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Mathematical Modelling in Engineering with Integral Transforms via Modified Adomian Decomposition Method

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ABSTRACT

In this work three integral transforms through modified Adomian decomposition method (ADM) are proposed to obtain the approximate analytical solution of different types of mathematical models arising in physical problems. These transformations are applied for both homogeneous and non-homogeneous linear differential equations. The efficiency and accuracy of the proposed methods are implemented through higher order non-homogeneous ordinary differential equations. Numerical tests are reported for applicability of the current scheme based on different transformations and compared with exact solutions.

1. INTRODUCTION

Mathematical modeling's of physical problems help us to understand the phenomena in an efficient way. These models express the problems in the form of linear and nonlinear differential equations having initial -boundary conditions and these are useful as they accommodate variations in the physical problems as per the demand of the situations. In the last three decades a number of integral transform techniques both analytical and computational have been devised by many researchers to obtain exact and approximate solutions to the differential equations. Some methods that are based on the extension of Laplace transformation namely Sumudu transform [1], Elzaki transform [2, 3], Aboodh transform [4, 5], New integral transform [6], Mohand transform [7], Kamal transform [8] contribute the analytical as well as the approximate solution of initial value problems (IVPs). G.K.Watugala formulated Sumudu transform in the year 1993 to solve problems in control engineering. T.M. Elzaki formulated Elzaki transformation method from classical Fourier transform to solve differential equations with variable coefficients which was then beyond the scope of Sumudu transform. Aboodh and Mohand transforms were introduced in the year 2013 and 2017 respectively to facilitate the solution process in time domain. Three integral transforms Elzaki, Abboodh and Mohand are discussed in the present work to solve linear initial value problems (IVPs) and boundary value problems (BVPs). These techniques are useful for both homogeneous and non-homogeneous linear differential equations which results in exact analytical solution but to obtain approximate solution and to solve nonlinear differential equations intervention of some other methods like Adomian decomposition method [9, 10], Differential transformation method [11-18], FDTD Method [19], ARA transform [20], New transform iterative method [21], Coupling Elzaki transform and Homotopy perturbation method [22], Polynomial integral transform [23], modified Adomian decomposition method [24], numerical quadrature for real and analytic functions [25-47], B-spline collocation [48-51] and its subsequent modification rules are essential. Our work comprises the analytic and approximate solution of higher-order IVPs in electrical circuits, mass-spring system, and beam theory. In case of RLC circuit the exact solution is obtained by direct application of the methods. In other two problems the integral transform methods coupled with MADM are used to acquire approximate results. Even though the methods discussed here are rudimentary its simple execution, effective and useful properties can be implemented in solving intricate IVPs in applied mathematics and many engineering problems.

The existing method on these three problems (Electric Circuits, mass-spring system, and beam theory) based on initial value problem, and Laplace transformation. But here we applied various modified form of Laplace transformation (Elzaki, Aboodh, Mohand transforms) with modified Adomian decomposition method (MADM) to obtain better approximate results to analytical solutions.

The article is presented as per the following plan: Section-1 is an Introductory. Section-2 deals with the basic properties and derivations of three transformations. In Section-3 three modeling problems are explained through different transformations and numerical results are verified. Some remarks and conclusions are reported in Sec-4.

2. DEFINITION AND DERIVATIONS OF THE TRANSFORMS

All the three transforms discussed in this article are defined for the piecewise continuous function in the set.

$$A=\{f(t):\exists M,k_1,k_2>0\,,|f(t)|<Me^{\frac{|t|}{k_j}},t\in(-1)^j\times[0,\infty)\}.$$

where, M is a finite constant and k_1, k_2 may or may not be finite.

The properties, transformations and inverse transformations for various functions are reported in Table 1 and Table 2 respectively.

Table 1. Three transformations for various functions

Function $\boldsymbol{f(t)}$	**Elzaki Transform** $\boldsymbol{E\{f(t)\} = T(v)}$	**Mohand Transform** $\boldsymbol{M\{f(t)\} = R(v)}$	**Aboodh Transform** $\boldsymbol{A\{f(t)\} = K(v)}$
1	v^2	v	$1/v^2$
t	v^3	1	$1/v^3$
t^2	$2v^4$	$2!/v$	$2!/v^4$
$t^n, n \in N$	$n!\, v^{n+2}$	$n!/v^{n-1}$	$n!/v^{n+2}$
$t^n, n < -1$	$\Gamma(\mathrm{n}+1) v^{n+2}$	$\Gamma(\mathrm{n}+1)/v^{n-1}$	$\Gamma(\mathrm{n}+1)//v^{n+2}$
e^{at}	$v^2/1 - av$	$v^2/v - a$	$1/v^2 - a$
$\sin at$	$av^3/1 + a^2\, v^2$	$av^2/(v^2 + a^2)$	$a/v(v^2 + a^2)$
$\cos at$	$v^2/1 + a^2\, v^2$	$v^3/v^2 + a^2$	$1/(v^2 + a^2)$
$\sinh at$	$av^3/1 - a^2\, v^2$	$av^2/(v^2 - a^2)$	$a/v(v^2 - a^2)$
$\cosh at$	$v^2/1 - a^2\, v^2$	$v^3/v^2 - a^2$	$1/(v^2 - a^2)$

