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Derivative Operator of Order $\varepsilon+\rho$-1 Associated with Differential Subordination and Superordination

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ABSTRACT

Professors Miller and Mocanu established the theory of differential subordination and its twin, the theory of differential super ordination, which are both based on reinterpreting fundamental inequalities for real-valued functions for the situation of complex-valued functions. Using different types of operators to study subordination and super ordination characteristics is a technique that is still extensively employed, with some investigations leading to sandwich-type theorems, as is the case in the current work. The objective of this work is to derive differential Subordination and Super ordination outcomes using the derivative operator of order E+-1. Differential subordination and super ordination results are achieved for analytic functions connected with the integral operator in the open unit disc. These findings are achieved by examining relevant types of admissible functions, differential supremacy theorem, several operator differential hyperboloids requiring partial integration of a stacking suprageometric function are produced, as well as the best subordinates. The result of a sandwich type links the outcomes of dependency and dependency using Theorem 9. Keep track of intriguing corollaries for certain occupations by using the best subordinate and dominant skills. Presented in this paper may be used to motivate the usage of alternative hyper-geometric functions related to partial integration.

1. INTRODUCTION

The concept of differential subordination evolved from the fact that, given a real valued function f that is twice continuously differentiable on the interval I=(1, 1) and assuming that the differential operator is a function of the differentiation operator [1].

Ibrahim and Darus [2] established the existence and uniqueness of univalent solution for fractional differential equation. Moreover, the study illustrated some properties of this solution containing differential and integral subordination properties. For a generalized fractional differintegral operator associated with *p*- valent functions, Aouf et al. [3] studied different properties of differential subordination and superordination related to this operator.

Agarwal et al. [4] introduced a unified subclass of analytic functions by making use of the principle of subordination, involving generalized Ruscheweyh Derivative operator. The properties such as inclusion relationships, distortion theorems, coefficient inequalities and differential sandwich theorem have been discussed.

Cho et al. [5] introduced an operator defined on the family of analytic functions in the open unit disk by using the generalized fractional derivative and integral operator with convolution. For this operator, the authors studied the subordination-preserving properties and their dual problems. Differential sandwich-type results for this operator are also investigated.

Hameed and Shihab [6] investigated some of the features of differential subordination of analytic univalent functions in an open unit disc. In addition, it has shed light on geometric features such as coefficient inequality, Hadamard product qualities, and the Komatu integral operator. furthermore, several results for third order differential subordination in the open unit disk using generalized hypergeometric function have been addressed using the convolution operator.

Wanas [7] defined new class of analytic functions in the upper half-plane as well as investigated appropriate classes of admissible functions, we obtain differential subordination results for functions belongs to this new class. Shenan [8] obtained some subordination and superordination-preserving results of analytic functions associated with the fractional differintegral operator. Sandwich-type result involving this operator is also derived.

The purpose of this work is to generate differential Subordination and Superordination outcomes by employing the order E+-1 derivative operator. For analytic functions coupled to the integral operator in the open unit disc, differential subordination and superordination results are obtained.

2. METHODS

The study considers differentiation as an abstract process that receives a function and returns another function in mathematics (in the style of a higher-order function in computer science). The current research examined relevant types of admissible functions, the differential supremacy theorem, and several operator differential hyperboloids requiring partial integration of a stacking suprageometric function are produced, as well as the best subordinates.

3. RESULTS AND DISCUSSION

To begin, consider differentiation as an abstract process in mathematics that accepts one function and returns another (in the style of a higher-order function in computer science [9].

Let the symbolize of the class of analytic functions in the disk $\odot=\{z\epsilon C:|z|<1\}$ is $\mathfrak{T}$. And let $\mathcal{S}[d,\rho]$ is the subclass of the function $\mho \in \mathfrak{T}$ like,

$$\mho(z) = d + d_\rho z^{\rho} + d_{\rho+1}z^{\rho+1} + \cdots (d \in \mathbb{C}; \rho \in \mathrm{N} = 1,2, \ldots..)$$

Also, let $\mathbb{O}(\rho)$ be in the subclass of $\mathfrak{T}$ consisting of functions:

$$\mho(z) = z^{\rho} + \sum_{\tau=1}^{\infty} d_{\tau+\rho}\, z^{\tau+\rho} \tag{1}$$

Let $\rho, h \in \mathfrak{T}$ and $\omega: C^3 x\odot \to \mathbb{C}$. If ρ satisfies the superordination of the second – order and $\left(\rho(z), z\,\rho'(z), z^2\rho''(z); z\right)$, ρ are univalent function in $\odot$ and if

$$h(z) \prec \omega\left(\rho(z), z\,\rho'(z), z^2\rho''(z); z\right) \tag{2}$$

then ρ is invited a solution of the differential superordination (2).

They discovered the following characteristics of functions ρ that satisfy the second-order differential subordination: If $\mho$ is subordinate to Ω, then Ω is supordinate to $\mho$. An analytic function g is invited a subordinant of (2), if $\varsigma \prec \rho$, $\forall$ function ρ satisfying (2). A univalent subordinate $\bar{\varsigma}$ that satisfies $\varsigma \prec \bar{\varsigma}$ $\forall$ subordinates ς of (2) is called the best subordinate. In recent years, several academics have presented and discussed the concept of second-order differential subordination and superordination, for example [10-17]. The second order will have to be investigated in the current work. They discovered the following characteristics of functions p that satisfy the second-order differential subordination:

