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New Class of M-Polar Fuzzy Measure Ideals Algebra in *BCK2/BCK1/BCI2*

Shadia Majeed Noori[1,2*], Abd Ghafur Ahmad[1]

[1] School of Mathematical Sciences, Faculty of Science and Technology, Universiti Kebangsaan Malaysia, 43600 UKM Bangi, Selangor, Malaysia
[2] Department of Mathematics, College of Education for Pure Science, University of Tikrit, Tikrit 34001, Iraq

Corresponding Author Email: shadianoori1980@gmail.com

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ABSTRACT

In this work, we introduced the concepts of fuzzy measure algebra of the *M*-polar electrode ambiguous ideals, and many of them have been investigated properties. Characterizations of the blurry M-polar measure sub-algebra and fuzzy (commutative) ideals of polarity are also looked at. Also, the relationships between *M*-polar fuzzy measure subalgebras, and M-polar ambiguous and ambiguous pole reciprocal ideals have been discussed. A new Concepts suggested here can be expanded to different types of ideals in *BCK2*, *BCK1* and *BCI2*-algebras, for instance, a-ideal, implicated, n-fold and n-fold ideals, and commutative ideals. Besides, the properties of *BCK2* (resp, *BCK1* and *BCI2*) M-polar fuzzy measure algebra are discussed. Finally, the study also investigates the relationships between the mysterious *BCK2* (resp, *BCK1* and *BCI2*) M-polar fuzzy measure ideal. Some examples related to it are also given.

1. INTRODUCTION

BCK/BCI-algebras first appeared in the mathematical literature in 1966, as a ramification of general algebra, in work by Iséki and were later formalized in other works [1]. In order to arrive at these concepts, two distinct methodologies were used: propositional calculi and set theory. *BCK/BC I*-algebras are algebraic patterns of the BCK/BCI-system, which are used in combinatory logic. The name *BCK/BC I*-algebras is derived from the use of the combinatories *B*, *C*, *K*, and I in combination to form the algebraic structure [1].

Chen et al. [2] expanded the view of bipolar fuzzy groups to get the idea of polar M fuzzy groups and confirmed that polar fuzzy groups and dipolar fuzzy groups are cryptographic mathematical tools. Multipolar information, the theory goes, is consistent with the evolution of value pickers.

BCI/BCK-algebras have been studied by Liu et al. [3] who have demonstrated the extension property of *BCI*-implicative ideals and described implicative *BCI* algebras in detail. Borzooei et al. [4] have researched the topic of generalized neutrosophic and suggested a novel concept. Similarly, Jun et al. [5] analysed a neutrosophic quadruple *BCK/BCI*-number in the context of an established collection.

Al-Masarwah [6] considered the ideal theory of *BCK/BCI*-algebras, defining and exploring several features. A similar study by Al-Masarwah & Ahmad [7] revealed that these ideals are related to doubt bipolar fuzzy H-ideals. Al-Masarwah [8] supported this, mentioning that bipolar fuzzy H-beliefs with specific homes typically play a crucial position withinside the shape concept of a *BCK/BCI* algebra. Also, *BCK/BCI* algebraic notions of homomorphic preimages, and doubt images, were studied by Al-Masarwah & Ahmad [9].

A unified derivation of summation, multiplication, and complex numbers in quantum theory was offered by Skilling & Knuth [10]. Akram [11] addressed the homomorphisms between Lie subalgebras, as well as how they relate to the domains and codomains of M-polar fuzzy Lie subalgebras. According to Ghorai and Pal [12], M-polar fuzzy planar graphs have features that allow for edge crossings that are not allowed in a crisp planar graph as shown in Figure 1. Furthermore, to characterize the relationships between individuals, Ghorai and Pal [13] used M-polar fuzzy set theory as well as to formulate these graphs. Also, an arc of an m-polar fuzzy graph tree is only strong if it is an M-polar fuzzy graph bridge, according to Mandal et al. [14]. In same regard, on topological surfaces, Mandal et al. [15] discussed isomorphism features of the M-polar fuzzy genus graph, as well as an application of this graph. Moreover, Farouk et al. [16] employed the view of the M-polar group to fuzzy graph theory.

