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The Use of Fuzzy Linear Regression with Trapezoidal Fuzzy Numbers to Predict the Compressive Strength of Lightweight Foamed Concrete

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ABSTRACT

Lightweight foamed concrete is defined as one of the most broadly implemented sustainable material in the construction of buildings. Due to its properties, it has been commonly applied in structural design providing energy conservation and excellent durability and functional properties. This paper describes the characteristics of lightweight foamed concrete and its properties for application in constructions. Also presents the prediction of its compressive strength by using Fuzzy Linear Regression (FLR) method with trapezoidal fuzzy numbers. Particularly, many approaches were applied in calculating the compressive strength of foamed concrete, such as multivariable nonlinear regression method, single or hybrid machine learning models and FLR method with trapezoidal fuzzy numbers. By applying them and analyzing the calculated values, it was concluded that although the last method did not have the smallest predictive accuracy criteria among the other methods, it provides a specific relation to calculate the compressive strength. In contrast to the other black box methods, FLR method with trapezoidal fuzzy numbers can be proposed as an efficient modelling tool in construction industry.

1. INTRODUCTION

The lightweight foamed concrete is recognized as an environmentally friendly material which consists of binders, water, fine aggregates and a foaming agent that forms the air pores [1, 2]. Due to its distinctive qualities, the major advantages of lightweight foamed concrete in contrast with the conventional concrete are the low density that reduces the dead load of the structure and thermal conductivity which makes it capable of being used in thermal insulation [3, 4]. Also, its high durability in resisting in external agents, which can cause the deterioration of the structure, as well as the excellent flowability, have the potential to be applied in the construction of buildings. In addition, the lightweight foamed concrete is characterized as an economical constructive material as the cost of the building is reduced by enhancing the low dead load of the foundation [5].

The aforementioned properties are responsible for defining the workability of the material and evaluating its mechanical properties. Therefore, the determination of the compressive strength of the lightweight foamed concrete is an important procedure as it provides the designing mixture proportions and the strength of the structure, avoiding construction failures. Many researchers were interested in examining the effects of foamed concrete constituents on the compressive strength, as this process may be very complex.

For example, in order to verify the compressive strength of foamed concrete, a series of experiments were performed using cement, water, foaming agent and fly ash in some mixes [3]. For calculating the compressive strength, cubes with dimension 150x150x150 mm were used and the type of cement was CEM I 42.5 R, according to PN-EN 197-1: 2011. The results showed that the compressive strength increased when the content of foaming agent was also increased. Moreover, an experimental study [6] has combined a set of specimens which were composed of three foaming agents named Foamin C, synthetic and FoamTek, three dry densities namely 400, 600 and 800 kg/m^3, two types of cement like CEM I 52,5 R and CEM II A-L 42,5 R according to EN 197-1 (2006) standards and superplasticizer. The results have demonstrated that the Foamin C increased the concrete compressive strength when the water to cement ratio (w/c) was equal to 0.3. Also, CEM I 52,5 R yielded higher values of strength than the other type of cement. It is also proved that the parameter w/c had the major effect in the experimental results. In addition, for examining the importance of w/c in calculating concrete compressive strength the following equation was proposed [7]:

$$f_c = 88.04 + 6.569 \ln t - 130.5 \frac{W}{C} \tag{1}$$

in which, f_c is the compressive strength (MPa) and t is the casting time (days). This equation was derived from the study which was carried out by Smith [8]. By evaluating the results, it was concluded that the ash content increased the strength of concrete.

Some of them submitted that the major factor that affects the compressive strength of foamed concrete is the density whilst others claimed that is the air-void distribution which interferes the strength of the material. In order to determine the exact relationship between the compressive strength of lightweight foamed concrete and its components, empirical methods were proposed that combined them with functions [9].

For instance, the compressive strength is determined as a function of the binder ratio [3], with the following form:

$$f_{cc} = 1.172\, f_c \alpha_b^{3.7} \quad (2)$$

where, f_c is the compressive strength of the cement paste and α_b is the binder ratio. Also, the compressive strength of the lightweight foamed concrete can be calculated by the dry density ratio [10], using the following equation:

$$f_{cc} = f_c(-0.324 + 1.325\alpha_d)^2 \quad (3)$$

where, f_c is the compressive strength of the cement paste and α_d is the dry density ratio.

However, these methods do not always lead to reliable results as the determination of the compressive strength of lightweight foamed concrete is a difficult task affecting many factors. These factors regard to all the constitutes of foam concrete and range from the cement type to the water to cement ratio. Each type of cement as well as the supplementary components that it comprised, affect the foamed concrete conductivity differently. Moreover, the proportion of w/c and the appropriate type of sand are highly complex factors which have to be properly set for increasing the compressive strength. Furthermore, the proportion of foaming agent is an important factor as it determines the air bubbles into the foamed concrete. These parameters make the prediction of the compressive strength a difficult task as they have to be controlled during the manufacturing process.

Therefore, by developing accurate predictive models, the level of quality assurance of the material would be ensured. In previous studies [11, 12] several methods, based on support machine learning models are proposed for predicting the compressive strength of lightweight foamed concrete. The results indicated that the methods which were based on nonlinear models yielded higher predictive accuracy than the linear models. This is because nonlinear models can consider the nonlinear relation between the parameters. It was also demonstrated that the method which has determined its hyperparameter settings during the learning period, have provided the most valid predictive effect in contrast to the other approaches in which these settings were defined as default.