The derivative operator [18] of order $\varepsilon+\rho$-1, is denoted by $\mathbb{D}^{\mathcal{E}+\rho-1}$ and defined as following:

$$\mathbb{D}^{\mathcal{E}+\rho-1}\mho(z) = \frac{z^{\rho}}{\mathbb{D}^{\mathcal{E}+\rho-1}\mho(z)} * \mho(z) = z^{\rho} + \sum_{\tau=1}^{\infty}\frac{\Gamma(\mathcal{E}+\tau+\rho)}{\Gamma(\mathcal{E}+\rho)\tau!} d_{\tau+\rho} z^{\tau+\rho}\ (\mathcal{E} > -\rho) \tag{3}$$

By (3) we get:

$$z\left(\mathbb{D}^{\mathcal{E}+\rho-1}\mho(z)\right)' = (\mathcal{E}+\rho)\mathbb{D}^{\mathcal{E}+\rho}\mho(z) - \mathcal{E}\mathbb{D}^{\mathcal{E}+\rho-1}\mho(z) \tag{4}$$

Definition (1) [19]: η symbolize set of all functions $\mho$ that are one to one and analytic on $\overline{\odot} \setminus E(\mho)$, where $E(\mho) = \left\{\vartheta \in \partial\odot\ : \lim_{z\to\vartheta} \mho(z) = \infty\right\}$ and are s.t $\mho'(\vartheta) \neq o\ \forall \vartheta \in \partial\odot \setminus E(\mho)$.

Lemma (1) [20]: Let ς be convex univalent function in $\odot$ and let $\sigma\in C$, $\beta\in C\backslash\{o\}$ with $Re\left\{1+\frac{\varsigma''(z)z}{\varsigma'(z)}\right\} > \max\left\{0, -Re\left(\frac{\sigma}{\beta}\right)\right\}$. If the analytic function ρ in $\odot$ and

$$\sigma\rho(\mathrm{Z}) + \beta z\rho'(\mathrm{Z}) \prec \sigma\varsigma(\mathrm{Z}) + \beta z\varsigma'(\mathrm{Z}), \tag{5}$$

Then ς is the good dominant of (5) and $\rho\prec\varsigma$.

Lemma (2) [21]: Let k_1 and k_2 be analytic in a domain $\mathfrak{D}$ contain q $(\odot)$ with $k_2(w)\neq 0$ when $w\in\varsigma(u)$ and let ς be univalent in the unit disk $\odot$.The Set $\eta(z)=z\varsigma'(z)k_2(\varsigma(z))$ and $h(z)=k_1(\varsigma(z))+\eta(z)$. Suppose that

i) $\eta(z)$ is starlike univalent in $\odot$,

ii) $\mathrm{Re}\left\{z\frac{h'(z)}{\eta(z)}\right\} > 0, \forall\, z \in \odot$. If ρ is analytic in $\odot$, with $\rho(o)=\varsigma(o)$, $\rho(\odot)\subset D$ and

$$\left(\rho(z)\right)k_1 + Z\rho'(z)\left(\rho(z)\right)k_2 \prec \left(\varsigma(z)\right)k_1 + z\,\varsigma'(z)\left(\varsigma(z)\right)k_2, \tag{6}$$

Then $\rho\prec\varsigma$ and ς is the good dominant of (6).

Lemma (3) [22]: Suppose ς be a univalent convex function in $\odot$, and let $\beta\in C$. And assume that Real $(\beta)>o$. If $\rho \in \mathcal{S}[\varsigma(o),1] \cap \eta$ and $\rho(z) + \beta z\,\rho'(z)$ is univalent in $\odot$, then

$$\varsigma(z) + \beta z\,\varsigma'(z) \prec \rho(z) + Z\beta\rho'(z), \tag{7}$$

and $\rho\prec\varsigma$, ς is the *best dominant* of (7).

Lemma (4) [23]: Let k_1 and k_2 be analytic in a domain $\mathfrak{D}$ containing ς $(\odot)$ such that ς be convex univalent in the unit disk $\odot$. Let

i) Real $\left\{\frac{k_1'(\varsigma(z))}{k_2(\varsigma(z))}\right\} > 0.\ for\ all\ z \in \odot.$

ii) $\eta(z)=z\varsigma'(z)k_2\left(\varsigma(z)\right)$ is starlike in $\odot$. If $\rho \in \mathcal{S}[\varsigma(o),1] \cap \eta$,and $\rho(\odot)\subset \mathfrak{D}$, $k_1\left(\rho(z)\right) + z\,\rho'(z)k_2\left(\rho(z)\right)$ is univalent in $\odot$ and

$$k_1\left(\varsigma(z)\right) + z\varsigma'(z)k_2\left(\rho(z)\right) \prec k_2\left(\varsigma(z)\right) + z\rho'(z)\emptyset\left(\rho(z)\right), \tag{8}$$

Then q is the best dominant of (8) and $\varsigma\prec\rho$,

Theorem (1): Suppose

$$Re\left\{1+\frac{z\varsigma''(z)}{\varsigma'(z)}\right\} > \max\left\{0, -Re\left(\frac{\mathcal{M}(\mathcal{E}+\rho)}{\mathcal{U}}\right)\right\} \text{ and if } V_1(z) = (\mathcal{U}+1)\left(\frac{z^{\rho}}{\mathbb{D}^{\mathcal{E}+\rho-1}\mho(z)}\right)^{\mathcal{M}} - \mathcal{U}\left(\frac{z^{\rho}}{\mathbb{D}^{\mathcal{E}+\rho-1}\mho(z)}\right)^{\mathcal{M}}\left(\frac{\mathbb{D}^{\mathcal{E}+\rho}\mho(z)}{\mathbb{D}^{\mathcal{E}+\rho-1}\mho(z)}\right) \tag{9}$$

$$and\ V_1(z) \prec \varsigma(z) + \frac{\mathcal{U}}{\mathcal{M}(\mathcal{E}+\rho)} z\,\varsigma'(z), \tag{10}$$

when $\varsigma(z)$ be univalent convex in $\odot$ with $\mathcal{U} \in \mathbb{C}/\{0\}, \mathcal{M} > o\ and\ \varsigma(o) = 1,$

$$then\ \left(\frac{z^{\rho}}{\mathbb{D}^{\mathcal{E}+\rho-1}\mho(z)}\right)^{\mathcal{M}} \prec \varsigma(z) \tag{11}$$

and $\varsigma(z)$is the *best dominant* of (10).