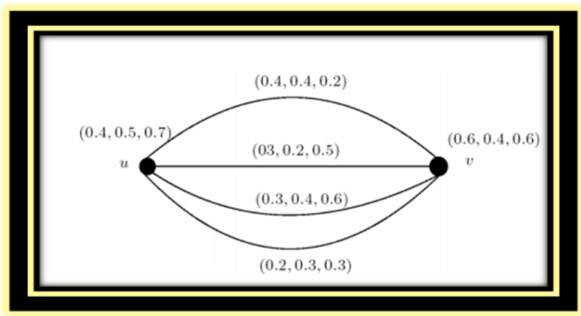

Figure 1. 3-polar fuzzy graph [12]

The current study discusses an idea for perfect M-polar fuzzy scaling groups with BCK2 (resp, BCK1 and BCI2)-algebras, and introduces concepts for fuzzy M-polar scaling algebras. Then, it investigates several properties and gives M-polar descriptions of fuzzy algebra and the fuzzy (mutual) ideals of the pole. Their relations are also considered. Finally, the study combines the ideas of M-polar haze clusters and M-polar haze points to introduce a new concept in BCK2, BCK1, and BCI2-algebras termed M-polar (α, β)-ambiguous ideals.

2. PRELIMINARIES

Definition 2.1.[8]

A functional $\mu: T \rightarrow R^{+}$ is called a σ-additive measure if whenever a set $A \in T$ is a disjoint union of an at most countable sequence $\{A_k\}_{k=1}^{M}$ (where N is either finite or M = ∞) then $u(A)=\sum_{k=1}^{M} u(A_k)$. If M = ∞ Then the above sum is understood as a string. If this property applies only to the finite values of M, then μ is a final additive measure.

Definition 2.2.[9]

If X represents a universe of discourse, then A represents a fuzzy set A that is characterized by a membership function that accepts values in the range [0, 1].

Definition 2.3.[4]

Let $J \neq \varnothing \subseteq F$, where F is BCK/BCI algebra. Then J is a sub algebra of F if $\forall \zeta, \eta \in J$ then $\zeta * \eta \in J$.

Definition 2.4.[4]

Let $J \neq \varnothing \subseteq F$, where F is BCK/BCI algebra. Then J is an ideal of F If it achieves:

1) $0 \in F$
2) $\forall \zeta, \eta \in F, \zeta * \eta \in J, \eta \in J \Longrightarrow \zeta \in J$.

Definition 2.5.[11]

Let $F \neq \varnothing$. An M-polar fuzzy set G on F is a map $\psi: F \rightarrow [0,1]^{z}$. Then, $\forall \zeta \in F$ is characterized by:

$\psi(\zeta)=(P_1 \circ \psi(\zeta), P_2 \circ \psi(\zeta), \ldots, P_z \circ \psi(\zeta))$

where $P_k \circ \psi(\zeta):[0,1]^{z} \rightarrow [0,1]$ is identified as the k-th function of projection.

3. BCK2, BCK1 AND BCI2 IN M-POLAR FUZZY MEASURE SUB-ALGEBRAS

Three concepts of BCK2, BCK1 and BCI2 are given in fuzzy measure algebra and with a study of its most prominent characteristics.

Definition 3.1.

Let $J \neq \varnothing \subseteq F$, where F is fuzzy measure algebra. Then J is a BCK2-sub algebra of F if $\zeta * \eta \in J \; \forall \zeta, \eta \in J$.

Definition 3.2.

Let $J \neq \varnothing \subseteq F$, where F is fuzzy measure algebra. Then J is a BCK1-sub algebra of F if $\eta \in J, \zeta * \eta \in J \; \forall \zeta \in J$.

Definition 3.3.

Let $J \neq \varnothing \subseteq F$, where F is fuzzy measure algebra. Then J is a BCI2-sub algebra of F if $\eta \in J, (\zeta * \eta) * \zeta \in J \; \forall \zeta \in J$.

Definition 3.4.