The data applied in the model construction was 150 sets of concrete cubes consisting of cement, water, sand and foam. The cement used in this study was ordinary Portland cement Type I that complies according to the British Standard (BS EN 197-1: 2000) and was mixed with tap water. The proportion of w/c was 0.3, 0.35, 0.40, 0.45 and 0.5. Also, fine silica sand with small density was applied in different sizes (600 mm, 1.18 and 2 mm), while the ratio of sand to cement was 1.0 for all the sets. The inputs that were used in all the methods were: the density of concrete (kg/m^3), cement (kg/m^3), sand (kg/m^3), sand to cement ratio, water to cement ratio, the size of sand (μm), foaming agent, foam (Vm^3) and the compressive strength at 7 days (MPa). The output was the compressive strength of foamed concrete at 28 days (MPa). The aforementioned factors were chosen as inputs as they are strongly correlative with the compressive strength of foamed concrete. The methods that were included in machine learning models were: support vector regression (SVR), artificial neural networks (ANNs), Random Forests (RF), M5rules, and the multiple nonlinear regression (LR).

Support vector regression [13, 14] provides an approach for independent and dependent variables which is formed as:

$$y = f(x) + \text{noise} \quad (4)$$

where, f(x) is a deterministic function.

In order to effectively train the model, in previous study regression support vector regression Type I was used and the error function was defined as:

$$\frac{1}{2}w^T w + C\sum_{i=1}^{N}\xi_i + C\sum_{i=1}^{N}\dot{\xi}_i \quad (5)$$

in which the error function was minimized using the following form:

$$w^T\varphi_{(x_i)} + b - y_i \leq \varepsilon + \dot{\xi}_i \quad (6)$$

$$y_i - w^T\varphi_{(x_i)} - b_i \leq \varepsilon + \xi_i \quad (7)$$

$$\xi_i, \dot{\xi}_i \geq 0, i = 1, \dots N \quad (8)$$

where, w is a vector in the feature space, ξi are vectors of slack variables, c and ε are the input parameters and b is a scalar threshold. It is preferable to replace the dot product of input data $\varphi_{(x_i)}$ with kernel function in several numbers like linear (LKF), polynomial (PKF), radial basis function (RBF), and sigmoid (SKF). These functions are expressed below, respectively:

$$K(X_i, X_j) = \begin{cases} X_i \cdot X_j \\ (\gamma X_i \cdot X_j + C)^d \\ \exp(-\gamma|X_i - X_j|)^2 \\ \tanh(\gamma X_i \cdot X_j + C) \end{cases} \quad (9)$$

where, γ is a factor of kernel functions and $K(X_i, X_j)=\varphi(X_i)\varphi(X_j)$.

Artificial neural network is powerful machine learning technique which is used for model predictions. Its structure consists of one input layer, one or more hidden layers and the output. The training of the model was carried out by minimizing the error value of the calculated and the target output. This can be achieved by optimizing the weight vector and modifying them using the following equation:

$$v_{ij}^{new} = v_{ij}^{old} - \delta\frac{dE}{dv_{ij}} \quad (10)$$

More information about its mathematical theory can be retrieved from the studies [15-17].

Random Forests is an effective machine learning method which integrates trees in order to decrease the variance resulting from the combination of bagging classifier with random subset of features. The algorithm for random decision forests was developed by Breiman [18]. In addition, M5Rules model provides a reliable way for predictions as it extracts rules from the model trees. More specifically, it follows a procedure in order to train a pruned tree with a tree learner and then to make the best leaf into a rule. The purpose of the method is to accomplish rules from the elite leaf which increases the accuracy of the model and reduces the over-pruning.

Also, multiple nonlinear regression is a popular technique that uses the input variables in order to measure the output. In the previous study its equation was formed as follows:

$$\mathrm{Var}_{10} = \mathrm{a}_0 \cdot \mathrm{v}_1^{\mathrm{a}_1} \cdot \mathrm{v}_2^{\mathrm{a}_2} \cdot \mathrm{v}_3^{\mathrm{a}_3} \cdot \mathrm{v}_4^{\mathrm{a}_4} \cdot \mathrm{v}_5^{\mathrm{a}_5} \cdot \mathrm{v}_6^{\mathrm{a}_6} \cdot \mathrm{v}_7^{\mathrm{a}_7} \cdot \mathrm{v}_8^{\mathrm{a}_8} \cdot \mathrm{v}_9^{\mathrm{a}_9} \quad (11)$$

where, Var_{10} was the compressive strength of foamed concrete, $v_1 \ldots v_9$ were the inputs and $a_0 \ldots a_1$ were the parameters of the regression.

However, a hybrid artificial intelligence model was proposed, which combines the least squares support vector regression (LSSVR) method with the grey wolf optimization (GWO). This method was used for evaluating the compressive strength of lightweight foamed concrete in two scenarios. In the first one all the aforementioned values were considered for the prediction of the output whilst in the second scenario the compressive strength at 7 days was not included in the input variables.

Taking all the methods into account by comparing the predicting values with the corresponding observed ones, it was concluded that the most effective predictive method of the compressive strength of foamed concrete was the combination of the least squares support vector regression method with the grey wolf optimization. This method had the most satisfactory results as it combines two techniques for improving the predictive accuracy. By using the least squares support vector regression method, the hyperparameters settings were defined while the grey wolf optimization method was responsible for the optimization of these settings. Also, the parameters were optimized over the learning phase in contrast to the other machine learning methods in which these settings had defined values.

This study proposed a more effective fuzzy model to determine the compressive strength of lightweight foamed concrete. For modelling the relationship between lightweight foamed concrete constituents and its compressive strength, Fuzzy Linear Regression with trapezoidal fuzzy numbers was applied in the same experimental data, except from the last 6 observed strength values the sand size of which was not mentioned in the experimental previous work [11]. By evaluating the results of all the methods, it was indicated that, although FLR method with trapezoidal fuzzy numbers revealed higher error among some of the other methods, it is the most accurate predictive method. This is because it provides a standard modelling equation to understand how each input variable effect the compressive strength.

2. MATERIALS USED IN FOAMED CONCRETE MANUFACTURE

Lightweight foamed concrete composed of four basic key factors affecting its compressive strength which are cement, water, sand and foam. The aforementioned materials will be analyzed in the following sub-sections [19-23].

