



Optimal Design of a Proportional-Derivative State Feedback Controller Based on Meta-Heuristic Optimization for a Quarter Car Suspension System

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ABSTRACT

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This paper presents a Proportional-Derivative State Feedback (PDSF) controller approach to design an active suspension system for quarter car. The objective of the PDSF controller is to eliminate the effects of road disturbances to achieve ride comfort of the driver and passengers. Finding the optimal feedback gain matrix of the PDSF controller is formulated as an optimization problem. Then, two meta-heuristic optimizations named Bees Algorithm (BA) and Grey Wolf Optimization (GWO) are employed to optimize the feedback gain matrix of the PDSF controller based on the Integral Time of Absolute Error (ITAE) index. The results show the superiority of the BA-based PDSF controller in terms of reducing the ITAE index in comparison with the results obtained from GWO based PDSF.

1. INTRODUCTION

Suspensions systems have been widely studied in the literature. The main objective of the suspension system is to isolate the car body from road disturbances [1]. The suspension systems can be categorized broadly as passive, semiactive and active. In the passive suspension system, a conventional method by utilizing a spring and a damper is used. The passive suspension system cannot handle the changes in road conditions [2]. In the case of a semiactive suspension system as compared to the passive suspension system, the conventional damper is replaced by a controllable damper. However, the active suspension system has an actuator parallel with the damper and the spring to improve passenger comfort [3]. The objective of the actuator is to reduce the vertical force that is transmitted to the car body by designing a suitable controller.

Various controller strategies for active suspensions systems have been designed and investigated to ensure better ride comfort. For example, Sam et al. [4] presented a controlling strategy based on Proportional-Integral Sliding Mode Control (PI-SMC) for active suspensions systems. Kumar and Vijayarangan [5] developed a Linear Quadratic Controller (LQR) active suspensions system for 3 degrees of freedom quarter car. The performance of the LQR active suspensions system is compared with the passive suspensions systems based on the Root Mean Square (RMS) index. The results show the superiority of the LQR active suspensions system. A robust control based on Sliding Mode Control (SMC) that allows avoiding the road disturbance for a quarter-car suspensions system is proposed by Alvarez-Sánchez [3]. The mass of the quarter-car was estimated using an algebraic estimator based on the identification method. Yakub et al. [6] presented a comparative study between the Proportional-Integral-Derivative (PID) controller and State Feedback (SF) controller to design a quarter car suspension system. The outcomes of this study show that the vibration in the

suspension system has been significantly improved by using the SF controller as compared to the PID controller.

Due to the usefulness of using optimization techniques instead of try and error method, many authors prefer to utilize meta-heuristic optimizations to determine the optimal tuning parameters of the controller. In this direction, Nagarkar and Vikhe [7] optimized the weighting matrix of the LQR controller using a Genetic Algorithm (GA). The performance of the GA-LQR controller for the quarter car suspension system is evaluated based on the RMS index. Romsai et al. [8] presented an application of tuned the designed parameters of the PID controller using Lévy-flight intensified current search (LFICuS) to improve the performance of the active suspension system. Parallel to these studies, this paper aims to investigate the ability of two meta-heuristic optimizations, Bees Algorithm (BA) and Grey Wolf Optimization (GWO), to optimize the feedback gains matrix of the Proportional-Derivative State Feedback (PDSF) controller based on Integral Time of Absolute Error (ITAE) index. The simulation results indicate that tuning the PDSF by BA was able to reduce the ITAE by 14.3% in comparison with tuning the PDSF by GWO.

The arrangement of the paper is as follows. Section 2 presents the dynamic modeling of the active suspension system. The PDSF controller is introduced in Section 3. Section 4 explains the two meta-heuristic optimizations. The simulation results are given in Section 5. Lastly, Section 6 will summarize the conclusion of this paper.

2. SYSTEM MODELLING

In order to examine the impact of the design of a state feedback controller on the active suspension system on the quarter car, an accurate mathematical model of the system based on the state-space approach must be established. The two degrees of freedom (2DOF) active suspension system that is considered in this study is illustrated in Figure 1 [9] where

m_s (Kg) is the sprung mass (chassis) and m_u (Kg) is the unsprung mass (wheel). An active suspension system can be represented as a passive suspension system with an actuator. The passive suspension system consists of a spring with a stiffness factor (k_s) N/m and a damper with a damping factor (b_s) Ns/m. A hydraulic actuator with force (F_a) N is added in parallel to the passive suspension. The tire has a stiffness factor (k_t) N/m. The state variables of the system are the displacement (x_1) m and (x_2) m and velocity (\dot{x}_1) $\frac{m}{s}$ and (\dot{x}_2) $\frac{m}{s}$ of the sprung and unsprung mass respectively. The vertical displacement of the road (d) m is represented the road disturbance.