Table 2. Determination of functions by three inverse transformations

$T(v)$	$R(v)$	$K(v)$	$f(t) = E^{-1}(T(v)) = M^{-1}(R(v)) = A^{-1}(K(v))$
v^2	v	$1/\mathrm{v}^2$	1
v^3	1	$1/\mathrm{v}^3$	t
v^4	$1/\mathrm{v}$	$1/\mathrm{v}^4$	$\mathrm{t}^2/2$
$\mathrm{v}^{\mathrm{n}+2}$	$1/\mathrm{v}^{\mathrm{n}-1}$	$1/\mathrm{v}^{\mathrm{n}+2}$	$\frac{\mathrm{t}^\mathrm{n}}{\mathrm{n}!}, \mathrm{n} \in \mathrm{N}$
$\mathrm{v}^{\mathrm{n}+2}$	$1/\mathrm{v}^{\mathrm{n}-1}$	$1/\mathrm{v}^{\mathrm{n}+2}$	$\frac{\mathrm{t}^\mathrm{n}}{\mathrm{n}!}, \mathrm{n} < -1$
$\mathrm{v}^2/1 - \mathrm{av}$	$\mathrm{v}^2/\mathrm{v} - \mathrm{a}$	$1/\mathrm{v}^2 - \mathrm{a}$	e^{at}
$\mathrm{av}^3/1 + \mathrm{a}^2\,\mathrm{v}^2$	$\mathrm{av}^2/(\mathrm{v}^2 + \mathrm{a}^2)$	$\mathrm{a}/\mathrm{v}(\mathrm{v}^2 + \mathrm{a}^2)$	sin at
$\mathrm{v}^2/1 + \mathrm{a}^2\,\mathrm{v}^2$	$\mathrm{v}^3/\mathrm{v}^2 + \mathrm{a}^2$	$1/(\mathrm{v}^2 + \mathrm{a}^2)$	cos at
$\mathrm{av}^3/1 - \mathrm{a}^2\,\mathrm{v}^2$	$\mathrm{av}^2/(\mathrm{v}^2 - \mathrm{a}^2)$	$\mathrm{a}/\mathrm{v}(\mathrm{v}^2 - \mathrm{a}^2)$	sinh at
$\mathrm{v}^2/1 - \mathrm{a}^2\,\mathrm{v}^2$	$\mathrm{v}^3/\mathrm{v}^2 - \mathrm{a}^2$	$1/(\mathrm{v}^2 - \mathrm{a}^2)$	cosh at

2.1 Elzaki transform

The Elzaki transform denoted by the operator $E(.)$ is defined as:

$$E\big(f(t)\big) = T(v) = v\int_0^\infty e^{-\frac{t}{v}} f(t)dt,\, k_1 \leq\, v < k_2, t \geq 0.$$

The properties of Elzaki transform are given by

Let $E(f(t))=T(v)$ then integration by parts gives the following results.

i) $E(f'(t)) = \frac{T(v)}{v} - f(0)$

ii) $E(f''(t)) = \frac{T(v)}{v^2} - f(0) - vf'(0)$

iii) $E(f^n(t)) = \frac{T(v)}{v^n} - \sum_{k=0}^{n-1} v^k f^{(k)}(0)$

iv) $E(sinx) = \sum_{n=0}^{\infty}(-1)^n \frac{1}{v}$

v) $E(e^{-x}) = \sum_{n=0}^{\infty}(-1)^n v^{n+2}$

2.2 Aboodh transform

Aboodh transform is denoted by the operator $A(\cdot)$ and defined as

$$A\big(f(t)\big) = k(v) = \frac{1}{v}\int_0^\infty e^{-vt} f(t)dt.$$

where, $k_1 \leq v < k_2, t \geq 0$.

