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A Novel Gibbs Entropy Model Based upon Cross-Efficiency Measurement for Ranking Decision Making Units

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ABSTRACT

Cross-efficiency measurement in data envelopment analysis (DEA) was developed to overcome the main disadvantage of DEA in discriminating decision making units (DMUs). However, the results obtained from each cross-efficiency model (Benevolent and aggressive models) may not generally be the same for similar problems, and each model may provide different viewpoints that we should take each model into account at the same time. Since Gibbs entropy is one of powerful tools to measure uncertainty, in this paper a novel linear programming model based on the concepts of Gibbs entropy (GE model) has been offered to combine cross-efficiency scores, which are obtained from the viewpoints of benevolent and aggressive models, for ranking DMUs. In order to validate the proposed GE model, it is tested with two examples, including the performance assessment problem and the relative efficiency of seven Thai provinces. The main advantages of the GE model are that it can be used to tackle large size problems with uncertainty, and it can be used to combine other models for ranking DMUs. In addition, the set of multiple solutions of optimal weights for each model can be ignored. By using the proposed model, decision-makers can achieve more reliable decision than individual models.

1. INTRODUCTION

The concept of data envelopment analysis (DEA) was first described by Farrel [1]. However, a mathematical model, called CCR model, was offered by Charnes et al. [2]. In the DEA model, the objective function o DEA is to maximize the ratio of the weighted sum of outputs to the weighted sum of inputs for each DMU, efficiency values of each DMU cannot be greater than one [3]. The CCR model has been proven to be a useful method for measuring the relative efficiency of a set of DMUs with multiple inputs and outputs. If a DMU with relative efficiency of one, the DMU is an efficient DMU; otherwise DMU, which the relative efficiency is less than one, is defined as inefficient DMU [4]. The main advantages of using DEA to calculate efficiency are as follows [5]. Firstly, there is no need to consider the weights of inputs and outputs. Secondly, it allows inefficient factor analysis by comparing inefficient DMUs and effective DMUs. Thirdly, there is no need for the form of a production function. Over the past four decades, several scholars have proposed relative efficiency evaluation issues in different fields [6, 7]. Although the DEA-CCR model is an effective technique to evaluate the relative efficiency of a set of homogenous DMUs, one of the disadvantages of CCR model and others traditional DEA models is that efficient DMUs are indistinguishable. Hence, the cross-efficiency measurement, an extension of traditional DEA models, has been developed to overcome the main drawback. The cross-efficiency measurement is a useful and effective method to provide a ranking for all DMUs [8]. Sexton et al. [9] have taken the concept of cross-efficiency measurement into DEA model. However, ranking results obtained from the cross-efficiency measurement may be not the same for similar problems because the optimal weights obtained by the DEA model are not unique. To overcome the main drawback, Doyle and Green [10] have proposed the well-known models, called aggressive and benevolent models, to tackle the main drawback of the cross-efficiency measurement by adding a secondary goal into the cross-efficiency model. Although, the aggressive and benevolent models have been proven to be two effective tools for ranking all DMUs, the ranking results obtained from the two models may not generally be the same for similar problems. So the question arises from decision makers which one is more suitable or better? Undoubtedly, these two viewpoints should not be ignored, and to achieve maximum benefit, it is wise to try effective cross-efficiency methods and integrate the efficiency scores of each method in order to rank DMUs.

There are other directions of studies that consider cross-efficiency intervals to transform into crisp results for ranking the DMU. Yang et al. [11] have proposed an effective method to consider all sets of possible weights in calculating the interval cross-efficiencies. In the interval cross-efficiency matrix (ICEM), the acceptability index based on SMAA-2 method is calculated to obtain the ranking results of each DMU. Alcaraz et al. [12] have offered an effective approach to achieve the cross-efficiency measurement without the need to generate any specific alternative for weights of DEA. Ramón et al. [13] have proposed two models that allow for all the possible DEA weights simultaneously to produce individual lower and upper bounds for the cross-efficiencies of the different units. These methods perform cross-efficiency assessments without selecting DEA weights.