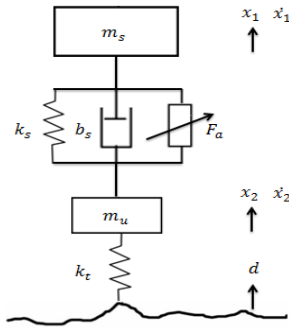


Figure 1. 2DOF active suspension system of the quarter car

Based on Newton's laws, the equations of motion for the sprung mass are given by [8]:

$$m_s \times \ddot{x}_1 = -(k_s \times (x_1 - x_2)) - (b_s \times (\dot{x}_1 - \dot{x}_2)) + F_a \quad (1)$$

Rearrange of Eq. (1) yields:

$$\ddot{x}_1 = -\left(\frac{k_s}{m_s} \times x_1\right) + \left(\frac{k_s}{m_s} \times x_2\right) - \left(\frac{b_s}{m_s} \times \dot{x}_1\right) + \left(\frac{b_s}{m_s} \times \dot{x}_2\right) + \left(\frac{1}{m_s} \times F_a\right) \quad (2)$$

In the same way, the equations of motion for the unsprung mass are given by [8]:

$$m_u \times \ddot{x}_2 = (k_s \times (x_1 - x_2)) + (b_s \times (\dot{x}_1 - \dot{x}_2)) - (k_t \times (x_2 - d)) - F_a \quad (3)$$

Rearrange of Eq. (3) yields:

$$\ddot{x}_2 = \left(\frac{k_s}{m_u} \times x_1\right) - \left(\left(\frac{k_s}{m_u} + \frac{k_t}{m_u}\right) \times x_2\right) + \left(\frac{b_s}{m_u} \times \dot{x}_1\right) - \left(\frac{b_s}{m_u} \times \dot{x}_2\right) + \left(\frac{k_t}{m_u} \times d\right) - \left(\frac{1}{m_u} \times F_a\right) \quad (4)$$

Based on Eq. (2) and Eq. (4) the state variable equations of the active quarter car suspension system are given by [8]:

$$\begin{bmatrix} \dot{x}_1 \\ \ddot{x}_1 \\ \dot{x}_2 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_s}{m_s} & -\frac{b_s}{m_s} & \frac{k_s}{m_s} & \frac{b_s}{m_s} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_u} & \frac{b_s}{m_u} & -\frac{(k_s+k_t)}{m_u} & -\frac{b_s}{m_u} \end{bmatrix} \times \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m_s} \\ 0 \\ -\frac{1}{m_u} \end{bmatrix} \times F_a + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{k_t}{m_u} \end{bmatrix} \times d \quad (5)$$

$$y = [1 \ 0 \ 0 \ 0] \times \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} \quad (6)$$

3. CONTROLLER DESIGN

In this paper, a Proportional-Derivative State Feedback (PDSF) controller is used to control the hydraulic actuator of the active suspension system. Consider the state-space model of a continuous Linear Time-Invariant (LTI) system with disturbance as follow [10]:

$$\dot{x}(t) = (A \times x(t)) + (B \times u(t)) + (W \times dis(t)) \quad (7)$$

$$y(t) = (C \times x(t)) + (D \times u(t)) \quad (8)$$

where, $x(t)$ and $\dot{x}(t)$ are the states and the derivative states of the system. A, B, C and D are dynamic matrix, input matrix, output matrix and feedforward matrix of the system. $u(t)$ is the control signal to the system whereas $y(t)$ is the output [11]. Due to the presence of the disturbance, W is the disturbance matrix and $dis(t)$ is the disturbance input.

