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Development of New Admittance Matrix for Newton-Raphson Power Flow in Distribution Networks

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ABSTRACT

Network modelling is a critical step in the analysis of distribution networks. It is used to relate the input currents and voltages with the output currents and voltages. This work aims to construct a new admittance matrix for distribution networks. The new admittance matrix is derived based on ABCD matrix, which presents the exact model of different types of power distribution networks considering the unbalance/balanced and single/three phase characteristics. The model takes into account shunt admittances to reflect the accurate performances of the components in the distribution networks, especially in the presence of distributed generation units. The importance of considering shunt admittance is due to the presence of capacitive charging current which can affect the node voltages. Application of the new admittance matrix of distribution networks in Newton-Raphson power flow analysis is performed. The standard IEEE 37 bus system is used to test the validity of the proposed approach. MATLAB environment is used to confirm the results. Simulation results show the accuracy of new network admittance matrix in power flow analysis.

1. INTRODUCTION

Electric power networks are one of the most complex systems [1]. Traditionally, power is produced by large power generation units and is transferred to users through transmission lines in a one-way direction. The distribution power systems have been considered as passive networks since the power flow is unidirectional (from the substation to the users). They generally rely on the transmission network for system control. However, the future electrical networks are hosting many loads, distributed generations and ancillary service devices. Such utilization makes distribution systems active due to the power generation inside their networks. Indeed, active networks can supply the excessive power to the high-voltage side. This creates bidirectional power flow in the system. Thus, the network complexity and size are significantly increased. This means that electrical networks are quickly changing and becoming harder to analysis and control [2].

Network modelling is a crucial stage in analysis of distribution networks. This is due to fact that network modelling can reflect the performance of network components in power system operation [3]. It is used to relate the input currents and voltages with the output currents and voltages [4]. The importance of selecting an appropriate and exact model for a particular power network can be understood in different issues such as power flow analysis, sensitivity analysis, and short circuit analysis. In power system literature, several methods were developed for power flow calculation [5-41], sensitivity analysis [42, 43], and short circuit analysis [44, 45] on distribution networks.

In this work, the authors focus on the issue of power flow computation. Indeed, power flow is an important analysis for future distribution systems. Many network issues like energy management, optimization, state estimation, and network reconfiguration depend on power flow results. Such issues are the heart of system operation, control and planning. Power flow calculation determines the steady state behaviour of the network by computing the voltages magnitudes and phase angles of power networks.

An efficient power flow approach is needed for three-phase unbalanced distribution systems in the presence of distributed generations and other active resources. The traditional power flow methods for transmission networks, such as Gauss Seidel, Newton-Raphson, and fast decoupled power flow are widely used for network analysis. However, these methods are associated with convergence problems when they are performed on distribution networks due to their special characteristics. Thus, many researchers have developed Backward–Forward sweep-based algorithms [11-18]. These methods can handle the radial networks but they are not suitable for meshed networks. This approach also depends on iterative processes in which two computational steps are required. Other researchers have modified the conventional methods (Newton-Raphson and Gauss–Seidel) to handle the characteristic of distribution networks [19-41]. A decoupled load-flow approach for distribution system is developed [19]. In the study [23], the Newton-Raphson method in complex form was studied. The Newton-Raphson method was studied based on current injections [24-26]. According to the study [27-29], the Newton-Raphson method was extended to include the unbalanced distribution networks. Loop frame of reference based three-phase power flow for unbalanced radial distribution networks is proposed [30]. A new power flow algorithm for islanded microgrids based on Newton trust region method is developed in Refs. [33-35]. The load flow computation is improved with an optimization factor [36]. A new method for solving the power flow problem in the ill-conditioned systems is developed [39]. Some of them show that the convergence characteristics are improved, and they are suitable to be implemented in distribution networks. However, this type of methods doesn't include the exact models of network components in formulating system admittance matrix and thus in network analysis. They are usually formulating the system using the series admittances and mutual coupling between phases ignoring the shunt admittances. The effect of the shunt admittances is significant in power flow analysis in distribution systems, especially with the incorporation of distributed generation. The integration of distributed generation can change the unidirectional characteristics of power flow, and thus the impact of shunt admittances may be changed. Taking into account that considering the shunt admittance increases the accuracy of power flow results. Therefore, system stability limits can be accurately determined under different loading conditions. Moreover, the allowable upper and lower voltage limits of system buses can be precisely set. This, in turn, would increase the accuracy of knowing the voltage regulation (V. R) on each bus.

In this regard, this work aims to formulate a new system admittance matrix for distribution networks considering the exact modelling of system components. It deals with the representation of distribution network components under normal operating conditions. The representation can be done by an equivalent model with appropriate fundamental circuit parameters. The component modelling is presented through ABCD matrix, which could simplify the analysis procedure of power network. The new admittance matrix is then utilized in Newton-Raphson based power flow.