The entropy formulation is one of the effective weighting methods used to measure the uncertainty of information. According to the concept of entropy, the information quality is a main determining factor in making the right decision [14]. The Shannon's entropy can be utilized to discriminate DMUs as in the literature [15-17]. Although Shannon's entropy is widely used in DEA, the application of entropy to intervals of DEA cross-efficiency values has been proposed recently, and it has become a topic of interest. Wang et al. [17] first applied the DEA model based on entropy to transform the cross-efficiency intervals into crisp relative efficiencies of all DMUs, and each DMU can be ranked based upon the positive ideal distance. Lu and Liu [18] have offered a new mathematical model based on Gibbs entropy to calculate the optimal entropy values for ranking all DMUs. This mathematical model can be used to transform cross-efficiency intervals into crisp entropy values for ranking all DMUs, and it is easy to apply in computing using optimization software. However, the original model, Lu and Liu [18], classified as a nonlinear programming model, the optimal solutions for large size problems with uncertainty may be very hard to obtain using optimization software/exact method. Hence, the original model should be transformed to linear programming model for solving efficiency intervals of each DMU. This is the reason why Gibbs entropy should be adapted as an alternative tool for discrimination among DMUs in this research.

From the above reasons, this paper offers a new linear programming model based on the concepts of Gibbs entropy (GE model) in aggregating the benevolent and aggressive viewpoints for ranking all DMUs. The proposed GE model has been adapted from the Lu and Liu [18] to achieve more reliable decision than individual models.

The rest of this paper unfolds as follows. In the sections that follow, we first present some cross-efficiency models. After that, Section 3 presents the new solution procedure based on the concept of Gibbs entropy for ranking DMUs. Then the ideal proposed in this paper in Section 4 will be illustrated with numerical examples. Finally, Section 5 is a conclusion.

2. BACKGROUND

2.1 DEA-CCR model

The DEA-CCR model, first formulated by Charnes et al. [2], is utilized to measure the relative efficiency of a set of homogenous DMUs with multiple outputs and inputs. Several scholars [19-23] have applied the DEA-CCR model for measuring the performance of DMUs in various fields.

Consider a number of DMU$_j$ with the inputs (x_{ij}, i = 1,...,m) and outputs of DMU$_j$ (y_{rj}, r = 1,..., s). Let u_{rk} be the weights of outputs, and v_{ik} is the weights of inputs. The CCR model for measuring the performance of a set of DMU$_k$ ($1 \le k \le n$) can be defined in Eq. (1).

$$
\begin{aligned}
& Max\, E_{kk} = \sum_{r=1}^{s} u_{rk} y_{rk} \\
& \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \le 0, \forall j, j = 1,2,3,...,n \\
& \sum_{i=1}^{m} v_{ik} x_{ij} = 1, \forall j \\
& v_{ik} \ge 0, \forall i, i = 1,2,3,...,m \\
& u_{rk} \ge 0, \forall r, r = 1,2,3,...,s
\end{aligned} \quad (1)
$$

For DMU$_k$, a set of relative efficiency scores can be obtained by solving Eq. (1).

2.2 The concept of cross-efficiency measurement

The cross-efficiency method has been offered to overcome the main drawback of the traditional CCR model in discriminating efficient DMUs. The cross-efficiency formulations are:

$$
E_{kj} = \frac{\sum_{r=1}^{s} u_{rk} y_{rk}}{\sum_{i=1}^{m} v_{ik} x_{ik}}, k, j = 1,2,3,\dots,n \quad (2)
$$

where E_{kj} is the efficiency value of each DMU$_k$ and target DMU$_j$. As a result, the average cross-efficiency of DMU$_j$ ($\bar{E}_j$) is as follows.

$$
\bar{E}_j = \frac{1}{n} \sum_{k=1}^{n} E_{kj}, k, j = 1,2,3,...,n \quad (3)
$$