In order to apply the PDSF controller, the states of the system need to be completely controllable and measurable. The basic idea of the PDSF controller is to measure the states of the system and to generate appropriate control action that regulates the system to achieve desirable performance. In the active suspension system, the objective of the controller is to regulate the states for any perturbation that happens in the road. The control action $u(t)$ of the PDSF is selected as given in Eq. (9) [12]:

$$u(t) = -(k_p x(t)) - (K_d \dot{x}(t)) \quad (9)$$

The control action $u(t)$ of the PDSF controller represents the control force (F_a) N of the active suspension system, k_p is the proportional gain matrix of the state feedback, K_d is the derivative gain matrix of the state feedback. Several approaches were used for finding the optimal feedback gain matrix of the PDSF controller [12]. To attain better control performance with less time consumption, two meta-heuristic optimizations named Bees Algorithm (BA) and Grey Wolf Optimization (GWO) are chosen for tuning the feedback gain matrix of the PDSF controller.

4. META-HEURISTIC OPTIMIZATIONS

As mention above, the PDSF controller has tuning parameters that need to be adjusted to obtain the desired performance. Instead of using the trial and error method which is considered a time-consuming method, the tuning process in this study is formulated as an optimization problem. Then, two meta-heuristic optimizations, Bees Algorithm (BA) and Grey Wolf Optimization (GWO) are utilized to solve the problem.

4.1 Bees Algorithm

The Bees Algorithm (BA) is a swarm-based optimization algorithm developed by Pham et al. in 2005 [13]. The algorithm mimics the way that the honey bee when it is

foraging for food. It consists of two methods of search: local search and random search. The optimization procedure of the BA begins with randomly initialized sites of a population of N scout bees within the search space as given in Eq. (10) [14]:

$$p_i = p_1 + (r \times (p_u - p_l)) \quad (10)$$

where, p_i is the initial site for each individual in the algorithm. i is the counter ($i=1,2,3,\dots,N$). r is a round value generated between $[0,1]$. p_l and p_u are the lower and upper bounds of the search space of the problem.

Then, the objective value of each scout bee is evaluated. The next step is to select an elite bee (eli) based on the best objective value was found by the scout bees. After that, a number of sites (m) are selected to perform the local search. A local search towards the site of the elite bee (p^{eli}) is performed based on the patch size (h) as given in Eq. (11) [14]:

$$p_i^{new} = p_i^{old} + h \times (p^{eli} - p_i^{old}) \quad (11)$$

where, p_i^{new} and p_i^{old} are the new site and old site of the bee i respectively. The remaining bees ($N-m$) assign to search randomly as given in Eq. (10). The new population is evaluated and greedy selection is used for the best solution. The pseudo code of the BA is given in Figure 2.

1. Input
 - Objective function (fitness function), population size (N), number of sites (m), patch size (pz), number of iteration (T)
2. Initialization
 - Initialize population N based on Eq. (10)
 - Evaluate Objective function
3. Loop:
 - For $t = 1: T$
 - ❖ Select elite bee (eli) (bees with the best objective value)
 - ❖ Determine sites m
 - ❖ Recruit bees for selected sites based on Eq. (11)
 - ❖ Assign remaining bees ($N - m$) for random search
 - ❖ Evaluate the objective function of the new population
 - ❖ Perform greedy selection for the best solution
 - ❖ If there is no convergence of the current solution & if $t > T$ go to Loop
4. Print the optimal solution

Figure 2. Pseudo code of the BA

BA has been used by many authors as a controller-tuning technique. For example, Bilgic et al. [15] demonstrated the efficiency of the tuning of the LQR controller by using BA to stabilize an inverted pendulum. Moreover, Hameed et al. [16] presented a comparative study of tuning the parameters of the PID controller using BA and Firefly Algorithm (FA). Two different transfer functions were used in the study. Both algorithms achieve good dynamic performance when the system is subject to a step input.

