



LQR Control with the New Triple In-Loops Algorithm for Optimization of the Tuning Parameters

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ABSTRACT

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The suspension system plays a role in ensuring the stability of the vehicle when traveling on the road. On many modern vehicles, the active suspension system has been proposed to replace the conventional passive suspension system. The performance of the controller for the active suspension system depends on its control method. In this paper, a half dynamics model of the vehicle is established. Besides, the LQR control method is also used. The parameters of the control matrix are calculated through the triple in-loop optimization algorithm, which has been shown in the research. This is a completely novel algorithm. This algorithm helps to choose the most optimal parameters. Thus, it ensures the efficiency and stability of the controller. The calculation and comparison process are done automatically. When the loop ends, the optimal parameters are explicitly indicated. The simulation process is done in the MATLAB-Simulink environment. The results of the research showed that when the LQR controller, which was optimized through the triple in-loop algorithm used, the vehicle's oscillation was significantly reduced. In the three survey situations, the values of the roll angle and the angular acceleration of the sprung mass are guaranteed to be stable. Besides, when using this controller, the phenomenon of "chattering" after the excitation ends does not appear. This topic can be further developed in the future.

1. INTRODUCTION

1.1 Oscillation of the vehicle

The oscillation of the vehicle is one of the important issues. It can affect the stability and smoothness of the vehicle. Besides, the problems of oscillation are also related to the durability and longevity of the vehicle. There are many causes of oscillation, in which, the stimulation from the road surface is the direct cause of this phenomenon.

The suspension system is an important part of the vehicle, and it helps to regulate and extinguish the oscillations transmitted from the road surface to the vehicle body. Normally, the suspension system will consist of three elements corresponding to their three functions: the elastic element (spring, leaf spring), the oscillation damping element (damper), and the guiding element (lever arm). The suspension system separates the vehicle into two parts. All assemblies located above the suspension are called sprung masses. In contrast, what lies below the suspension system is known as the unsprung mass. These masses have a great influence on the vehicle's oscillation [1]. In today's popular vehicles, passive suspension (mechanical) is often used. For this suspension system, the stiffness of the spring and damper is unchanged. Therefore, in many special cases, the smoothness and comfort of the vehicle may not be guaranteed. To improve the system's instability, several modern suspension systems that are controlled automatically have been proposed. On some high-

end models, the air suspension system has been equipped. For this system, the stiffness of the air spring can be changed continuously. This change depends on the traveling conditions of the vehicle [2, 3]. Thus, the smoothness can be enhanced. In addition, the damping coefficient can also be changed through the semi-active suspension. Inside the electronic damper, extremely small iron filings are arranged through a magnetic field, which is generated by the current supplied from the controller [4-6]. Besides, the active suspension system, which uses hydraulic actuators, was introduced, and used on some luxury models [7, 8]. The effect the active suspension system brings is very positive, and the vehicle's oscillations can be significantly improved.

In some research about vehicle oscillation, the quarter dynamics and half dynamics models are often used. For the quarter dynamics model, it describes the vehicle's oscillation through a position of a suspension system. It is quite simple. The oscillation criteria in this model include displacement and acceleration of the sprung mass. However, this model cannot completely describe the effects of the oscillation on the vehicle. Therefore, the half dynamics model is used to replace the other model. The half dynamics model describes the vehicle's oscillations through the two sides of the suspension system. In this model, the roll angle was taken into consideration. The oscillation criteria can be the vertical displacement and acceleration of the sprung mass, or the roll angle and angular acceleration of the sprung mass. The half dynamics model provides more accurate results. However, when the controller

is integrated with this model, the control process becomes more complicated. When evaluating the value of the output data, which is mentioned above, their maximum and average values will be considered. In fact, the average value of these parameters can be calculated according to the RMS [9].

