



Dispatching Rules for Minimizing Deviation from JIT Schedule Using the Earliness - Tardiness Scheduling Problem with Due Windows Approach

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ABSTRACT

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In Earliness-Tardiness (E/T) scheduling approach, the Just-In-Time (JIT) schedule is a schedule with zero earliness and zero tardiness. However, this is an optimal schedule and even notional in some instances where tardiness and earliness are inevitable. However, minimizing the deviation at the upper region (tardiness) and the lower region (earliness) from the JIT schedule is a challenge. This work proposes solutions. Two proposed heuristics; TA1 and TA2 as well as some existing heuristics were explored to solve simulated problems ranging from $5 \leq n \leq 400$ and the results obtained were benchmarked against the JIT schedule. The results obtained show that one of the heuristics, TA2 yielded JIT schedules for many problem sizes at the lower and upper deviation than other solution methods.

1. INTRODUCTION

There exist global economic meltdown and most production firms are faced with the challenges of optimizing profit. Therefore, minimizing sources of leakages and income losses like overproduction, high inventory cost, waiting and down tool time is a prerequisite. Just-In-Time (JIT) production has proved to be an essential requirement of world-class manufacturing concept [1]. This has made researchers to explore different variants of Earliness-Tardiness (E/T) scheduling problems to support the realization of JIT environment characterized by zero earliness, zero tardiness and zero inventories. However, JIT schedule is either an optimal schedule which is NP hard or even notional schedule and thus deviation is inevitable. In a due window approach, the due date has three components: The earliest due date, the original due date and the latest due date. The interval between the earliest and latest due date is called the due window [2]. This work proposes solution methods to minimize the deviation from the JIT schedule using the E/T scheduling problem with due window approach.

2. LITERATURE REVIEW

In a general scheduling system, penalty is usually associated with tardy and late jobs while early jobs are compensated. However, this is not always valid. Akande and Ajisegiri [3] discussed extensively three classes of Earliness – Tardiness scheduling problems and highlighted the variation of associated penalty with early jobs. The class three, as defined by the authors, associated penalty with both early and the tardy jobs. This is the concept of JIT production system. There is a global growing interest in JIT production, because the system ensures that all the jobs are completed at exactly due date or

within the due windows and thus zero inventory is achieved [4]. Though several researchers have explored the Earliness-Tardiness scheduling problems but literature is sparse in which the problem is used to measure the deviation from the JIT schedule. This is the basis for this work. Nevertheless, Sourd [5] explored a dynamic programming procedure for solving the scheduling problems of minimizing the penalties associated with not delivering on time (JIT) and the idleness cost using the deviation in earliness–tardiness function. Also, authors like [6, 7] considered the problem of minimizing the weighted earliness and tardiness on a single machine. The dynamic variant of E/T scheduling problems was explored by Mazzini and Armentano [8] and the results obtained were used as a benchmark by Oyetunji and Oluleye [9] for two proposed two heuristics named ETA1 and ETA2. In this work, static variant of E/T scheduling problem with the due windows approach is considered and explored to measure the deviation from the JIT schedule. Liman et al; Janiak and Marek; Zhu, et al. [10-12] among others are some of researchers that have also explored various functions of due windows for scheduling problems.

3. MATERIALS AND METHOD

3.1 Assumptions and notations

The assumptions made in solving the problem are outlined for clarity as follows:

- The problem is deterministic with each job has a known processing time and a due window of known size.
- The problem is subjected to a static constraint.
- The machine is available continually and can process a job at a time with all the jobs are available at the same time.

iv. Pre-emption is not allowed, and all the data are integer. Some notations are also employed, and they are presented in Table 1.

Table 1. Notations used for solving the problem

I	Job position for $i = 1, 2, \dots, n$
N	Number of Jobs
p_i	Processing time of job i
C_i	Completion time of Job i
d_i	Original Due date of Job i
D_i^e	Earliest due date of job i
D_i^l	Latest due date of job i
E_i	Earliness of Job i
T_i	Tardiness of Job i
L_i	Lateness of Job i
N_{JIT}	Number of Just in Time Jobs
UD	Upward Deviation
DD	Downward Deviation
DOD	Degree of Deviation
NSG	Deviation value less than 1 (unit)
SG	Deviation value greater than 1 (unit)
SPT	Shortest Processing Time
MDD	Modified Due Date
FCFS	First Come First Schedule
EDD	Earliest Due Date

3.2 Problem definition

Consider an automobile servicing firm with only one technician working on a service bay and one service advisor for customers scheduling and appointment. For every job completed after the allocated due window given to the customer, there is always a penalty either in terms of goodwill or reduction in the service charge. Also, for all the jobs completed before the due window, there is an associated cost with parking the car (inventory cost) [13, 14]. Therefore, a system where the completed vehicle is released to the customer at the point of completion will eliminate not only the inventory cost but also the penalty cost associated with lateness or tardiness. Such a system is a typical JIT system. At this point, it is expected that $d_i = C_i$.