4.2 Grey Wolf Optimization

Grey Wolf Optimization (GWO) is a swarm-based meta-

heuristic optimization technique proposed by Mirjalili et al. in 2014. GWO simulates the social predatory behavior of gray wolves. Mirjalili et al. [17] categorized the individuals of gray wolves into four levels. The first category is the alpha wolf (α), these wolves either males or females are responsible to make decisions and lead the pack. The second category is the beta wolf (β), these wolves are lower level than the alphas which provide consultants to the alphas. The third category is the delta wolf (δ), these wolves submit information to alphas and betas. The last category is the omega wolf (ω) [18]. The simulation of the hunting process in GWO can be divided into three processes which are named: searching for prey, encircling prey and attacking prey. The general mathematical model of the movement of wolves towards the prey is formulated as follows [19]:

$$C_{wo} = 2 \times r \quad (12)$$

$$A_{wo} = (2 \times a_{wo} \times r) - a \quad (13)$$

$$D_{wo} = |C_{wo} \times W_{wop}(t) - W_{wo}(t)| \quad (14)$$

$$W_{wo}(t+1) = W_{wop}(t) - (A_{wo} \times D_{wo}) \quad (15)$$

where, a_{wo} is a coefficient its value is linearly decreased from 2 to 0 for each iteration. C_{wo} , A_{wo} and D_{wo} are coefficients their value are calculated as given in Eq. (12), Eq. (13) and Eq. (14) respectively. r is a random value generated between $[0,1]$. t refers to the current iteration in the algorithm. W_{wop} and W_{wo} are the position of the prey and the wolf respectively.

The hunting process is guided by the first three categories of wolves (α , β and δ). Hence, all wolves should adjust their position and movement based on the information that is provided by these three categories of wolves. The mathematical model of the hunting process can be represented as follows [17]:

The movement of individual wolf with respect to alphas α :

$$D_{wo\alpha} = |(C_{wo1} \times W_{wo\alpha}) - W_{wo}(t)| \quad (16)$$

$$W_{wo1} = W_{wo\alpha} - (A_{wo1} \times D_{wo\alpha}) \quad (17)$$

The movement of individual wolf with respect to betas β :

$$D_{wo\beta} = |(C_{wo2} \times W_{wo\beta}) - W_{wo}(t)| \quad (18)$$

$$W_{wo2} = W_{wo\beta} - (A_{wo2} \times D_{wo\beta}) \quad (19)$$

The movement of individual wolf with respect to deltas δ :

$$D_{wo\delta} = |C_{wo3} \cdot W_{wo\delta} - W_{wo}(t)| \quad (20)$$

$$W_{wo3} = W_{wo\delta} - (A_{wo3} \times D_{wo\delta}) \quad (21)$$

The new position of individual wolf:

$$W_{wo}^{New}(t+1) = \frac{W_{wo1} + W_{wo2} + W_{wo3}}{3} \quad (22)$$

where, $W_{wo\alpha}$, $W_{wo\beta}$ and $W_{wo\delta}$ are the best solution found by alpha, beta and delta respectively. The Pseudo code of the GWO algorithm is given in Figure 3. In the context of control engineering, it has been observed that the GWO is successfully handling the tuning process of the parameters of controllers in

a number of papers such as for Proportional-Integral (PI) controller [19], Proportional-Integral-Derivative (PID) [20], and Fractional Order PID (FOPID) controller [21].

1. Input
 - Objective function (fitness function), population size (N), coefficient value (a), number of iteration (T)
2. Initialization
 - Initialize population based on Eq. (10)
 - Evaluate Objective function
3. Loop:
 - For $t = 1:T$
 - Find $W_{wo\alpha}$, $W_{wo\beta}$ and $W_{wo\delta}$
 - For $i = 1:N$
 - ❖ Calculate C_{wo} as in Eq. (12) and A_{wo} as in Eq. (13)
 - ❖ Find the movement of alphas α based on Eq. (16) and (17)
 - ❖ Find the movement of betas β based on Eq. (18) and Eq. (19)
 - ❖ Find the movement of deltas δ based on Eq. (20) and Eq. (21)
 - ❖ Update position of the population based on Eq. (22)
 - Evaluate the objective function
 - If there is no convergence of the current solution & if $t > T$ go to Loop
4. Print the optimal solution

Figure 3. Pseudo code of the GWO

5. SIMULATION RESULTS

This section presents simulations of the PDSF controller approach to design an active quarter car suspension system. The Matlab software is used to conduct simulations and analyze the performance. The parameters of the quarter car suspension system are selected as given in Table 1 [8].