Let $J \neq \varnothing \subseteq F$, where F is BCK2, BCK1 and BCI2 fuzzy measure algebra. Then, J is an ideal of F If it achieves:

1) $0,1 \in F$
2) $\forall \zeta, \eta \in F, \zeta * \eta \in J, \eta \in J \Longrightarrow \zeta \in J$.

Definition 3.5.

Let $F \neq \varnothing$. An M-polar fuzzy measure set ψ on F is a mapping $\psi: F \rightarrow [0,1]^{z}$. The membership value of $\forall \zeta \in F$ is defined by:

$\psi(\zeta)=(P_1 \circ \psi(\zeta), P_2 \circ \psi(\zeta), \ldots, P_z \circ \psi(\zeta))$

where, $P_k \circ \psi(\zeta):[0,1]^{z} \rightarrow [0,1]$ is identified as the k-th function of projection.

Definition 3.6.

A fuzzy measure effect algebra is a system (F,M,O, u,⊕) consisting of a set F,M is fuzzy measure on bolean algebra, special elements 0_F called the zero and the unit respectively, and a totally defined binary operation ⊕ on F, called the ortho sum if for all $h, l, ƛ \in F$:

If $h \oplus l$ and $(h \oplus l) \oplus ƛ$ are defined, then $l \oplus ƛ$ and $p \oplus (l \oplus ƛ)$ are defined and $h \oplus (l \oplus ƛ) = (l \oplus q) \oplus ƛ$.

If $h \oplus l$, then $h \oplus l = l \oplus h$, also $l \oplus h$ is fuzzy. $\forall h \in F$, there is a unique $l \in F$ such that $h \oplus l$ is fuzzy and $h \oplus l = u$.

If $h \oplus u$ is fuzzy defined, then h = 0_F.

Definition 3.7.

Consider the case of $\check{\Theta}$ an M-polar fuzzy measure. Set of F is referred to as an M-polar fuzzy measure sub-algebra if and only if the following conditions are met:

$\forall \mu, \nu \in F \; (\check{\Theta}(\mu * \nu)) \geq inf \{\check{\Theta}(\mu), \check{\Theta}(\nu)\}$,

where $\check{\Theta}(\mu)$, $\check{\Theta}(\nu)$ are fuzzy measure point of μ and y. So $\forall \mu, \nu \in F$.

$p_i \circ \check{\Theta}(\mu * \nu) \geq inf \{p_i \circ \check{\Theta}(\mu), p_i \circ \check{\Theta}(\nu)\} \; \forall i = 1, 2 \ldots, \zeta$.

Example 3.8.

Let F= {0, ι,κ} be BCK2,BCK1 and BCI2- fuzzy measure algebra.

Define a mapping $\check{\Theta}: F \longrightarrow [0,1]^{3}$ by:

$\check{\Theta}(\mu)$={((0.1,0.6,0.7) if μ=0 @(0.3,0.4,0.5) if μ=ι @(0.4,0.5,0.2) if μ=κ)┤

Theorem 3.9.

Assume $\check{\Theta}$ is an M-polar fuzzy measure set of F. Also, $\check{\Theta}$ is an M-polar fuzzy measure sub-algebra of F if $\check{\Theta}_{[\check{\sigma}]} \neq \varnothing$ is a fuzzy measure sub- algebra of F for all σ̌ 〖= {σ_1, σ_2…, σ_m} ∈ [0,1] 〗 ^m.

Proof. Let $\check{\Theta}$ is an M-polar fuzzy measure sub-algebra of F and

σ̌ 〖 ∈[0,1]〗 ^m be $\check{\Theta}_{[\check{\sigma}]} \neq \varnothing$. Let $\mu, \nu \in \check{\Theta}_{[\check{\sigma}]}$.

Then $\check{\Theta}(\mu) \geq \tilde{\sigma}$. It follows that $\check{\Theta}(\mu * \nu) \geq inf \{\check{\Theta}(\mu), \check{\Theta}(\nu)\} \geq \tilde{\sigma}$, so that $\mu * \nu \in \check{\Theta}_{[\check{\sigma}]}$. Therefore $\check{\Theta}_{[\check{\sigma}]}$ is a fuzzy measure sub-algebra of F.