$$RMS(\zeta) = \sqrt{\frac{1}{T} \int_0^T f^2(\zeta) d\zeta} \quad (1)$$

1.2 Literature review

The effectiveness of the active suspension system depends entirely on its control method. Recently, there have been many types of research on the control problem for the active suspension system published. Among them, many linear control methods have been used. Bello et al. [10] introduced the state feedback controller, which is used for the active suspension system. The two in-loop PID control method has also been proposed by Shafie et al. [11]. This method consists of an inner in-loop and an outer in-loop, and the outer in-loop being the setpoint signal for the inner in-loop. The parameters of the PID controller can be self-tuning in the MATLAB environment. This has been shown in the paper of Talib and Darus [12]. In many situations, the conventional linear controller is not able to meet the vehicle's stability requirements. Therefore, Nguyen proposed the use of double-integrated controllers for each side of the suspension system [13]. According to his paper, two PID controllers take care of two different objects, including displacement and acceleration of the sprung mass. Besides, the PID control method can also be combined with many other control methods to improve the stability efficiency [14-16]. Assuming that the vehicle is a MIMO system, the LQR control method is proposed to control the output parameters of the object [17]. According to the study [18], the LQR control method can minimize the cost function of the system. The parameters of the controlled object can be significantly reduced by using this method [19]. However, the selection of the coefficients of the control matrix is extremely complicated. In Refs. [20, 21], these authors just gave the controller parameters without mentioning the optimal selection of these values. Based on the theory of the LQR control method, Chai and Sun integrated a Gaussian filter for this controller, and it became the LQG controller [22]. This method is also used in many studies [23-26].

Besides the traditional control methods, many modern control algorithms have also been used for the active suspension system. In their study, Bai and Guo [27] introduced the Sliding Mode Control method for the active suspension. In this paper, hydraulic actuators are mentioned. However, this mention is not clear. According to Nguyen [28], the Sliding Mode Control method can be highly effective for SISO nonlinear systems. This system is stable when the sliding surface is selected appropriately [29]. However, the "chattering" phenomenon may occur when using this controller. To improve this situation, the Sliding Mode Control method can be combined with several intelligent control methods [30]. The control delay was also considered in the study of Alves et al. [31]. In addition, the Robust control method also brings high efficiency to the system [32]. Yamada et al. [33] used the Robust controller for the half dynamics model. For this method, the equations describing the vehicle's oscillation are rewritten in the form of state matrices [34]. At the same time, the control signal is stable against the change of external factors [35-39]. Even the active suspension system

controlled by the Robust controller has been used on many current electric vehicle models [40]. In the studies [41, 42], the theories of H_2 and H_∞ optimization were also clearly analysed. In contrast, the Adaptive control method can generate control signals suitable to the vehicle's moving conditions [43]. According to Huang et al. [44], the Adaptive control method is perfectly suitable for complex nonlinear systems. This method can be combined with some intelligent control algorithms. This was shown in the paper of Fu et al. [45]. Besides, the Fuzzy control method is also commonly used for active suspension models [46]. The logic modes of this method can respond well to external stimuli [47, 48]. Besides, many other intelligent control methods have also been applied to the active suspension system [49-53].

Table 1. Advantages and disadvantages of the control methods

Methods	Advantages	Disadvantages
PID	Simple, durable, low cost	Only applicable to the SISO system
LQR	Can be applied to the MIMO system	Oscillation matrix needs to be established
SMC	Can be applied to nonlinear system	Very complex
Robust, Adaptive, Fuzzy	High performance	Complex

The control methods mentioned above all have their advantages and disadvantages. These contents are shown in Table 1. In this research, the author proposes the LQR control method for the active suspension system. This method is used to control MIMO systems. Besides, the triple in-loop optimization algorithm used to optimize the parameters of the control matrix has been demonstrated. These parameters will be calculated specifically, for each case. This is the new point of the paper. The content of the paper consists of four parts. The first part deals with problem analysis and literature review. The second part covers the establishment of the dynamics model and optimal algorithms. In the third part, the results of the simulation are shown. In the conclusion, some ideas are suggested.

2. MATERIALS AND METHODS

2.1 Half dynamics model

In this research, a half dynamics model is used to describe the vehicle's oscillations (Figure 1). Separating the sprung mass and the un-sprung mass, this model consists of four degrees of freedom.