The properties Aboodh of transform are as follows:

If $A\big(f(t)\big) = k(v)$,then

vi) $A(f'(t)) = vk(v) - \frac{f(0)}{v}$

vii)$A(f''(t)) = v^2K(v) - f(0) - \frac{f'(0)}{v}$

viii)$A(f^n(t)) = v^nK(v) - \sum_{k=0}^{n-1}\frac{f^k(0)}{v^{2-n+k}}$

ix)$A(\sin x) = \sum_{n=0}^{\infty}(-1)^n \frac{1}{v^{2n+3}}$

x)$A(e^{-x}) = \sum_{n-=0}^{\infty}(-1)^n \frac{1}{v^{n+2}}$

2.3 Mohand transform

Mohand transform is denoted by the operator $M(\cdot)$ and defined as

$$M\big(f(t)\big) = R(v) = v^2\int_0^\infty e^{-vt} f(t)dt,$$
$$where, k_1 \leq v < k_2\,, t \geq 0$$

The properties of Mohand transform are represented as:

Let $M(f(t))=R(v)$, then

xi)$M\big(f'(t)\big) = vR(v) - v^2f(0)$.

xii)$M\big(f''(t)\big) = v^2R(v) - v^3f(0) - v^2f'(0)$.

xiii)$M\left(f^{(n)}(t)\right) = v^nR(v) - \sum_{k=0}^{n-1} v^{n-k+1}f^{(k)}(0)$.

xiv)$M(sinx) = \sum_{n-0}^{\infty}(-1)^n v^{-2n}$

xv) $M(e^{-x}) = \sum_{n=0}^{\infty}(-1)^n \frac{1}{v^{n-1}}$.

3. MATHEMATICAL MODELING OF PHYSICAL PROBLEMS

In this section three mathematical models (electrical network problem, mass spring system, and elastic beam) are suggested with three transformations, i.e., Elzaki, Aboodh and Mohand transformation to obtain their exact and approximate salutations. Moreover, the first model (RLC circuit) is best fit to its analytical solution and the rest two models (Mass Spring System, and Elastic beam problems) are numerically in good agreement to their exact solutions with introduction of MADM.

3.1 RLC circuit

The RLC circuit with resistance(R), inductance, capacitance(C) with electromotive force(v(t)) is depicted in Figure 1 [12].

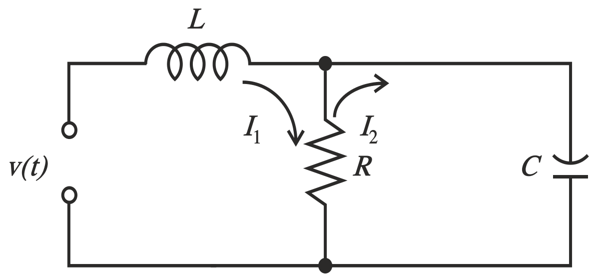

Figure 1. RLC circuit

Let us consider a network system as given in the Figure 1, where L=20 henry, R=10 ohms, C=0.05 farad, v=20 volts with, $I_1(0)=I_2(0)=0$. By Kirchoff"s voltage law (KVL) [9], the mathematical model of the network is $LI_1{}' + R(I_1 - I_2) = v(t)$ and $R(I_2{}' - I_1{}') + \frac{1}{C}I_2 = 0$.

Hence substituting the respective values into the system of equations we have:

$$\begin{gathered} 20I_1{}' + 10(I_1 - I_2) = 20. \\ 2I_1{}' + (I_1 - I_2) = 2 \end{gathered} \tag{1}$$

$$\begin{gathered} 10(I_2{}' - I_1{}') + 20I_2 = 0 \\ \text{Therefore,} (I_2{}' - I_1{}') + 2I_2 = 0 \end{gathered} \tag{2}$$

Three transforms are illustrated for analytical solution of circuit as discussed below.

3.1.1 Elzaki transformation

Let $E(I_1(t))=T_1(v)$ and $E(I_2(t))=T_2(v)$. Taking Elzaki transformation of Eqns. (1) and (2) and applying the properties(i)-(v).

$$T_2(v) = \left(\frac{2+v}{v}\right)T_1(v) - 2v^2 \tag{3}$$

$$T_2(v) = \left(\frac{1}{2v+1}\right)T_1(v) + \frac{2v^2}{1+2v} \tag{4}$$

Comparing Eqns. (3) and (4),

$$T_1(v) = \frac{2v^3}{1+v} = 2\left(v^2 - \frac{v^2}{1+v}\right)$$
$$T_2(v) = \frac{2v^3}{(v+1)(2v+1)} + \frac{2v^2}{2v+1} = \frac{2v^2}{1+v}$$

Taking the inverse Elzaki transform, we obtain:

$$I_1(t) = 2(1 - e^{-t}), I_2(t) = 2(e^{-t}).$$

as the solutions to the system.

It is observed that this result is same to exact solution.

3.1.2 Aboodh transformation

Taking Aboodh transformation of Eqns. (1) and (2) with all properties(vi)-(x). The following results are obtained.

$$K_2(v) = (2v+1)K_1(v) - \frac{2}{v^2} \tag{5}$$

$$K_2(v) = \left(\frac{1}{v+2}\right)\left(\frac{2}{v} + vK_1(v)\right) \tag{6}$$

Comparison of the two Eqns. (5) and (6):

$$K_1(v) = \frac{2v}{v^2(v^2+v)} = 2\left(\frac{1}{v^2} - \frac{1}{v^2+v}\right)$$
$$K_2(v) = \frac{2v(v+2)}{v^2(v+1)(v+2)} = 2\left(\frac{1}{v^2+v}\right)$$

Taking the inverse Aboodh transform the solutions are obtained as;

$$I_1(t) = 2(1 - e^{-t}), I_2(t) = 2(e^{-t}),$$

which coincides with the exact solution of RLC circuit.

