A novel dynamic identification model for small unmanned helicopters

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ABSTRACT. *Targeting small unmanned helicopters with complex dynamic features, this paper puts forward a dynamic modelling method based on frequency domain identification. Firstly, the angular dynamic model structure of the small unmanned helicopter was determined by mechanism modeling, and the modelling parameters were obtained through frequency identification of partial coherence analysis. After that, the system model was constructed in light of the model structure and modelling parameters. The proposed model was then subjected to a flight test, and compared against time-domain data. The results show that the model demonstrated the dynamic features of the target helicopter with high accuracy and adaptability. The research findings provide new insights into the detection of dynamic features of small aircrafts.*

RÉSUMÉ. *En ciblant les petits hélicoptères sans pilote dotés de fonctionnalités dynamiques complexes, cet article propose une méthode de modélisation dynamique basée sur l'identification du domaine de fréquence. Premièrement, la structure du modèle dynamique angulaire du petit hélicoptère sans pilote a été déterminée par la modélisation mécanique, et les paramètres de modélisation qui ont été obtenus par l'identification fréquentielle d'analyses de cohérence partielle. Après cela, le modèle de système a été construit selon la structure du modèle ainsi que les paramètres de modélisation. Le modèle proposé a ensuite été soumis à un vol de test et comparé à des données temporelles. Les résultats montrent que le modèle a démontré les foncitonnalités dynamiques de l'hélicoptère cible avec une précision et une adaptabilité élevées. Les résultats de la recherche fournissent de nouvelles visions sur la détection des fonctionnalités dynamiques des petits aéronefs.*

KEYWORDS: *small unmanned helicopter, frequency domain identification, dynamic modeling, time domain verification.*

MOTS-CLÉS: *petit hélicoptère sans pilote, identification du domaine de fréquence, modélisation dynamique, vérification du domaine temporel.*

DOI:10.3166/JESA.50.379-390

1. Introduction

Small unmanned helicopter is small in size, complex in structure and non-linear, through mathematical model of mechanism analysis we can't accurately describe the dynamic characteristics of its system. Frequency domain identification has been widely applied in many areas as an effective modeling method (Castillo *et al*., 2005; Ljung, 1993). Scholars at home and abroad hope to identify the dynamic characteristic parameters of small unmanned helicopter by using frequency domain identification method, these parameters were hard to get through theoretical analysis and calculation. Using frequency domain identification method in the mathematical model of the helicopter system has made great progress, in 1980s, the U.S. NASA helicopter department developed a modeling method based on frequency domain identification, and an application software CIFER was developed as well. Then this method has been identified in many mathematical models of full sized helicopter in engineering application, for example: Black Hawk helicopters (Black Hawk), BO-105 helicopters, SH-2G helicopter and tilt-rotor helicopter XV-15 (Budiyono *et al*., 2009).

This paper focus on self-developed small unmanned helicopter system (Zhou *et al*., 2014), studies identification of its dynamic model, then obtains the model by combining mechanism modeling and frequency domain identification, collects data which is to be identified via actual flight test, and verifies in time domain of the identification method's effectiveness and accuracy of the model.

2. Small unmanned helicopter system

The small unmanned helicopter is a typical multiple input & multiple output (MIMO) system, by controlling input to manipulate the actuator and generate the corresponding rudder deflection, it alters size and direction of force and torque on the helicopter to change its flight state. Compare with normal helicopter with a pilot, the small unmanned helicopter is more difficult to control, since it's small in size, high rotor speed and the rotor is rigid connected to the propeller hub, all these features make the helicopter more sensitive to input-controlling and outside disturbance. To reduce difficulty in controlling and overcome disadvantages, most small unmanned helicopters are equipped with hub-mounted Bell-Hiller stabilized ailerons which are longitudinal to main rotor. The Bell-Hiller stabilized ailerons act as a damper for the rotor system, producing a delayed angular velocity feedback that improves maneuverability and stability of the helicopter.

Small unmanned helicopter system is shown in Figure 1, Raptor-50 model aircraft helicopter is selected as the body part, and is modified and equipped with electronic equipment so as to complete the objective helicopter this paper studies. Figure 1 shows the hovering flight test photo of the objective helicopter with avionics installed. Figure 2 shows the feature size of the Raptor-50 model helicopter body. The onboard electronic device is a flight control computer with a TMS320F28335 as its central processing unit. This computer can process the data measured by sensors such as inertial measurement unit (IMU), GPS, magnetic heading sensor and altitude sensor in real time. When designing the overall of the small unmanned helicopter system, it is necessary to consider both the system model identification and the flight control method to verify the performance requirements of the system.