The total number of jobs completed within the due window is given by

$$N_{JIT} = \sum_{i=1}^n JIT_i \quad (1)$$

where

$$JIT_i = \begin{cases} 1 & \text{if } d_i = C_i \\ 0 & \text{Otherwise} \end{cases} \quad (2)$$

If JIT system is achieved, for $i = 1, 2, \dots, n$

$$d_i = C_i \quad (3)$$

$$n = N_{JIT} = \sum_{i=1}^n JIT_i \quad (4)$$

Also, the tardiness of each jobs, i and total tardiness of all the jobs will be given by

$$T_i = \max \{0, (C_i - d_i)\} = 0 \quad (5)$$

$$T_{tot} = \sum_{i=1}^n T_i = 0 \quad (6)$$

Similarly, the earliness of each jobs, i and the total earliness of all the jobs will be given by

$$E_i = \max \{-L_i, 0\} = 0 \quad (7)$$

$$E_{tot} = \sum_{i=1}^n E_i = 0 \quad (8)$$

However, it is not feasible to always attain JIT condition, thus

$$n \gg N_{JIT} \quad (9)$$

$$T_i \geq 0 \quad (10)$$

$$T_{tot} \geq 0 \quad (11)$$

$$E_i \geq 0 \quad (12)$$

$$E_{tot} \geq 0 \quad (13)$$

However, the target is to minimize these inevitable deviations which are the upward curve deviation associated with the tardiness (jobs completed after the due date) and the downward curve deviation associated with the earliness (job completed earlier than the due dates). Therefore, the deviation in JIT can be described as a piecewise function with the T_{tot} in the upper region domain and the E_{tot} in the lower region domain. This condition is illustrated in Figure 1.

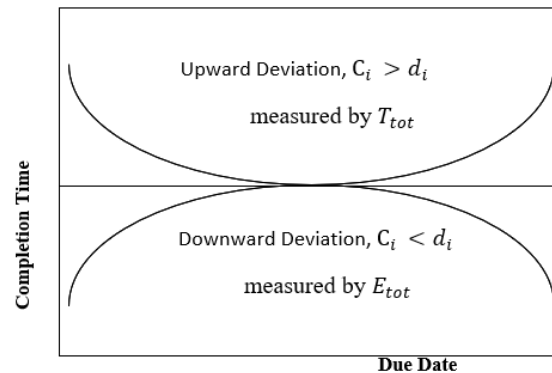


Figure 1. Completion time against due date to show deviation from JIT

Therefore, if the total tardiness (upward deviation) and the total earliness (downward deviation) are minimized simultaneously with respect to the equilibrium or JIT values, then the total deviation is also minimized. The two solution methods proposed are hereby discussed.

3.3 Proposed solution methods

The steps of the heuristics are stated as follows.

TA1 ALGORITHM: This algorithm is based on the theorem stated below:

When solving the $\sum_{i=1}^n E_i + \sum_{i=1}^n T_i$ for any two jobs say j and k , there exists an optimal sequence for which j appear before k if the following conditions hold:

$$I. P_j \leq P_k \quad (14)$$

$$II. D_j \leq D_k \quad (15)$$

For due windows, this corollary is still valid.

If the Eqns. (14) and (15) are valid then, it can be deduced that: $P_j + D_j \leq P_k + D_k$.

Therefore, the steps of the TA1 heuristics are outlined as follows:

STEP 1: Compute the factor time (P+D) for the jobs in job set A.

STEP 2: Arrange the jobs set A in order of increasing Factor Time (P+D) and put the same in Job set C. If there is a tie set, break the tie arbitrarily.

STEP 3: Set $i=1$ where i is the index number of the job in job set C and update Job Set B with job, i and remove the same job from Job set C. If there is a tie, update Job Set B with the job that has the lowest due date among the tie jobs. Continue until all the jobs have been updated. Job set B is the required sequence.

STEP 5: Compute the earliness and the tardiness of each of the job in the Job Set B.

STEP 8: Stop

TA 2 ALGORITHM

The statements of the TA2 heuristics are outlined as follows:

STEP 1: Arrange the jobs Set A in the order of increasing Factor Time (P + D) and put the same in Job Set C. if there is a tie break the tie arbitrarily.

STEP 2: Arrange Job Set A using the MDD and put same in job Set B.

STEP 3: Compute the earliness and the tardiness of each of the jobs in the Job set B and Job set C.

STEP 4: Compute the FUNCTION $E_i + T_i$ of each of the jobs in the Job set B and Job set C.

STEP 5: Combine the two schedules by scheduling the job at the same level and with the minimum FUNCTION $E_i + T_i$ and which has not been previously scheduled.

STEP 6: Remove any jobs that existed more than once. Compute the length of the resultant schedule. Called the schedule Job Set C.

STEP 7: If the length of Job Set C is equal to the length of the job set A. Then Job Set C is the required schedule Then go to **step 9**. Else go to **step 8**.

STEP 8: Subtract Job Set C from Job Set A to obtain the jobs that have not been scheduled. Thus, Job Set H = Job Set C – Job Set A.

STEP 9: Arrange job set H at the back of job set C in increasing order of due date. This is called Job set P.

STEP 10: Compute the tardiness and the total tardiness of the optimal sequence schedule.