Vise versa, assume that $\check{\Theta}_{[\check{\sigma}]}$ is a fuzzy measure sub-algebra of F. Suppose that ∃

ι, κ ∈ F s.t, $\check{\Theta}(\iota * \kappa) < inf \{\check{\Theta}(\iota), \check{\Theta}(\kappa)\}$. Thus ∃ σ̌ 〖= {σ_1, σ_2…, σ_m} ∈ [0,1] 〗 ^m

s.t, $\check{\Theta}(\iota * \kappa) < \tilde{\sigma} \leq inf \{\check{\Theta}(\iota), \check{\Theta}(\kappa)\}$.

Hence ι, κ∈Θˇ_ [σˇ], but ι,κ∉Θˇ_[σˇ] and its contradiction, so that Θˇ(μ*ν)≥ inf {Θˇ(μ),Θˇ(ν)},∀ μ,ν∈F. Thus Θˇis an M-polar fuzzy measure sub- algebra of F.

Lemma 3.10.

Let M-polar fuzzy measure sub-algebra Θˇof F satisfies the following inequality:

∀ μ∈F, Θˇ(0) ≥Θˇ(μ)

Proof: Since μ*μ=0 ∀ μ ∈ F. So,

Θˇ(0)=Θˇ(μ*μ)≥inf {Θˇ(μ),Θˇ(μ)}=Θˇ(μ) ∀ μ∈F.

Proposition 3.11.

If M-polar fuzzy measure sub-algebra Θˇof F satisfies:

∀ μ, ν∈F, Θˇ(μ*ν) ≥Θˇ(ν), then Θˇ(x)=Θˇ(0).

Proof. Let μ∈F, so Θˇ(μ)≥Θˇ(μ*0) ≥Θˇ(0). Thus Θˇ(μ)=Θˇ(0).

Definition 3.12.

An M-polar fuzzy measure set Θˇof F is named an M-polar fuzzy measure ideal if satisfies:

∀ μ, ν∈F, (p_i ∘Θˇ(0) ≥p_i ∘Θˇ(x)≥inf {p_i ∘Θˇ(μ*ν), p_i ∘Θˇ(ν)})

∀ i= 1, 2..., ζ.

Example 3.13.

Let F= {0, ι,8,9} be BCK2, BCK1 and BCI2-fuzzy measure algebra with Cayley table defines a mapping Θˇ: F⟶ [0,1] ^3 by:

Θˇ(μ)={((0.6,0.7,0.4) if μ=0 @(0.3,0.5,0.6) if μ=ι,8 @(0.1,0.4,0.5) if μ=9)┤

then Θˇis a 3-polar fuzzy measure ideal of F. because for any M-polar fuzzy measure set

Θˇon F and σˇ〖= {σ_1, σ_2…, σ_m} ∈ [0,1]〗^m, the following satisfies:

Table 1. BCK2, BCK1 and BCI2-*-operation

*	0	a	1	2
0	0	0	2	2
a	a	0	2	1
1	1	1	0	2
2	2	2	0	1

Proposition 3.14.

If Θˇis an M-polar fuzzy measure ideal of F:

∀ μ, ν∈F,μ≤ν⇒Θˇ(μ)≥Θˇ(ν).

Proof. Let μ, ν ∈ F be s.t, μ ≤ ν. Then μ * ν=0 and so

Θˇ(μ)≥inf {Θˇ(μ * ν),Θˇ(ν)}=inf {Θˇ(0),Θˇ(ν)}=Θˇ(ν). Thus Θˇ(μ)≥Θˇ(ν).

Theorem 3.15.

Let ω ∈F. If Θˇis an M-polar fuzzy measure ideal of F, then F_ω is a fuzzy measure ideal of F.

Proof. Let ω ∈F_ω. Let μ, ν ∈F be s.t, μ * ν∈F_ω and ν ∈F_ω.

Then Θˇ(μ * ν) ≥Θˇ(ω) and Θˇ(ν)≥Θˇ(ω). Since Θˇis an M-polar fuzzy measure ideal of F, so Θˇ(μ)≥inf {Θˇ(μ * ν), Θˇ(ν)} ≥Θˇ(ω), ω ∈F_ω. Hence, F_ω is a fuzzy measure ideal of F.