Proof: We define the analytic function.

$$\rho(\mathbb{z}) = \left(\frac{\mathbb{z}^{\rho}}{\mathbb{D}^{\mathcal{E}+\rho-1}\mho(\mathbb{z})}\right)^{\mathcal{M}} \quad (12)$$

With respect to $\mathbb{z}$, differentiating (12).

$$\frac{\rho'(\mathbb{z})\mathbb{z}}{\rho'(\mathbb{z})} = \mathcal{M}\left[\rho - \frac{\mathbb{z}\left(\mathbb{D}^{\mathcal{E}+\rho-1}\mho(\mathbb{z})\right)'}{\mathbb{D}^{\mathcal{E}+\rho-1}\mho(\mathbb{z})}\right] \quad (13)$$

Now, by (4), we get:

$$\frac{\rho'(\mathbb{z})\mathbb{z}}{\rho'(\mathbb{z})} = \mathcal{M}(\mathcal{E}+\rho)\left[1 - \mathbb{z}\,\frac{\left(\mathbb{D}^{\mathcal{E}+\rho}\mho(\mathbb{z})\right)}{\mathbb{D}^{\mathcal{E}+\rho-1}\mho(\mathbb{z})}\right] \quad (14)$$

Then $\frac{\rho'(\mathbb{z})\mathbb{z}}{\mu(\mathcal{E}+\rho)} = \left(\frac{\mathbb{z}^{\rho}}{\mathbb{D}^{\mathcal{E}+\rho-1}\mho(\mathbb{z})}\right)^{\mathcal{M}}\left(1 - \frac{Z\left(\mathbb{D}^{\mathcal{E}+\rho}\mho(\mathbb{z})\right)}{\mathbb{D}^{\mathcal{E}+\rho-1}\mho(\mathbb{z})}\right) \quad (15)$

The assertion (10) is equivalent to:

$$\rho(\mathbb{z}) + \frac{\mathcal{U}}{\mathcal{M}(\mathcal{E}+\rho)}\mathbb{z}\,\rho'(\mathbb{z}) \prec \varsigma(\mathbb{z}) + \frac{\mathcal{U}}{\mathcal{M}(\mathcal{E}+\rho)}\mathbb{z}\,\varsigma'(\mathbb{z}).$$

By Lemma (1) with $\sigma = 1$, and $\beta = \frac{\mathcal{U}}{\mathcal{M}(\mathcal{E}+\rho)}$,we get (11).■

In theorem (1), Putting

$$\Omega(z) = \frac{1+\mathcal{D}_1\mathbb{z}}{1+\mathcal{D}_2\mathbb{z}}\ (-1 \le \mathcal{D}_2 < \mathcal{D}_1 \le 1),$$

we obtain the corollary (1).

Corollary (1): Suppose $Re\left(\frac{1-\mathcal{D}_2\mathbb{z}}{1+\mathcal{D}_2\mathbb{z}}\right) > \max\left\{0, -Re\left(\frac{\mathcal{M}(\mathcal{E}+\rho)}{\mathcal{U}}\right)\right\}$ when $\mathcal{U} \in \mathrm{C}/\{0\}$ and $-1 \le \mathcal{D}_2 < \mathcal{D}_1 \le 1$. If $\mho \in \mathbb{O}(\rho)$ satisfies the following subordination condition:

$$V_1(z) \prec \frac{1+\mathcal{D}_1\mathbb{z}}{1+\mathcal{D}_2\mathbb{z}} + \frac{\mathcal{U}}{\mathcal{M}(\mathcal{E}+\rho)}\frac{(\mathcal{D}_1-\mathcal{D}_2)\mathbb{z}}{(1+\mathcal{D}_2\mathbb{z})^2}$$

where, $V_1(z)$ given by (9), then

$$\left(\frac{\mathbb{z}^{\rho}}{\mathbb{D}^{\mathcal{E}+\rho-1}\mho(\mathbb{z})}\right)^{\mu} \prec \frac{1+\mathbb{z}}{1-\mathbb{z}}$$

and $\frac{1+\mathbb{z}}{1-\mathbb{z}}$ is the best dominant.■

Corollary (2): Suppose $Re\left(\frac{1-\mathcal{D}_2\mathbb{z}}{1+\mathcal{D}_2\mathbb{z}}\right) > \max\left\{0, -Re\left(\frac{\mathcal{M}(\mathcal{E}+\rho)}{\mathcal{U}}\right)\right\}$.
when $\mathcal{U} \in \mathrm{C}/\{0\}$ and

If $\mho \in \mathbb{O}(\rho)$ satisfies the following subordination condition:

$$\vee_1(\mathbb{z}) \prec \frac{\mathbb{z}+1}{-\mathbb{z}+1} + \frac{\mathcal{U}}{\mathcal{M}(\mathcal{E}+\rho)}\frac{(\mathbb{z})2}{(-\mathbb{z}+1)^2}$$

where, $V_1(\mathbb{z})$ defined by (9), then

$$\left(\frac{\mathbb{z}^{\rho}}{\mathbb{D}^{\mathcal{E}+\rho-1}\mho(\mathbb{z})}\right)^{\mu} \prec \frac{\mathbb{z}+1}{-\mathbb{z}+1}$$

and $\frac{1+\mathbb{z}}{1-\mathbb{z}}$ is the best dominant.■

Theorem (2): Suppose $\mho$ and *Ω* satisfy (16) and (17) conditions.