2.3 The benevolent and aggressive models

Doyle and Green [10] have offered the benevolent and aggressive models to generate the average cross-efficiency for ranking of all DMUs. Details of the benevolent and aggressive formulations are:

$$
\begin{aligned}
& Max\, E_{kj} = \sum_{r=1}^{s} u_{rk} \sum_{j=1,\, j\ne k}^{n} y_{rj} \\
& \sum_{i=1}^{m} v_{ik} \sum_{j=1,\, j\ne k}^{n} x_{ij} = 1, \\
& \sum_{r=1}^{s} u_{rk} y_{rj} - E_{kk} \sum_{i=1}^{m} v_{ik} x_{ij} = 0, \forall j, j \ne k, j = 1,2,...,n, \\
& \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \le 0, \forall j, j \ne k, j = 1,2,...,n, \\
& v_{ik} \ge 0, \forall i, i = 1,2,...,m, \\
& u_{rk} \ge 0, \forall r, r = 1,2,...,s,
\end{aligned} \quad (4)
$$

and

$$
Min\, E_{kj} = \sum_{r=1}^{s} u_{rk} \sum_{j=1,\, j\ne k}^{n} y_{rj} \quad (5)
$$

Subject to: the same constraints as in Eq. (4)

Eq. (4) is benevolent model which aims to maximize the efficiencies of the other (n-1) DMUs. Eq. (5) is aggressive model which aims to minimize the efficiencies of the other (n-1) DMUs. Since the viewpoints of the two models are different, the same ranking results may be not guaranteed. Hence, the idea of integrating benevolent and aggressive viewpoints in order to rank DMUs is an attractive way in applied DEA.

2.4 Gibbs entropy formulation for cross-efficiency intervals

Table 1 reports a cross-efficiency interval matrix (C-EI matrix) based on the viewpoints of the benevolent and aggressive models.

Table 1. Generalized C-EI matrix

DMU	Target DMU1	…	Target DMU*n*	Average
1	$[E_{11}^L, E_{11}^U]$	…	$[E_{1n}^L, E_{1n}^U]$	$[\bar{E}_1^L, \bar{E}_1^U]$
2	$[E_{21}^L, E_{21}^U]$	…	$[E_{2n}^L, E_{2n}^U]$	$[\bar{E}_2^L, \bar{E}_2^U]$
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
m	$[E_{m1}^L, E_{m1}^U]$	…	$[E_{mn}^L, E_{mn}^U]$	$[\bar{E}_m^L, \bar{E}_m^U]$

To deal with this problem, Lu and Liu [18] proposed a nonlinear programming model based on the concepts of Gibbs entropy to transform cross-efficiency intervals into crisp entropy values for ranking DMUs. In the C-EI matrix, DMUs and target DMUs were viewed as alternatives and criteria respectively. The values of E_{ij}^L and E_{ij}^U solved from Eq. (4) and Eq. (5), respectively, are the lower and upper values of the interval cross-efficiencies between DMU$_i$ and target DMU$_j$.

Let P_j be the probability P_j ($j = 1, 2,\dots, n$) and K is a constant value, then the entropy formulation is:

$$H = -K\sum_{j=1}^{n} P_j \ln P_j \tag{6}$$

where, $0 \le P_j \le 1, \sum_{j=1}^{n} P_j = 1$.

The entropy value of DMU$_i$ (H_i) for cross-efficiency can be formulated as:

$$\begin{aligned} H_i &= -K_i\sum_{j=1}^{n} G_{ij} \ln G_{ij} \\ &= -K_i\sum_{j=1}^{n}\left(\frac{E_{ij}}{\sum_{j=1}^{n} E_{ij}} \ln \frac{E_{ij}}{\sum_{j=1}^{n} E_{ij}}\right) \end{aligned} \tag{7}$$

where, $G_{ij} = E_{ij}/\sum_{j=1}^{n} E_{ij}$ and $K_i = (\bar{E}_i^L + \bar{E}_i^U)/2$ and the $\bar{E}_i^L$ and $\bar{E}_i^U$ are the average efficiency scores of aggressive and benevolent formulations respectively. Then Eq. (7) can be transformed as:

$$\hat{H}_i = -K_i\sum_{j=1}^{n}\left(\frac{\hat{E}_{ij}}{\sum_{j=1}^{n}\hat{E}_{ij}}\, ln\, \frac{\hat{E}_{ij}}{\sum_{j=1}^{n}\hat{E}_{ij}}\right) \tag{8}$$

where, $E_{ij}^L \le \hat{E}_{ij} \le E_{ij}^U, i = 1,2,\dots m, j = 1,2,\dots,n.$ To obtain the optimal value of $\hat{H}_i$ (Lowest uncertainty), the minimum value of $\hat{H}_i$ can be formulated as in Eq. (9).