Proposition 3.16.

Assume Θˇ is an M-polar fuzzy measure ideal of F. If F satisfies then:

∀ μ, ν,ξ∈F,μ * ν≤ξ,

then Θˇ(μ)≥inf {Θˇ(ν), Θˇ(ξ)} ∀ μ,ν,ξ∈F.

Proof.

μ * ν≤ξ is satisfied in F ∀ μ,ν,ξ∈F, . So

Θˇ(μ * ν)≥inf {Θˇ((μ * ν) *ξ),Θˇ(ξ)}=inf {Θˇ(0),Θˇ(ξ)}=Θˇ(ξ) ∀ μ,ν,ξ∈F.

It follows that Θˇ(μ)≥inf {Θˇ(μ * ν), Θˇ(ν)} ≥inf {Θˇ(ν), Θˇ(ξ)} ∀ μ, ν,ξ∈F. Therefore,

Θˇ(μ)≥inf {Θˇ(ν),Θˇ(ξ)}

Theorem 3.17.

For any BCK2, BCK1 and BCI2 fuzzy measure algebra F, then each measure ideal is M-polar fuzzy measure sub-algebra.

Proof.

Θˇis an M-polar fuzzy measure ideal of BCK2,BCK1 and BCI2- fuzzy measure algebra F and let μ ,ν ∈ F. Then,

Θˇ(μ * ν)≥inf {Θˇ((μ * ν)*μ),Θˇ(μ)}=inf {Θˇ((μ * μ)*ν),Θˇ(μ)}

=inf {Θˇ(0*ν), Θˇ(μ)} =inf {Θˇ(0), Θˇ(μ)} ≥inf {Θˇ(μ), Θˇ(ν)}. Thus, Θˇis an M-polar fuzzy measure sub-algebra of F.

Example 3.18.

Consider BCK2, BCK1 and BCI2 -fuzzy measure algebra F= {0, ι,κ} which is characterize a 2-polar fuzzy measure set Θˇ:F→[0,1]^2 by:

Θˇ(μ)={((0.2,0.9) if μ=0@(0.5,0.3) if μ=ι)┤

Then Θˇis a 3-polar fuzzy measure sub-algebra of F. But it is not a 2-polar fuzzy measure ideal of F, because Θˇ(μ)= (0.5,0.3) < (0.2,0.9) = inf {Θˇ(ι*κ), Θˇ(κ)}.

Remark 3.19. If F is a BCI2-fuzzy measure algebra, then M-polar fuzzy measure ideal. Then, Θˇ: F⟶ [0,1] ^m by:

Θˇ(μ)={ ((0.3,0.3,0.3) ,μ∈Γ@ (0.1,0.1,0.1), μ∉Γ)┤

Then Θˇis M-polar fuzzy measure ideal of F. And μ= (0,0) and ν= (0,1/5), then ξ=μ * ν= (0,0) *(0,1/5) = (0, -1/5), thus Θˇ(μ * ν) =Θˇ(ξ)= (0.1,0. 1…,0.1) < (0.3,0. 3…,0.3) =inf {Θˇ(μ), Θˇ(ν)}. Hence, Θˇis not measure sub-algebra of F.

Definition 3.20.

Assume that F is a fuzzy measure algebra composed of BCK2, BCK1, and BCI2. The closed state of an M-polar fuzzy measure ideal Θˇof F is achieved when the ideal of F is an M-polar fuzzy measure sub-algebra of F.

Example 3.21.

Let BCK2, BCK1 and BCI2 fuzzy measure algebra, F= {0, ι,8,9} which is define a mapping Θˇ: F⟶ [0,1] ^3 by:

Θˇ(μ)={((0.5,0.6,.0.8) if μ=0 @(0.3,0.4,0.6) if μ=ι,9 @(0.2,0.3,.0.5) if μ=1,8)┤

And, Θˇis an ideal of F with a closed 3-polar fuzzy measure.

Theorem 3.22.