2.1 Cement

Cement is the most important binder material in concrete composition. There are many types of cement that can be used in lightweight foamed concrete construction such as Portland, high alumina and calcium and sulphoaluminate cement. In order to enhance the mechanical properties of cement, supplementary components like fly ash, silica fume and microsilica are applied in specific proportion. For instance, fly ash application causes the deterioration of bubble size in the composition of foamed cement and hence the increment of its early-age compressive strength. Also, due to its pozzolanic activity, silica fume contributes to the hydration process and to cement strengthening in short time. In addition, microsilica purpose is to improve the thermal insulation and the strength of concrete by making more rounded the form of the air pores. Therefore, each supplementary material has different purpose in improving cement's conductivity and should be applied in partial replacement according to cement requirements.

2.2 Water

The amount of water used in lightweight foamed concrete is conditional on the desirable properties of the mixture. More specifically, water content in low ratio causes the increment of the stiffness and the density of the mix which lead to the breaking of the bubbles. Likewise, water content in high ratio results in degeneration of the foam which also causes the increment of the density. Also, in order to avoid the negative impact of organic elements on foamed concrete, water is suggested to be normal consumable and clean.

2.3 Sand

Sand is recognized as the most used fine aggregate in the lightweight foamed concrete mixture. There are many types of sand that are appropriate in composing this concrete such as fine silica sand, ground quartz sand, M-sand and eco sand. However, fineness of the sand particles affects the strength and the workability of the concrete. It is also demonstrated that by applying different size of sand particles, the consistency of the lightweight foamed concrete will be enhanced.

2.4 Foam agent

Foam agent is used to manage the density of concrete by incorporating air bubbles into the lightweight foamed concrete mixture. The quality of foam has an impact on the final compressive strength and the stiffness of concrete as it affects the distribution and the size of its pores. In order to ensure its quality, foam agent must be maintained in airtight containers under the temperature of 25 degree centigrade. Also, foam agent is suggested to have strong structure for resisting the pressure of mortal as long as it develops a high strength skeleton around the air pores.

3. PROPERTIES OF FOAMED CONCRETE

The application of foamed concrete in structures has significant influence on its constructional behavior. The most important properties [20, 23-25] of lightweight foamed concrete are categorized into fresh, which regards consistency, rheology, stability and workability of concrete, mechanical, that are the compressive, flexural and tensile strengths, and the modulus of elasticity and physical, which are the density, drying shrinkage and porosity of foamed concrete.

3.1 Fresh properties

Fresh properties consist of many factors which are mostly affected by water to cement ratio, supplementary materials and the type of aggregates added. For instance, consistency and rheology are measured in order to determine the performance of the mixture. In addition, stability is also an important parameter which is included in fresh properties of foamed concrete. Also, the appropriate workability of foamed concrete reflects the acceptable viscosity of the mix.

3.2 Mechanical properties

Mechanical properties are very important factors to evaluate the quality of lightweight foamed concrete. The estimation of the compressive strength of concrete is a necessary process which is affected by the proportion of foam agent, water to cement and cement to sand into the mixture, the density of the final concrete mix and the sand type.

Except from the compressive strength, major controlling factors that enable the evaluation of the efficiency of foamed concrete are flexural and tensile strength and the modulus of elasticity.

3.3 Physical properties

Physical properties are associated with density, drying shrinkage and porosity of lightweight foamed concrete. The density is classified into fresh and dry density. The aim of determining the first one is to control the casting and the volume of the mixture whilst the dry density is used for managing the properties of the final concrete mix. In addition, the drying shrinkage which appears in the first days of casting time, is in higher levels in contrast to normal concrete due to the composition of foamed concrete. Finally, the porosity of concrete is an important factor of evaluating its properties as it measures the volume of voids inside the concrete.

4. METHODOLOGY

Fuzzy Linear regression model is used for determining a relationship between the dependent and the independent variables in a fuzzy environment. There are many studies [26, 27] which have applied fuzzy linear regression with triangular fuzzy numbers in order to minimizing the fuzziness of the model by using the following equation [28-34]:

$$\mathrm{Y_j = A_0 + A_1x_1 + A_2x_2 + \cdots + A_nx_n} \tag{12}$$

where, A_n are triangular fuzzy numbers. However, these models have some restrictions ensuring the inclusion of experimental values in the estimated model. More specifically, the identification of the obtained model to the experimental data is set at a selected level without guarantee the inclusion at any other levels [35]. The aim of this study is to use fuzzy linear regression with trapezoidal membership functions whilst the experimental inputs and outputs are crisp and fuzzy triangular, respectively. This model verifies that thanks to the linearity of the membership function the inclusion is guaranteed in every level of confidence optimizing the fuzziness of the model:

$$[\mathrm{y_j}]_\mathrm{h} \subseteq [\widetilde{\mathrm{Y}}_\mathrm{j}]_\mathrm{h},\ \forall\ \mathrm{h} \in [0,1] \tag{13}$$

where, y_j is the output with triangular membership function, $\widetilde{\mathrm{Y}}_\mathrm{j}$ is the calculated output with trapezoidal membership function and h is the level of confidence. This can be ensured by taking into account the following constrains:

$$\text{For } \alpha\text{=1: } [\mathrm{y_j}]_{\mathrm{h=1}} \subseteq [\widetilde{\mathrm{Y}}_\mathrm{j}]_{\mathrm{h=1}} \Leftrightarrow \mathrm{K_{Y_j}} \in \left[\mathrm{K_{\widetilde{Y}_j}}^-, \mathrm{K_{\widetilde{Y}_j}}^+\right] \tag{14}$$

$$\text{For } \alpha\text{=0: } [\mathrm{y_j}]_{\mathrm{h=0}} \subseteq [\widetilde{\mathrm{Y}}_\mathrm{j}]_{\mathrm{h=0}} \Leftrightarrow \left[\mathrm{K_{Y_j}} - \mathrm{R_{Y_j}}, \mathrm{K_{Y_j}} + \mathrm{R_{Y_j}}\right] \subseteq \left[\mathrm{S_{\widetilde{Y}_j}}^-, \mathrm{S_{\widetilde{Y}_j}}^+\right] \tag{15}$$

A two-phase possibilistic model [36] was used with nine crisp measured independent parameters and one fuzzy measured dependent value. In the first phase, Tanaka's method was applied in order to estimate the supports of the triangular fuzzy numbers. In the second phase, the aforementioned supports were coincided with the kernel of the trapezoidal fuzzy numbers and the supports of the trapezoidal fuzzy membership functions were calculated.