STEP 11: Stop

3.4 Problem generation for simulation

The utilities of the proposed solution methods were

demonstrated by simulated some single processor scheduling problems using the desktop tool module (editor) from MATLAB R2010 programming language. Equations explored by Suer et al. [15]; Akande and Ajisegiri, [16]; Sunday et al. [17]; Bedhief and Dridi [18]; and Joshi and Satpathy [19] were modified to generate the required parameters which include the processing time and the due windows as follows:

$$D_j = R_j + KP_j \quad (16)$$

$$D_j^{L/E} = D_j \pm (A_j \times D_j) \quad (17)$$

$$D_j^e = D_j - (D_j \times A_j) \quad (18)$$

$$D_j^l = D_j + (D_j \times A_j) \quad (19)$$

where,

D_j^e : is the earliest due date for job j .

D_j : is the original due date.

D_j^l : is the latest due date for job j .

A_j : is the flow allowance assigned to job j at time zero. Is set at (20% - 40% of D_j).

In this simulating experiment, the proposed solution methods as well as the existing ones (SPT, EDD, MDD and FCFS) were tested with the three components of the due windows.

4. RESULTS AND DISCUSSION

The results of the computational experiment are grouped into two classes. Tables 2 – 12 show the results of the total earliness and the tardiness and the Degree of Deviation (DOD) for each of the considered problem sizes for all the solution methods for the three due window components. Table 13, Table 14 and Table 15 show the value of the tardiness for the early due window, original due window, and the latest due window respectively. Table 16, Table 17, and Table 18 show the value of earliness for the early due window, original due window, and the latest due window respectively. However, it should be noted that the value of the earliness and the tardiness for the notional JIT schedule is zero for all the problem sizes. Furthermore, Figure 2, Figure 3, and Figure 4 illustrate the degree of deviation of the total tardiness (Upper deviation) from the notional JIT in the early due window, original due window, and the latest due window respectively, while Figure 5, Figure 6 and Figure 7 show the lower deviation (Total earliness) in the three windows respectively.

Table 2. Results of the total Earliness and Tardiness and the degree of deviation for 5 x 1 problem size

Solution Method	Earliest Due Date				Original Due Date				Latest Due Date			
	$\sum_{i=1}^n T_i$	$\sum_{i=1}^n E_i$	UD	LD	$\sum_{i=1}^n T_i$	$\sum_{i=1}^n E_i$	UD	LD	$\sum_{i=1}^n T_i$	$\sum_{i=1}^n E_i$	UD	LD
SPT	29.65	2.85	SG	NSG	0.00	118.00	JIT	SG	0.00	35.85	JIT	SG
MDD	28.35	0.55	SG	NSG	0.00	8.00	JIT	SG	0.00	35.85	JIT	SG
TA1	28.35	0.55	SG	NSG	0.00	117.00	JIT	SG	0.00	35.85	JIT	SG
TA2	28.35	0.55	SG	NSG	0.00	47.00	JIT	SG	0.00	35.85	JIT	SG
FCFS	35.35	0.55	SG	NSG	12.00	15.00	SG	SG	3.50	34.35	SG	SG
EDD	30.35	0.55	SG	NSG	0.00	117.00	JIT	SG	0.00	35.85	JIT	SG

Table 3. Results of the total Earliness and Tardiness and the degree of deviation for 10 x 1 problem size

Solution Method	Earliest Due Date				Original Due Date				Latest Due Date					
	$\sum_{i=1}^n T_i$		$\sum_{i=1}^n E_i$		DOD		DOD		$\sum_{i=1}^n T_i$		$\sum_{i=1}^n E_i$		DOD	
			UD	LD			UD	LD			UD	LD		
SPT	135.0	0.96	SG	NSG	9.0	158.00	SG	SG	82.0	25.60	SG	SG		
MDD	135.0	0.95	SG	NSG	29.0	3.00	SG	SG	67.0	6.20	SG	SG		
TA1	139.0	0.95	SG	NSG	0.00	118.00	JIT	SG	73.0	8.35	SG	SG		
TA2	135.0	0.95	SG	NSG	29.0	3.00	SG	SG	67.0	6.20	SG	SG		
FCFS	225.0	6.65	SG	SG	20.0	32.00	SG	SG	162.0	42.80	SG	SG		
EDD	139.0	0.95	SG	NSG	0.00	104.00	JIT	SG	80.0	6.20	SG	SG		

Table 4. Results of the total Earliness and Tardiness and the degree of deviation for 15 x 1 problem size

Solution Method	Earliest Due Date				Original Due Date				Latest Due Date					
	$\sum_{i=1}^n T_i$		$\sum_{i=1}^n E_i$		DOD		DOD		$\sum_{i=1}^n T_i$		$\sum_{i=1}^n E_i$		DOD	
			UD	LD			UD	LD			UD	LD		
SPT	350.0	0.00	SG	JIT	107.0	107.0	SG	SG	203.0	14.70	SG	SG		
MDD	350.0	0.00	SG	JIT	200.0	7.0	SG	SG	1.97.0	6.00	SG	SG		
TA1	359.0	0.00	SG	JIT	105.0	40.0	SG	SG	222.0	3.15	SG	SG		
TA2	350.0	0.00	SG	JIT	118.0	34.0	SG	SG	197.0	6.00	SG	SG		
FCFS	543.0	6.30	SG	SG	196.0	25.0	SG	SG	401.0	26.80	SG	SG		
EDD	366.0	0.00	SG	JIT	118.0	34.0	SG	SG	244.0	2.75	SG	SG		