The main contribution of this work is to include the exact models of network components in formulating the system admittance matrix. Compared with well-known methods, this method includes the shunt admittances in the analysis.

The rest of this work is organized as follows. Section 2 present the component modelling of different types of distribution power networks. Section 3 presents the new system admittance matrix. Section 4 shows simulation results. The conclusions and future works are stated in Section 5.

2. MODELING OF NETWORK COMPONENTS

This section deals with the representation of distribution network components under normal operating conditions. The representation can be done by an equivalent model with appropriate fundamental circuit parameters. A general form of representing a segment (component) is shown in Figure 1. The segment is between two nodes "*n*" and "*m*". The current passes from node "*n*" to node "*m*".

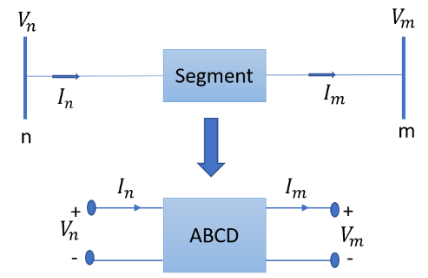

Figure 1. Two-port representation of a network segment

By applying Kirchhoff's voltage and current laws (KVL and KCL), we can obtain the following general equations:

$$V_n = AV_m + BI_m \quad (1)$$

$$I_n = CV_m + DI_m \quad (2)$$

In matrix form:

$$\begin{bmatrix} V_n \\ I_n \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_m \\ I_m \end{bmatrix} \quad (3)$$

where, *n*: sending end of the segment; *m*: receiving end of the segment; V_n: input voltage; V_m: output voltage; I_n: input current; I_m: output current; *ABCD*: segment parameters.

Eqns. (1) and (2) represent the relation between the input (voltage & current) and the output (voltage & current) of a segment. The equations can be written in terms of the circuit coefficients (i.e. ABCD constants). Thus, the segment can be represented by a two-port system with ABCD model. According to the type (or characteristics) of distribution networks, network modeling can be classified as presented in the next subsections.

2.1 Single-phase networks (Approximate model)

The representation of single-phase distribution networks can be done by an equivalent model on a per phase basis. The terminal voltage is represented as line to neutral and the current is for one phase.

2.1.1 Distribution line modeling

The distribution overhead and underground line segments in a single-phase distribution network can be approximately modeled by series impedances as shown in Figure 2. This approximation is due to fact that shunt admittances of line segments can be ignored in some cases.

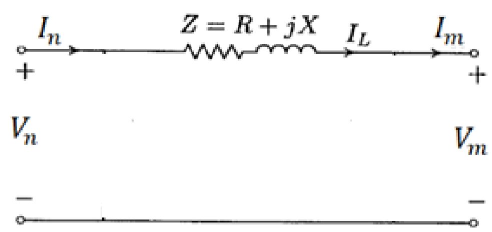

Figure 2. Approximate model of a single-phase line

where, *Z* is the total impedance of the line. *R* and *X* represent the total resistance and reactance of the line. The total line impedance can be represented as $(r+jwL)l=R+jX$. The values; *r* and L are the per phase resistance and inductance per unit length, respectively. l is the line length. V_n and I_n are the phase voltage and current at the sending end of the line, and V_m and I_m are the phase voltage and current at the receiving end of the line. I_L represents the line current.

Set of equations can be developed to model the line segment. For the line segment of Figure 2, the equations that relate the input (node n) voltage and current to the output (node m) voltage and current are developed as follows:

By applying KVL, the phase voltage at the sending end is:

$$V_n = V_m + ZI_L \quad (4)$$

By applying KCL, the phase current at the sending end is:

$$I_n = I_m \quad (5)$$

By comparing (4) and (5) with the general model Eqns. (1) and (2), we obtain that A=D=1; C=0; B=Z. In matrix form, we obtain:

$$\begin{bmatrix} V_n \\ I_n \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_m \\ I_m \end{bmatrix} \quad (6)$$

2.1.2 Distribution transformer modeling

The equivalent circuit for a two-winding transformer in a single-phase system can be approximately represented as shown in Figure 3. This approximation is due to fact that shunt admittances of transformers can be ignored in some cases. The equivalent circuit of Figure 3 can be modified by referring the primary impedance (Z_1) to the secondary side as shown in Figure 4. It is clear that the equivalent circuit of single-phase transformer consists of an ideal transformer together with elements which represent its real performance.