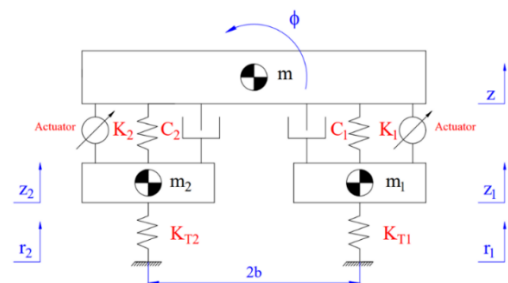


Figure 1. Half dynamics model

The equations describing the vehicle's oscillation is given below:

$$m\ddot{z} = F_{K1} + F_{C1} - F_{A1} + F_{K2} + F_{C2} + F_{A2} \quad (2)$$

$$J\ddot{\phi} = (F_{C1} + F_{K1} - F_{C2} - F_{K2} - F_{A1} - F_{A2})b \quad (3)$$

$$m_1\ddot{z}_1 = F_{KT1} - F_{K1} - F_{C1} + F_{A1} \quad (4)$$

$$m_2\ddot{z}_2 = F_{KT2} - F_{K2} - F_{C2} - F_{A2} \quad (5)$$

where,

Force of the spring:

$$F_{K1} = K_1(z_1 - z - b\phi) \quad (6)$$

$$F_{K2} = K_2(z_2 - z + b\phi) \quad (7)$$

Force of the damper:

$$F_{C1} = C_1(\dot{z}_1 - \dot{z} - b\dot{\phi}) \quad (8)$$

$$F_{C2} = C_2(\dot{z}_2 - \dot{z} + b\dot{\phi}) \quad (9)$$

Force of the tire:

$$F_{KT1} = K_{T1}(r_1 - z_1) \quad (10)$$

$$F_{KT2} = K_{T2}(r_2 - z_2) \quad (11)$$

For the oscillation system to be stable, the force generated by the actuators on both sides of the suspension system needs to be of equal value. Therefore:

$$A = \begin{bmatrix} 0 & I & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{(K_1+K_2)}{m} & \frac{(C_1+C_2)}{m} & \frac{(K_1-K_2)b}{m} & \frac{(C_1-C_2)b}{m} & \frac{K_1}{m} & \frac{C_1}{m} & \frac{K_2}{m} & \frac{C_2}{m} \\ 0 & 0 & 0 & I & 0 & 0 & 0 & 0 \\ \frac{(K_1-K_2)b}{J} & \frac{(C_1-C_2)b}{J} & \frac{(K_1+K_2)b^2}{J} & \frac{(C_1+C_2)b^2}{J} & \frac{K_1b}{J} & \frac{C_1b}{J} & \frac{K_2b}{J} & \frac{C_2b}{J} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{K_1}{m_1} & \frac{C_1}{m_1} & \frac{K_1b}{m_1} & \frac{C_1b}{m_1} & \frac{(K_1+K_{T1})}{m_1} & \frac{C_1}{m_1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{K_2}{m_2} & \frac{C_2}{m_2} & \frac{K_2b}{m_2} & \frac{C_2b}{m_2} & 0 & 0 & \frac{(K_2+K_{T2})}{m_2} & \frac{C_2}{m_2} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & -\frac{2b}{J} & 0 & \frac{1}{m_1} & 0 & -\frac{1}{m_2} \end{bmatrix}^T$$

The control signal is determined based on the state feedback controller R [54].

$$u = -Rx \quad (18)$$

$$F_{A1} = F_{A2} = u \quad (12)$$

Substituting the values of the binding force into the equations above, and it can be rewritten as:

$$m\ddot{z} = -(K_1 + K_2)z - (C_1 + C_2)\dot{z} - (K_1 - K_2)b\phi - (C_1 - C_2)b\dot{\phi} + K_1z_1 + C_1\dot{z}_1 + K_2z_2 + C_2\dot{z}_2 \quad (13)$$

$$J\ddot{\phi} = -(K_1 - K_2)bz - (C_1 - C_2)b\dot{z} - (K_1 + K_2)b^2\phi - (C_1 + C_2)b^2\dot{\phi} + K_1bz_1 + C_1b\dot{z}_1 - K_2bz_2 - C_2b\dot{z}_2 - 2bu \quad (14)$$

$$m_1\ddot{z}_1 = K_1z + C_1\dot{z} + K_1b\phi + C_1b\dot{\phi} - (K_1 + K_{T1})z_1 - C_1\dot{z}_1 + u \quad (15)$$

$$m_2\ddot{z}_2 = K_2z + C_2\dot{z} - K_2b\phi - C_2b\dot{\phi} - (K_2 + K_{T2})z_2 - C_2\dot{z}_2 - u \quad (16)$$

2.2 LQR controller and triple in-loops algorithm

Let:

$$\begin{aligned} x_1 &= z & x_2 &= \dot{z} & x_3 &= \phi & x_4 &= \dot{\phi} \\ x_5 &= z_1 & x_6 &= \dot{z}_1 & x_7 &= z_2 & x_8 &= \dot{z}_2 \end{aligned}$$

The equations that have been established can be rewritten in matrix form as follows:

$$\dot{x} = Ax + Bu + Cr \quad (17)$$

where, A is the state matrix and B is the control signal matrix.