4.1 First phase

In this study, the equation of FLR method with trapezoidal fuzzy numbers was defined as:

$$\widetilde{\mathrm{Y}}_\mathrm{j} = \widetilde{\mathrm{A}_0} + \widetilde{\mathrm{A}_1}\mathrm{x_1} + \widetilde{\mathrm{A}_2}\mathrm{x_2} + \widetilde{\mathrm{A}_3}\mathrm{x_3} + \widetilde{\mathrm{A}_4}\mathrm{x_4} + \widetilde{\mathrm{A}_5}\mathrm{x_5} + \widetilde{\mathrm{A}_6}\mathrm{x_6} + \widetilde{\mathrm{A}_7}\mathrm{x_7} + \widetilde{\mathrm{A}_8}\mathrm{x_8} + \widetilde{\mathrm{A}_9}\mathrm{x_9} \tag{16}$$

where, $\widetilde{A}_\iota$ were trapezoidal fuzzy numbers [28] that were expressed as:

$$\widetilde{\mathrm{A}} = ([\mathrm{K_A^-}, \mathrm{K_A^+}], [\mathrm{S_A^-}, \mathrm{S_A^+}]) \tag{17}$$

in which, $\mathrm{K_A}$, $\mathrm{S_A}$ were the kernel and the supports of the trapezoidal membership functions [28] respectively, that were formed as:

$$\mathrm{K_{\widetilde{A}}} = \mathrm{kernel}(\widetilde{\mathrm{A}}) = [\mathrm{K_A^-}, \mathrm{K_A^+}] \tag{18}$$

$$\mathrm{S_{\widetilde{A}}} = \mathrm{supports}(\widetilde{\mathrm{A}}) = [\mathrm{S_A^-}, \mathrm{S_A^+}] \tag{19}$$

In the first step according to the study [35], the kernel inclusion constrains were defined as:

$$\mathrm{k_{\widetilde{y}_j}} \in \left[\mathrm{K_{\widetilde{y}_j}^-}, \mathrm{K_{\widetilde{y}_j}^+}\right] \tag{20}$$

which illustrated that the kernel of the measured parameters encircled the kernel of obtained values. By applying the Tanaka's method in the range $\left[\mathrm{Y_j^-}, \mathrm{Y_j^+}\right]^{\mathrm{h=0}}$ with triangular membership functions, the possibilistic model had the following form:

$$\mathrm{Y_j = A_0 + A_1x_1 + A_2x_2 + A_3x_3 + A_4x_4 + A_5x_5 + A_6x_6 + A_7x_7 + A_8x_8 + A_9x_9} \tag{21}$$

where, A_i=(r_i, c_i) were triangular functions and the following linear programming problem was turned out:

$$\min(\mathrm{c}) = \mathrm{mc_0} + \sum_{\mathrm{j=1}}^{144}\sum_{\mathrm{i=1}}^{9} \mathrm{c_i}|\mathrm{x_{ij}}| \tag{22}$$

$$Y^r = \sum_{i=0}^{9} r_i x_{ij} + \sum_{i=0}^{9} c_i x_{ij} \geq y_j \quad (23)$$

$$Y^l = \sum_{i=0}^{9} r_i x_{ij} - \sum_{i=0}^{9} c_i x_{ij} \leq y_j \ , x_{0j} = 1 \quad (24)$$

$$c_0, c_1 \geq 0 \quad (25)$$

where, the relations (22)-(24) were applied for minimizing the objective function (21).

From the relations (21)-(24) the surroundings were defined as:

$$\begin{aligned} Y^r = (r_0 + c_0) + (r_1 + c_1)x_j + \ldots (r_9 + c_9)x_j = \\ = 918.816 - 1.142x_1 + 0.942x_2 \\ + 0.833x_3 - 14.102x_4 \\ + 88.451x_5 - 0.010x_6 - 0.056x_7 \\ - 0.992x_8 + 0.196x_9 \end{aligned} \quad (26)$$

$$\begin{aligned} Y^l = (r_0 - c_0) + (r_1 - c_1)x_j + \ldots (r_9 - c_9)x_j = \\ = 918.816 - 1.146x_1 + 0.940x_2 \\ + 0.833x_3 - 14.102x_4 \\ + 88.451x_5 - 0.010x_6 - 0.056x_7 \\ - 0.992x_8 + 0.132x_9 \end{aligned} \quad (27)$$

Since the same restrictions were satisfied, the surroundings of the fuzzy triangular membership functions were coincided with the kernel of the fuzzy trapezoidal membership functions, using the equations:

$$\begin{aligned} \widetilde{Y^r} = K_j^+ = K_{\widetilde{A_0}}^+ + K_{\widetilde{A_1}}^+ x_{1j} + \ldots + K_{\widetilde{A_9}}^+ x_{9j}, \\ j = 1, \ldots 144 \end{aligned} \quad (28)$$

$$\begin{aligned} \widetilde{Y^l} = K_j^- = K_{\widetilde{A_0}}^- + K_{\widetilde{A_1}}^- x_{1j} + \ldots + K_{\widetilde{A_9}}^- x_{9j}, \\ j = 1, \ldots 144 \end{aligned} \quad (29)$$

