Dissipativity criteria for digital filters with saturation nonlinearity

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ABSTRACT. *This paper analyzes the dissipativity of the direct form digital filters with saturation nonlinearity. First a (Q,S,R)-α dissipativity of direct form digital filters with saturation nonlinearity has been studied. Based on this existing criterion a new (Q,S,R)-α dissipativity criterion of direct form digital filters has been established and verified with some general characterization of nonlinearity.*

This paper also deals under what conditions the asymptotic stability of the digital filters can be assured which is very crucial for the design of robust controllers. Some numerical examples have been employed to demonstrate the usefulness of the theorems. The theorems in this paper have been verified using the suitable Lyapunov and dissipative functions.

RÉSUMÉ. *Cet article analyse la dissipativité des filtres numériques à forme directe avec une non-linéarité de saturation. Tout d'abord, une (Q,S,R) - une dissipativité de filtres numériques à forme directe avec non-linéarité de saturation a été étudiée. Sur la base de ce critère existant, un nouveau (Q,S,R)– un nouveau critère de dissipativité a des filtres numériques de forme directe a été établi et vérifié avec certain caractérisation générale de non-linéarité.*

Cet article traite également des conditions dans lesquelles la stabilité asymptotique des filtres numériques peut être assurée, ce qui est essentiel pour la conception de contrôleurs robustes. Quelques exemples numériques ont été utilisés pour démontrer l'utilité des théorèmes. Les théorèmes de cet article ont été vérifiés en utilisant les fonctions de Lyapunov et dissipatives appropriées.

KEYWORDS: *dissipativity, digital filters, direct form, Lyapunov.*

MOTS-CLÉS: *dissipativité, filtres numériques, forme directe, Lyapunov.*

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1. Introduction

In control systems we deal with stationary as well as the dynamical systems. The analysis of dynamical systems is not so easy as the state of such systems can change with the input supply or disturbances. Hence, the design of robust controllers for dynamical systems is a challenging task. The concept of dissipativity (Willems, 1972) can be used to synthesize robust controllers.

The dissipative analysis for linear systems with quadratic supply rates were done in (Willems, 1972). Further the concepts were generalized for nonlinear systems (Hill and Moylan, 1977; Hill and Moylan, 1980). Also the dissipative analysis have been done for continuous (Xie *et al*., 1998) and discrete (Tan *et al*., 1999) systems. Further the concepts were extended to stochastic systems (Zhang *et al.*, 2010). Recently, the dissipativity analysis has been done for static neural networks with time delay (Wu *et al*., 2012) and fuzzy – delayed systems (Su *et al*., 2014). In this paper we focus on the application of dissipativity in designing robust controllers and filters.

Before digital filters are implemented based on computer software or digital hardware, it has to be divided into small filters (Tsividis, 2002; Monteiro and Leuken, 2010). C.K Ahn handled this problem of disturbances and derived results for interfered 1-D systems (Ahn, 2015; Ahn and Shi, 2015) and 2-D systems (Ahn, 2013; Ahn, 2014; Ahn, 2014; Ahn, 2014; Ahn and Kar, 2015; Ahn and Kar, 2015). (Ahn and Shi, 2016) described the dissipativity criteria of direct form of digital filters with saturation nonlinearity and also investigated whether the dissipativity criteria of interconnected digital filters can be assured. However, in his work system matrices of the digital filters were of particular form such that the output was scalar in nature.

Also, the State space model for linear image processing has been discussed in (Roesse, 1975). Dissipativity analysis of 2-D systems has been discussed in (Hinamoto, 1997; Bisiacco, 1995; Du and Xie, 2002; Kar and Singh, 1999; Kar and Singh, 2004; Ahn *et al*., 2015). Improved stability results for the uncertain discrete state-delayed systems have been described in (Tadepalli and Kandanvli, 2014). Dissipativity analysis for the discrete singular systems with time varying delay is given in (Feng *et al*., 2016).The effects of limit cycles have been described in (Ghaffari, 2009). In this paper it has been investigated whether dissipativity criteria of digital filters can be assured with some general form of nonlinearity ? How we can assure the asymptotic stability of the systems? Motivated by the recent works (Ahn and Shi, 2015) and (Ahn *et al.*, 2015), this paper attempts to answer these questions.