Table 5. Results of the total Earliness and Tardiness and the degree of deviation for 20 x 1 problem size

Solution Method	Earliest Due Date				Original Due Date				Latest Due Date					
	$\sum_{i=1}^n T_i$		$\sum_{i=1}^n E_i$		DOD		DOD		$\sum_{i=1}^n T_i$		$\sum_{i=1}^n E_i$		DOD	
			UD	LD			UD	LD			UD	LD		
SPT	8.08e+2	1.25	SG	SG	2.97e+2	147.00	SG	SG	3.67e+2	5.50	SG	SG		
MDD	8.07e+2	0.00	SG	JIT	5.23e+2	2.00	SG	SG	3.69e+2	1.75	SG	SG		
TA1	8.30e+2	0.00	SG	JIT	3.03e+2	50.00	SG	SG	3.94e+2	4.80	SG	SG		
TA2	8.07e+2	0.00	SG	JIT	4.39e+2	13.00	SG	SG	3.69e+2	1.75	SG	SG		
FCFS	11.89e+2	0.00	SG	JIT	4.49e+2	28.00	SG	SG	6.83e+2	12.25	SG	SG		
EDD	8.71e+2	0.00	SG	JIT	3.47e+2	41.00	SG	SG	4.07e+2	1.75	SG	SG		

Table 6. Results of the total Earliness and Tardiness and the degree of deviation for 40 x 1 problem size

Solution Method	Earliest Due Date				Original Due Date				Latest Due Date					
	$\sum_{i=1}^n T_i$		$\sum_{i=1}^n E_i$		DOD		DOD		$\sum_{i=1}^n T_i$		$\sum_{i=1}^n E_i$		DOD	
			UD	LD			UD	LD			UD	LD		
SPT	2.84e+3	0.00	SG	JIT	2.29e+3	159.00	SG	SG	1.64e+3	1.40	SG	SG		
MDD	2.84e+3	0.00	SG	JIT	2.98e+3	3.00	SG	SG	1.64e+3	0.35	SG	NSG		
TA1	2.89e+3	0.00	SG	JIT	2.64e+3	32.00	SG	SG	1.70e+3	0.35	SG	NSG		
TA2	2.84e+3	0.00	SG	JIT	2.87e+3	6.00	SG	SG	1.64e+3	0.35	SG	NSG		
FCFS	4.00e+3	5.40	SG	SG	3.31e+3	24.00	SG	SG	2.70e+3	17.15	SG	SG		
EDD	2.96e+3	0.00	SG	JIT	2.76e+3	25.00	SG	SG	1.73e+3	0.35	SG	NSG		

Table 7. Results of the total Earliness and Tardiness and the degree of deviation for 50 x 1 problem size

Solution Method	Earliest Due Date				Original Due Date				Latest Due Date					
	$\sum_{i=1}^n T_i$		$\sum_{i=1}^n E_i$		DOD		DOD		$\sum_{i=1}^n T_i$		$\sum_{i=1}^n E_i$		DOD	
			UD	LD			UD	LD			UD	LD		
SPT	3790	0.95	SG	NSG	3360	144	SG	SG	3020	3.05	SG	SG		
MDD	3790	0.00	SG	JIT	4430	7.00	SG	SG	3010	0.35	SG	NSG		
TA1	3910	0.00	SG	JIT	3990	29.00	SG	SG	3170	0.35	SG	NSG		
TA2	3790	0.00	SG	JIT	4340	14.00	SG	SG	3010	0.35	SG	NSG		
FCFS	6120	0.00	SG	JIT	5340	29.00	SG	SG	4640	20.1	SG	SG		
EDD	4050	0.00	SG	JIT	4300	26.00	SG	SG	3270	0.35	SG	NSG		

Tables 2-12 reveal that the two proposed performance measures yielded better results than the selected existing solution methods. This is because the results of TA1 and TA2 yielded a more optimal JIT schedule in the three windows at both the upper deviation (total Tardiness) and the lower

deviation (Total earliness) in the three windows components. Also, FCFS yielded the worst results among all the solution methods for each of the problem sizes. However, to measure the performance of the solution over the range of the considered problem sizes (5-400), Tables 13-18 also shows the

values of the total earliness and the tardiness at each of the window's components. Figure 2 - Figure 7 also shows the deviation graphically.

Figure 2 shows that FCFS has the highest upper deviation.

The SPT also has a higher deviation especially for the lower number of jobs. As the number of jobs increases, the performance of SPT, EDD, and the two proposed solution methods (TA1 and TA2) coincides.