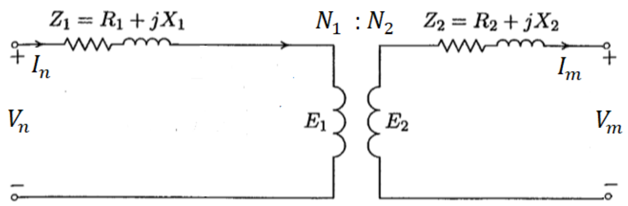

Figure 3. Approximate model of a single-phase transformer

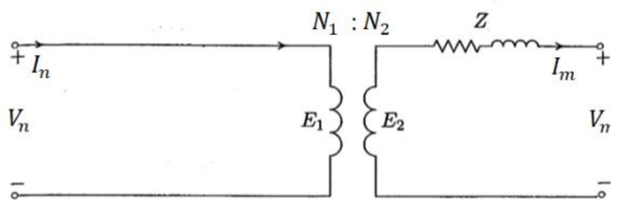

Figure 4. Modified approximate model of a single-phase transformer

The total impedance Z of the transformer is given by:

$$Z = n_t^2 Z_1 + Z_2$$

where, n_t represent the turn ratio of the transformer and can be calculated as:

$$n_t = \frac{N_2}{N_1}$$

where, Z_1: impedance of the primary winding; Z_2: impedance of the secondary winding; R_1: resistance of the primary winding; R_2: resistance of the secondary winding; X_1: reactance of the primary winding; X_2: reactance of the secondary winding; N_1: number of turns of the primary winding; N_2: number of turns of the secondary winding.

Referring to Figure 4, the equation for the ideal transformer becomes:

$$E_2 = n_t\, E_1$$

Applying KVL in the secondary circuit, we obtain

$$E_2 = V_m + ZI_m$$

The input voltage of the transformer is:

$$V_n = E_1 = \frac{1}{n_t} E_2$$

$$\text{Thus, } V_n = \frac{1}{n_t} V_m + \frac{Z}{n_t} I_m \quad (7)$$

The input current to the transformer is:

$$I_n = n_t\, I_m \quad (8)$$

By comparing (7) and (8) with the general model equations: we obtain that:

$$\begin{bmatrix} V_n \\ I_n \end{bmatrix} = \begin{bmatrix} \frac{1}{n_t} & \frac{Z}{n_t} \\ 0 & n_t \end{bmatrix} \begin{bmatrix} V_m \\ I_m \end{bmatrix} \quad (9)$$

2.2 Single-phase networks (Exact model)

The exact model of single-phase networks can be obtained by including the shunt admittances of line segments and transformers as shown in Figure 5 and Figure 6, respectively. The only difference between the exact and the approximate models is the parameters of ABCD matrix.

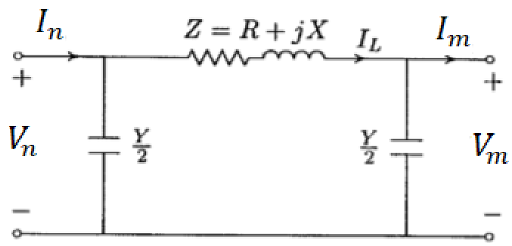

Figure 5. Exact model of a single-phase line

where, Y is the total shunt admittance of the line and can be represented as:

$$Y = (jwc)l$$

where, c is the per phase capacitance per unit length, respectively.

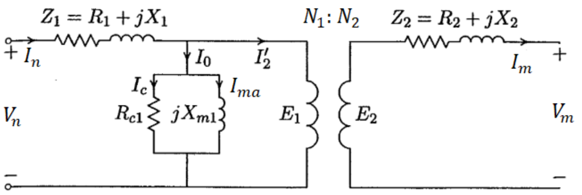

Figure 6. Exact model of a single-phase transformer

where, R_{c1} and X_{m1} are the core resistance and magnetizing reactance, respectively. The transformer shunt admittance Y can be obtained as $R_{c1}//X_{m1}$.

By repeating the same procedure presented in section 2.1.1, we obtain that the matrix ABCD of single-phase lines is changed as:

$$\begin{bmatrix} V_n \\ I_n \end{bmatrix} = \begin{bmatrix} 1+\frac{ZY}{2} & Z \\ Y\left(1+\frac{ZY}{4}\right) & 1+\frac{ZY}{2} \end{bmatrix} \begin{bmatrix} V_m \\ I_m \end{bmatrix} \tag{10}$$

Similarly, by repeating the same procedure presented in section 2.1.2, we also obtain that the matrix ABCD of single-phase transformers is changed as:

$$\begin{bmatrix} V_n \\ I_n \end{bmatrix} = \begin{bmatrix} \frac{1}{n_t} & \frac{Z}{n_t} \\ \frac{Y}{n_t} & \frac{YZ}{n_t}+n_t \end{bmatrix} \begin{bmatrix} V_m \\ I_m \end{bmatrix} \tag{11}$$

2.3 Balanced three-phase networks

The representation of balanced three-phase distribution networks can be done by an equivalent model on a "per-phase" basis, similar to models presented in section 2.1. The parameters (Z and Y) are represented per phase. The terminal voltage is presented from line to neutral and the current is for one phase. Thus, the relations expressed in (6), (9) are used for approximate modelling of balanced three phase networks while the expressions illustrated in (10) and (11) are used for exact modelling of balanced three phase networks.