4.2 Second phase

According to the supports inclusion it is resulted the following relation:

$$\left[S_{\widetilde{y_j}}^-, S_{\widetilde{y_j}}^+\right] \subseteq \left[S_{\widetilde{y_j}}^-, S_{\widetilde{y_j}}^+\right] \quad (30)$$

where, $S_{\widetilde{y_j}}^-, S_{\widetilde{y_j}}^+$ were determined as:

$$\left[S_{\widetilde{y_j}}^- \ = \ y_j^- = y_j - e_j, \ S_{\widetilde{y_j}}^+ = y_j^+ = y_j + e_j \ \right] \quad (31)$$

where, e is the spread of the data outputs. As a result, the previous linear programming problem was turned into the following:

$$\min(c) = M\left(c_0^l + c_0^r\right) + \sum_{j=1}^{144} \sum_{i=1}^{9} \left(c_i^l + c_i^r\right)\left|x_{ij}\right| \quad (32)$$

$$K_{\widetilde{Y_j}}^- - \sum_{i=0}^{9} c_i^l x_{ij} \leq \widetilde{y_j^-} - e_j, \qquad x_{0j} = 1 \quad (33)$$

$$\begin{aligned} K_{\widetilde{Y_j}}^+ + \sum_{i=0}^{9} c_i^r x_{ij} \leq \widetilde{y_j^-} + e_j, \qquad x_{0j} = 1, j \\ = 1,2 \ldots ,144 \end{aligned} \quad (34)$$

$$c_0, c_1 \geq 0 \quad (35)$$

Solving the aforementioned linear programming problem, we concluded that the equations of the supports were written as follow:

$$\begin{aligned} \widetilde{Y^r} = 918.816 - 1.140x_1 + 0.943x_2 + 0.833x_3 \\ - 14.102x_4 + 88.451x_5 \\ - 0.010x_6 - 0.056x_7 \\ -0.992x_8 + 0.228x_9 \end{aligned} \quad (36)$$

$$\begin{aligned} \widetilde{Y^l} = 918.816 - 1.148x_1 + 0.939x_2 + 0.833x_3 \\ - 14.102x_4 + 88.451x_5 \\ - 0.010x_6 - 0.056x_7 - 0.992x_8 \\ + 0.100x_9 \end{aligned} \quad (37)$$

5. MODEL APPLICATION

The prediction of 144 data of the compressive strength of lightweight foamed concrete was studied by using fuzzy linear regression with trapezoidal fuzzy numbers. The input parameters were the density of foamed concrete (kg/m^3), the cement content (kg/m^3), the sand content (kg/m^3), the sand to cement ratio, the water to cement ratio, the sand size (μm), the foaming agent, the foam (l/m^3) and the compressive strength at 7 days (MPa). The output value was the compressive strength at 28 days (MPa). The calculations of this model were represented in the Table 1 (Appendix). In order to ensure the predictive accuracy of the model the root mean square error (RMSE) and the mean absolute percentage error (MAPE) were calculated as:

$$RMSE = \sqrt{\frac{\sum_{t=1}^{n} (e_t)^2}{n}} = 2.64 \quad (38)$$

$$\begin{aligned} MAPE = 100\% \ \frac{1}{n}\sum_{t=1}^{n}\left|\frac{e_t}{A_t}\right| = 100\% \frac{1}{n}\sum_{t=1}^{n}\left|\frac{A_t - F_t}{A_t}\right| \\ = 9.21\% \end{aligned} \quad (39)$$

where, A_t are the experimental and F_t are the obtained values and n is the number of data. The values of these parameters of the previous methods as well as the FLR method are summarized in Table 2.

Table 2. The RMSE and MAPE of all the methods

Metric	RMSE	MAPE
SVR with LKF	5.03	-
SVR with RKF	2.31	-
SVR with RBF	1.81	8.65
SVR with SKF	8.17	-
ANNs	1.79	13.27
Random Forest	1.85	12.77
M5Rules	2.00	13.60
Non linear reg.	2.40	9.07
GWO-LSSVR1	1.39	3.54
GWO-LSSVR2	1.62	4.86
FLR	2.64	9.21

As it proved, each predictive model had satisfactory performance, as the deviations of the compressive strength of foamed concrete from the experimental results were low. The deviations came up from the results of MAPE parameter which indicates the mean of the dispersion between the predicted and the experimental values. According to Table 2, the most accurate approach in compressive strength calculation was the least squares support vector regression method combined with the grey wolf optimization. Its values of the root mean square error (RMSE) and the mean absolute percentage error (MAPE) were lower than the other methods which indicates the high accuracy of this model. Also, in this model the necessary hyperparameter setting were defined at the learning phase contrary to the other machine learning methods.

However, the Fuzzy Linear Regression method uses a specific relation between the input parameters in order to estimate the output values. This standard equation provides a reliable way for determining the coefficients that affect the generating results in contrast to the machine learning methods which are black box models. As a result, by using this method, it is clear the effect of every input in the compressive strength of lightweight foamed concrete. In addition, the inclusion of the FLR method with trapezoidal fuzzy numbers is ensured in every level of confidence, evaluating an effective way of predicting the compressive strength. As a result, although FLR method did not have the smallest values of RMSE factor, it can be characterized as the most valid method for carrying out the perceptual uncertainties of compressive strength predictions.

6. CONCLUSIONS

The estimation of concrete compressive strength is not an easy process as it includes many processing factors that determine its performance. In this study Fuzzy Linear Regression method with trapezoidal fuzzy numbers was applied for developing more accurate predictive model to evaluate the compressive strength of lightweight foamed concrete. More specifically, nine quality control parameters were used as inputs to estimate the output value that was the compressive strength at 28 days. These parameters were determined for calculating the output since, as indicated, they are affecting mostly the compressive strength of lightweight foamed concrete. By applying this method and comparing the results with the multivariable non linear regression, support vector machine, artificial neural networks, random forest, M5rules and the least squares support vector regression with the grey wolf optimization algorithm methods, which were analyzed in previous studies [11, 12], we concluded that the smallest RMSE value had the GWO-LSSVR method.