3.1.3 Mohand transformation

Let $M(I_1(t)) = R_1(v)$and $M(I_2(t)) = R_2(v)$.

Operation of Mohand transformation on Eqns. (1) and (2) and simplifying the following results with properties(xi)-(xv) are obtained.

$$R_1(v) = \frac{2v}{v+1} = 2\left(v - \frac{v^2}{v+1}\right) \tag{7}$$

$$R_2(v) = \frac{2v^2}{v+1} \tag{8}$$

Inverse Mohand transform of the Eqns. (7) and (8), gives the same solution as in the previous cases.

3.2 Mass spring system

Forced motion in a mass spring system with periodic input is given by the general second order equation.

$$my'' + cy' + ky = F_0 \cos \omega t, F_0 > 0, \omega > 0$$

where, m is the mass of the spring, c,k are the damping and spring constants respectively and the solution of the system $y(t)$ that is the displacement of the body at any time t.

Let us determine the motion of the undamped forced mass spring system corresponding to Eq. (9).

$$y'' + 25y = 24sint\ , y(0) = y'(0) = 1 \tag{9}$$

All the three transforms are discussed in this section to get the approximate solution of Eq. (9).

3.2.1 Aboodh transform

Aboodh transformation of Eq. (9) results:

$$A(y'') = 24A(sint) - 25A(y)$$
$$v^2K(v) - \frac{y'(0)}{v} - y(0) = 24A(sint) - 25A(y)$$
$$v^2K(v) = 1 + \frac{1}{v} + 24A(sint) - 25A(y)$$
$$K(v) = \frac{1}{v^2} + \frac{1}{v^3} + \frac{24}{v^2}A(sint) - \frac{1}{v^2}25A(y)$$

Operating Aboodh inverse on both sides.

$$y(t) = 1 + t + 24A^{-1}\left[\frac{1}{v^2}\sum_{r=0}^{\infty}(-1)^r \frac{1}{v^{2r+3}}\right] - 25A^{-1}\left[\frac{1}{v^2}A(y)\right]$$

$$y(t) = 1 + t + 24\sum_{r=0}^{\infty}(-1)^r \frac{t^{2r+3}}{(2r+3)!} - 25A^{-1}\left(\frac{1}{v^2}A(y)\right) \quad (10)$$

Eq. (11) is obtained by taking the series solution for the Aboodh transform.

$$y(t) = \sum_{n=0}^{\infty} y_n(t) \quad (11)$$

Substituting Eq. (11) into Eq. (10):

$$\sum_{n=0}^{\infty} y_n(t) = 1 + t + 24\sum_{r=0}^{\infty}(-1)^r \frac{t^{2r+3}}{(2r+3)!} - 25A^{-1}\left(\frac{1}{v^2}A(y)\right) \quad (12)$$

Further, using modified Adomain decomposition method (MADM), we can decompose Eq. (12) into two parts as:

$$y_0 = 1 + t + 24\sum_{r=0}^{\infty}(-1)^r \frac{t^{2r+3}}{(2r+3)!}$$

The recurrence relation is generated as:

$$y_{n+1} = -25A^{-1}\left(\frac{1}{v^2}A(y_n)\right), \text{for } n \geq 0. \quad (13)$$

Hence for $n = 0,1,2,3$ in Eq. (13), the corresponding y_1, y_2, y_3, y_4 are:

$$y_1 = -25A^{-1}\left(\frac{1}{v^2}A(y_0)\right)$$

$$= -25A^{-1}\left(\frac{1}{v^2}A\left(1 + t + 24\sum_{r=0}^{\infty}(-1)^r \frac{t^{2r+3}}{(2r+3)!}\right)\right)$$

$$= -25A^{-1}\left(\frac{1}{v^2}\left(\frac{1}{v^2} + \frac{1}{v^3} + 24\sum_{r=0}^{\infty}(-1)^r \frac{1}{v^{2r+5}}\right)\right)$$

$$= -25A^{-1}\left(\frac{1}{v^4} + \frac{1}{v^5} + 24\sum_{r=0}^{\infty}(-1)^r \frac{1}{v^{2r+7}}\right)$$

$$= -\frac{25}{2!}t^2 - \frac{25}{3!}t^3 - 25 \times 24\sum_{r=0}^{\infty}(-1)^r \frac{t^{2r+5}}{(2r+5)!}$$

$$= -25\left(\frac{t^2}{2!} + \frac{t^3}{3!} + 24\sum_{r=0}^{\infty}(-1)^r \frac{t^{2r+5}}{(2r+5)!}\right)$$