Table 8. Results of the total Earliness and Tardiness and the degree of deviation for 100 x 1 problem size

Solution Method	Earliest Due Date				Original Due Date				Latest Due Date			
	$\sum_{i=1}^n T_i$	$\sum_{i=1}^n E_i$	DOD		$\sum_{i=1}^n T_i$	$\sum_{i=1}^n E_i$	DOD		$\sum_{i=1}^n T_i$	$\sum_{i=1}^n E_i$	LCOF	
			UD	LD			UD	LD			UD	LD
SPT	17900	0.30	SG	NSG	16400	260.00	SG	NSG	16200	1.05	SG	SG
MDD	17900	0.00	SG	JIT	20200	1.00	SG	NSG	16204	0.35	SG	NSG
TA1	18500	0.00	SG	JIT	19400	6.00	SG	NSG	17100	0.35	SG	NSG
TA2	17900	0.00	SG	JIT	19900	4.00	SG	NSG	16200	0.35	SG	NSG
FCFS	25900	4.50	SG	SG	23300	17.00	SG	NSG	22600	23.0	SG	SG
EDD	19300	0.00	SG	JIT	20200	4.00	SG	NSG	17400	0.35	SG	NSG

Table 9. Results of the total Earliness and Tardiness and the degree of deviation for 150 x 1 problem size

Solution Method	Earliest Due Date				Original Due Date				Latest Due Date			
	$\sum_{i=1}^n T_i$	$\sum_{i=1}^n E_i$	DOD		$\sum_{i=1}^n T_i$	$\sum_{i=1}^n E_i$	DOD		$\sum_{i=1}^n T_i$	$\sum_{i=1}^n E_i$	DOD	
			UD	LD			UD	LD			UD	LD
SPT	3.819e+4	0.95	SG	NSG	3.606e+4	312	SG	SG	3.344e+4	2.80	SG	SG
MDD	3.819e+4	0.00	SG	JIT	4.386e+4	1.00	SG	SG	3.343e+4	0.35	SG	NSG
TA1	3.924e+4	0.00	SG	JIT	4.176e+4	1.00	SG	SG	3.500e+4	0.35	SG	NSG
TA2	3.819e+4	0.00	SG	JIT	4.386e+4	1.00	SG	SG	3.343e+4	0.35	SG	NSG
FCFS	5.468e+4	8.80	SG	SG	5.152e+4	13.00	SG	SG	5.092e+4	46.15	SG	SG
EDD	4.067e+4	0.00	SG	JIT	4.417e+4	1.00	SG	SG	3.596e+4	0.35	SG	NSG

Table 10. Results of total Earliness and Tardiness and the degree of deviation for 200 x 1 problem size

Solution Method	Earliest Due Date				Original Due Date				Latest Due Date			
	$\sum_{i=1}^n T_i$	$\sum_{i=1}^n E_i$	DOD		$\sum_{i=1}^n T_i$	$\sum_{i=1}^n E_i$	DOD		$\sum_{i=1}^n T_i$	$\sum_{i=1}^n E_i$	DOD	
			UD	LD			UD	LD			UD	LD
SPT	6.679e+4	0.30	SG	NSG	6.051e+4	356.00	SG	SG	6.721e+4	0.35	SG	NSG
MDD	6.679e+4	0.00	SG	JIT	7.728e+4	2.00	SG	SG	6.721e+4	0.35	SG	NSG
TA1	6.860e+4	0.00	SG	JIT	7.335e+4	15.00	SG	SG	7.095e+4	0.35	SG	NSG
TA2	6.679e+4	0.34	SG	NSG	7.689e+4	5.00	SG	SG	6.721e+4	0.35	SG	NSG
FCFS	9.690e+4	5.35	SG	SG	9.220e+4	19.00	SG	SG	9.815e+4	33.65	SG	SG
EDD	7.134e+4	0.00	SG	JIT	7.734e+4	8.00	SG	SG	7.279e+4	0.35	SG	NSG

Table 11. Results of total Earliness and Tardiness and the degree of deviation for 300 x 1 problem size

Solution Method	Earliest Due Date				Original Due Date				Latest Due Date			
	$\sum_{i=1}^n T_i$	$\sum_{i=1}^n E_i$	DOD		$\sum_{i=1}^n T_i$	$\sum_{i=1}^n E_i$	DOD		$\sum_{i=1}^n T_i$	$\sum_{i=1}^n E_i$	DOD	
			UD	LD			UD	LD			UD	LD
SPT	1.661e+5	0.30	SG	NSG	1.560e+5	303.00	SG	SG	1.605e+5	4.80	SG	SG
MDD	1.661e+5	0.30	SG	NSG	1.870e+5	0.00	SG	JIT	1.605e+5	0.00	SG	JIT
TA1	1.715e+5	0.00	SG	JIT	1.812e+5	0.00	SG	JIT	1.673e+5	0.35	SG	NSG
TA2	1.661e+5	0.00	SG	JIT	1.870e+5	0.00	SG	JIT	1.605e+5	0.00	SG	JIT
FCFS	2.342e+5	4.45	SG	SG	2.157e+5	24.00	SG	SG	2.267e+5	29.50	SG	SG
EDD	1.779e+5	0.00	SG	JIT	1.894e+5	0.00	SG	JIT	1.712e+5	0.35	SG	NSG

Table 12. Results of total Earliness and Tardiness and the degree of deviation for 400 x 1 problem size