2.4 Unbalanced three-phase networks (Exact model)

For unbalanced systems, it is necessary to involve the actual phasing and the correct spacing between segments (or windings). The impedances and admittances parameters are represented per phase.

2.4.1 Distribution line modelling

The distribution overhead and underground line segments in an unbalanced three-phase distribution network can be represented as shown in Figure 7.

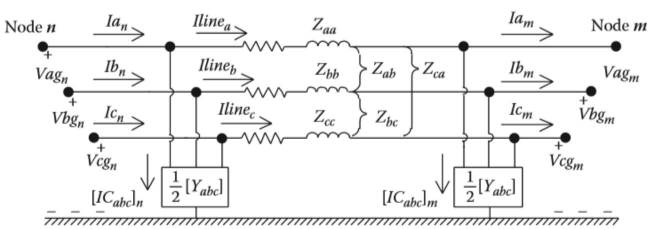

Figure 7. Unbalanced three- phase distribution line model

where, Z_{aa}, Z_{bb}, and Z_{cc} are self-impedances for the line a, b and c respectively. Z_{ab}, Z_{bc}, and Z_{ca} are mutual impedances between the lines. Thus, the impedance matrix Z_{abc} and shunt admittance matrix Y_{abc} for the three phases can be obtained as:

$$Z_{abc} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix}, Y_{abc} = \begin{bmatrix} Y_{aa} & Y_{ab} & Y_{ac} \\ Y_{ba} & Y_{bb} & Y_{bc} \\ Y_{ca} & Y_{cb} & Y_{cc} \end{bmatrix}$$

By applying KCL and KVL concepts, we can obtain the ABCD matrix of three-phase lines is as presented in (12)-(15) conclude that:

$$\begin{bmatrix} V_n^{abc} \\ I_n^{abc} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_m^{abc} \\ I_m^{abc} \end{bmatrix} \tag{12}$$

where, $V_n^{abc} = \begin{bmatrix} V_n^{ag} \\ V_n^{bg} \\ V_n^{cg} \end{bmatrix}$, $V_m^{abc} = \begin{bmatrix} V_m^{ag} \\ V_m^{bg} \\ V_m^{cg} \end{bmatrix}$, $I_n^{abc} = \begin{bmatrix} I_n^{a} \\ I_n^{b} \\ I_n^{c} \end{bmatrix}$, $I_m^{abc} = \begin{bmatrix} I_m^{a} \\ I_m^{b} \\ I_m^{c} \end{bmatrix}$.

From the analysis of KVL and KCL concepts ABCD parameters can be found as:

$$A = D = U + \frac{1}{2} Z_{abc} Y_{abc} \tag{13}$$

$$B = Z_{abc} \tag{14}$$

$$C = Y_{abc} + \frac{1}{4} Y_{abc} Z_{abc} Y_{abc} \tag{15}$$

where, U is a diagonal matrix and their elements equal to 1.

2.4.2 Distribution transformer modeling

The distribution transformers can be divided into several types based on their winding's connection. The transformer modelling depends on the winding connection. A general three-phase transformer is shown in Figure 8.

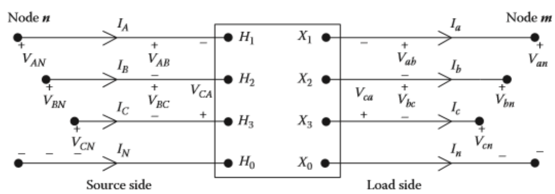

Figure 8. General three-phase transformer bank

By applying KVL and KCL on the transformer terminals, we conclude that:

$$\begin{bmatrix} V_n^{abc} \\ I_n^{abc} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_m^{abc} \\ I_m^{abc} \end{bmatrix} \tag{16}$$

where,

$$V_n^{abc} = \begin{bmatrix} V_n^{AN} \\ V_n^{BN} \\ V_n^{CN} \end{bmatrix}, V_m^{abc} = \begin{bmatrix} V_m^{an} \\ V_m^{bn} \\ V_m^{cn} \end{bmatrix}, I_n^{abc} = \begin{bmatrix} I_n^{A} \\ I_n^{B} \\ I_n^{C} \end{bmatrix}, I_m^{abc} = \begin{bmatrix} I_m^{a} \\ I_m^{b} \\ I_m^{c} \end{bmatrix}.$$

The ABCD parameters of different types of distribution transformer are summarized in Table 1.
where,

$$Z_{tabc} = \begin{bmatrix} Z_{ta} & 0 & 0 \\ 0 & Z_{tb} & 0 \\ 0 & 0 & Z_{tc} \end{bmatrix} \quad (17)$$

$$WV^{-1} = \frac{1}{3}\begin{bmatrix} 4 & -2 & 1 \\ 1 & 4 & -2 \\ -2 & 1 & 4 \end{bmatrix} \quad (18)$$

$$WI^{-1} = \frac{1}{3}\begin{bmatrix} 2 & 0 & 1 \\ 1 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix} \quad (19)$$

The terms V_{LL} *and* V_{LN} denote the line- line voltage and line-neutral voltages, respectively. The subscripts p and s represent the primary and secondary winding, respectively. n_t is the effective turn ratio. The impedances Z_{ta}, Z_{tb}, and Z_{tc} are impedances for *Y* connected windings. The impedances Z_{tab}, Z_{tbc}, and Z_{tca} are impedances for *Δ* connected windings.