$$\hat{H}_i = Min(-K_i\sum_{j=1}^{n}\left(\frac{\hat{E}_{ij}}{\sum_{j=1}^{n}\hat{E}_{ij}}\, ln\, \frac{\hat{E}_{ij}}{\sum_{j=1}^{n}\hat{E}_{ij}}\right)), \forall i \tag{9}$$

$$s.t.\ E_{ij}^L \le \hat{E}_{ij} \le E_{ij}^U, i = 1,2,\dots m,\ j = 1,2,\dots,n$$

Eq. (9) is a nonlinear fractional programming model; using the concept of Charnes and Cooper [24], set t_i as $t_i = 1/\hat{E}_{ij}$ and $\omega_{ij} = t_i\hat{E}_{ij}$. This model can be transformed to Eq. (10).

$$\begin{aligned} &\hat{H}_i = Min\ (-K_i\sum_{j=1}^{n}\omega_{ij} \ln \omega_{ij}) \\ &s.t. \sum_{j=1}^{n}\omega_{ij} = 1, \forall i = 1,2,3,\dots,m \\ &E_{ij}^L t_i \le \omega_{ij} \le E_{ij}^U t_i,\ j = 1,2,\dots,n, and\ t > 0 \end{aligned} \tag{10}$$

where, $\hat{H}_i$ is the entropy value of DMU$_i$, K_i can be defined as $K_i = (\bar{E}_i^L + \bar{E}_i^U)/2, \forall i = 1,2,3,\dots,n$. ω_{ij} is an increasing function for DMU$_i$ and target DMU$_j$, E_{ij}^L can be obtained from the cross-efficiencies of aggressive model that are defined in Eq. (4). E_{ij}^U can be obtained from the cross-efficiencies of benevolent model that are defined in Eq. (5). In C-EI matrix, E_{ij}^L and E_{ij}^U are the minimum and maximum efficiency scores, respectively, between DMU$_i$ and target DMU$_j$. Based on the optimal solution of $\hat{H}_i$, each DMU can be ranked. The higher value of $\hat{H}_i$ means a better ranking of the DMU.

3. PROPOSED METHOD

This section offers a new Gibb entropy models, called GE model for ranking all DMUs. The proposed framework for this study is shown in Figure 1.

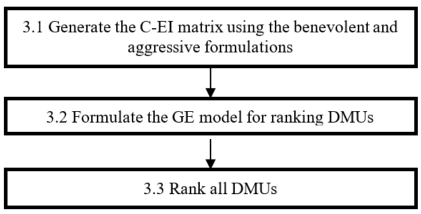

Figure 1. The proposed framework for this study

3.1 Generating the C-EI matrix

Based on viewpoints of the aggressive and benevolent formulations [10], the benevolent and aggressive models must be evaluated first using Eqns. (4) to (5). As a result, cross-efficiency matrices (CEMs) for benevolent (E^B) and aggressive (E^A) can be generated as

$$E^B = \begin{bmatrix} e_{11}^B & e_{12}^B & e_{13}^B & \cdots & e_{1n}^B \\ e_{21}^B & e_{22}^B & e_{23}^B & \cdots & e_{2n}^B \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ e_{m1}^B & e_{m2}^B & e_{m3}^B & \cdots & e_{mn}^B \end{bmatrix} \text{ and } E^A = \begin{bmatrix} e_{11}^A & e_{12}^A & e_{13}^A & \cdots & e_{1n}^A \\ e_{21}^A & e_{22}^A & e_{23}^A & \cdots & e_{2n}^A \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ e_{m1}^A & e_{m2}^A & e_{m3}^A & \cdots & e_{mn}^A \end{bmatrix} \tag{11}$$

where $E_{ij}^L = min\{e_{ij}^B, e_{ij}^A\}, \forall i = 1,2,\dots,m, \forall j = 1,2,\dots,n$ and $E_{ij}^U = max\{e_{ij}^B, e_{ij}^A\}, \forall i = 1,2,\dots,m, \forall j = 1,2,\dots,n$.