Solution Method	Earliest Due Date				Original Due Date				Latest Due Date			
	$\sum_{i=1}^n T_i$	$\sum_{i=1}^n E_i$	DOD		$\sum_{i=1}^n T_i$	$\sum_{i=1}^n E_i$	DOD		$\sum_{i=1}^n T_i$	$\sum_{i=1}^n E_i$	DOD	
			UD	LD			UD	LD			UD	LD
SPT	2.677e+5	0.00	SG	JIT	2.708e+5	304	SG	SG	2.749e+5	3.05	SG	SG
MDD	2.677e+5	0.00	SG	JIT	3.240e+5	0.00	SG	JIT	2.749e+5	0.35	SG	NSG
TA1	2.750e+5	0.00	SG	JIT	3.161e+5	1.00	SG	SG	2.876e+5	0.35	SG	NSG
TA2	2.677e+5	0.00	SG	JIT	3.240e+5	0.00	SG	JIT	2.749e+5	0.35	SG	NSG
FCFS	3.846e+5	2.85	SG	SG	3.826e+5	10.00	SG	SG	3.876e+5	28.85	SG	SG
EDD	2.869e+5	0.00	SG	JIT	3.302e+5	1.00	SG	SG	2.946e+5	0.35	SG	NSG

In the original due date window, as revealed in Figure 3, all the solution methods except FCFS yielded the JIT schedule for 5x1 problem size while only EDD, TA1, and TA2 also yielded JIT schedule for 10x1 problem sizes. However, as the job size increases, the upper deviation increase with SPT has the lowest deviation at 400x1 problem sizes.

In the latest due date window, as revealed in Figure 4, all the solution methods except FCFS yielded the JIT schedule for the 5x1 problem size. However, as the job size increases, the upper deviation increases with SPT, MDD, and TA2 coincides

with the lowest deviation at 400x1 problem sizes while FCFS and TA1 has the highest deviation.

The results of lower deviation (Total earliness) were erratic. Though most of the solution method yielded a JIT schedule for most of the problem sizes, the results of FCFS and SPT shows the highest deviation.

Figure 6 reveals that SPT yielded the highest deviation from the JIT schedule for all the problem sizes.

Also, TA1 and EDD show high deviation for problem sizes, $n \leq 50$.

Table 13. The Tardiness value for the Earliest due date window for all the solution methods

Problem size	JIT SCHEDULE	SPT	MDD	EDD	FCFS	TA1	TA2
5X1	0	29.65	28.35	30.35	35.35	28.35	28.35
10X1	0	135	135	139	225	139	135
15 X1	0	350	350	366	543	359	350
20X1	0	808	807	871	1190	830	807
40x1	0	2840	2840	2960	40000	2890	2840
50x1	0	3790	3790	4050	6120	3910	3790
100x1	0	17900	17900	19300	25900	18500	17900
200x1	0	66790	66790	71340	96900	68600	66790
300x1	0	166100	166100	177900	234200	171500	166100
400x1	0	267700	267700	286900	384600	275000	267700

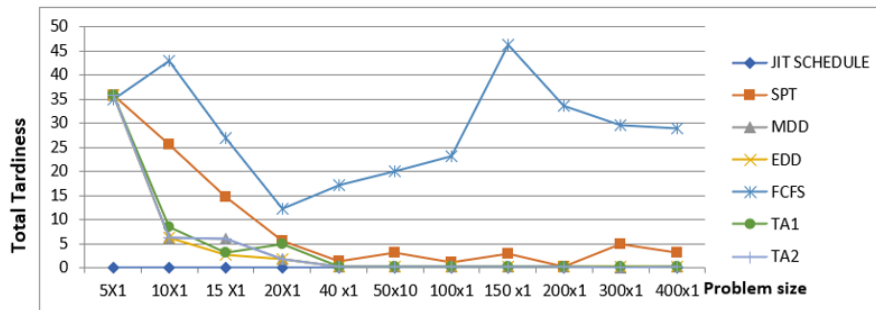


Figure 2. Plot of Total tardiness against the Problem sizes for the early due date window

Table 14. The Tardiness value for the Original due date window for all the solution methods

Problem size	JIT SCHEDULE	SPT	MDD	EDD	FCFS	TA1	TA2
5X1	0	0	0	0	12	0	0
10X1	0	9	29	0	20	0	0
15 X1	0	107	200	118	196	118	105
20X1	0	297	523	347	449	303	439
40x1	0	2290	2980	2760	3310	2640	2870
50x1	0	3790	3790	4050	6120	3910	3790
100x1	0	16400	20200	20200	23300	19400	19900
200x1	0	60510	77280	77340	92200	73350	76890
300x1	0	156000	187000	189400	215700	181200	187000
400x1	0	270800	324000	330200	382600	316100	324000

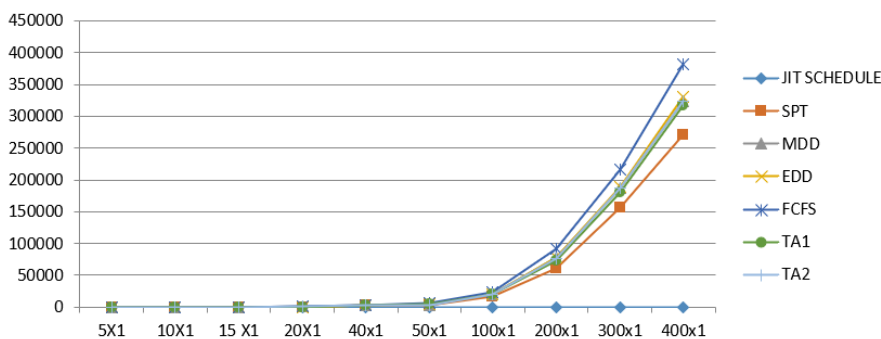


Figure 3. Plot of Total tardiness against the Problem sizes for the original due date window