Table 1. ABCD parameters of different types of distribution transformer

Transformer connection			ABCD parameters			
Node *n*	Node *m*	n_t	*A*	B^{-1}	*C*	*D*
Y-G	Y-G	$\frac{V_{LN,p}}{V_{LN,s}}$	$n_t\,U$	$\frac{1}{n_t}\begin{bmatrix} Z_{ta} & 0 & 0 \\ 0 & Z_{tb} & 0 \\ 0 & 0 & Z_{tc} \end{bmatrix}^{-1}$	0	$\frac{1}{n_t}\,U$
Y-G	Δ	$\frac{V_{LN,p}}{V_{LL,s}}$	$n_t\,WV^{-1}$	$\frac{1}{n_t}\begin{bmatrix} \frac{1}{Z_{tab}} & 0 & \frac{-1}{Z_{tca}} \\ \frac{-1}{Z_{tab}} & \frac{1}{Z_{tbc}} & 0 \\ 0 & \frac{-1}{Z_{tbc}} & \frac{1}{Z_{tca}} \end{bmatrix}$	0	$\frac{1}{n_t}\,WI^{-1}$
Y	Δ	$\frac{V_{LN,p}}{V_{LL,s}}$	$n_t\,WV^{-1}$	$\frac{1}{n_t}\begin{bmatrix} \frac{1}{Z_{tab}} & 0 & \frac{-1}{Z_{tca}} \\ \frac{-1}{Z_{tab}} & \frac{1}{Z_{tbc}} & 0 \\ 0 & \frac{-1}{Z_{tbc}} & \frac{1}{Z_{tca}} \end{bmatrix}$	0	$\frac{1}{3n_t}\begin{bmatrix} 2 & 0 & 1 \\ 2 & 3 & 1 \\ -1 & 0 & 1 \end{bmatrix}$
Δ	Y-G	$\frac{V_{LL,p}}{V_{LN,s}}$	$\frac{-n_t}{3}\begin{bmatrix} 0 & 2 & 1 \\ 1 & 0 & 2 \\ 2 & 1 & 0 \end{bmatrix}$	$\frac{-1}{3n_t}\begin{bmatrix} \frac{-2}{Z_{ta}} & \frac{1}{Z_{ta}} & \frac{4}{Z_{ta}} \\ \frac{4}{Z_{tb}} & \frac{-2}{Z_{tb}} & \frac{1}{Z_{tb}} \\ \frac{1}{Z_{tc}} & \frac{4}{Z_{tc}} & \frac{-2}{Z_{tc}} \end{bmatrix}$	0	$\frac{1}{n_t}\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$
Δ	Δ	$\frac{V_{LL,p}}{V_{LL,s}}$	$n_t\,U$	WV^{-1}	0	$\frac{1}{n_t}\,U$
Open Y-G (phase c)	Open Δ (phase c)	$\frac{V_{LN,p}}{V_{LL,s}}$	$n_t\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$	$\frac{-1}{3n_t}\begin{bmatrix} \frac{1}{Z_{ta}} & 0 & 0 \\ \frac{-1}{Z_{ta}} & \frac{1}{Z_{tb}} & 0 \\ 0 & \frac{-1}{Z_{tb}} & 0 \end{bmatrix}$		$\frac{1}{n_t}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$

3. NEW NETWORK ADMITTANCE MATRIX OF DISTRIBUTION NETWORKS

Figure 9 presents a one-line diagram of a balanced three-phase distribution system. The network consists of four buses. The voltage magnitude V_n, phase angle δ_n, active and reactive power P_n, Q_n are the common quantities linked with every bus.