The C-EI matrix based on values of E^B and E^A is:

$$E = \begin{bmatrix} [E_{11}^L, E_{11}^U] & [E_{12}^L, E_{12}^U] & \cdots & [E_{1n}^L, E_{1n}^U] \\ [E_{21}^L, E_{21}^U] & [E_{22}^L, E_{22}^U] & \cdots & [E_{2n}^L, E_{2n}^U] \\ \vdots & \vdots \quad \vdots & \vdots & \vdots \\ [E_{m1}^L, E_{m1}^U] & [E_{m2}^L, E_{m2}^U] & \cdots & [E_{mn}^L, E_{mn}^U] \end{bmatrix} \quad (12)$$

3.2 Formulating the GE model

Since the original model, Eq. (10), is classified as a non-linear programming model, the optimal solutions may be difficult to obtain using optimization software. Hence, the original model should be transformed to linear programming model. This paper offers a novel GE formulation based upon the benevolent and aggressive models for ranking all DMUs. The details of the proposed GE model are as follows.

After obtaining the C-EI matrix from Eq. (12), k_{ij} values must be calculated first. The values of k_{ij} can be calculated as in Table 2.

Table 2. The values of k_{ij}

DMU	Target DMU$_1$	…	Target DMUn
1	$k_{11} = (E_{11}^L, E_{11}^U)/2$	…	$k_{1n} = (E_{1n}^L, E_{1n}^U)/2$
2	$k_{21} = (E_{21}^L, E_{21}^U)/2$	…	$k_{2n} = (E_{2n}^L, E_{2n}^U)/2$
.	.	.	.
.	.	.	.
m	$k_{m1} = (E_{m1}^L, E_{m1}^U)/2$	…	$k_{mn} = (E_{mn}^L, E_{mn}^U)/2$

In the original model/Eq. (10), a constant value (K_i) can be estimated into k_{ij} and $\omega_{ij}=t_i \cdot k_{ij}$, so the entropy of DMU$i$ ($\hat{H}_i$) can be defined as:

$$Min\, \hat{H}_i = -\sum_{j=1}^{n} k_{ij}(t_i k_{ij}) \ln(t_i k_{ij})$$
$$\sum_{j=1}^{n}(t_i k_{ij}) = 1, \forall i = 1,2,\dots,m \quad (13)$$
$$k_{ij} = (E_{ij}^L + E_{ij}^U)/2, and\ t_i > 0, \forall i, \forall j$$

3.3 Ranking all DMUs

In this section, the proposed GE model is examined for validity with two examples, including the performance assessment problem and a case study of relative efficiency of seven Thai provinces. Details of the calculation procedure for each problem are shown in Sections 4.1 and 4.2 respectively.

4. NUMERICAL EXAMPLES

4.1 Solving the performance assessment problem

Andersen and Petersen [25] have proposed a performance assessment problem that has five DMUs with two inputs (x_1 and x_2) and one output (y_1). Table 3 provides the data set of this problem, together with the efficiency scores based on the CCR model of each DMU. In Table 3, DMU$_1$, DMU$_2$, DMU$_3$ and DMU$_4$ are efficient DMUs, and it cannot discriminate among them.

Table 3. Data set of performance assessment problem

DMU	x_1	x_2	y_1	CCR
1	2.0	12.0	1.0	**1.000**
2	2.0	8.0	1.0	**1.000**
3	5.0	5.0	1.0	**1.000**
4	10.0	4.0	1.0	**1.000**
5	10.0	6.0	1.0	**0.750**

The efficiency scores of each DMU based on Eq. (1) must be calculated first. Next, the benevolent and aggressive formulations, as shown in Eq. (4) and Eq. (5) respectively, were solved using LINGO. As a result, the cross-efficiency matrices of benevolent and aggressive models (Benevolent CEM and aggressive CEM) were generated as listed in Table 4 and Table 5 respectively.