$$\frac{\gamma\mathbb{D}^{\mathcal{E}+\rho}\mho(\mathbb{z})+(1-\gamma)\mathbb{D}^{\mathcal{E}+\rho-1}\mho(\mathbb{z})}{\mathbb{z}^{\rho}} \neq 0\ (\mathbb{z} \in \odot, 0 \le \gamma \le 1) \quad (16)$$

and

$$Re\left\{1 + \frac{r_2}{\mathcal{U}}\varsigma(\mathbb{z}) + \frac{2r_3}{\mathcal{U}}[\varsigma(\mathbb{z})]^2 - \frac{\varsigma'(\mathbb{z})\mathbb{z}}{\varsigma(\mathbb{z})} + \frac{\varsigma''(\mathbb{z})\mathbb{z}}{\varsigma'(\mathbb{z})}\right\} > o, \quad (17)$$

When $\varsigma(\mathbb{z}) \neq o$ be univalent function in $\odot$, ς(o)=1 and $\frac{\varsigma'(\mathbb{z})}{\varsigma(\mathbb{z})}\mathbb{z}$ is starlike in $\odot$ and suppose $\mathcal{M}, \mathcal{U} \in \mathrm{C}/\{o\}$, $r_1, r_2, r_3 \in \mathrm{C}$ *and* $\mho \in \mathbb{O}(\rho)$.

If $\vee_2(\mathbb{z}) = r_1 + r_2\left(\frac{\gamma\mathbb{D}^{\mathcal{E}+\rho}\mho(z)+(1-\gamma)\mathbb{D}^{\mathcal{E}+\rho-1}\mho(z)}{z^{\rho}}\right)^{\mathcal{M}} + r_3\left(\frac{\gamma\mathbb{D}^{\mathcal{E}+\rho}\mho(z)+(1-\gamma)\mathbb{D}^{\mathcal{E}+\rho-1}\mho(z)}{z^{\rho}}\right)^{2\mathcal{M}} + \mathcal{U}\mathcal{M}\left[\frac{\gamma z\left(\mathbb{D}^{\mathcal{E}+\rho 1}\mho(z)\right)'+(1-\gamma)z\left(\mathbb{D}^{\mathcal{E}+\rho-1}\mho(Z)\right)'}{\gamma\mathbb{D}^{\mathcal{E}+\rho}\mho(Z)+(1-\gamma)\mathbb{D}^{\mathcal{E}+\rho-1}\mho(z)} - \rho\right]. \quad (18)$

and $\vee_2(\mathbb{z}) \prec r_1 + 1 + r_2\varsigma(\mathbb{z}) + r_3[\varsigma(\mathbb{z})]^2 + \mathcal{U}\frac{\varsigma'(\mathbb{z})\mathbb{z}}{\varsigma(\mathbb{z})}, \quad (19)$

then

$$\left(\frac{\gamma\mathbb{D}^{\mathcal{E}+\rho}\mho(\mathbb{z})+(1-\gamma)\mathbb{D}^{\mathcal{E}+\rho-1}\mho(\mathbb{z})}{\mathbb{z}^{\rho}}\right)^{\mathcal{M}} \prec \varsigma(\mathbb{z}).$$

And ς is the *best dominant* of (19).

Proof: ρ is analytic defined by:

$$\rho(\mathbb{z}) = \left(\frac{\gamma\mathbb{D}^{\mathcal{E}+\rho}\mho(\mathbb{z})+(1-\gamma)\mathbb{D}^{\mathcal{E}+\rho-1}\mho(\mathbb{z})}{\mathbb{z}^{\rho}}\right)^{\mathcal{M}} \quad (20)$$

Then $\rho(0)=1$, and ρ is analytic in $\odot$, logarithmically (20) with respect to $\mathbb{z}$, we get:

$$\frac{\rho'(\mathbb{z})\mathbb{z}}{\rho'(\mathbb{z})} = \mathcal{M}\left[\frac{\gamma\mathbb{z}\left(\mathbb{D}^{\mathcal{E}+\rho}\mho(\mathbb{z})\right)'+(1-\gamma)\mathbb{z}\left(\mathbb{D}^{\mathcal{E}+\rho-1}\mho(\mathbb{z})\right)'}{\gamma\mathbb{D}^{\mathcal{E}+\rho}\mho(\mathbb{z})+(1-\gamma)\mathbb{D}^{\mathcal{E}+\rho-1}\mho(\mathbb{z})} - \rho\right] \quad (21)$$

By setting $k_2(w) = \frac{\mathcal{U}}{w}$ and $k_1(w) = r_1 + r_2 w + r_3 w^2\ (w \in \mathrm{C}/\{0\})$,
we get $k_2(w)$ is analytic in C/{0}, $k_1(w)$ is analytic in C and that $k_2(w) \neq o$, $w \in C/\{o\}$. Also, we have,

$$\eta(z) = \mathbb{z}\,\varsigma'(Z)k_2(\varsigma(\mathbb{z})) = \mathcal{U}\frac{\mathbb{z}\,\varsigma'(\mathbb{z})}{\varsigma(\mathbb{z})}, (\mathbb{z} \in \odot),$$

$$and\ h(\mathbb{z}) = k_1(\varsigma(\mathbb{z})) + \eta(\mathbb{z}) = r_1 + r_2 q(\mathbb{z}) + r_3[q(\mathbb{z})]^2 + \mathcal{U}\frac{\mathbb{z}\,\varsigma'(\mathbb{z})}{\varsigma(\mathbb{z})},$$

where, $\eta(z)$ is starlike in $\odot$ then

$$Re\,\frac{h'(\mathbb{z})\mathbb{z}}{\eta(\mathbb{z})} = Re\left\{1 + \frac{r_2}{\mathcal{U}}q(\mathbb{z}) + \frac{2r_3}{\mathcal{U}}[\varsigma(\mathbb{z})]^2 - \frac{\varsigma'(\mathbb{z})\mathbb{z}}{q(\mathbb{z})} + \frac{\varsigma''(\mathbb{z})\mathbb{z}}{\varsigma'(\mathbb{z})}\right\} > o. (\mathbb{z} \in \odot).$$