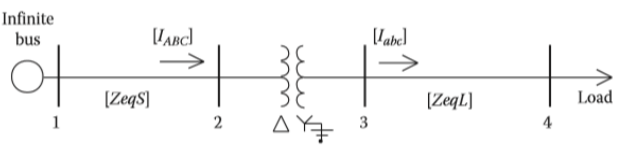

Figure 9. One line diagram of a balanced three phase distribution feeder

By applying KCL at each node and considering the general formula illustrated in (2), we obtain:

At node 1 (Where 0 denotes the ground):

$$I_1 = (C_{10}V_1) + (C_{12}V_2 + D_{12}I_{12}) \quad (20)$$

At node 2:

$$0 = (C_{21}V_1 + D_{21}I_{21}) + (C_{23}V_3 + D_{23}I_{23}) \quad (21)$$

At node 3:

$$0 = (C_{32}V_2 + D_{32}I_{32}) + (C_{34}V_4 + D_{34}I_{34}) \quad (22)$$

At node 4:

$$-I_4 = (C_{43}V_3 + D_{43}I_{43}) \quad (23)$$

where, the subscripts "n" & "m" of the terms (C_{nm}, D_{nm}, and I_{nm}) denote the branch n-m. the subscript "0" denotes the ground. Since the buses 2 and 3 are tie buses, their current injections are zero. Bus 4 is a load bus and therefore it was assigned with a negative value.

The currents I_{nm} can be obtained from the general equation illustrated in (2) as:

$$I_{nm} = \frac{V_n - A_{nm}V_m}{B_{nm}} \tag{24}$$

By substituting (24) into (20), (21), (22) and (23), we obtain

$$I_1 = \left(C_{10} + D_{12}{B_{12}}^{-1}\right)V_1 + \left(C_{12} - D_{12}{B_{12}}^{-1}A_{12}\right)V_2 \tag{25}$$

$$\begin{aligned} 0 = &\left(C_{21} - D_{21}{B_{21}}^{-1}A_{21}\right)V_1 \\ &+ \left(D_{21}{B_{21}}^{-1} + D_{23}{B_{23}}^{-1}\right)V_2 \\ &+ \left(C_{23} - D_{23}{B_{23}}^{-1}A_{23}\right)V_3 \end{aligned} \tag{26}$$

$$\begin{aligned} 0 = &\left(C_{32} - D_{32}{B_{32}}^{-1}A_{32}\right)V_2 \\ &+ \left(D_{32}{B_{32}}^{-1} + D_{34}{B_{34}}^{-1}\right)V_3 \\ &+ \left(C_{34} - D_{34}{B_{34}}^{-1}A_{34}\right)V_4 \end{aligned} \tag{27}$$

$$-I_4 = \left(C_{43} - D_{43}{B_{43}}^{-1}A_{43}\right)V_3 + D_{43}{B_{43}}^{-1}V_4 \tag{28}$$

or

$$I_1 = Y_{11}V_1 + Y_{13}V_2 + 0V_3 + 0V_4 \tag{29}$$

$$0 = Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + 0V_4 \tag{30}$$

$$0 = 0V_1 + Y_{32}V_2 + Y_{33}V_3 + Y_{34}V_4 \tag{31}$$

$$-I_4 = 0V_1 + 0V_2 + Y_{43}V_3 + Y_{44}V_4 \tag{32}$$

By comparing (29)-(32) with (25)-(28), we obtain:

$$Y_{11} = C_{10} + D_{12}{B_{12}}^{-1} \tag{33.a}$$

$$Y_{22} = D_{21}{B_{21}}^{-1} + D_{23}{B_{23}}^{-1} \tag{33.b}$$

$$Y_{33} = D_{32}{B_{32}}^{-1} + D_{34}{B_{34}}^{-1} \tag{33.c}$$

$$Y_{44} = D_{43}{B_{43}}^{-1} \tag{33.d}$$

$$Y_{12} = Y_{21} = C_{12} - D_{12}{B_{12}}^{-1}A_{12} \tag{34.a}$$

$$Y_{23} = Y_{32} = C_{23} - D_{23}{B_{23}}^{-1}A_{23} \tag{34.b}$$

$$Y_{34} = Y_{43} = C_{34} - D_{34}{B_{34}}^{-1}A_{34} \tag{34.c}$$

$$Y_{24} = Y_{42} = Y_{13} = Y_{31} = Y_{14} = Y_{41} = 0 \tag{35}$$

From (33)-(35), we can conclude a general formula for the elements of system impedance matrix. The diagonal elements can be obtained as:

$$Y_{nn} = C_{n0} + \sum_{m=1}^{N} D_{nm}{B_{nm}}^{-1} \tag{36}$$

The non-diagonal elements can be obtained as:

$$Y_{nm} = Y_{mn} = C_{nm} - D_{nm}{B_{nm}}^{-1}A_{nm} \tag{37}$$

If there is no connection between the any two buses, then:

$$Y_{nm} = 0 \tag{38}$$

Thus, in general form, the nodal current equations can be written as:

$$\begin{bmatrix} I_1 \\ \vdots \\ I_i \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} Y_{11} & \cdots & Y_{1i} & \cdots & Y_{1N} \\ \vdots & & \vdots & & \vdots \\ Y_{i1} & \cdots & Y_{ii} & \cdots & Y_{iN} \\ \vdots & & \vdots & & \vdots \\ Y_{N1} & \cdots & Y_{Ni} & \cdots & Y_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ \vdots \\ V_i \\ \vdots \\ V_N \end{bmatrix} \tag{39}$$