Table 15. The Tardiness value for the Latest due date window for all the solution methods

Problem size	JIT SCHEDULE	SPT	MDD	EDD	FCFS	TA1	TA2
5X1	0	0	0	0	3.5	0	0
10X1	0	82	67	80	162	73	67
15 X1	0	203	197	244	401	222	197
20X1	0	367	369	407	683	394	369
40 x1	0	1640	1640	1730	2700	1700	1640
50x10	0	3020	3010	3270	4640	3170	3010
100x1	0	16200	16200	17400	22600	17100	16200
150 x1	0	33440	33430	35960	50920	35000	33430
200x1	0	67210	67210	72790	98150	150000	67210
300x1	0	160500	160500	171200	226700	167300	160500
400x1	0	274900	274900	294600	387600	287600	274900

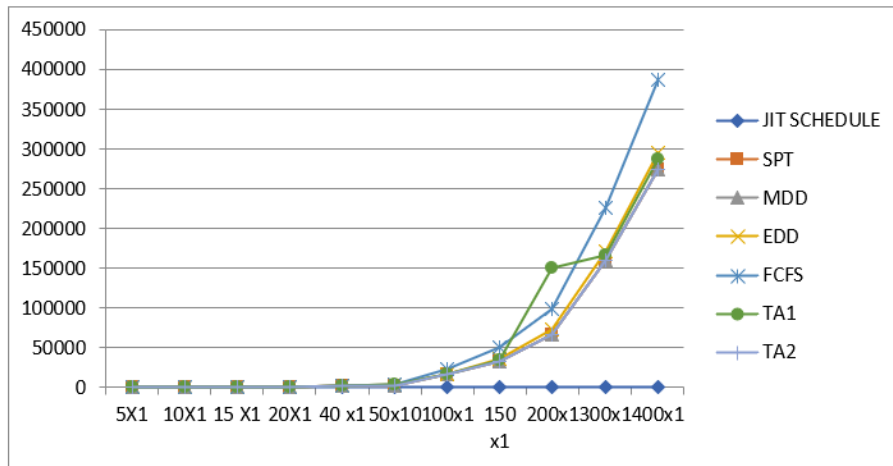


Figure 4. Plot of Total tardiness against the Problem sizes for the latest due date window

Table 16. The total Earliness value for Early Due Date Earliness for all the solution methods

Problem size	JIT SCHEDULE	SPT	MDD	EDD	FCFS	TA1	TA2
5X1	0	2.85	0.55	0.55	0.55	0.55	0.55
10X1	0	0.96	0.95	0.95	6.65	0.95	0.95
15 X1	0	0	0	0	6.3	0	0
20X1	0	1.25	0	0	0	0	0
40x1	0	0	0	0	5.4	0	0
50 x 1	0	0.95	0	0	0	0	0
100 x 1	0	0.3	0	0	4.5	0	0
150 x 1	0	0.95	0	0	8	0	0
200 x 1	0	0.3	0	0	5.35	0	0.34
300 x 1	0	0.3	0.3	0	4.45	0	0
400 x 1	0	0	0	0	2.85	0	0

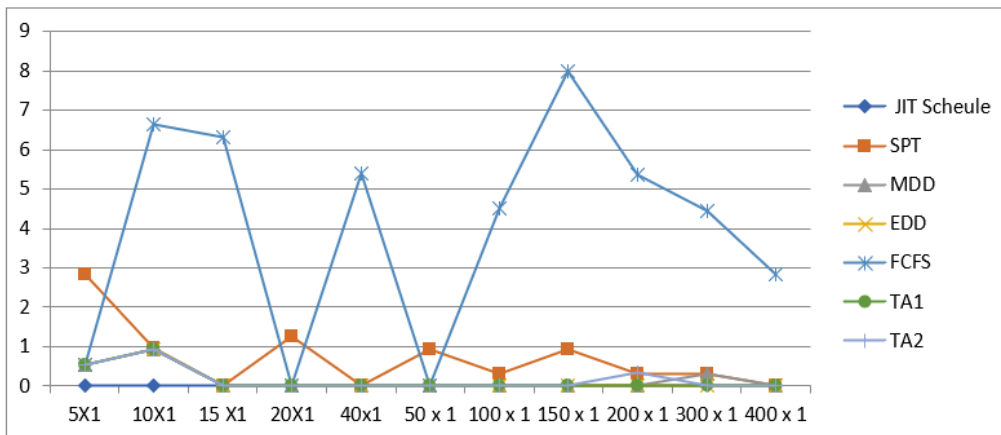


Figure 5. Plot of Total earliness against the Problem sizes for the early due date window

Table 17. The Earliness value for the Original due date window for all the solution methods

Problem size	JIT Schedule	SPT	MDD	EDD	FCFS	TA1	TA2
5X1	0	118	8.00	117	15	117.0	47
10X1	0	158.0	3.0	104	32	118	3.0
15 X1	0	107	7.00	34	25	40	34
20X1	0	147	2.00	41	28	50	13
40 x1	0	159	3.00	25.00	24.00	32.00	6.00
50x10	0	144.00	7.00	26.00	29.00	29.00	14.00
100x1	0	260.00	1.00	4.00	17.00	6.00	4.00
150 x1	0	312	1.00	1.00	13.00	1.00	1.00
200x1	0	356.00	2.00	8.00	19.00	15.00	5.00
300x1	0	303.00	0.00	0.00	24.00	0.00	0.00
400x1	0	304	0.00	10.00	1.00	1.00	0.00

