω for S_c=0.3, λ=1, t=0.3

8. CONCLUSIONS

1. High mass diffusivity accelerates the flow.
2. There is a substantial fall in the magnitude of the secondary velocity under the effect of rotation.
3. Thickness of the concentration boundary layer gets enhanced due to mass diffusivity as well as internal friction of the fluid.
4. The pressure gradient has significant contribution for increasing the viscous drag in the mean flow direction.
5. The magnitude of Sherwood number increases for increasing values of S_c or R_e
6. The study of the present problem finds wide scope of applications in civil engineering, chemical engineering, a mechanical engineering, geophysics, astrophysics etc.

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NOMENCLATURE

$\vec{B}$	magnetic induction vector,
B_o	magnetic inductance, Tesla
C_o	mean concentration, K. mol. m^{-3}
C_p	specific heat at constant pressure, J. kg^{-1}. K^{-1}
C'	dimensional concentration, K. mol. m^{-3}

D	molar mass diffusivity, $m^2 . s^{-1}$		u'	x – component of $\vec{V}$, $m. s^{-1}$
$\vec{E}$	electric field,		u	non dimensional velocity in x direction
e	electron charge, ampere		v'	y – component of $\vec{V}$, $m. s^{-1}$
G_m	solutal Grashof number,		v	non dimensional velocity in y direction
G_r	thermal Grashof number,		w_o	injection/suction velocity, $m. s^{-1}$
g	acceleration due to gravity, $m. s^{-2}$		$\vec{\Omega}$	angular velocity of the field,
h	slip flow parameter,		Ω'	uniform angular velocity, sec^{-1}
$\vec{J}$	current density vector,		Ω	rotation parameter,
k	thermal conductivity, $W. m^{-1}. K^{-1}$		ρ	density, $kg. m^{-3}$
M	Hartmann number,		μ	coefficient of viscosity, $kg. (m. s)^{-1}$
m	Hall parameter,		υ	kinematic viscosity, $m^2. s^{-1}$
N	radiation parameter,		σ	electrical conductivity, $(ohm. m)^{-1}$
P	pressure gradient, $N. m^{-3}$		ω_e	electron frequency, sec^{-1}
P_r	Prandtl number,		τ_e	electron collision time, sec
p	pressure, $N. m^{-2}$.		η_e	number density of electron, m^{-3}
p_e	electron pressure,		λ	injection/suction parameter,
q	radiative heat, $W. m^{-2}$		ω'	dimensional frequency of oscillation, sec^{-1}
R_e	Reynolds number,		ω	frequency of oscillation.
S_c	Schmidt number,		β	volumetric coefficient of thermal expansion, K^{-1}
T_o	mean temperature, K		$\bar{\beta}$	volumetric coefficient for solutal expansion, $(K. mol)^{-1}$
T'	dimensional temperature, K			
t'	time, sec			
U	mean axial velocity, $m. s^{-1}$			