For the system to be stable, the value of the cost function must be minimal [55].

$$Q = \frac{1}{2} \int_0^{\infty} (x^T E x + u^T F u) dt \xrightarrow{R} \min \quad (19)$$

where, E is the output matrix, F is the navigation matrix.

$$E = \begin{bmatrix} \gamma_1 \times 10^{\lambda_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \gamma_2 \times 10^{\lambda_2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \gamma_3 \times 10^{\lambda_3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma_4 \times 10^{\lambda_4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma_5 \times 10^{\lambda_5} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \gamma_6 \times 10^{\lambda_6} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \gamma_7 \times 10^{\lambda_7} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_8 \times 10^{\lambda_8} \end{bmatrix} \quad F = [\delta]$$

According to Lyapunov's theory, the matrix R can be defined as follows:

$$R = F^{-1} B^T P \quad (20)$$

where, P is the solution of the Riccati algebraic equation:

$$PBF^{-1}B^TP - PA - A^TP = E \quad (21)$$

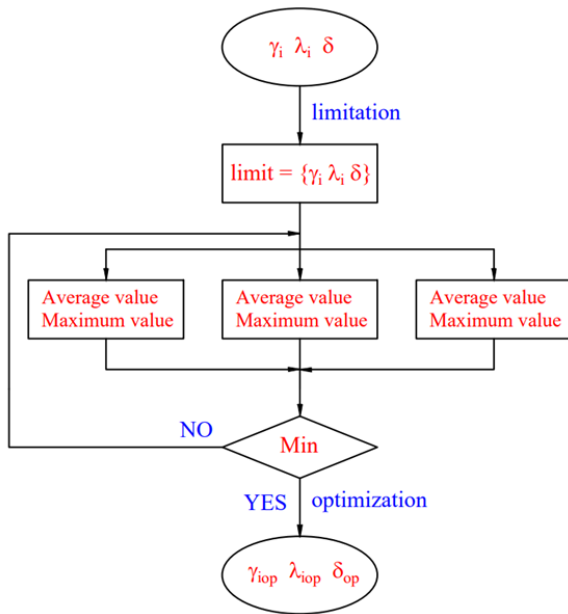


Figure 2. Optimal algorithm diagram

Based on the presented formulas, if the coefficients of matrix E and matrix F can be selected appropriately, the LQR controller will achieve the highest efficiency. Therefore, optimization of controller parameters needs to be performed.

The optimal algorithm diagram is shown in Figure 2. According to this diagram, three parameters are optimized, including γ_i , λ_i , and δ . The process of optimizing parameters is carried out in three stages. At the first stage, the limits of the parameters will be determined. This limit is determined based on the stability conditions of the system. The results of the first stage will help the optimization process go faster. In the second stage, the simulation is performed corresponding to the parameters, which change continuously closed in-loops. The results of the second stage are the average and maximum values of the roll angle of the sprung mass and the angular acceleration of the sprung mass. The results obtained from the second stage will be compared with each other. In the final stage, the parameters will be selected, so that, the oscillation of the vehicle is minimal. After the controller parameters have

been optimized, the simulation will be performed again to verify the obtained results.

2.3 Simulation conditions

The simulation takes place when the controller parameters have been determined. In this paper, the vehicle's oscillations will be compared in three cases:

In the first case, the vehicle uses a conventional passive suspension system.

In the second case, the vehicle uses an active suspension system, and the controller's optimization is done through double in-loops (determining λ_i and δ).

In the third case, the vehicle uses an active suspension system, the controller's optimization is done through triple in-loops (determining γ_i , λ_i , and δ).

The vehicle specifications are shown in Table 2.