Similarly, for unbalanced three phase distribution networks, The diagonal, non-diagonal elements, and the nodal current equation can be obtained as:

$$Y_{nn}^{abc} = C_{n0} + \sum_{m=1}^{x} D_{nm}{B_{nm}}^{-1} \tag{40}$$

$$Y_{nm}^{abc} = Y_{mn} = C_{nm} - D_{nm}{B_{nm}}^{-1}A_{nm} \tag{41}$$

$$\begin{bmatrix} I_1^{abc} \\ \vdots \\ I_i^{abc} \\ \vdots \\ I_N^{abc} \end{bmatrix} = \begin{bmatrix} Y_{11}^{abc} & \cdots & Y_{1i}^{abc} & \cdots & Y_{1N}^{abc} \\ \vdots & & \vdots & & \vdots \\ Y_{i1}^{abc} & \cdots & Y_{ii}^{abc} & \cdots & Y_{iN}^{abc} \\ \vdots & & \vdots & & \vdots \\ Y_{N1}^{abc} & \cdots & Y_{Ni}^{abc} & \cdots & Y_{NN}^{abc} \end{bmatrix} \begin{bmatrix} V_1^{abc} \\ \vdots \\ V_i^{abc} \\ \vdots \\ V_N^{abc} \end{bmatrix} \tag{42}$$

where, V_i^{abc} and I_i^{abc} are vectors of the three phase voltages and current, respectively, of node *i*.

The new admittance matrix can be utilized Newton-Raphson power flow for accurate results.

4. SIMULATION RESULTS

To validate the proposed approach, the new admittance matrix is used for power flow analysis via Newton-Raphson power flow. The new admittance matrix considers the exact model of networks (i.e. series impedance, shunt admittances, ant transformer turn ratio).

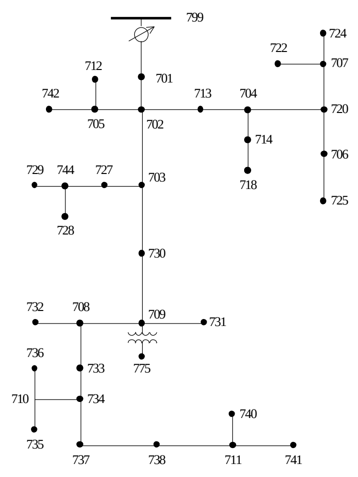

Figure 10. The standard IEEE 37 bus network

The standard IEEE 37-bus network shown in Figure 10 was chosen as an unbalanced test network. The power flow and the new admittance matrix algorithms were implemented in MATLAB software. Without loss of generality, only voltage magnitudes are considered for check the accuracy of the ne admittance matrix. Two cases are considered for the analysis as shown in the next subsections.

4.1 At no distributed generation

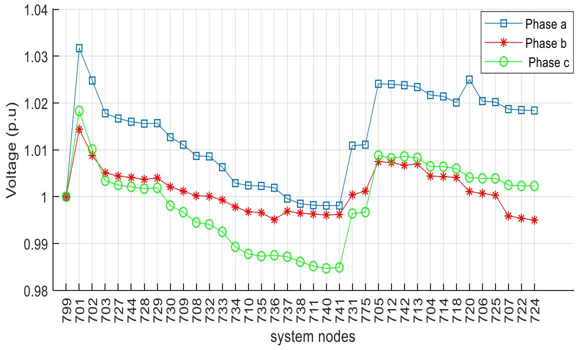

Figure 11. Voltage profile obtained from power flow results using the new admittance matrix at no distributed generation

Table 2. The percent errors in the voltages at no distributed generation ($\times 10^{-3}$)

Bus	Phase a	Phase b	Phase c
799	0.00	0.00	0.00
701	0.00	0.00	0.00
702	0.14	0.23	0.34
703	0.76	0.51	0.71
727	0.25	0.49	0.77
744	0.22	0.32	0.69
728	0.15	0.14	1.12
729	0.49	0.57	1.02
730	0.11	0.55	0.34
709	0.36	0.65	0.60
708	1.05	1.14	0.66
732	1.20	1.25	1.68
733	0.95	0.56	0.78
734	0.85	0.28	0.85
710	1.80	0.48	1.81
735	1.60	0.48	0.80
736	1.68	1.24	0.91
737	1.19	0.72	0.22
738	1.66	1.08	0.55
711	2.00	0.94	1.08
740	0.46	0.61	0.89
741	1.51	1.69	1.02
731	1.90	2.03	1.93
775	1.28	0.80	0.98
705	0.16	0.12	0.23
712	0.37	0.58	0.43
742	0.24	0.03	0.16
713	0.12	0.04	0.68
704	0.99	0.44	0.27
714	0.97	0.59	0.36
718	0.36	0.74	0.82
720	1.04	0.84	1.00
706	0.67	0.82	0.37
725	0.61	0.12	0.49
707	0.45	0.35	0.86
722	1.66	1.23	1.58
724	1.92	1.80	2.00