Assume F is a BCK2, BCK1, and BCI2 -fuzzy measure algebra, and that is the M-polar fuzzy measure set of F's:

Θˇ(μ)={(tˇ=(t_1,t_2,…,t_m), μ∈F @sˇ=(s_1,s_2,…,s_m), otherwise)┤

where tˇ, sˇ∈ [0,1] ^m with tˇ>sˇand F= {μ∈ F:0 ≤μ}. Then Θˇis the ideal of F's closed M-polar fuzzy measure.

Proof. Let 0∈F, then Θˇ(0) = tˇ= (t_1, t_2…, t_m) ≥Θˇ(μ) ∀ μ∈F.

Let μ, ν∈F. If μ∈F, then Θˇ(μ)= tˇ= (t_1, t_2…, t_m) ≥inf {Θˇ(μ*ν), Θˇ(ν)}.

Suppose that, μ∉F. If μ * ν∈F, then ν∉F; if ν∈F, then μ, ν ∉F. In other hand,

we get Θˇ(μ)= sˇ= (s_1, s_2…, s_m) ≥inf {Θˇ(μ * ν), Θˇ(ν)}. For any μ, ν∈F, if

μ or ν∉ F, then Θˇ(μ * ν) ≥ sˇ= (s_1, s_2…, s_m) = inf {Θˇ(μ), Θˇ(ν)}. If

μ, ν∈F, then μ * ν∈F, and so Θˇ(μ)= tˇ= (t_1, t_2…, t_m) ≥inf {Θˇ(μ), Θˇ(ν)}.

Therefore, Θˇis a closed M- polar fuzzy measure ideal of F.

Proposition 3.23. Specifically, each closed M-polar fuzzy measure ideal(Θ)ˇof a BCK2-fuzzy measure algebra F meets the following conditions:

∀ μ∈F Θˇ(0*μ) ≤Θˇ(μ).

Proof. For any μ ∈ F, we have Θˇ(0*μ) ≤inf {Θˇ(0), Θˇ(μ)} ≤inf {Θˇ(x), Θˇ(μ)} =Θˇ(μ). Therefore, Θˇ(0*μ) ≤Θˇ(μ).

Proposition 3.24. Let F be BCK1 and BCI2-fuzzy measure algebra.

Proof. (μ * ν) *μ≤0*ν ∀ μ, ν∈F. Thus,

Θˇ(μ * ν) ≥inf {Θˇ(μ), Θˇ(0*ν)} ≥inf {Θˇ(μ), Θˇ(ν)}.

So, Θˇ is M-polar fuzzy measure sub-algebra of F and therefore Θˇis a closed M-polar fuzzy measure ideal of F.

4. M-POLAR (A, B)-FUZZY MEASURE IDEALS

Herein, it suggests and discussion this concept M-polar (α, β)- BCK2, BCK1 and BCI2 fuzzy measure ideals, where:

$$\alpha,\beta \in \{\in,\delta,\in \vee\delta,\in \wedge\delta\},\alpha \neq \in \wedge q.$$

Proposition 4-1. Let ℘ be an M-pfm of F, the set 〖℘〗_1≠∅ ∀ ιˆ∈ 〖(0.25,1]〗^m is an ideal of F,

then,

(1) inf {℘ (0), (0.25)ˇ} ≤ ℘(x),

(2) inf{℘(x), (0.25)ˇ} ≤ inf {℘ (x * y), ℘(y)}.

Proof.

〖℘〗_1≠∅ be an ideal of F. Let υ ∈ F such that

sup {℘ (0), (0.25)ˇ} < ℘(ν). Then, ℘(ν) ∈ 〖(0.25,1]〗^m,so υ ∈℘_(℘(ν)),hence

℘(0)>℘(ν), thus 0 ∉℘_(℘(ν)) and it's a contradiction. So that (1) holds.

Now, Assume sup{℘(x), (0.25)ˇ}> inf {℘ (x * y), ℘(y)} =ιˆ for some x, y ∈ F.

So,

ιˆ∈ 〖(0.25,1]〗^m m and y,x * y ∈ 〖℘〗_1.

Let, x∉ 〖℘〗_1 since ℘(x) >ιˆ, a contradiction. Hence, (2) holds.