Table 1. Quarter car active suspension system parameters

Parameters	Symbol	Values	Units
Sprung mass	m_s	300	Kg
Unsprung mass	m_u	60	Kg
Stiffness of sprung spring	k_s	16000	N/m
Stiffness of tire spring	k_t	19000	N/m
Damping factor of sprung damper	b_s	1000	Ns/m

Initially, the performance of the Passive Suspension System (PSS) was tested to a road disturbance. The simulation was performed for a period of 5 seconds. The profile of the road disturbance d was chosen as a double bump in the form of:

$$d = \begin{cases} \gamma_1 \times (1 - \cos(4 \times \pi \times t_{sim})), & 0 \leq t_{sim} \leq 0.5 \\ \gamma_2 \times (1 - \cos(4 \times \pi \times t_{sim})), & 2 \leq t_{sim} \leq 2.5 \\ 0, & \text{otherwise} \end{cases} \quad (23)$$

where the value of γ_1 and γ_2 were chosen to be 0.02 and 0.03 to simulate bumps with height of 0.04 m and 0.03 m.

The investigation of the dynamic performance of the PSS is presented in Figure 4. Figure 4 shows that the system has an oscillation frequency response (unstable). Therefore, an Active Suspension System (ASS) using an optimal PDSF controller based on BE (BE-PDSF) and GWO (GWO-PDSF)

are implemented to improve the dynamics of the suspension system. The Integral Time of Absolute Errors (ITAE) criterion was selected as a cost function for the BA and the GWO to tune the feedback gain matrix of the PDSF controller. The ITAE is given by [22]:

$$ITAE = \int_0^{t_{sim}} t_{sim} \times |e(t)| dt \quad (24)$$

where, t_{sim} denotes the period time of the simulation and e is the error between the displacement of the sprung mass x_1 and the initial value (i.e. zero). Therefore, the optimization problem for the PDSF controller approach to design an active suspension system for the quarter car is defined as:

$$\text{minimize } \{ITAE(\text{var})\} \quad (25)$$

where, ITAE is the objective functions that need to be minimized. The decision vector var is the four proportional feedback gain matrix for each state and one derivative feedback gain of the output of the PDSF controller.

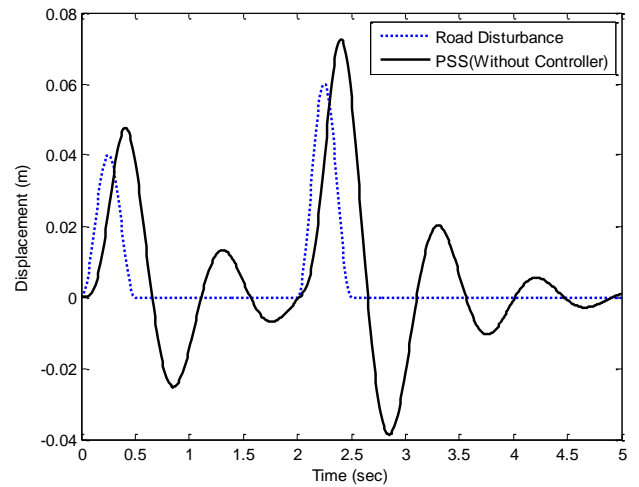


Figure 4. Road disturbance profile

The parameters of the BA and GWO algorithms are given in Table 2. These values are obtained experimentally. The experiments are performed repeatedly until the solution quality of both algorithms is improved. The simulations are carried out for BA- PDSF and GWO- PDSF models as shown in Figure 5. The performance measured in terms of ITAE is summarized in Table 3. The set of the controller's parameters for both BA- PDSF and the GWO- PDSF controlled system are given in Table 4.

Table 2. BA and GWO algorithms parameters

Parameters	BA	GWO
Number of population (N)	50	50
Number of iteration (T)	100	100
Patch size (h)	2	-
Number of sites (m)	10	-
Coefficient value (a)	-	2

Table 3. Performance comparison between BA-PDSF and GWO-PDSF

Performance Index	BA-PDSF	GWO-PDSF
TIAE	0.24	0.28

Table 4. Set of controller's parameters for BA-PDSF and GWO-PDSF

Controller's Parameters	BA-PDSF	GWO-PDSF
K_p matrix	[1125 1470 713 -1090]	[-267 656 -236 -1219]
K_d	1225	957

Table 3 and Figure 5 show that BA-PDSF and GWO-PDSF controllers were able to stabilize the suspension system effectively in comparison with the passive suspension system. However, the response achieved by the BA-PDSF controller is slightly better in comparison with the response of the GWO-PDSF controller where the ITAE of the system with the BA-PDSF controller is reduced by 14.3% in comparison with the ITAE of the system with the GWO-PDSF controller.