The paper is organized as follows:

(1) First the dissipativity criteria for direct form digital filters is presented

(2) Based on this criterion a new dissipativity criterion of digital filters has been established and verified with general form of nonlinearity

(3) The asymptotic stability of the system has also been verified.

(4) Finally some numerical examples have been employed.

2. Direct form digital filters

During hardware implementation, digital filters suffer from nonlinearities this eventually leads the filters towards instability and poor performances.

Consider the digital filter of the direct form given by (Ahn and Shi, 2015):

$$G(z) = h_0 z^{-n} + h_1 z^{-(n-1)} + \cdots + h_n \tag{1}$$

$G(z)$: transfer function of the digital filter

where:

$y(r)$: Output signal of $G(z)$

$f(y(r))$+$u(r)$*: input signal of* $G(z)$

$u(r)$*: External input*

$f(\cdot): \mathfrak{R} \rightarrow \mathfrak{R}$ Saturation overflow arithmetic employed on the output signal of $G(z)$

Saturation overflow arithmetic has been defined as

$$f(y(r)) = \begin{cases} 1 \ , if \ y(r) > 1 \\ y(r) \ , if -1 \le y(r) \le 1 \\ -1 \ , if \ y(r) < -1 \end{cases} \tag{2}$$

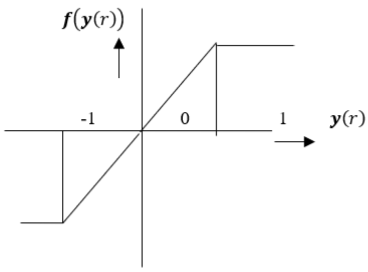

Figure 1. Saturation overflow arithmetic

where

$$f(0) = 0, and\ 0 \le \frac{f(y(r))}{y(r)} \le 1 \tag{3}$$

If we assume the stability of the infinite –precision counterpart of filter (1) then

The digital filter (1) can also be expressed as:

$$x(r+1) = Ax(r) + Bf(y(r)) + Bu(r) \tag{4}$$

$$\text{And } \mathrm{y(r)} = \mathrm{H^T x(r)} + \mathrm{h_n u(r)} \tag{5}$$

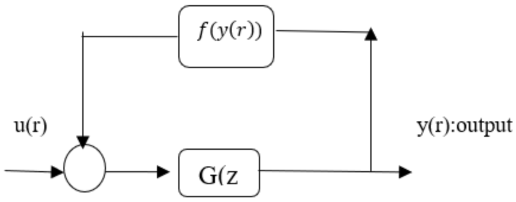

Figure 2. Saturation nonlinearity over output

Where A, B, H and u(r) are matrices of appropriate dimensions:

$$A = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & I_{n-1} & \vdots \\ 0 & \cdots & \cdots \end{bmatrix}_{nxn} \quad B = \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix}_{nx1}$$

$$H = \begin{bmatrix} h_0 \\ \vdots \\ h_{n-1} \end{bmatrix}_{nx1} \quad x(r) = \begin{bmatrix} x_1(r) \\ \vdots \\ x_n(r) \end{bmatrix}_{nx1} \tag{6}$$

I_{n-1}: represents an Identity matrix.

The dimension of $x(r+1)$ will be nx1 and that of $\mathrm{y(r)}$ will be 1x1$\forall n \in Z$.

2.1. Dissipativity criteria for direct form digital filters

For the given constant scalars $\alpha \geq 0$ and matrices Q, S and R, digital filter (4)-(5) is (Q,S,R)-α dissipative if it satisfies the following inequality:

$$\sum_{r=0}^{T} Q\, y^2(r) + 2\sum_{r=0}^{T} Sy(r)u(r) + \sum_{r=0}^{T} Ru^2\,(r) \geq \alpha \sum_{r=0}^{T} u^2\,(r) \tag{7}$$

under the zero initial condition, where *T>0 and* α is the Dissipativity Performance bound.