Table 2. Specification of the vehicle

Symbol	Description	Value	Unit
m	Sprung mass	900	kg
m_i	Un-sprung mass	65	kg
K_i	Stiffness of spring	40000	Nm ⁻¹
C_i	Damping coefficient	3500	Nsm ⁻¹
K_{Ti}	Stiffness of tire	170000	Nm ⁻¹

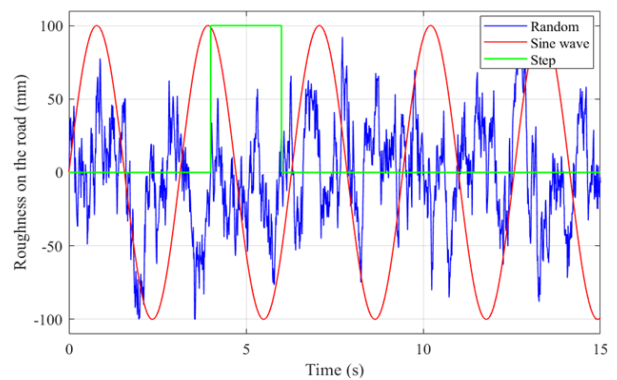


Figure 3. Roughness on the road

The excitement from the road surface is the direct cause of the vehicle's oscillations. In this research, three types of stimulation were used corresponding to the three survey situations (Figure 3), including:

The first situation: Sine wave stimulation:

$$r(t) = R \sin(2\pi ft + \varphi) \quad (22)$$

The second situation: Random stimulation:

$$r(t) = \int_0^t \left[2\pi\sqrt{Gv\omega(\tau)} - 2\pi fr(\tau) \right] d\tau \quad (23)$$

The third situation: Step stimulation:

$$r(t) = \begin{cases} 0, t < t_0 \\ h, t_0 \leq t \leq t_i \\ 0, t > t_i \end{cases} \quad (24)$$

3. RESULTS AND DISCUSSIONS

3.1 Sine wave

Sine wave excitation is commonly used in control problems. This excitation has a cyclic form with a low frequency. With this type of excitation, the periodic oscillation is repeated according to a certain law.

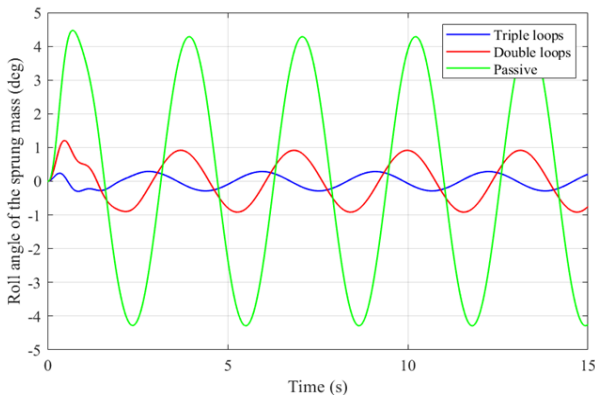


Figure 4. Roll angle of the sprung mass

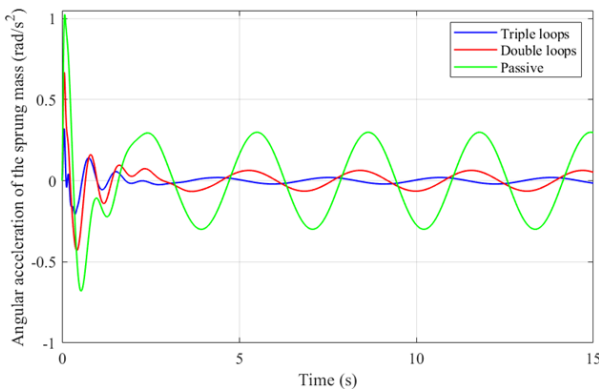


Figure 5. Angular acceleration of the sprung mass

Figure 4 shows the change in the roll angle of the sprung mass over time. In the case of the vehicle using a passive suspension system, the roll angle of the vehicle body is large. Its maximum value can reach 4.47°. If this system is replaced by an active suspension system, the maximum value of the roll angle can be significantly reduced. This value reaches 1.21° and 0.30° respectively for the double in-loops control algorithm and the triple in-loops control algorithm. Besides, the average value of the roll angle, which is calculated according to (1), is also shown. In this way, its average value is 3.03°, 0.66°, and 0.20° respectively for the three simulation situations.

The angular acceleration of the sprung mass is a characteristic of the vehicle's oscillations. If this value is large, the vehicle's stability cannot be guaranteed. The value of the angular acceleration of the sprung mass is shown in Figure 5. The maximum value of the acceleration is 1.03 (rad/s²), 0.67 (rad/s²) and 0.32 (rad/s²), respectively for the above three cases. Also, their average values in terms of RMS are 0.25 (rad/s²), 0.09 (rad/s²), and 0.04 (rad/s²). Obviously, the difference between using passive suspension and active suspension is huge.