For, $n = 1$

$$y_2 = -25A^{-1}\left(\frac{1}{v^2}A(y_1)\right)$$

$$= -25A^{-1}\left(\frac{1}{v^2}A\left(-\frac{25}{2!}t^2 - \frac{25}{3!}t^3 - 25 \times 24\sum_{r=0}^{\infty}(-1)^r \frac{t^{2r+5}}{(2r+5)!}\right)\right)$$

$$= 25^2A^{-1}\left(\frac{1}{v^2}\left(\frac{1}{v^4} + \frac{1}{v^5} + 24\sum_{r=0}^{\infty}(-1)^r \frac{1}{v^{2r+7}}\right)\right)$$

$$= 25^2A^{-1}\left(\frac{1}{v^6} + \frac{1}{v^7} + 24\sum_{r=0}^{\infty}(-1)^r \frac{1}{v^{2r+9}}\right)$$

$$= 25^2A^{-1}\left(\frac{t^4}{4!} + \frac{t^5}{5!} + 24\sum_{r=0}^{\infty}(-1)^r \frac{t^{2r+7}}{(2r+7)!}\right)$$

Similarly for, $n = 2,3$,

$$y_3 = -25^3\left(\frac{t^6}{6!} + \frac{t^7}{7!} + 24\sum_{r=0}^{\infty}(-1)^r \frac{t^{2r+9}}{(2r+9)!}\right)$$

$$y_4 = 25^4\left(\frac{t^8}{8!} + \frac{t^9}{9!}\right) + O(10)$$

Therefore, using first five terms of the series and neglecting rest of $O(10)$,

$$y(t) = y_0 + y_1 + y_2 + y_3 + y_4$$

$$\begin{aligned} y(t) &= 1 + t + 24\left(\frac{t^3}{3!} - \frac{t^5}{5!} + \frac{t^7}{7!} - \frac{t^9}{9!}\right) \\ &-25\left(\frac{t^2}{2!} + \frac{t^3}{3!} + 24\left(\frac{t^5}{5!} - \frac{t^7}{7!} + \frac{t^9}{9!}\right)\right) \\ &+25^2\left(\frac{t^4}{4!} + \frac{t^5}{5!} + 24\left(\frac{t^7}{7!} - \frac{t^9}{9!}\right)\right) \\ &-25^3\left(\frac{t^6}{6!} + \frac{t^7}{7!} + 24\frac{t^9}{9!}\right) + 25^4\left(\frac{t^8}{8!} + \frac{t^9}{9!}\right) + O(10) \\ y(t) &= 1 + t - \frac{(5t)^2}{2!} - \frac{t^3}{3!} + \frac{(5t)^4}{4!} \\ &+ \frac{t^5}{5!} - \frac{(5t)^6}{6!} - \frac{t^7}{7!} + \frac{(5t)^8}{8!} + \frac{t^9}{9!} + \cdots \end{aligned} \quad (14)$$

The exact solution of the problem is $y(t)$=cos 5(t)+sin t [12] and the series solution in Eq. (14) is nothing but the Maclaurin series of the exact solution.

3.2.2 Mohand transform

Taking Mohand transformation of Eq. (9).

$$M(y'') = 24M(\sin t) - 25M(y)$$

$$R(v) = 1 + v + 24M^{-1}\left[\frac{1}{v^2}\sum_{r=0}^{\infty}(-1)^r \frac{1}{v^{2r}}\right] - 25M^{-1}\left[\frac{1}{v^2}M(y)\right] \quad (15)$$

Application of inverse Mohand transform on Eq. (15)

$$y(t) = t + 1 + 24\sum_{r=0}^{\infty}(-1)^r \frac{t^{2r+3}}{(2r+3)!} - 25\,M^{-1}\left[\frac{1}{v^2}M(y)\right] \quad (16)$$

The solution $y(t)$ can be defined as an infinite series as in Eq. (11).

Hence putting Eq. (11) in Eq. (16) the series is obtained as:

$$\sum_{n=0}^{\infty} y_n(t) = 1 + t + 24\sum_{r=0}^{\infty}(-1)^r \frac{t^{2r+3}}{(2r+3)!} - 25\, M^{-1}\left[\frac{1}{v^2}M(y_n)\right] \quad (17)$$

$$y_0 = 1 + t + 24\sum_{r=0}^{\infty}(-1)^r \frac{t^{2r+3}}{(2r+3)!}$$

The recurrence relation, $y_{n+1} = -25M^{-1}\left[\frac{1}{v^2}M(y_n)\right]$ $for\ n \geq 0$.