Assume (1) and (2) hold. And, ιˆ∈ 〖(0.25,1]〗^m be such that 〖℘〗_1≠∅

For any x ∈ ℘, then (0.25)ˇ> ιˆ≤ ℘(x) ≥ sup{℘(x), (0.25)ˇ}. Also, ℘ (0) =

sup{℘(x), (0.25)ˇ} ≤ιˆ. Thus, 0 ∈ 〖℘〗_ιˆ. Let x, y ∈F be such that x * y,y ∈ 〖℘〗_ιˆ.

Therefore, sup{℘(x), (0.25)ˇ} ≤ inf {℘ (x * y), ℘(y)} ≤ιˆ< (0.25)ˇ

hence, ℘(x)= sup{℘(x), (0.25)ˇ} ≤ ιˆ, that is, x ∈ 〖℘〗_ιˆ. Thus 〖℘〗_ιˆis an ideal of F.

Definition 4.2.

Let ℘ be an M-pfm-ideal of F. Then ℘ is named an (α, β)-BCK2-fuzzy measure ideal (M-polar) of F if for all x, y ∈ F and ιˆ, κˆ∈ 〖(0.5,1]〗^m

If x_ιˆα℘ then 〖0.5〗_ιˆβ℘,

If 〖(x * y)〗_ιˆ α℘ and 〖y〗_κˆ α℘ then x_sup{ιˆ,κˆ}β℘.

Definition 4.3.

Let ℘ be an M-pfm-ideal of F. Then ℘ is named an (α, β)-BCK1- fuzzy measure ideal (M-polar) of F if for all x, y ∈F and ιˆ, κˆ∈ 〖(0.5,1]〗^m

If x_ιˆα℘ then 〖0.075〗_ιˆβ℘,

If 〖(x * y)〗_ιˆα℘ and 〖y〗_κˆα℘ then x_inf {ιˆ, κˆ} β℘.

Definition 4.4.

Let ℘ be an M-pfm-ideal of F. Then ℘ is named an (α, β)-BCK2-fuzzy measure ideal (M-polar) of F if for all x, y ∈ F and ιˆ, κˆ∈ 〖(0.5,1]〗^m

If x_ιˆα℘ then 〖0.25〗_ιˆβ℘,

If 〖(x * y)〗_ιˆα℘ and 〖y〗_κˆα℘ then x_inf {ιˆ, κˆ} β℘.

Theorem 4.5.

Let ℘ be an M-pfm-ideal, and:

(1) ℘(x) = (0.5)ˇ, for all x ∉ ξ,

(2) ℘(x) ≥ 0.5, for all x ∈ J.

Then, ℘(x) is an M-polar (α, ∈ ∨q)- BCK2- fuzzy measure ideal of F.

Proof. (1) (For α = q) Let x ∈F and ιˆ∈ 〖(0.5,1]〗^m such that x_ιˆq℘.

Then, ℘(x) +ιˆ> 1ˇ. Since 0.5 ∈ J, so ℘ (0.5) ≥ (0.75)ˇ. If ιˆ ≤ (0.75)ˇ, then

℘ (0.5) ≤ιˆand so 0.5 ∈℘. ιˆ≥ (0.75)ˇ, then ℘ (0.5) +ιˆ<1ˇ.

Hence, 〖0.5〗_ιˆ∈ ∨q℘.

Let x, y ∈ F and ιˆ, κˆ∈ 〖(0.5,1]〗^m be such that 〖(x * y)〗_ιˆq℘ and y_κˆq℘ thus,

℘ (x * y) +ιˆ<1ˇand ℘(y)+ κˆ< 1ˇ.

Therefore x * y, y ∈ J, and x ∈ J,

℘(x) ≤ (0.75)ˇ. If inf {ιˆ, κˆ} ≥ (0.75)ˇ, then ℘(x) ≤ (0.75)ˇ≤ inf {ιˆ, κˆ}

and so, x_inf {ιˆ, κˆ} ∈q℘. If inf {ιˆ, κˆ} < (0.75)ˇ, then ℘(x) +inf {ιˆ, κˆ} < (1)ˇ

and we have x_inf {ιˆ, κˆ} ∈∨q℘. Therefore, ℘(x) is an M-polar (α, ∈ ∨q)- BCK2-fuzzy measure ideal of F.