3.2 Random

Random excitation usually occurs when the vehicle is moving on the actual road. With this excitation, the amplitude and frequency of the oscillation change continuously. As a result, the vehicle may experience strong oscillations.

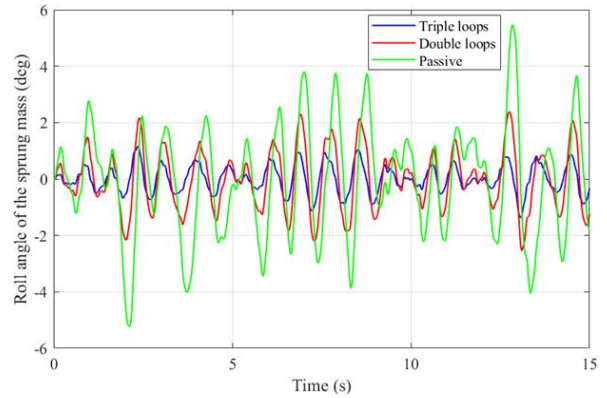


Figure 6. Roll angle of the sprung mass

In Figure 6, the variation of the roll angle of the vehicle body is shown. Accordingly, the maximum roll angle can be up to 5.47°, 2.54°, and 1.37°, respectively, for the cases introduced in the paper. Besides, the average value of oscillation is also larger than the case of using cyclic excitation. These values are 1.97°, 1.06°, and 0.49°, respectively. According to this result, if the vehicle uses a conventional passive suspension, the value of the roll angle can be 4.02 times that of using an active suspension system controlled by a triple in-loops algorithm.

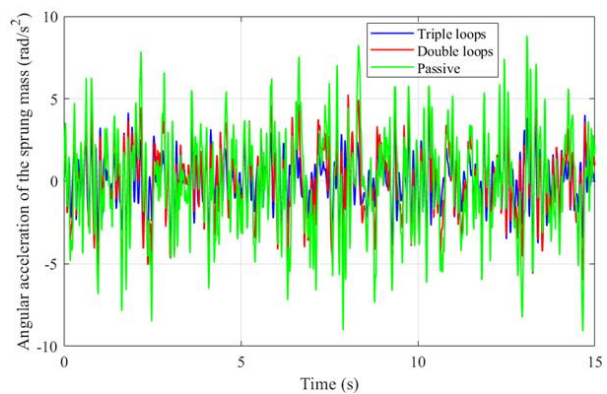


Figure 7. Angular acceleration of the sprung mass

In this case, the angular acceleration of the sprung mass can be very large (Figure 7). At some point, the angular acceleration was as high as 9.10 (rad/s²). However, this value can be reduced to only 7.95 (rad/s²) and 4.71 (rad/s²) when the

mechanical suspension system is replaced by the active suspension system. In addition, the average value of the angular acceleration that is calculated according to the RMS also has a very large difference, and they can be 1.93 times each other.

3.3 Step

Step excitation is also commonly used in oscillation problems. Its frequency is low. However, the change of the stimulus signal is abrupt. So, it can cause big oscillation for the vehicle. In this paper, step excitation with a single pulse is proposed. At time $t = 4$ (s), the vehicle starts to oscillate (Figure 8). The roll angle of the sprung mass increases gradually to the maximum value. If a passive suspension is fitted on the vehicle, this value can go up to 6.11° . At the same time, it will cause the vehicle's acceleration to increase rapidly, reaching 11.81 (rad/s^2). The LQR controller with a triple in-loops algorithm can effectively reduce these values. When this controller is used, the value of the oscillation is only 0.78° and 4.32 (rad/s^2) (Figure 9).

Some control methods can lead to continuous oscillations after the excitation ceases to exist. This can greatly affect the smoothness of the vehicle. With the designed triple in-loops algorithm, the end time of the control process will be equivalent to the end time of the passive suspension system. At time $t = 9.03$ (s), 9.19 (s), and 9.08 (s), the roll angle's value is approximately 0.001° , the control process can be considered stable.

The results of the simulation are shown in Table 3 and Table 4. In which the maximum and average values of the roll angle of the sprung mass and the angular acceleration of the sprung mass are calculated exactly. This data corresponds to three

simulation cases and three survey conditions.