For, $n = 0$,

$$y_1 = -25M^{-1}\left[\frac{1}{v^2}M(y_0)\right]$$

$$= -25M^{-1}\left[\frac{1}{v^2}M\left[1 + t + 24\sum_{r=0}^{\infty}(-1)^r \frac{t^{2r+3}}{(2r+3)!}\right]\right]$$

$$= -25M^{-1}\left[\frac{1}{v^2}\left[v^2 + v^3 + 24\sum_{r=0}^{\infty}(-1)^r \frac{1}{v^{2r+2}}\right]\right]$$

$$= -25M^{-1}\left[v + 1 + 24\sum_{r=0}^{\infty}(-1)^r \frac{1}{v^{2r+4}}\right]$$

$$= -25M^{-1}\left[\frac{1}{v^2} + \frac{1}{v} + 24\sum_{r=0}^{\infty}(-1)^r \frac{1}{v^{2r+4}}\right]$$

$$= -25\left[\frac{t^2}{2!} + \frac{t^3}{3!} + 24\sum_{r=0}^{\infty}(-1)^r \frac{t^{2r+5}}{(2r+5)!}\right]$$

Similarly, at $n = 0,1,2,3$ the results occur as it is in case of Aboodh transform.

These components in Eq. (11) obtain the desired result.

3.2.3 Elzaki transformation

The Elzaki transform of Eq. (9).

$$E(y'') = 24ME(\sin t) - 25E(y)$$

$$v^{-2}T(v) - y(0) - vy'(0) = 24\sum_{r=0}^{\infty}(-1)^r v^{2r+3} - 25E(y) \quad (18)$$

$$T(v) = v^2 + v^3 + 24\sum_{r=0}^{\infty}(-1)^r v^{2r+5} - 25v^2E(y)$$

Taking inverse Elzaki transform of Eq. (18),

$$y(t) = 1 + t + 24\sum_{r=0}^{\infty}(-1)^r \frac{t^{2r+3}}{(2r+3)!} - 25\, E^{-1}(v^2E(y))$$

Further, using MADM we can decompose Eq. (18) into two parts as:

$$y_0 = 1 + t + 24\sum_{r=0}^{\infty}(-1)^r \frac{t^{2r+3}}{(2r+3)!}$$

The recurrence relation is given by:

$$y_{n+1} = -25E^{-1}\left(v^2E(y_n)\right) \quad (19)$$

Putting values of $n = 0,1,2,3$ the components as well as the series solution for the given differential equation repeats itself as it is in previous two cases.

The absolute errors for mass spring system are reported in Table 3 and depicted in Figure 2.

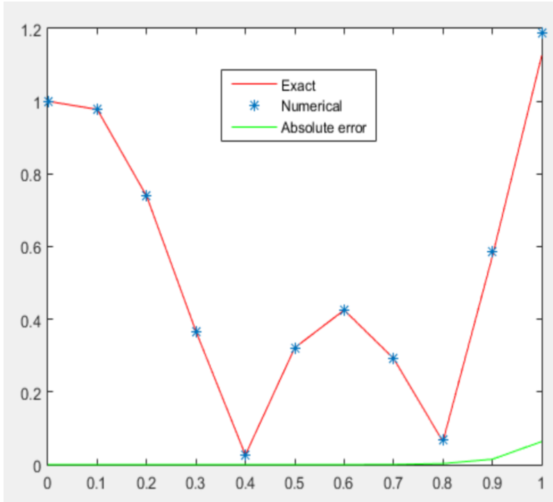

Figure 2. Comparisons for exact, approximate and absolute errors for different mesh points for mass spring system

3.3 Elastic beam

For a beam free at both ends on an elastic foundation under the action of a distributed load $w(x)$, the flexurat deflection $y(x)$ is governed by the equation.

$$Ely^4 + ky = w(x) \text{with the boundary conditions } y''(0) = y'''(0) = 0 = y''(L) = y'''(L)$$

In particular, let us consider the following example:

$$y^{(4)} + y = e^{-x}, 0 \leq x \leq 1, y''(0) = y'''(0) = y''(1) = y'''(1) = 0 \quad (20)$$

3.3.1 Elzaki transform

The Elzaki transform of Eq. (20), obtains:

$$v^{-4}T(v) - v^{-2}y(0) - v^{-1}y'(0) - y''(0) - vy'''(0) = E(e^{-x} - y) \quad (21)$$
$$T(v) = C_1v^2 + C_2v^3 + v^4E(e^{-x} - y)$$

The value of $C_1 = y(0), C_2 = y'(0)$ will be determined after the series solution as in Eq. (6) is obtained.