The (Q, S, R)-α dissipativity criteria for the direct digital filters (3)-(4) is given as following:

Theorem1 (Ahn and Shi, 2016):

Digital filter (1) is (Q,S,R)-α dissipative if for given scalar α ≥0 there exist a matrix variable

$P = P^T > 0$ *and scalar variables* $\delta > 0$ *and* $m > 0$

Such that the LMI,

$$\begin{bmatrix} \Gamma_{1,1} & * & * \\ \Gamma_{2,1} & \Gamma_{2,2} & * \\ \Gamma_{3,1} & \Gamma_{3,2} & \Gamma_{3,3} \end{bmatrix} < 0 \tag{8}$$

Where the matrix terms

$$\Gamma_{1,1} = A^T PA - P - QHH^T + \delta HH^T$$
$$\Gamma_{2,1} = B^T PA + mH^T$$
$$\Gamma_{2,2} = B^T PB - 2m - \delta$$
$$\Gamma_{3,1} = B^T PA - Qh_n H^T - SH^T + \delta h_n H^T$$
$$\Gamma_{3,2} = B^T PB + mh_n$$
$$\Gamma_{3,3} = B^T PB - Qh_n{}^2 - 2Sh_n - [R - \alpha] + \delta h_n{}^2$$

Remark 1: It is possible to obtain the Optimal Dissipativity Performance Bound α^* by optimizing the following problem (Ahn and Shi, 2015)

Maximize α subject to $\Gamma < 0, P > 0, \delta > 0, m > 0$

Remark 2: If the system is unforced then

$$u(r) = 0$$

$\Delta V(x(r)) - Qy^2(r) < 0$

This implies

$\Delta V(x(r)) < Qy^2(r)$

Now if $Q \leq 0 \Rightarrow \Delta V(x(r)) \leq 0$

Hence, the system is asymptotically stable.

3. Saturation nonlinearity of the general form

We can say that the direct form of digital filter (4)-(5) is dissipative. Also we can say that with zero input the system is asymptotically stable. So with this result we can obtain the robust controllers for the filters to withstand various nonlinearities.

Now we are going to test whether the direct form of digital filters (4)-(5) is dissipative and stable against some general form of nonlinearity.

Let χ be the nonlinear characterization for the saturation nonlinearity to be imposed on the system (4)-(5) (Tadepalli and Kandanvli, 2014)

$$\chi = \sum_{i=1}^{n} 2[y_i\,(k) - f(y_i(k))]\,[\sum_{i=1;j\neq 1}^{n} \{(\alpha_{ij} + \beta_{ij}) f_i(y_i(k) + (\alpha_{ij} - \beta_{ij})\, f_j(y_j(k)\}]$$

$$= y^T(k)Cf(y(k)) + f^T(y(k))C^T(y(k)) - f^T(y(k))(C + C^T)f(y(k)) \tag{9}$$

Where the saturation non-linearity is given as:

$$f_i(y_i(k)) = \begin{cases} 1 \ , if \ y_i(k) > 1 \\ y_i(k) \ , if \ -1 \leq y_i(k) \leq 1 \\ -1 \ , if \ y_i(k) < -1 \end{cases} \tag{10}$$

Let $C = [C_{ij}] \in R^{nxn}$ denote a matrix (Kandanvli and Kar, 2013; Kar 2007; Tadepalli *et al.*, 2014)

Where

$$c_{ii} = \sum_{j=1; j \neq i}^{n} (\alpha_{ij} + \beta_{ij}), i = 1,2,3 \ldots . n \tag{11}$$

$$c_{ij} = (\alpha_{ij} - \beta_{ij}); i, j = 1,2 \ldots . n(i \neq j) \tag{12}$$

$$\alpha_{ij} > 0; \ \beta_{ij} > 0; i, j = 1,2 . . n(i \neq j) \tag{13}$$

For $n = 1$ C corresponds to a scalar $\mu > 0$

We now propose a new Dissipativity Criteria with Nonlinearity given by (9)-(13).