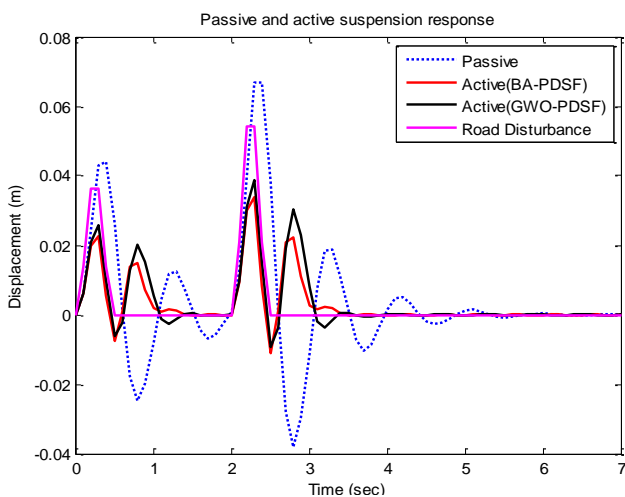


Figure 5. Passive and active suspension response to road disturbance profile

6. CONCLUSIONS

In this paper, a Proportional-Derivative State Feedback (PDSF) controller based on two meta-heuristic optimizations named Bees Algorithm (BA) and Grey Wolf Optimization (GWO) is successfully designed for active suspensions systems of a quarter car. The problem of finding the optimal value of the feedback gain matrix of the PDSF controller to make the system reach stabilization and reduce the oscillation is formulated as an optimization problem. Both controllers, BA-PDSF and GWO-PDSF, were able to stabilize the suspension system effectively in comparison with the passive suspension system. However, the simulation results show that the BA-PDSF was able to reduce the TIAE in comparison with the GWO-PDSF by 14.3%. These results show that adopting BA as a tuning technique for PDSF controller is more promising as compared with the results obtained from the GWO algorithm. This work could be extended by utilizing another optimization technique to handle the tuning process.

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NOMENCLATURE

a_{wo}	Coefficient value linearly decreased from 2 to 0 for each iteration
A	System or dynamic matrix
A_{wo}	Coefficient value calculated as given in Eq. (13)
b_s	Damping factor of the damper, Ns/m
B	Input matrix
C	Output matrix
C_{wo}	Coefficient value calculated as given in Eq. (12)
D	Vertical displacement of the road, m
dis(t)	Disturbance input

D	Feed-forward matrix
D_{wo}	Coefficient value calculated as given in Eq. (14)
e	Error
eli	Elite bee
F_a	Actuator force, N
h	Patch size
i	Counter ($i = 1, 2, 3, \dots, N$)
ITAE	Integral Time of Absolute Errors
k_p	Proportional gain
k_d	Derivative gain
k_s	Stiffness factor of the spring, (k_s) N/m
k_t	Stiffness factor of the tire, (k_s) N/m
m	Number of sites
m_s	Sprung mass (chassis), Kg
m_u	Unsprung mass (wheel), Kg
N	Size of the population
p_i	Initial site of the scout bees i
p_i^{new}	New site for bees i
p_i^{old}	Old site for bees i
p_i^{eli}	Site of the elite bees
p_l	Lower bound of the search space
p_u	Upper bound of the search space
r	Random value between [0,1]
t	Current iteration
t_{sim}	Simulation time
u(t)	Control input
Var	Decision vector
W	Disturbance matrix
W_{wo}	Position of the wolf in WOA
W_{wop}	Position of the prey
$W_{wo\alpha}$	The best solution found by alpha wolf
$W_{wo\beta}$	The best solution found by beta wolf
$W_{wo\delta}$	The best solution found by delta wolf
W_{wo}^{New}	The new position of the wolf in WOA
x(t)	States of the system
x_1	Displacement of the sprung mass, m
x_2	Displacement of the unsprung mass, m
$\dot{x}(t)$	Derivative states of the system
\dot{x}_1	Velocity of the sprung mass, m/s
\dot{x}_2	Velocity of the unsprung mass, m/s
y(t)	System output

Greek symbols

α	Alpha wolf
β	Beta wolf
δ	Delta wolf
ω	Omega wolf
γ_1, γ_2	Coefficient to form the bumps