Table 4. Benevolent CEM of performance assessment

DMU	Target DMU$_1$	Target DMU$_2$	Target DMU$_3$	Target DMU$_4$	Target DMU$_5$
1	**1.000**	0.714	0.714	0.484	0.484
2	1.000	**1.000**	1.000	0.714	0.714
3	0.400	1.000	**1.000**	1.000	1.000
4	0.200	0.714	0.714	**1.000**	1.000
5	0.200	0.625	0.625	0.750	**0.750**

Table 5. Aggressive CEM of performance assessment

DMU	Target DMU$_1$	Target DMU$_2$	Target DMU$_3$	Target DMU$_4$	Target DMU$_5$
1	**1.000**	1.000	0.484	0.333	0.484
2	1.000	**1.000**	0.714	0.500	0.714
3	0.400	0.400	**1.000**	0.800	1.000
4	0.200	0.200	1.000	**1.000**	1.000
5	0.200	0.200	0.750	0.667	**0.750**

After obtaining the benevolent CEM and aggressive CEM, the C-EI matrix was obtained using Eqns. (11) to (12). Details of C-EI matrix of performance assessment problem were shown in Table 6.

Table 6. The C-EI matrix of performance assessment

DMU	Target DMU$_1$	Target DMU$_2$	Target DMU$_3$	Target DMU$_4$	Target DMU$_5$
1	[1.000, 1.000]	[0.714, 1.000]	[0.484, 0.714]	[0.333, 0.484]	[0.484, 0.484]
2	[1.000, 1.000]	[1.000, 1.000]	[0.714, 1.000]	[0.500, 0.714]	[0.714, 0.714]
3	[0.400, 0.400]	[0.400, 1.000]	[1.000, 1.000]	[0.800, 1.000]	[1.000, 1.000]
4	[0.200, 0.200]	[0.200, 0.714]	[0.714, 1.000]	[1.000, 1.000]	[1.000, 1.000]
5	[0.200, 0.200]	[0.200, 0.625]	[0.625, 0.750]	[0.667, 0.750]	[0.750, 0.750]

In the proposed GE model/ Eq. (13), the values of k_{ij} were obtained using the average efficiency score of each element in the C-EI matrix (Table 6), for example, k_{11} = (1.000 + 1.000)/2 = 1.000, k_{12} = (0.714 + 1.000)/2 = 0.857, k_{13} = (0.484 + 0.714)/2 = 0.599, k_{14} = (0.333+ 0.484)/2 = 0.409 and k_{15} = (0.484 + 0.484)/2 = 0.484. Details of k_{ij} are shown in Table 7.

After obtaining the values of k_{ij}, to find the optimal entropy value of $\hat{H}_i$, for DMU$_i$, the data listed in Table 6 and Table 7 were taken into Eq. (13). For example, the linear programming model for $\hat{H}_1$ is:

$$Min\,\hat{H}_1 = -\begin{bmatrix} (1.000(1.000t_1)\ln(1.000t_1) + \\ 0.857(0.857t_1)\ln(0.857t_1) + \\ 0.599(0.599t_1)\ln(0.599t_1) + \\ 0.409(0.409t_1)\ln(0.409t_1) + \\ 0.484(0.484t_1)\ln(0.484t_1) \end{bmatrix}$$

$$s.t.\ 3.349t_1 = 1,$$

$$t_1 > 0$$

Table 7. The values of k_{ij} for performance assessment

DMU	Target DMU$_1$	Target DMU$_2$	Target DMU$_3$	Target DMU$_4$	Target DMU$_5$
1	**1.000**	0.857	0.599	0.409	0.484
2	1.000	**1.000**	0.857	0.607	0.714
3	0.400	0.700	**1.000**	0.900	1.000
4	0.200	0.457	0.857	**1.000**	1.000
5	0.200	0.413	0.688	0.708	**0.750**

This, as a linear programming model, is easy to solve using any optimization solver. In this paper, the proposed GE model was coded using LINGO software. The optimal entropy value of $\hat{H}_1$ is solved as 1.0845 occurring at t_1^*= 0.2986239. With the same solution procedure, the optimal entropy values for$\hat{H}_2$, $\hat{H}_3$, $\hat{H}_4$ and $\hat{H}_5$ were 1.3488, 1.3008, 1.1642 and 0.9062 respectively. As a result, the ranking comparisons for all DMUs are provided in Table 8.