The inverse Elzaki Transformation of Eq. (21) gives the solution as follows:

$$y(x) = C_1 + C_2x + E^{-1}[v^4E(e^{-x} - y)]$$
$$y(x) = e^{-x} + C_1 - 1 + (C_2 + 1)x - \frac{x^2}{2} + \frac{x^3}{3!} - E^{-1}[v^4E(y)] \quad (22)$$

By MADM and Taylor's series approximation, Eq. (22) is decomposed as:

$$y_0 = C_1 + C_2x + \frac{x^4}{4!} - \frac{x^5}{5!} + \frac{x^6}{6!} - \frac{x^7}{7!} + \frac{x^8}{8!} - \frac{x^9}{9!} + \frac{x^{10}}{10!} + O(11)$$

The recurrence relation is given by:

$$y_{n+1} = -E^{-1}[v^4 E(y_n)] \; for \; n \geq 0$$

For $n = 0$, $y_1 = -E^{-1}[v^4 E(y_0)]$

$$= -E^{-1}\left(v^4 E\left(C_1 + C_2 x + \frac{x^4}{4!} - \frac{x^5}{5!} + \frac{x^6}{6!} - \frac{x^7}{7!} + \frac{x^8}{8!} - \frac{x^9}{9!} + \frac{x^{10}}{10!}\right)\right)$$

$$= -E^{-1}\left(v^4 (C_1 v^2 + C_2 v^3 + v^6 - v^7 + v^8 - v^9 + v^{10} - v^{11} + v^{12})\right)$$

$$= -\left(C_1 \frac{x^4}{4!} + C_2 \frac{x^5}{5!} + \frac{x^8}{8!} - \frac{x^9}{9!} + \frac{x^{10}}{10!} - \frac{x^{11}}{11!} + \frac{x^{12}}{12!} - \frac{x^{13}}{13!} + \frac{x^{14}}{14!}\right)$$

For $n = 1$, $y_2 = C_1 \frac{x^8}{8!} + C_2 \frac{x^9}{9!} + \frac{x^{12}}{12!} - \frac{x^{13}}{13!} + \frac{x^{14}}{14!}$

For $n = 2$, $y_3 = -C_1 \frac{x^{12}}{12!} - C_2 \frac{x^{13}}{13!}$

Substitution of the values of these components in Eq. (11), the approximate solution is:

$$y(x) = C_1 + C_2 x + (1 - C_1)\frac{x^4}{4!} - (1 + C_2)\frac{x^5}{5!} + \frac{x^6}{6!} - \frac{x^7}{7!} + C_1 \frac{x^8}{8!} + C_2 \frac{x^9}{9!} + (1 - C_1)\frac{x^{12}}{12!} + C_2 \frac{x^{13}}{13!} + \cdots \quad (23)$$

The boundary condition in Eq. (20) are substituted in Eq. (23), provides:

$$C_1 = 0.943127854229432,$$

$$C_2 = -0.622264767724301$$

Hence,

$$y = \frac{8494940505740811}{9007199254740992} - \frac{1401215688024475}{2251799813685248}x + \frac{512258748000181}{216172782113783808}x^4 - \frac{850584125660773}{270215977642229760}x^5 + \frac{1}{720}x^6 - \frac{1}{5040}x^7 + \frac{2831646835246937}{121056757983718932480}x^8 - \frac{280243137604895}{163426623278020558848}x^9 + \frac{512258749000181}{4314462854539742753587200}x^{12} - \frac{56048627520979}{560880171090166557966336}x^{13} + \cdots$$

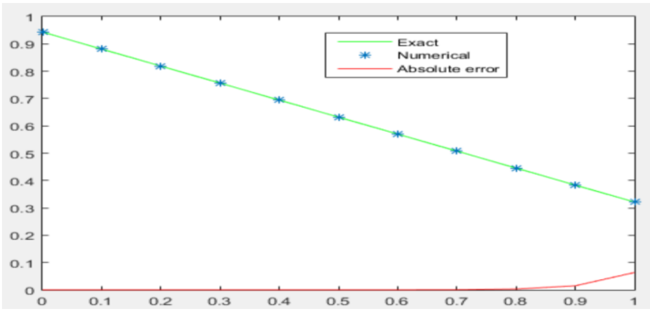

Figure 3. Comparisons of exact, approximate and absolute errors for different mesh points for elastic beam modelling

Table 3. Absolute errors for mass spring system at different mesh points

x	Exact value	Approximate value	Absolute error
0	1.000000000000000	1.000000000000000	0.000000000
0.1	0.977415978537201	0.977415978557043	1.984201691840326e-11
0.2	0.738971636663201	0.738971639214306	2.551104993919751e-09
0.3	0.366257408329042	0.366257455039382	4.671033998482344e-08
0.4	0.026728494238492	0.026727984314194	5.099242979984819e-07
0.5	0.321718076942731	0.321712362702702	5.714240029042195e-06
0.6	0.425350023205410	0.425291603001488	5.842020392199387e-05
0.7	0.292239000053105	0.291771119899731	4.678801533740118e-04
0.8	0.063712470035911	0.066638254725620	2.925784689709 e-03
0.9	0.572531110196704	0.587375957831336	1.4844847634632 e-02
1	1.125133170271123	1.188609209077959	6.3476038806836 e-02