Use (21), the hypothesis (19) can be equivalently that

$$(\rho(\mathbb{z}))k_1 + \rho'(\mathbb{z})\mathbb{z}(\rho(\mathbb{z}))k_2 \prec (\varsigma(\mathbb{z}))k_2 + \varsigma'(\mathbb{z})\mathbb{z}(\varsigma(\mathbb{z}))k_2.$$

Now by application the Lemma (2).■

Theorem (3): Suppose that $\mho$ and *Ω* satisfy the conditions (22) and (23):

$$\frac{\gamma\mathbb{D}^{\mathcal{E}+\rho}\mho(\mathbb{z})+(1-\gamma)\mathbb{D}^{\mathcal{E}+\rho-1}\mho(\mathbb{z})}{\mathbb{z}^{\rho}} \neq o, (\mathbb{z} \in \odot, o \leq \gamma \leq 1) \quad (22)$$

$$and\ Re\left\{1+\frac{\mathbb{z}\varsigma''(\mathbb{z})}{\varsigma'(\mathbb{z})}\right\} > \max\left\{o, -Re\left(\frac{r_2}{\mathcal{U}}\right)\right\} \quad (23)$$

Such that the function ς (ℤ) univalent in ⊙ by ς(o)=1 and suppose $\mathcal{M}, \mathcal{U} \in \mathbb{C}/\{o\}, r_1, r_2, r_3 \in \mathbb{C}, \mho(\mathbb{z}) \in \mathbb{O}(\rho)$. And, also

$$\text{if } V_3(\mathbb{z}) = \left(\frac{\gamma\mathbb{D}^{\mathcal{E}+\rho}\mho(\mathbb{z})+(1-\gamma)\mathbb{D}^{\mathcal{E}+\rho-1}\mho(\mathbb{z})}{\mathbb{z}^{\rho}}\right)^{\mathcal{M}} \times \left[r_2 + \mathcal{U}\mathcal{M}\left(\frac{\gamma z\left(\mathbb{D}^{\mathcal{E}+\rho}\mho(\mathbb{z})\right)'+(1-\gamma)\mathbb{z}\left(\mathbb{D}^{\mathcal{E}+\rho-1}\mho(\mathbb{z})\right)'}{\gamma\mathbb{D}^{\mathcal{E}+\rho}\mho(\mathbb{z})+(1-\gamma)\mathbb{D}^{\mathcal{E}+\rho-1}\mho(\mathbb{z})} - \rho\right)\right] + r_3 \quad (24)$$

$$\text{and } V_3(\mathbb{z}) \prec r_2(\mathbb{z}) + \mathcal{U}\mathbb{z}\varsigma'(\mathbb{z}) + r_3. \quad (25)$$

Then $\left(\frac{\gamma\mathbb{D}^{\mathcal{E}+\rho}\mho(\mathbb{z})+(1-\gamma)\mathbb{D}^{\mathcal{E}+\rho-1}\mho(\mathbb{z})}{\mathbb{z}^{\rho}}\right)^{\mathcal{M}} \prec \varsigma(z)$, and ς is the best dominant of (25).

Proof: Let the function ρ be defined on ⊙ by (13).

$$\mathbb{z}\rho'(z) = \mathcal{M}\left(\frac{\gamma\mathbb{D}^{\mathcal{E}+\rho}\mho(\mathbb{z})+(1-\gamma)\mathbb{D}^{\mathcal{E}+\rho-1}\mho(\mathbb{z})}{\mathbb{z}^{\rho}}\right)^{\mathcal{M}} \times \left[\left(\frac{\gamma z\left(\mathbb{D}^{\mathcal{E}+\rho}\mho(\mathbb{z})\right)'+(1-\gamma)\mathbb{z}\left(\mathbb{D}^{\mathcal{E}+\rho-1}\mho(\mathbb{z})\right)'}{\gamma\mathbb{D}^{\mathcal{E}+\rho}\mho(\mathbb{z})+(1-\gamma)\mathbb{D}^{\mathcal{E}+\rho-1}\mho(\mathbb{z})}\rho\right)\right]$$

By setting $k_1(w) = r_2 w + r_3,\ k_2(w) = \mathcal{U}, (w \in \mathbb{C})$

we have $k_1(w),\ k_2(w) \in \mathbb{C}$ and $k_2(w) \neq o$,
$\eta(Z) = z\varsigma'(\mathbb{z})k_2(\varsigma(\mathbb{z})) = \mathcal{U}z\varsigma'(\mathbb{z}), (\mathbb{z} \in \odot)$

$$and\ h(\mathbb{z}) = k_1(\varsigma(\mathbb{z})) + \eta(\mathbb{z}) = r_1 + r_2\varsigma(\mathbb{z}) + \mathcal{U}\,z\varsigma'(\mathbb{z}) + r_3, (\mathbb{z} \in \odot)$$

by the assertion (23) we see $\eta(\mathbb{z})$ is starlike in ⊙ and

$$Re\frac{\mathbb{z}h'(\mathbb{z})}{\eta(\mathbb{z})} = Re\left\{\frac{r_2}{\mathcal{U}} + \frac{\varsigma\mathbb{z}''(\mathbb{z}).}{\varsigma'(\mathbb{z})} + 1\right\} > o. (\mathbb{z} \in \odot)$$