As seen in Table 8, the rankings of all models problem were obtained. The benevolent, aggressive and GE models assess that DMU$_2$> DMU$_3$ > DMU$_4$ > DMU$_1$ > DMU$_5$, but Lu and Liu's model assesses that DMU$_2$> DMU$_3$ > DMU$_1$ > DMU$_4$ > DMU$_5$.

Finally, Spearman's rank correlation test was used for testing the correlation of each method (r_s). The details of each r_s value are shown in Table 9.

As seen in Table 9, the correlation coefficients (r_s) for proposed GE model and benevolent model, aggressive model and Lu and Liu's model are calculated as r_s = 1.00, 1.00 and 0.90 respectively. This is a guarantee that the proposed GE model has a higher correlation with the benevolent and aggressive models than Lu and Liu's model [18].

Table 8. The rankings of models for performance assessment

DMU	Benevolent	Rank	Aggressive	Rank	Lu and Liu's model	Rank	GE	Rank
1	0.6793	4	0.6602	4	1.0167	3	1.0845	4
2	0.8857	1	0.7857	1	1.3185	1	1.3488	1
3	0.8800	2	0.7200	2	1.2223	2	1.3008	2
4	0.7257	3	0.6800	3	0.9932	4	1.1642	3
5	0.5900	5	0.5133	5	0.8064	5	0.9062	5

Table 9. The correlation test for performance assessment

DMU	GE	Ben.	Agg.	Lu and Liu
GE	**1.00**	1.00	1.00	0.90
Benevolent	1.00	**1.00**	1.00	0.90
Aggressive	1.00	1.00	**1.00**	0.90
Lu and Liu	0.90	0.90	0.90	**1.00**

Table 10. Data set of seven Thai provinces [26, 27]

DMU	x_1	x_2	x_3	x_4	y_1	CCR
1	169.71	1813	209.4	206950	41515	1.000
2	108.37	2700	203.7	227477	30003	0.930
3	191.17	4342	220.4	307629	54985	1.000
4	72.57	2669	200.6	169656	24711	1.000
5	292.77	4957.0	267.1	389695	60737	0.931
6	162.80	3042	302.8	280643	45053	1.000
7	105.31	2062	185.4	190081	27316	0.929

4.2 Solving the relative efficiency of seven Thai provinces

The upper northeastern region of Thailand is the poorest region of Thailand. Agriculture is still the largest sector. Rice, sugarcane and cassava are the main agriculture crop. Measuring the relative efficiency and ranking of each province is one way to find ways to develop these provinces. The upper northeastern provinces have eight DMUs with four inputs (x_1, x_2, x_3 and x_4) and one output (y_1). The DMU$_1$, DMU$_2$, DMU$_3$, DMU$_4$, DMU$_5$, DMU$_6$ and DMU$_7$ are Nong Khai, Nong Bua Lamphu, Loei, Bueng Kan, Sakon Nakhon, Nakhon Phanom and Mukdahan respectively. The x_1, x_2, x_3, x_4 and y_1 are Energy consumption (Ktoe), Agricultural area (km^2), Annual budget for 2020 (million baht), Labor and Gross Provincial Product (baht) respectively. Table 10 provides the data set of this problem, together with the efficiency scores based on the CCR model of each DMU.

In Table 10, DMU$_1$, DMU$_3$, DMU$_4$ and DMU$_6$ are efficient DMUs, and it cannot discriminate among them.

Based on the same calculation step of Section 4.1, the C-EI matrix for efficiency of seven Thai provinces was shown in Table 11.

Finally, the optimal entropy values for $\hat{H}_1$, $\hat{H}_2$, $\hat{H}_3$, $\hat{H}_4$, $\hat{H}_5$, $\hat{H}_6$ and $\hat{H}_7$were 1.872, 1.613, 1.863, 1.676, 1.521, 1.744 and 1.628 respectively. As a result, the ranking comparisons for all DMUs are provided in Table 12.