Table 4. Absolute errors for elastic beam problem at different grid points

x	Exact value	Approximate value	Absolute error
0	0.943200000000000	0.943124474808975	7.552519102504984e-05
.1	0.880967246664593	0.880899363884991	6.788277960201317e-05
.2	0.818736870650345	0.818676710002571	6.016064777392138e-05
.3	0.756513311064063	0.756461038146504	5.227291755904862e-05
.4	0.694301142844698	0.694257015147056	4.412769764206015e-05
.5	0.632067866921078	0.632103494794425	3.562787334698836e-05
.6	0.569921209167234	0.569894537246701	2.667192053296130e-05
.7	0.507752671841497	0.507735517094754	1.715474674301998e-05
.8	0.445594249046082	0.445587280495720	6.968550362029813e-06
.9	0.383441272930632	0.383445269271236	3.996340603995563e-06
1.0	0.321289523969921	0.321305374716945	1.585074702398215e-05

All the numerical computations are carried on using Matlab and the comparison with the exact solution is reported in the Table 4 and also shown in Figure 3.

3.3.2 Mohand transforms

Taking the Mohand transform of Eq. (20).

$$v^4R(v) - v^5y(0) - v^4y'(0) - v^3y''(0) - v^2y'''(0) = M(e^{-x} - y)$$
$$v^4R(v) = C_1v^5 + C_2v^4 + M(e^{-x}) - M(y)$$
$$R(v) = C_1v + C_2 + \frac{1}{v^4}M(e^{-x}) - \frac{1}{v^4}M(y)$$

Taking the inverse Mohand transform and Taylor's series expansion for exponential function.

$$y(x) = C_1 + C_2x + \frac{x^4}{4!} - \frac{x^5}{5!} + \frac{x^6}{6!} - \frac{x^7}{7!} + \frac{x^8}{8!} - \frac{x^9}{9!} + \frac{x^{10}}{10!} - E^{-1}[v^4E(y)]$$

By decomposition method:

$$y_0 = C_1 + C_2x + \frac{x^4}{4!} - \frac{x^5}{5!} + \frac{x^6}{6!} - \frac{x^7}{7!} + \frac{x^8}{8!} - \frac{x^9}{9!} + \frac{x^{10}}{10!},$$

and, the recurrence relation $y_{n+1} = -M^{-1}\left[\frac{1}{v^4}M(y_n)\right],\ n \geq 0$ is obtained.

As earlier the components of Eq. (11) are calculated which agrees with the values calculated for Elzaki transformation. Therefore the solution derived by Mohand transformation is also same.

3.3.3 Aboodh transform

The Aboodh transform of Eq. (20), yields:

$$v^4K(v) - v^2y(0) - vy'(0) - y''(0) - \frac{1}{v}y'''(0) = A(e^{-x} - y)$$
$$v^4K(v) = C_1v^2 + C_2v + A(e^{-x}) - A(y)$$
$$K(v) = C_1\frac{1}{v^2} + C_2\frac{1}{v^3} + \frac{1}{v^4}\left(\sum_{r=0}^{\infty}(-1)^r v^{r+2}\right) - \frac{1}{v^4}A(y)$$

The inverse Aboodh transform of the above equation:

$$y(x) = C_1 + C_2x + \frac{x^4}{4!} - \frac{x^5}{5!} + \frac{x^6}{6!} - \frac{x^7}{7!} + \frac{x^8}{8!} - \frac{x^9}{9!} + \frac{x^{10}}{10!} - A^{-1}\left[\frac{1}{v^4}A(y)\right]$$

The repetition of the steps in former transforms evaluate the components of the infinite series of the Eq. (6) which are exactly equal to the ones derived by Elzaki and Mohand transform and so also the final solution.

4. CONCLUSIONS

The basic objective of this work is to implement the given transformations to linear non homogeneous differential equations in modeling occur in engineering, applied sciences and other physical phenomena The applicability and efficiency of the proposed scheme is executed by three test problems though three transformations numerically as well as graphically. The absolute error is merged with the abscissa except with some fluctuations at 0.9 and 1 as shown in Figure 2 and Figure 3 for mass spring system and elastic beam problems respectively. The present scheme can also be implemented to different branches of applied sciences for constructive modeling in the field of ordinary differential equation (ODEs) as well as partial deferential equations (PDE) in Mathematical Sciences. The proposed scheme with numerical quadrature can be implemented for electromagnetic field problems in Electronics engineering and Fracture mechanics in Civil and mechanical engineering.

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