From Lemma (2) get the subordination (25) implies ρ(ℤ) ≺ ς(ℤ), and ς is best dominant of (25).■

Theorem (4): Suppose $\mho \in \mathbb{O}(\rho)$ satisfies $\left(\frac{\mathbb{z}^{\rho}}{\mathbb{D}^{\mathcal{E}+\rho-1}\mho(\mathbb{z})}\right)^{\mathcal{M}} \in \mathcal{S}[\varsigma(o), 1] \cap \eta, \mu > 0$. And ς be convex in ⊙, $\text{Re}\{\mathcal{U}\} > 0$ and ς(o)=1, then

If $V_1(\mathbb{z})$ given by (9) is univalent in ⊙, and

$$\varsigma(\mathbb{z}) + \frac{\mathcal{U}}{\mathcal{M}(\mathcal{E}+\rho)}\varsigma'(\mathbb{z})\mathbb{z} \prec V_1(\mathbb{z}) \quad (26)$$

$$\text{then } \varsigma(\mathbb{z}) \prec \left(\frac{\mathbb{z}^{\rho}}{\mathbb{D}^{\mathcal{E}+\rho-1}\mho(\mathbb{z})}\right)^{\mathcal{M}}$$

And ς is best dominant of (26).

Proof: The analytic function $\rho(\mathbb{z})$ defined as:

$$\rho(\mathbb{z}) \prec \left(\frac{\mathbb{z}^{\rho}}{\mathbb{D}^{\mathcal{E}+\rho-1}\mho(\mathbb{z})}\right)^{\mathcal{M}} \quad (27)$$

Logarithmically (26) with respect to ℤ, we have

$$\frac{\mathbb{z}\rho'(\mathbb{z})}{\rho'(\mathbb{z})} = \mathcal{M}\left[\rho - \mathbb{z}\frac{\left(\mathbb{D}^{\mathcal{E}+\rho-1}\mho(\mathbb{z})\right)'}{\mathbb{D}^{\mathcal{E}+\rho-1}\mho(\mathbb{z})}\right] \quad (28)$$

By the identity (4), from (27), we get

$$V_1(\mathbb{z}) = \rho(\mathbb{z}) + \frac{\mathcal{U}}{\mathcal{M}(\mathcal{E}+\rho)}\mathbb{z}\rho'(\mathbb{z}),$$

Also, by Lemma (3).■

In theorem (4), Putting $\varsigma(z) = \frac{1+\mathcal{D}_1\mathbb{z}}{1+\mathcal{D}_2\mathbb{z}}\ (-1 \leq \mathcal{D}_2 < \mathcal{D}_1 \leq 1)$ we get the corollary (3).

Corollary (3): Let $\left(\frac{\mathbb{z}^{\rho}}{\mathbb{D}^{\mathcal{E}+\rho-1}\mho(\mathbb{z})}\right)^{\mathcal{M}} \in \mathcal{S}[\varsigma(0), 1] \cap \eta, (-1 \leq \mathcal{D}_2 < \mathcal{D}_1 \leq 1), \mu > o$ and $\text{Re}\{\mathcal{U}\} > o$. And if $V_1(z)$ given by (9) is univalent in ⊙, and $\mho \in \mathbb{O}(p)$ satisfies the superordination condition (28):

$$\frac{1+\mathcal{D}_1\mathbb{z}}{1+\mathcal{D}_2\mathbb{z}} + \frac{\mathcal{U}}{\mathcal{M}(\mathcal{E}+\rho)}\frac{(\mathcal{D}_1-\mathcal{D}_2)\mathbb{z}}{(1+\mathcal{D}_2\mathbb{z})^2} \prec V_1(z), \quad (29)$$

$$then\ \frac{1+\mathcal{D}_1\mathbb{z}}{1+\mathcal{D}_2\mathbb{z}} \prec \left(\frac{\mathbb{z}^{\rho}}{\mathbb{D}^{\mathcal{E}+\rho-1}\mho(\mathbb{z})}\right)^{\mathcal{M}},$$

and $\frac{1+\mathcal{D}_1\mathbb{z}}{1+\mathcal{D}_2\mathbb{z}}$ is the best subordinate.■

Theorem (5): Suppose ς (ℤ) ≠ 1, is univalent convex in ⊙ with ς(o)=1, and $\frac{\varsigma'(\mathbb{z})}{\varsigma(\mathbb{z})}\mathbb{z}$ is starlike in ⊙ and ς satisfies:

$$Re\left\{\left(r_2 + 2r_3\varsigma(\mathbb{z})\right)\frac{q(\mathbb{z})q'(\mathbb{z})}{\mathcal{U}}\right\} > 0\ (\mathbb{z} \in \odot), \quad (30)$$

also let $\mathcal{M}, \mathcal{U} \in \mathbb{C}/\{0\}$ and $r_1, r_2, r_3 \in \mathbb{C}$. Furthermore let $\mho(\mathbb{z}) \in \mathbb{O}(\rho)$ and let $\mho(\mathbb{z})$ satisfies (30) and (31):

$$\frac{\gamma\mathbb{D}^{\mathcal{E}+\rho}\mho(\mathbb{z})+(1-\gamma)\mathbb{D}^{\mathcal{E}+\rho-1}\mho(\mathbb{z})}{\mathbb{z}^{\rho}} \neq 0 (\mathbb{z} \in \odot, 0 \leq t \leq 1) \quad (31)$$

$$and\ \left(\frac{\gamma\mathbb{D}^{\mathcal{E}+\rho}\mho(\mathbb{z})+(1-\gamma)\mathbb{D}^{\mathcal{E}+\rho-1}\mho(\mathbb{z})}{\mathbb{z}^{\rho}}\right)^{\mu} \in \mathcal{S}[\varsigma(o), 1] \cap \eta. \quad (32)$$