As seen in Table 12, the aggressive, Lu and Liu and GE models assess that DMU$_1$> DMU$_3$ > DMU$_6$ > DMU$_4$ > DMU$_7$> DMU$_2$ > DMU$_5$, but benevolent model assesses that DMU$_3$> DMU$_1$ > DMU$_6$ > DMU$_4$ > DMU$_7$> DMU$_2$ > DMU$_5$. The correlation coefficients (r_s) for proposed GE model and benevolent model, aggressive model and Lu and Liu's model are calculated as r_s = 0.964, 1.00 and 1.00 respectively. This is a guarantee that the proposed GE model has a high correlation with the other methods.

Since the original model, Lu and Liu [18], is classified as a non-linear programming model, the optimal solutions for large size problems with uncertainty may be difficult to obtain using optimization software/exact method. Hence, the original model should be transformed to linear programming model for solving efficiency intervals of each DMU. The main advantages of the proposed GE model are that it can be used to deal with large size problems with uncertainty, and it is simple but powerful. In addition, the set of multiple solutions of optimal weights for each model can be ignored.

Table 11. The C-EI matrix of seven Thai provinces

DMU	Target DMU_1	Target DMU_2	Target DMU_3	Target DMU_4	Target DMU_5	Target DMU_6	Target DMU_7
1	[1.000,1.000]	[0.988,0.988]	[0.795,1.000]	[0.718,0.997]	[1.000,1.000]	[1.000,1.000]	[0.988,0.988]
2	[0.485,0.925]	[0.930,0.930]	[0.590,0.925]	[0.813,0.920]	[0.671,0.671]	[0.918,0.925]	[0.930,0.930]
3	[0.553,1.000]	[1.000,1.000]	[1.000,1.000]	[0.845,1.000]	[1.000,1.000]	[0.982,1.000]	[1.000,1.000]
4	[0.404,0.982]	[1.000,1.000]	[0.494,0.982]	[1.000,1.000]	[0.560,0.560]	[0.975,0.982]	[1.000,1.000]
5	[0.535,0.781]	[0.776,0.776]	[0.781,0.911]	[0.609,0.779]	[0.931,0.931]	[0.769,0.781]	[0.776,0.776]
6	[0.647,1.000]	[1.000,1.000]	[0.596,1.000]	[0.813,1.000]	[0.727,0.727]	[1.000,1.000]	[1.000,1.000]
7	[0.578,0.928]	[0.929,0.929]	[0.591,0.928]	[0.762,0.925]	[0.703,0.703]	[0.925,0.928]	[0.929,0.929]

Table 12. The rankings of models for seven Thai provinces

DMU	Benevolent	Rank	Aggressive	Rank	Lu and Liu	Rank	GE	Rank
1	0.996	2	0.927	1	1.864	1	1.872	1
2	0.889	6	0.762	6	1.585	6	1.613	6
3	1.000	1	0.911	2	1.844	2	1.863	2
4	0.930	4	0.776	4	1.610	4	1.676	4
5	0.801	7	0.758	7	1.504	7	1.521	7
6	0.961	3	0.826	3	1.720	3	1.744	3
7	0.896	5	0.774	5	1.608	5	1.628	5

5. CONCLUSIONS

Although the cross-efficiency evaluation method is an effective tool to calculate average cross-efficiency score for ranking DMUs with multiple inputs and outputs, rankings derived from each cross-efficiency model may not be the same for similar problems. The traditional models (Benevolent and aggressive models) may select a set of optimal weights based on their alternative secondary goals for performing cross-efficiency measurement. It is therefore possible that the optimal weight obtained from each model may not be the same for similar problems due to the different viewpoints of each model, and each model may provide valuable information that we should take each model into account. Differently from previous methods in the literature, this paper offers a novel linear programming model based on Gibbs entropy, called GE model, to measure uncertainty of cross-efficiency intervals for ranking all DMUs. Two numerical examples, including the performance assessment problem and the relative efficiency of seven Thai provinces, have illustrated the advantages, potential and applications of the proposed method. Exactly, the DMU discrimination performance is more reliable results than individual models.

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