3.1. Dissipativity criteria for direct form digital filters with general form of nonlinearity

For the given constant scalars $\alpha \geq 0$, identity matrix I and matrices Q, S and R digital filter (4)-(5) is (Q,S,R)-α dissipative if it satisfies the following inequality:

$$\sum_{r=0}^{T} Q \, y^2(r) + 2 \sum_{r=0}^{T} Sy(r)u(r) + \sum_{r=0}^{T} Ru^2 \, (r) \geq \alpha I \sum_{r=0}^{T} u^2 \, (r) \tag{14}$$

Under the zero initial condition, where *T>0* , *and* α is the Dissipativity Performance bound.

Let system matrices of digital filters (4)-(5):

$x(r + 1) = Ax(r) + Bf(y(r)) + Bu(r)$

And $\mathrm{y(r)} = \mathrm{H^T x(r)} + \mathrm{h_n u(r)}$

are given by:

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix}_{nxn} \quad B = \begin{bmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & \ddots & \vdots \\ b_{n1} & \cdots & b_{nn} \end{bmatrix}_{nxn}$$

$$H = \begin{bmatrix} H_{11} & \cdots & H_{1n} \\ \vdots & \ddots & \vdots \\ H_{n1} & \cdots & H_{nn} \end{bmatrix}_{nxn} \quad h_n = \begin{bmatrix} h_{11} & \cdots & h_{1n} \\ \vdots & \ddots & \vdots \\ h_{n1} & \cdots & h_{nn} \end{bmatrix}_{nxn} \quad x(r) = \begin{bmatrix} x_1(r) \\ \vdots \\ x_n(r) \end{bmatrix}_{nx1} \tag{15}$$

Clearly,

$y(r): nx1$; $f\big(y(r)\big): nx1$ are the respective dimensions.

Clearly with the general form of nonlinearity the criteria for dissipativity of the system (4)-(5) will be slightly changed. So we propose new criteria for the *(Q, S, R)* - α dissipativity of the system (4)-(5) in the following theorem:

Theorem 2: *Digital filter (4) –(5) is (Q, S, R)-α dissipative if for given scalar α ≥0 there exist a matrix variable* $P = P^T > 0$ *and variable matrix elements of C, i.e*

$$\alpha_{ij} > 0;\ \beta_{ij} > 0; i,j = 1,2 \ldots\ldots n(i \neq j)$$

Such that the LMI,

$$\Gamma = \begin{bmatrix} \Gamma_{1,1} & * & * \\ \Gamma_{2,1} & \Gamma_{2,2} & * \\ \Gamma_{3,1} & \Gamma_{3,2} & \Gamma_{3,3} \end{bmatrix} < 0 \tag{16}$$

Where:

$\mathit{\Gamma}_{1,1} = A^T PA - P - QHH^T$

$\mathit{\Gamma}_{2,1} = B^T PA + C^T H^T$

$\mathit{\Gamma}_{2,2} = B^T PB - C - C^T$

$\mathit{\Gamma}_{3,1} = B^T PA - Qh_n H^T - SH^T$

$\mathit{\Gamma}_{3,2} = B^T PB + Ch_n$

$\mathit{\Gamma}_{3,3} = B^T PB - Q{h_n}^2 - 2Sh_n - [R - \alpha I]$

$*$ represents *Symmetrical terms*

Proof: If we impose the nonlinearity as given by (9)-(13) in system

Then

$$J\big(x(r)\big) \triangleq \Delta V\big(x(r)\big) - Qy^2(r) - 2Sy(r)u(r) - [R - \alpha I]u^2(r) + \chi[20] \tag{17}$$