If $V_2(\mathbb{z})$ defined by (18) is univalent in ⊙,

$$r_1 + r_2\varsigma(\mathbb{z}) + r_3[\varsigma(\mathbb{z})]^2 + \mathcal{U}\mathbb{z}\frac{\varsigma'(\mathbb{z})}{\varsigma(\mathbb{z})} \prec V_2(\mathbb{z}) \quad (33)$$

$$then\ \varsigma(\mathbb{z}) \prec \left(\frac{\gamma\mathbb{D}^{\mathcal{E}+\rho}\mho(\mathbb{z})+(1-\gamma)\mathbb{D}^{\mathcal{E}+\rho-1}\mho(\mathbb{z})}{\mathbb{z}^{\rho}}\right)^{\mathcal{M}},$$

and ς is the *best dominant* of (32).

Proof: By (20), let the function $\rho(\mathbb{z})$ be defined on ⊙.Then shows that

$$\frac{\mathbb{z}\rho'(\mathbb{z})}{\rho'(\mathbb{z})} = \mathcal{M}\left(\frac{\gamma\mathbb{z}\left(\mathbb{D}^{\mathcal{E}+\rho}\mho(\mathbb{z})\right)'+(1-\gamma)\mathbb{z}\left(\mathbb{D}^{\mathcal{E}+\rho-1}\mho(\mathbb{z})\right)'}{\gamma\mathbb{D}^{\mathcal{E}+\rho}\mho(\mathbb{z})+(1-\gamma)\mathbb{D}^{\mathcal{E}+\rho-1}\mho(\mathbb{z})} - \rho\right), \quad (34)$$

By setting $k_1(w) = r_1 + r_2 w + r_3 w^2,\ k_2(w) = \frac{\mathcal{U}}{w}(w \in C/\{o\})$

We see that $k_2(w)$ is analytic function in $C/\{o\}$, $\theta(w)$ is analytic in C, w belong to $C/\{o\}$ and $k_2(w) \neq o$. And we have:

$$\eta(\mathbb{z}) = Z\varsigma'(\mathbb{z})(\varsigma(\mathbb{z}))k_2 = \mathcal{U}\frac{\varsigma'(\mathbb{z})\mathbb{z}}{\varsigma(\mathbb{z})}, \mathbb{z} \in \odot$$

The function $\eta(\mathbb{z})$ is starlike in ⊙ and that

$$Re\frac{k_1'(\varsigma(\mathbb{z}))}{k_2(\varsigma(\mathbb{z}))} = Re\left\{r_2 + 2r_3\varsigma(\mathbb{z})\frac{\varsigma(\mathbb{z})\varsigma'(\mathbb{z})}{\mathcal{U}}\right\} > o. \mathbb{z} \in \odot$$

By use of (32) the hypothesis (33) can be written as:

$$k_1(\varsigma(z)) + z\varsigma'(z)k_2(\varsigma(z)) \prec k_1(\rho(z)) + z\rho'(z)k_2(\rho(z)),$$

By Lemma (4). ■

Theorem (6): Let ς be convex in $\odot$ and $q(o)=1$, let $\mathcal{M}, \mathcal{U} \in C/\{o\}$ and $r_1, r_2, r_3 \in C$ and $Re\left\{r_2 \frac{\varsigma'(z)}{\mathcal{U}}\right\} > o$. Let $f \in \mathbb{O}(\rho)$ and let $\mho(z)$ satisfies the conditions (34) and (35):

$$\frac{\gamma\mathbb{D}^{\mathcal{E}+\rho}\mho(z)+(1-\gamma)\mathbb{D}^{\mathcal{E}+\rho-1}\mho(z)}{z^\rho} \neq 0, (z \in \odot, 0 \leq \gamma \leq 1) \quad (35)$$

$$and \left(\frac{\gamma\mathbb{D}^{\mathcal{E}+\rho}\mho(z)+(1-\gamma)\mathbb{D}^{\mathcal{E}+\rho-1}\mho(z)}{z^\rho}\right)^{\mathcal{M}} \in \mathcal{S}[\varsigma(0), 1] \cap \eta. \quad (36)$$

If the $V_3(z)$ given by (24) is univalent function in $\odot$, and

$$r_2\varsigma(z) + r_3 + \mathcal{U}z\varsigma'(z) \prec V_3(z), \quad (37)$$

$$then\ \varsigma(z) \prec \left(\frac{\gamma\mathbb{D}^{\mathcal{E}+\rho}\mho(z)+(1-\gamma)\mathbb{D}^{\mathcal{E}+\rho-1}\mho(z)}{z^\rho}\right)^{\mathcal{M}},$$

and ς is the best subordinate of (36). By using the theorem (3), and by Lemma (4), we obtain the result of theorem (6).■