However, as indicated FLR is more suitable predictive method because it provides a standard relation between the parameters which specifies the way that the results are calculated. This is in contrast to black box machine learning methods, that the transformation of input into an output is a complex process not visible to the audience. Also, although multivariable non linear regression method yielded lower RMSE (2.40) than the FLR method (2.64), its obtained equation is not valid in the case that some parameters have the value of zero. In addition, the supports of the fuzzy linear regression contain every equation of non linear models.

In conclusion, all models had provided properly results with small deviations from the obtained output. Nonetheless, FLR method is more qualified than the other models and can be used accurately as a fuzzy decision model in the field of engineering.

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NOMENCLATURE

t	casting time, days
W/C	water to cement ratio
f_{cc}	compressive strength of concrete, Mpa
f_c	compressive strength of cement paste, Mpa
y	output parameter
f(x)	deterministic function
w	vector in the feature space
c	input parameter
e	input parameter
b	scalar threshold
$K(X_i,X_j)$	kernel function
v_{ij}	weight
E	error level
Var_{10}	compressive strength of foamed concrete, Mpa
$v_1 ... v_9$	inputs
y_j	output with triangular fuzzy numbers
A	triangular fuzzy number
$\widetilde{Y}_j$	output with trapezoidal fuzzy numbers
$\widetilde{A_0} ... \widetilde{A_9}$	trapezoidal fuzzy numbers
h	level of confidence
$K_{\widetilde{Y}_j}$	kernel
$S_{\widetilde{Y}_j}$	support
r_i	center of triangular number
c_i	range of values
RMSE	root mean square error
MAPE	mean absolute percentage error
A_t	experimental output
F_t	obtained value of output

Greek symbols

α_b	binder ratio

α_d dry density ratio
ξ_i vector of slack variable
γ factor of kernel function
δ learning rate
$\alpha_0 \ldots \alpha_9$ parameters of regression

APPENDIX

Table 1. Compressive strength of foamed concrete at 28 days as determined by the FLR method across 144 sets

Density	Cement	sand	s/c	w/c	Sand size	Foaming agent	foam	7-compr. strength	28-compr. strength	FLR
1505.80	614.60	614.60	1.00	0.45	600.00	0.00	295.00	7.92	11.76	14.84
1490.96	608.60	608.60	1.00	0.45	600.00	0.00	302.00	8.40	11.88	14.31
1485.04	606.13	606.13	1.00	0.45	600.00	0.00	305.00	8.24	11.92	13.70
1440.00	587.80	587.80	1.00	0.45	600.00	0.00	326.00	8.00	12.30	11.83
1417.78	578.70	578.70	1.00	0.45	600.00	0.00	336.00	8.03	11.68	11.20
1425.19	581.70	581.70	1.00	0.45	600.00	0.00	333.00	8.28	13.45	11.06
1739.85	710.14	710.14	1.00	0.45	600.00	0.00	185.00	17.32	24.63	27.24
1746.96	713.00	713.00	1.00	0.45	600.00	0.00	182.00	18.42	24.53	27.33
1749.93	714.30	714.30	1.00	0.45	600.00	0.00	181.00	19.22	24.49	27.37
1723.85	703.60	703.60	1.00	0.45	600.00	0.00	193.00	17.25	24.87	25.99
1714.37	699.70	699.70	1.00	0.45	600.00	0.00	197.00	17.22	24.39	25.95
1720.59	702.30	702.30	1.00	0.45	600.00	0.00	194.00	17.48	25.61	26.46
1904.59	777.40	777.40	1.00	0.45	600.00	0.00	108.00	28.84	31.44	36.37
1898.67	775.00	775.00	1.00	0.45	600.00	0.00	111.00	23.64	32.04	35.05
1933.63	789.30	789.30	1.00	0.45	600.00	0.00	95.00	23.50	31.46	36.28
1892.74	772.50	772.50	1.00	0.45	600.00	0.00	114.00	13.66	32.16	32.79
1845.33	753.20	753.20	1.00	0.45	600.00	0.00	136.00	13.90	31.71	31.00
1863.70	760.70	760.70	1.00	0.45	600.00	0.00	127.00	14.73	37.83	32.36
1528.89	784.00	392.00	0.50	0.45	600.00	0.00	247.00	11.03	15.54	17.58
1525.04	782.00	391.00	0.50	0.45	600.00	0.00	249.00	11.51	15.24	17.37
1519.11	779.00	389.50	0.50	0.45	600.00	0.00	252.00	12.08	15.75	17.20
1476.15	757.00	378.50	0.50	0.45	600.00	0.00	273.00	12.26	16.05	15.67
1483.26	760.65	380.33	0.50	0.45	600.00	0.00	270.00	12.32	15.71	15.49
1460.15	748.80	374.40	0.50	0.45	600.00	0.00	281.00	12.13	16.16	14.89
1748.00	900.00	450.00	0.50	0.45	600.00	0.00	136.00	14.46	30.59	35.07
1759.11	902.00	451.00	0.50	0.45	600.00	0.00	134.00	18.32	30.40	27.69
1771.26	908.30	454.15	0.50	0.45	600.00	0.00	128.00	16.99	30.19	28.07
1723.56	883.90	441.95	0.50	0.45	600.00	0.00	152.00	26.00	32.60	27.19
1725.04	884.60	442.30	0.50	0.45	600.00	0.00	151.00	26.02	25.76	27.44
1739.26	892.00	446.00	0.50	0.45	600.00	0.00	144.00	26.23	26.61	28.20
1925.93	987.70	493.85	0.50	0.45	600.00	0.00	52.00	27.16	37.20	35.98
1936.00	992.80	496.40	0.50	0.45	600.00	0.00	47.00	26.98	36.96	36.31
1910.52	979.80	489.90	0.50	0.45	600.00	0.00	60.00	27.26	37.34	34.96
1915.26	982.20	491.10	0.50	0.45	600.00	0.00	57.00	24.94	34.63	35.39
1869.04	958.50	479.25	0.50	0.45	600.00	0.00	80.00	25.54	35.48	33.38
1921.19	985.20	492.60	0.50	0.45	600.00	0.00	54.00	26.09	36.24	35.85
1544.30	447.60	895.20	2.00	0.45	600.00	0.00	319.00	8.81	11.75	9.63
1553.78	450.37	900.74	2.00	0.45	600.00	0.00	315.00	8.97	11.96	10.00
1552.00	449.90	899.80	2.00	0.45	600.00	0.00	315.00	7.84	10.45	10.62
1515.26	439.20	878.40	2.00	0.45	600.00	0.00	332.00	9.00	12.51	8.08
1523.85	441.70	883.40	2.00	0.45	600.00	0.00	328.00	9.50	13.19	8.82
1519.11	440.32	880.64	2.00	0.45	600.00	0.00	330.00	8.27	11.48	8.46
1686.52	488.90	977.80	2.00	0.45	600.00	0.00	256.00	10.25	13.85	17.33
1673.48	485.00	970.00	2.00	0.45	600.00	0.00	262.00	9.86	13.33	16.06
1675.85	485.80	971.60	2.00	0.45	600.00	0.00	261.00	10.56	14.27	16.54
1643.56	476.40	952.80	2.00	0.45	600.00	0.00	275.00	8.96	13.18	14.83
1657.48	480.40	960.80	2.00	0.45	600.00	0.00	269.00	9.41	13.84	15.36
1652.74	479.00	958.00	2.00	0.45	600.00	0.00	271.00	8.60	12.65	15.01
1885.63	546.60	1093.20	2.00	0.45	600.00	0.00	168.00	23.45	33.99	29.43
1883.26	545.90	1091.80	2.00	0.45	600.00	0.00	169.00	21.00	30.43	28.92
1893.93	549.00	1098.00	2.00	0.45	600.00	0.00	165.00	23.92	34.66	29.24
1864.89	540.50	1081.00	2.00	0.45	600.00	0.00	177.00	21.50	30.28	28.01
1891.56	548.30	1096.60	2.00	0.45	600.00	0.00	166.00	22.51	31.70	28.91
1861.33	539.50	1079.00	2.00	0.45	600.00	0.00	179.00	19.96	28.11	27.24
2009.48	837.30	837.30	1.00	0.40	600.00	0.00	81.00	33.96	43.54	45.84
2001.19	833.80	833.80	1.00	0.40	600.00	0.00	85.00	34.15	43.79	45.17
1996.44	831.90	831.90	1.00	0.40	600.00	0.00	87.00	33.70	43.20	45.18
1982.22	826.00	826.00	1.00	0.40	600.00	0.00	94.00	31.10	39.87	43.61
1972.15	821.70	821.70	1.00	0.40	600.00	0.00	98.00	32.16	41.23	43.71
1963.26	818.00	818.00	1.00	0.40	600.00	0.00	103.00	31.95	40.96	42.32