Where the Lyapunov function:

$$V\big(x(r)\big) \triangleq x^T(r)Px(r)$$

Now the forward difference is:

$$\Delta V\big(x(r)\big) \triangleq V\big(x(r+1)\big) - V\big(x(r)\big) = x^T(r+1)Px(r+1) - x^T(r)Px(r)$$

By defining:

$$\psi(r) = [\, x^T(r)\, f^T\big(y(r)\big)\, u^T(r)]^T$$

Now similar to theorem (1) by putting the values of $x^T(r+1)$; $x(r+1)$;$y(r)$ and χ

from (4), (5) and (9) in (17) we have :

$$J\big(x(r)\big) = \psi^T(r)\Gamma\psi(r)$$

Now

$$\Gamma < 0 \Rightarrow J\big(x(r)\big) < 0$$

So we have:

$$J\big(x(r)\big) \triangleq \Delta V\big(x(r)\big) - Qy^2(r) - 2Sy(r)u(r) - [R - \alpha I]u^2(r) + \chi < 0$$

Hence

$$J\big(x(r)\big) \triangleq \Delta V\big(x(r)\big) - Qy^2(r) - 2Sy(r)u(r) - [R - \alpha I]u^2(r) < -\chi \quad (18)$$

Now as the quantity χ is non-negative (Tadepalli et al. 2014) [28]

So we have

$$\chi \geq 0 \quad (19)$$

So from (18) and (19) we have:

$$J\big(x(r)\big) \triangleq \Delta V\big(x(r)\big) - Qy^2(r) - 2Sy(r)u(r) - [R - \alpha I]u^2(r) < 0 \quad (20)$$

Similar to theorem (1) of (Ahn and Shi, 2015) if we take the summation from $0\ to\ (T-1)$ we have:

$$\sum_{r=0}^{T} Q\, y^2(r) + 2\sum_{r=0}^{T} Sy(r)u(r) + \sum_{r=0}^{T} [R - \alpha I]u^2\,(r) > V(x(T+1) - V(x(0))$$

Assuming zero initial conditions we obtain (14).

This completes the proof.

Remark3: we optimize the performance bound by maximizing α subject to

$\Gamma < 0, P > 0, a_{12} > 0, a_{21} > 0, b_{12} > 0, b_{21} > 0$

Where $a_{12}, a_{21}, b_{12}, b_{21}$

are the elements of matrix C.

3.2. Dissipativty in the case of scalar nonlinear function

If we Substitute

$$C = C^T = \mu \tag{21}$$

Now if we put the substitute value (21) in theorem 2 then we get the following result which is similar to Theorem1 which can be given as:

Corollary 1:

Digital filter (4) –(5) is (Q, S, R)-α dissipative if for given scalar $\alpha \geq 0$ *there exist a matrix variable* $P = P^T > 0$ *and scalar matrix* $\mu > 0$

Such that the LMI,

$$\Gamma = \begin{bmatrix} \Gamma_{1,1} & * & * \\ \Gamma_{2,1} & \Gamma_{2,2} & * \\ \Gamma_{3,1} & \Gamma_{3,2} & \Gamma_{3,3} \end{bmatrix} < 0 \tag{22}$$

Where:

$\Gamma_{1,1} = A^T PA - P - QHH^T$

$\Gamma_{2,1} = B^T PA + \mu\, H^T$

$\Gamma_{2,2} = B^T PB - 2\,\mu$

$\Gamma_{3,1} = B^T PA - Qh_n H^T - SH^T$

$\Gamma_{3,2} = B^T PB + \mu h_n$

$\Gamma_{3,3} = B^T PB - Q{h_n}^2 - 2Sh_n - [R - \alpha]$

Remark 4:

Optimize the performance bound α subject to $\Gamma < 0, P > 0,\ \mu > 0$, then we obtain the optimal dissipativity performance bound α^*

Remark 5:

If the system is unforced then

$$u(r) = 0$$

So from (20)

$$\Delta V\big(x(r)\big) - Qy^2(r) < 0$$

$$\Rightarrow \Delta V\big(x(r)\big) < Qy^2(r)$$

Now if $Q \le 0 \Rightarrow \Delta V\big(x(r)\big) < 0$ so the system is asymptotically stable with zero input.