Theorem (7): Let $\varsigma_1(0)=\varsigma_2(0)=1$ be two convex functions in $\odot$ and ς_2 satisfies (11), $\mathcal{M} > 0, \mathcal{U} \in C$ with $Re\{\mathcal{U}\} > 0$. If $\mho \in \mathbb{O}(\rho)$ such that $\left(\frac{z^\rho}{\mathbb{D}^{\mathcal{E}+\rho-1}\mho(z)}\right)^{\mathcal{M}} \in \mathcal{S}[\varsigma(o), 1] \cap \eta$, $V_1(z)$ is univalent function in $\odot$ and satisfies:

$$\varsigma_1(z) + \frac{\mathcal{U}}{\mathcal{M}(\mathcal{E}+\rho)}\varsigma_1'(z)z \prec V_1(z) \prec \varsigma_2(z) + \frac{\mathcal{U}}{\mathcal{M}(\mathcal{E}+\rho)}\varsigma_2'(z)z \quad (38)$$

where, $V_1(z)$ is given by (9), then

$$\varsigma_1(z) \prec \left(\frac{z^\rho}{\mathbb{D}^{\mathcal{E}+\rho-1}\mho(z)}\right)^{\mathcal{M}} \prec \varsigma_2(z)$$

The ς_1, ς_2 are best subordinate respectively and the best dominant of (37). ■ The following sandwich theorem obtain by theorem (2) with theorem (5).

Theorem (8): Let $\varsigma_j \neq 0$ be two convex functions in $\odot$ s.t, $\varsigma(o)=1$ $\frac{z\varsigma'_j(z)}{\varsigma_j(z)}$ $(j = 1,2)$ is starlike in $\odot$, $let r_1, r_2, r_3 \in C$, $\mathcal{M}, \mathcal{U} \in C/\{0\}$ further suppose ς_1 satisfies (29), and ς_2 satisfies (17). Let $\mho \in \mathbb{O}(p)$ and let $\mho$ satisfies the following conditions:

$$\frac{\gamma\mathbb{D}^{\mathcal{E}+\rho}\mho(z)+(1-\gamma)\mathbb{D}^{\mathcal{E}+\rho-1}\mho(z)}{z^\rho} \neq 0 (z \in \odot, 0 \leq \gamma \leq 1)$$

$$and \left(\frac{\gamma\mathbb{D}^{\mathcal{E}+\rho}\mho(z)+(1-\gamma)\mathbb{D}^{\mathcal{E}+\rho-1}\mho(z)}{z^\rho}\right)^{\mu} \in \mathcal{S}[\varsigma(o), 1] \cap \eta$$

If the $V_2(z)$ given by (18) is univalent in $\odot$,

$$r_1 + r_2\varsigma_1(z) + r_3[\varsigma_1(z)]^2 + \frac{\mathcal{U}z\varsigma_1'(z)}{\varsigma_1(z)} \prec V_2(z) \prec r_1 + r_2\varsigma_2(z) + r_3[\varsigma_2(z)]^2 + \frac{\mathcal{U}z\varsigma_2'(z)}{\varsigma_2(z)}, \quad (39)$$

$$then\ \varsigma_1(z) \prec \left(\frac{\gamma\mathbb{D}^{\mathcal{E}+\rho}\mho(z) + (1-\gamma)\mathbb{D}^{\mathcal{E}+\rho-1}\mho(z)}{z^\rho}\right) \prec \varsigma_2(z).$$

The best subordinant and dominant of (38), where, ς_1 and ς_2 are, respectively ■

We obtain the sandwich theorem by combing theorem (3) with theorem (6).

Theorem (9): Let $\varsigma_1(0)=\varsigma_2(0)=1$ be two convex functions in $\odot$, and $Let r_1, r_2, r_3 \in C$ and $\mathcal{M}, \mathcal{U} \in C/\{0\}$ and with $Re\left\{r_2 \frac{\varsigma'_1(z)}{\mathcal{U}}\right\} > 0$and ς_2 satisfies (23). $\mho \in \mathbb{O}(\rho)$ and let that $\mho$ satisfies the next conditions:

$$\frac{\gamma\mathbb{D}^{\mathcal{E}+\rho}\mho(z)+(1-\gamma)\mathbb{D}^{\mathcal{E}+\rho-1}\mho(z)}{z^\rho} \neq 0 (z \in \odot, 0 \leq \gamma \leq 1)$$

$$and \left(\frac{\gamma\mathbb{D}^{\mathcal{E}+\rho}\mho(z)+(1-\gamma)\mathbb{D}^{\mathcal{E}+\rho-1}\mho(z)}{z^\rho}\right)^{\mathcal{M}} \in \mathcal{S}[\varsigma(o), 1] \cap \eta$$

If the $V_3(z)$ given by (24) is univalent function in $\odot$,

$$r_2\varsigma_1(z) + r_3 + \mathcal{U}z\varsigma_1'(z) \prec V_3(z) \prec r_2\varsigma_2(z) + r_3 + \mathcal{U}z\varsigma_2'(z), \quad (40)$$

$$then\ \varsigma_1(z) \prec \left(\frac{\gamma\mathbb{D}^{\mathcal{E}+\rho}\mho(z)+(1-\gamma)\mathbb{D}^{\mathcal{E}+\rho-1}\mho(z)}{z^\rho}\right) \prec \varsigma_2(z),$$

The best subordinate and best dominant of (39), where the function ς_1 and ς_2 are respectively.■

4. CONCLUSIONS

Following the differential supremacy theorem, several operator differential hyperboloids requiring partial integration of a stacking suprageometric function are produced, as well as the best subordinates. The result of a sandwich type links the outcomes of dependency and dependency using Theorem 9. Keep track of intriguing corollaries for certain occupations by using the best subordinate and dominant skills. The research presented in this paper may be used to motivate the usage of alternative hyper-geometric functions related to partial integration.

Relationships with other known classes may be verified, and parameter estimates can be created, since the classes acquired using this operator must be sufficiently interesting and distinct from any other previously obtained using various operators. The findings in Corollary may spark new ideas for furthering the study, which was designed with certain functions in mind.

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