1747.26	728.00	728.00	1.000.40	600.00	0.00	201.00	21.25	27.96	30.79
1739.26	724.70	724.70	1.000.40	600.00	0.00	205.00	21.37	28.12	30.14
1738.37	724.30	724.30	1.000.40	600.00	0.00	205.00	19.68	25.89	30.17
1742.22	726.00	726.00	1.000.40	600.00	0.00	204.00	20.78	27.34	29.96
1730.96	721.23	721.23	1.000.40	600.00	0.00	209.00	22.01	28.96	29.62
1737.48	724.00	724.00	1.000.40	600.00	0.00	206.00	21.49	28.27	29.97
1431.70	596.54	596.54	1.000.40	600.00	0.00	346.00	8.64	11.08	12.68
1432.59	596.90	596.90	1.000.40	600.00	0.00	345.00	8.38	10.74	13.25
1437.04	598.80	598.80	1.000.40	600.00	0.00	343.00	8.51	10.91	13.53
1406.81	586.20	586.20	1.000.40	600.00	0.00	357.00	6.44	9.20	11.53
1418.07	590.90	590.90	1.000.40	600.00	0.00	352.00	7.25	10.35	12.08
1411.56	588.15	588.15	1.000.40	600.00	0.00	355.00	6.53	9.33	11.56
1919.70	816.13	816.13	1.000.35	600.00	0.20	144.00	34.39	43.53	44.13
1951.11	829.50	829.50	1.000.35	600.00	0.20	130.00	35.24	44.61	45.94
1928.89	820.00	820.00	1.000.35	600.00	0.20	140.00	35.56	45.01	44.64
1905.78	810.20	810.20	1.000.35	600.00	0.20	150.00	30.25	43.21	42.90
1923.56	817.80	817.80	1.000.35	600.00	0.20	142.00	29.48	42.11	43.86
1926.81	819.15	819.15	1.000.35	600.00	0.20	141.00	31.07	44.38	43.79
1758.52	747.60	747.60	1.000.35	600.00	0.20	216.00	27.26	39.51	34.35
1777.19	755.50	755.50	1.000.35	600.00	0.20	207.00	28.94	41.94	36.21
1782.52	757.80	757.80	1.000.35	600.00	0.20	205.00	27.38	39.68	35.93
1760.00	748.20	748.20	1.000.35	600.00	0.20	215.00	24.55	36.10	34.27
1747.26	742.80	742.80	1.000.35	600.00	0.20	221.00	24.21	35.60	33.26
1746.07	742.30	742.30	1.000.35	600.00	0.20	221.00	23.70	34.86	33.65
1596.15	678.60	678.60	1.000.35	600.00	0.20	288.00	14.65	20.63	24.21
1607.70	683.50	683.50	1.000.35	600.00	0.20	283.00	16.94	23.86	25.02
1620.15	688.80	688.80	1.000.35	600.00	0.20	277.00	15.63	22.01	25.92
1584.00	673.40	673.40	1.000.35	600.00	0.20	294.00	15.19	21.10	23.02
1587.85	675.00	675.00	1.000.35	600.00	0.20	292.00	15.75	21.87	23.53
1589.04	675.60	675.60	1.000.35	600.00	0.20	291.00	16.82	23.36	24.40
1984.00	861.40	861.40	1.000.30	600.00	0.30	139.00	43.17	48.50	52.85
1978.07	858.80	858.80	1.000.30	600.00	0.30	141.00	42.11	47.31	52.87
1974.52	857.26	857.26	1.000.30	600.00	0.30	143.00	43.50	48.88	52.44
1960.30	851.10	851.10	1.000.30	600.00	0.30	149.00	40.40	47.53	51.32
1965.63	853.40	853.40	1.000.30	600.00	0.30	147.00	41.21	48.48	51.42
1962.07	851.90	851.90	1.000.30	600.00	0.30	148.00	41.13	48.39	51.82
1759.41	763.90	763.90	1.000.30	600.00	0.30	236.00	32.57	40.21	38.86
1809.78	785.70	785.70	1.000.30	600.00	0.30	214.00	33.21	41.00	41.83
1781.93	773.60	773.60	1.000.30	600.00	0.30	226.00	33.04	40.79	40.30
1774.81	770.60	770.60	1.000.30	600.00	0.30	229.00	29.12	38.32	39.50
1767.11	767.20	767.20	1.000.30	600.00	0.30	233.00	29.53	38.86	38.38
1786.07	775.40	775.40	1.000.30	600.00	0.30	224.00	28.57	37.59	40.01
1595.26	692.60	692.60	1.000.30	600.00	0.30	307.00	19.27	25.36	27.54
1608.30	698.30	698.30	1.000.30	600.00	0.30	302.00	19.84	26.11	27.79
1624.30	705.20	705.20	1.000.30	600.00	0.30	295.00	20.12	26.47	28.72
1587.56	689.30	689.30	1.000.30	600.00	0.30	311.00	18.53	24.70	26.41
1590.52	690.50	690.50	1.000.30	600.00	0.30	309.00	18.42	24.56	27.12
1577.48	684.90	684.90	1.000.30	600.00	0.30	315.00	17.47	23.29	25.99
2000.59	850.00	850.00	1.000.35	1180.00	0.40	107.00	38.34	41.23	43.22
2005.93	852.00	852.00	1.000.35	1180.00	0.40	104.00	37.84	40.69	43.55
2006.52	852.24	852.24	1.000.35	1180.00	0.40	104.00	36.15	38.87	43.02
1986.96	844.00	844.00	1.000.35	1180.00	0.40	113.00	36.41	40.91	41.90
1981.04	841.40	841.40	1.000.35	1180.00	0.40	116.00	35.40	39.77	40.92
1969.78	836.60	836.60	1.000.35	1180.00	0.40	121.00	36.04	40.49	40.43
1814.22	770.60	770.60	1.000.35	1180.00	0.40	190.00	24.70	30.12	31.00
1788.15	759.50	759.50	1.000.35	1180.00	0.40	202.00	27.30	33.30	29.65
1828.15	776.00	776.00	1.000.35	1180.00	0.40	184.00	27.74	33.83	31.09
1785.78	758.50	758.50	1.000.35	1180.00	0.40	203.00	22.23	29.25	28.77
1806.22	767.20	767.20	1.000.35	1180.00	0.40	194.00	23.47	30.88	29.95
1785.19	758.20	758.20	1.000.35	1180.00	0.40	203.00	24.26	31.92	29.24
1654.81	702.90	702.90	1.000.35	1180.00	0.40	261.00	18.29	22.86	21.78
1650.37	701.00	701.00	1.000.35	1180.00	0.40	263.00	18.58	23.23	21.55
1658.07	704.25	704.25	1.000.35	1180.00	0.40	260.00	19.78	24.73	21.68
1633.19	693.70	693.70	1.000.35	1180.00	0.40	271.00	16.98	21.77	20.06
1627.26	691.16	691.16	1.000.35	1180.00	0.40	274.00	16.82	21.56	19.33
1631.70	693.00	693.00	1.000.35	1180.00	0.40	272.00	16.91	21.68	19.52
1952.00	829.10	829.10	1.000.35	600.00	0.40	129.00	29.81	39.22	44.31
1956.74	831.10	831.10	1.000.35	600.00	0.40	126.00	32.37	42.60	45.83
1947.85	827.30	827.30	1.000.35	600.00	0.40	130.00	31.86	41.92	45.20
1927.11	818.50	818.50	1.000.35	600.00	0.40	140.00	29.13	39.37	42.95
1924.15	817.30	817.30	1.000.35	600.00	0.40	141.00	28.84	38.97	43.17