By performing the power flow on the test system and by considering the new admittance matrix, the three phase voltages are obtained. Figure 11 shows the three phase voltage profiles. It is clear that the voltage at slack bus equals 1 at the three phases. The results also show that the variation in the voltage from one bus to another bus is directly related to the physical connection between the nodes. The results of Newton-Raphson power flow that consider only the series impedances are also obtained. The percentage errors in the three phase voltages of network buses are presented in Table 2. All the relative errors are in the order of 10^{-3}-10^{-2}, which demonstrate the importance of including the shunt admittances in the analysis for accurate results. Indeed, the exact models of electric power components reflect the charging currents that can be supplied to the network.

4.2 With distributed generation

The effect on the three phase voltages in case of integrating distributed generation into the system is also studied in this work. Two distributed generation units are installed in the system. Their locations are at buses 724 and 722. The capacity of each unit is half its own load. By performing the power flow analysis considering the new admittance matrix, we obtain the three-phase voltage profiles shown in Figure 12. It is clear that the voltages are significantly improved at (and near) the buses 724 and 722. The results are also compared with power flow results without including the shunt admittances. The percent errors in the three phase voltages of network buses in the presence of distributed generation are presented in Table 3. The errors are also in the order of 10^{-3}-10^{-2}. This is due to the fact that including the shunt admittances in the analysis can affect the charging current injected into the network, and hence the bus voltages.

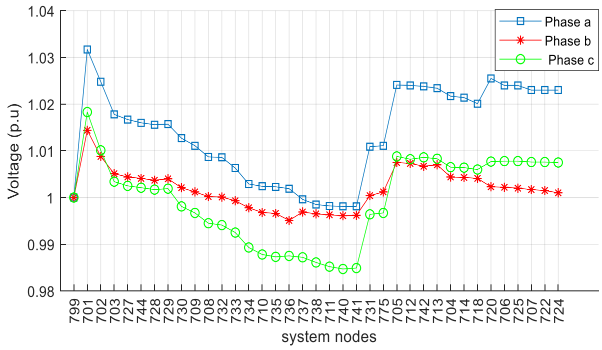

Figure 12. Voltage profile obtained from power flow results using the new admittance matrix in the presence of distributed generation

Table 3. The percent errors in the voltages in the presence of distributed generation ($\times 10^{-3}$)

Bus	Phase a	Phase b	Phase c
799	0.00	0.00	0.00
701	0.00	0.00	0.00
702	0.15	0.25	0.4
703	0.81	0.54	0.8
727	0.27	0.52	0.86
744	0.24	0.34	0.73
728	0.14	0.13	1.18
729	0.55	0.67	1.08

730	0.10	0.58	0.31
709	0.41	0.69	0.55
708	1.17	1.35	0.78
732	1.34	1.48	1.87
733	1.12	0.66	0.87
734	0.95	0.32	0.78
710	2.00	0.54	1.65
735	1.78	0.54	0.95
736	1.77	1.45	1.08
737	1.26	0.84	0.26
738	1.94	1.20	0.62
711	2.23	1.05	1.21
740	0.52	1.01	0.94
741	1.68	1.54	1.08
731	2.21	1.85	2.25
775	1.49	0.94	1.15
705	0.18	0.14	0.26
712	0.42	0.65	0.48
742	0.27	0.04	0.18
713	0.11	0.05	0.62
704	0.9	0.47	0.25
714	0.89	0.63	0.42
718	0.41	0.87	0.92
720	1.16	0.98	1.12
706	0.75	0.92	0.44
725	0.72	0.14	0.45
707	0.53	0.41	1
722	1.94	1.37	1.67
724	2.14	2.01	2.11

By obtaining the average percent errors for each phase voltage, we find that the average errors in case of the presence of distributed generation is higher than the normal case. This also demonstrates the necessity to include such new admittance matrix in the analysis, instead of the conventional matrix. It is worth mentioning that the errors can be higher in case of practical systems or large scale-systems.

5. CONCLUSION

This work developed a new admittance matrix for distribution network for Newton-Raphson power flow purposes. The new admittance matrix considers the exact models for network components. Newton-Raphson power flow was performed on standard IEEE 37 bus network. The results obtained using the proposed method showed that the voltage profiles among network buses coincide with the physical connection between the nodes. The results also showed that the percent errors (in the voltages obtained using the new admittance matrix and the conventional ones) are in the order of 10^{-3}-10^{-2}. This demonstrates the importance of including such new admittance matrix in the analysis to obtain accurate results. Besides, the average errors in case of presence of distributed generation were greater than in case of passive networks.

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