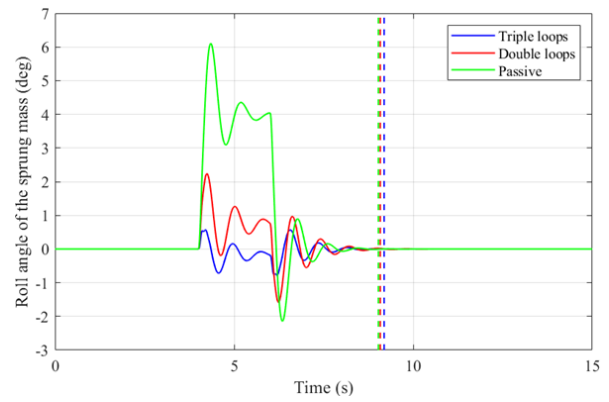


Figure 8. Roll angle of the sprung mass

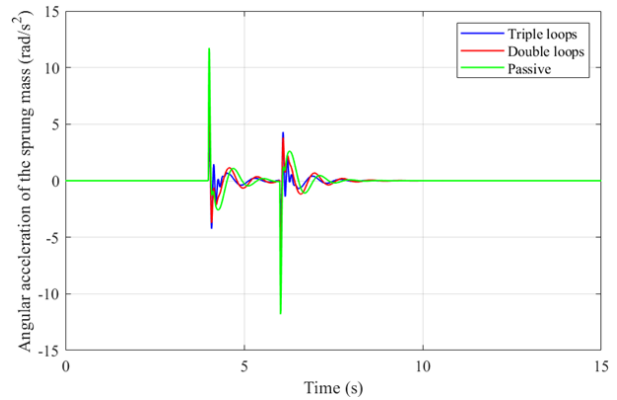


Figure 9. Angular acceleration of the sprung mass

Table 3. Average value

	Roll angle	Acceleration	Roll angle	Acceleration	Roll angle	Acceleration
Random	0.49	1.55	1.06	2.30	1.97	2.99
Sine wave	0.20	0.04	0.66	0.09	3.03	0.25
Step	0.17	0.45	0.43	0.73	1.55	0.98

Table 4. Maximum value

	Roll angle	Acceleration	Roll angle	Acceleration	Roll angle	Acceleration
Random	1.37	4.71	2.54	7.95	5.47	9.10
Sine wave	0.30	0.32	1.21	0.67	4.47	1.03
Step	0.78	4.32	2.24	8.35	6.11	11.81

4. CONCLUSION

The excitement from the road surface is what causes the vehicle to oscillate. The suspension system plays an important role in maintaining and ensuring the stability of the vehicle against this stimulation. In many cases, conventional mechanical suspensions are not able to meet the vehicle's comfort requirements. Therefore, the active suspension is used to replace the passive suspension. The performance of the active suspension system depends on its controller. In this paper, the LQR controller is used.

This paper has established a half dynamics model of the vehicle to describe its oscillations. Besides, the triple in-loops optimization algorithm is also introduced. The controller parameters can be optimally calculated based on the proposed

algorithm. This method brings high efficiency to the calculation process. In addition, the execution time is also very short. Simultaneously, the simulation is conducted for three survey cases and three stimulus situations from the road surface. The results of the research have shown the outstanding performance of the optimization algorithm, which has been shown in the paper. When the vehicle uses the active suspension system, the maximum and average values of oscillations are greatly reduced compared to using the passive suspension system. Besides, the triple in-loops optimization algorithm provides more stability and convenience than the double in-loops algorithm. The performance of the controller is very high, and it works stably in many situations. These are the positive results brought about by the new algorithm, which is mentioned in the paper.

This research only conducts calculations and simulations. There are many influencing factors that have been overlooked. Therefore, it is necessary to have an experimental process to verify the results of the paper. This is the basis to be able to conduct experiments based on this control method. In the future, intelligent control algorithms can be used to better optimize the parameters of the LQR controller.

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NOMENCLATURE

F_A	Actuator force, N
F_C	Damper force, N
F_K	Spring force, N
F_{KT}	Tire force, N
J	Moment of inertia, kgm ²
m	Sprung mass, kg
m_i	Un-sprung mass, kg
r_i	Roughness on the road, m
z	Displacement of the sprung mass, m
z_i	Displacement of the un-sprung mass, m

Greek symbols

ϕ	Roll angle, rad
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