1945.48	826.30	826.30	1.000.35	600.00	0.40	131.00	29.56	39.95	44.77
1833.48	778.80	778.80	1.000.35	600.00	0.40	181.00	29.11	36.85	38.96
1773.63	753.30	753.30	1.000.35	600.00	0.40	208.00	29.50	37.34	35.47
1771.26	752.30	752.30	1.000.35	600.00	0.40	209.00	28.62	36.23	35.27
1802.67	765.70	765.70	1.000.35	600.00	0.40	195.00	26.00	35.13	36.57
1761.78	748.30	748.30	1.000.35	600.00	0.40	213.00	26.01	35.15	34.63
1768.00	750.90	750.90	1.000.35	600.00	0.40	211.00	26.90	36.35	34.25
1595.26	677.60	677.60	1.000.35	600.00	0.40	288.00	21.17	26.80	24.51
1596.44	678.00	678.00	1.000.35	600.00	0.40	287.00	21.68	27.44	24.95
1569.48	666.70	666.70	1.000.35	600.00	0.40	299.00	20.78	26.30	23.69
1568.30	666.00	666.00	1.000.35	600.00	0.40	300.00	19.23	24.65	22.55
1556.15	661.00	661.00	1.000.35	600.00	0.40	305.00	20.76	26.62	22.87
1554.67	660.30	660.30	1.000.35	600.00	0.40	306.00	19.86	25.46	22.18