Theorem 4.6.

Let ℘ M-pfm-ideal, and:

(1) ℘(x) = (0.5)ˇ, for all x ∉ ξ,

(2) ℘(x) ≥ 0.5, for all x ∈ J.

And, ℘(x) is an M-polar (α, ∈ ∨q)-BCK1-fuzzy measure ideal of F.

Proof. (1) (For α = q) Let x ∈F and ιˆ∈ 〖(0.25,0.50]〗^m such that x_ιˆq℘.

Then, ℘(x) +ιˆ< (0.50)ˇ. Since 0.25 ∈ J, so ℘ (0.25) ≤

(0.35)ˇ. If ιˆ≥ (0.35)ˇ, then

℘ (0.5) ≤ιˆ and so 0.5 ∈℘. ιˆ≥ (0.75)ˇ, then ℘ (0.25) +ιˆ> (0.50)ˇ.

Hence, 〖0.25〗_ιˆ∈ ∨q℘.

Let x, y ∈ F and ιˆ, κˆ∈ 〖(0.25,0.50]〗^m be such that 〖(x * y)〗_ιˆq℘ and y_κˆq℘ thus,

℘ (x * y) +ιˆ>1ˇand ℘(y)+ κˆ> 1ˇ.

Therefore x * y, y ∈ J, and x ∈ J,

℘(x) ≥ (0.35)ˇ. If inf {ιˆ, κˆ} ≥ (0.35)ˇ, then ℘(x) ≤ (0.35)ˇ≥ inf {ιˆ, κˆ}

and so, x_inf {ιˆ, κˆ} ∈q℘. If inf {ιˆ, κˆ} < (0.75)ˇ, then ℘(x) +inf {ιˆ, κˆ} < (1)ˇ

and we have x_inf {ιˆ, κˆ} ∈∨q℘. Therefore, ℘(x) is an M-polar (α, ∈ ∨q)- BCK1-fuzzy measure ideal of F.

Theorem 4.7

Let ℘ M-pfm-ideal subset of F and ξ be an ideal of F such that

(1) ℘(x) = (0.5)ˇ, for all x ∉ ξ,

(2) ℘(x) ≥ 0.5, for all x ∈ J.

And, ℘(x) is an M-polar (α, ∈ ∨q)- BCI2- fuzzy measure ideal of F.

Proof. As same as Theorem 4.5.

Example 4.8.

Let F= {0,1,2, c,d} be BCK2, BCK1 and BCI2-fuzzy measure algebra with Cayley in Table 2.

Table 2. BCK2, BCK1 and BCI2-*-operation under **FUZZY MEASURE IDEALS**

*	0	1	2	c	d
0	0	0	0	d	d
1	1	0	1	c	d
2	2	2	0	d	d
c	c	d	c	0	1
d	d	d	d	1	2

Define a mapping Θˇ: F⟶ [0,1] ^3 by:

℘(x)= {((0.6,0.7,0.8) if μ=0,1 @ ((0.6,0.6,0.6) if μ=c @ (0.9,0.8,0.8) if μ=2 @ (0.6,0.8,0.7) if μ=d @)) ┤

Then, J = {0, d,1} is an ideal of F. Thus, ℘(x) is a 3-polar (α, ∈ ∨q)-BCK2, BCK1 and BCI2-fuzzy measure ideal of F.

5. CONCLUSIONS

In the recent study, novel concepts BCK2, BCK1, and BCI2 based entirely on M-polar fuzzy modules were studied and added. As well as some properties and ideas of the fuzzy algebra M-polar. The descriptions of the fuzzy M-polar sub-algebra and the ambiguous (mutual) beliefs of polarity were studied. In addition, their relationships were discussed. For example, a completely new idea known as m-polar (α, β)-BCK2, BCK1 and BCI2-fuzzy measure algebras was derived and some results related to these concepts were obtained. Finally, some results for the concepts BCK2, BCK1 and BCI2 were obtained.

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