4. Numerical examples

To demonstrate the usefulness of the theorems following examples have been employed:

Example 4.1

Consider the system (4) and (5) with n=2,

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; H = \begin{bmatrix} -0.43 \\ 0.37 \end{bmatrix} \; Q = -0.2\,, S = 0.1\,, R = 0.6$$

and $h_2 = 0.51$.

Optimization using Remark 1: Now we optimize the performance bound according to Remark 1

The Optimal Dissipativity performance Bound is obtained as

$$\alpha^* = 0.1631$$

Asymptotic stability using Remark 2: Also here

$$Q = -0.2 < 0,$$

so according to Remark 2 this ensures the asymptotic stability of the system under zero input.

Optimization using Remark4:

The Optimal Dissipativity performance Bound is obtained as

$$\alpha^* = 0.1631$$

Which is same as that of Remark 1

Asymptotic stability using Remark 5: Also here $Q = -0.2 < 0$

so according to Remark 5 this ensures the asymptotic stability of the system under zero input.

Example 4.2

Consider the system given by (4) and (5)

With $H = \begin{bmatrix} \Re^2 \\ -2\Re cos\omega T \end{bmatrix}$ $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$; $Q = -0.01, S = 0.2, R = 0.3$ $\Re = 0.6$ and $wT = \frac{\pi}{3}$

then we have: $H = \begin{bmatrix} 0.36 \\ -0.6 \end{bmatrix}$

Optimization using Remark1:

The optimal dissipativity performance bound is obtained

as

$$\alpha^* = 0.0183$$

Asymptotic Stability using Remark 2: Also here $Q = -0.01 < 0$, so according to remark 2 this ensures the asymptotic stability of the system under zero input.

Optimization using Remark4:

Again according to Remark 4

The Optimal Dissipativity performance Bound is obtained as

$$\alpha^* = 0.0183$$

same as that of Remark 1

Asymptotic stability using remark 5: Also here

$Q = -0.01 < 0$ so according to remark (5) this ensures the asymptotic stability of the system under zero input.

Example 4.3

Consider the direct form of digital filters (4) and (5)

Also consider the non-linearity of the form of (18)-(22)

Clearly with this form of non-linearity system matrices will have some new dimensions:

So according to (24) for n=2 we consider

$$A = \begin{bmatrix} 0.1 & 0.2 \\ 0.3 & 0.4 \end{bmatrix} \; B = \begin{bmatrix} 0.5 & 0.5 \\ 0.6 & 0.6 \end{bmatrix}$$

$$H = \begin{bmatrix} -0.4 & 0.3 \\ 0.3 & -0.4 \end{bmatrix} \; h_2 = \begin{bmatrix} 0.5 & 0.2 \\ 0.2 & 0.5 \end{bmatrix}$$

$$Q = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.1 \end{bmatrix} S = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$$

$$R = \begin{bmatrix} 0.6 & 0 \\ 0 & 0.6 \end{bmatrix} I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Optimization using Remark 3:

The optimal dissipativity Performance bound is obtained as: $\alpha = 0.6023$

Asymptotic stability using Remark 5: $Q = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.1 \end{bmatrix} < 0$ i.e.Q is negative definite matrix so according to Remark 5 the system is asymptotically stable under unforced condition.

The above simulation results have been obtained using YALMIP, SeDuMi and MATLAB.

7. Conclusion

A new theorem has been proposed with different characterization of nonlinearity and it has been found that the system is also dissipative and asymptotically stable.

The proposed theorem is more general with generalized system matrices which can be reduced to that of the existing theorem. The proposed theorem contains less terms so it has less numerical complexity than the existing theorem. Also the digital filters considered in the examples are dissipative with optimal dissipativity performance bound. Under no input conditions the digital filters are asymptotically stable which is very crucial for robust controller design.

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