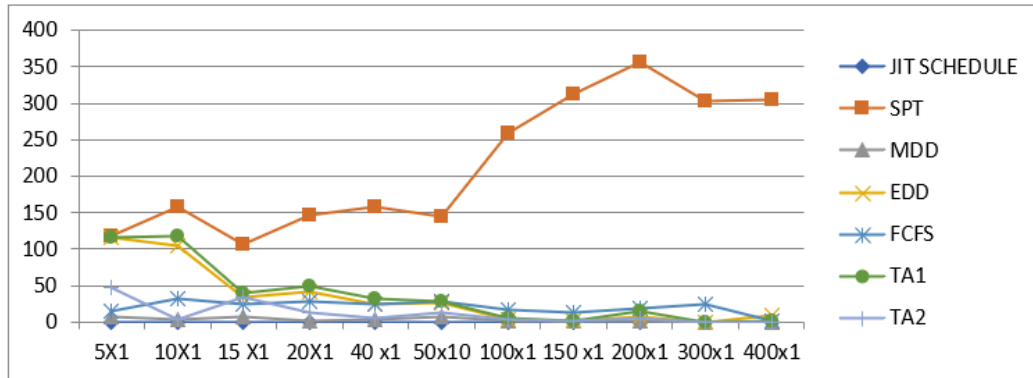


Figure 6. Plot of Total earliness against the Problem sizes for the original due date window

Table 18. The total earliness value for the Latest Due Date window for all the solution methods

Problem size	JIT SCHEDULE	SPT	MDD	EDD	FCFS	TA1	TA2
5X1	0	35.85	35.85	35.85	34.85	35.85	35.85
10X1	0	25.6	6.2	6.2	42.8	8.35	6.2
15 X1	0	14.7	6	2.75	26.8	3.15	6
20X1	0	5.5	1.75	1.75	12.25	4.8	1.75
40 x1	0	1.4	0.35	0.35	17.15	0.35	0.35
50x10	0	3.05	0.35	0.35	20.1	0.35	0.35
100x1	0	1.05	0.35	0.35	23.05	0.35	0.35
150 x1	0	2.8	0.35	0.35	46.15	0.35	0.35
200x1	0	0.35	0.35	0.35	33.65	0.35	0.35
300x1	0	4.8	0	0.35	29.5	0.35	0
400x1	0	3.05	0.35	0.35	28.85	0.35	0.35

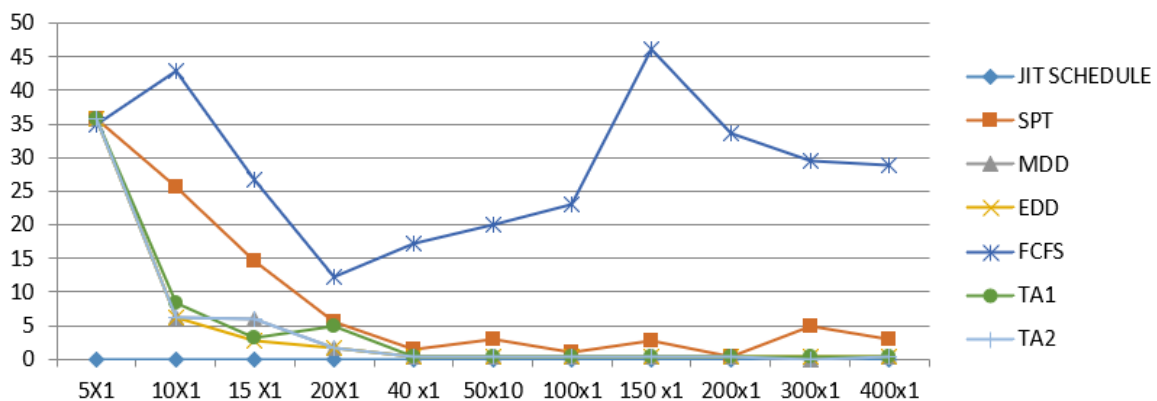


Figure 7. Plot of Total earliness against the Problem sizes for the latest due date window

Figure 7 reveals that FCFS and SPT yielded the highest deviation from the JIT schedule for all the problem sizes. Also, other solution methods show higher deviation at problem sizes;

$n \leq 40$. As the problem sizes exceeded this limit, the deviation becomes not significant for all the solution methods except FCFS and SPT.

5. CONCLUSIONS

This work proposed two heuristics for Earliness-Tardiness scheduling problems as a means of measuring the deviation of a schedule from the notional JIT Schedule. The results obtained revealed that one of the proposed heuristics TA2 yielded a lower deviation at both the upper and the lower region compared to other proposed heuristics (TA1) and other selected solution methods from the literature.

The work can be explored further by expressing the results of the total earliness and tardiness in composite function using any form of the mathematical expression be it linear, quadratic, or arbitrary equations. However, normalization must be carried for the LCOF using the notional JIT as the benchmark to avoid unbalanced and skewed normalized results.

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