Figure 1. The small single wing unmanned helicopter this study uses

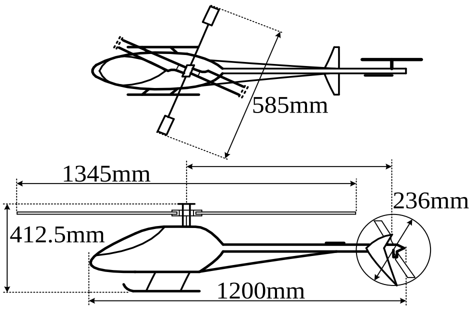

Figure 2. Feature size of Raptor-50 model helicopter

Therefore, the constructed small unmanned helicopter system not only has the ability to meet real-time flight control solution and processing capability, but also added equipment that can collect and record flight input/output signals in real-time to meet requirements of model identification method in flight input and output.

3. Dynamic mechanism modeling

3.1. Helicopter body dynamic modeling

The small unmanned helicopter is regarded as a rigid body (Tischler and Remple, 2006). According to the structural characteristics of the objective helicopter, its body is symmetrical in longitudinal direction. Therefore, the moment of inertia is zero; although its transverse plane is unsymmetrical, its moment of inertia is small and can be ignored as zero, that is: $I_{xy}=I_{yz}=I_{zx}=0$. And the inertia generated by cross-coupling is also ignored so the formula of the objective helicopter is:

$$\dot{u} = -wq + vr + g\sin\theta + X/m \tag{1}$$

$$\dot{v} = -ur + wp - g\sin\phi\cos\theta + Y/m \tag{2}$$

$$\dot{w} = -vp + uq - g\cos\phi\cos\theta + Z/m \tag{3}$$

$$\dot{p} = -qr(I_{yy} - I_{zz})/I_{xx} + L/I_{xx} \tag{4}$$

$$\dot{q} = pr(I_{zz} - I_{xx})/I_{yy} + M/I_{yy} \tag{5}$$

$$\dot{r} = pq(I_{xx} - I_{yy})/I_{zz} + N/I_{zz} \tag{6}$$

Objective helicopter aerodynamic force F and aerodynamic torque M are:

$$X = X_m + X_f \tag{7}$$

$$Y = Y_m + Y_f + Y_t + Y_v \tag{8}$$

$$Z = Z_m + Z_f + Z_h \tag{9}$$

$$L = L_m + L_v + L_t \tag{10}$$

$$M = M_m + M_h \tag{11}$$

$$N = N_m + N_t + N_v \tag{12}$$

Where, $()_m$ is the force and torque which is generated by the main rotor; $()_f$ is the force and torque which is generated by the helicopter body; $()_t$ is the force and torque which is generated by the tail rotor; $()_v$ is the force and torque which is generated by the lateral tail; and $()_h$ is the force and torque which is generated by the longitudinal tail.

3.2. Main rotor and stabilizer rod dynamic modeling

The self-developed small unmanned helicopter rotor system consists of two main rotors and Bell-Hiller stabilized ailerons which are perpendicular to the main rotor. The special structure of the rotor makes the objective helicopter different from normal helicopters in the rotor dynamic modeling. In this case, we need to set up dynamic model to analyze the main rotor and the stabilizer rod. The two main rotors of the objective helicopter are mounted to the propeller hub, the rotor and hub rotate together when the engine is on. The mass of the main rotor is uniformly distributed on the reference line of the rotor, and the main rotor has no radial extension or elastic deformation. The aerodynamic analysis of the main rotor is carried out by using the blade-element analysis method. When the objective helicopter hovers in the air, its main rotor flapping equation can be simplified as:

$$\tau_m \dot{a}_1 = -a_1 - \tau_m q + A_b b_1 + B_1 \tag{13}$$

$$\tau_m \dot{b}_1 = -b_1 - \tau_m p - B_a a_1 - A_1 \tag{14}$$

Where: $A_b = \frac{8}{\gamma\Omega^2}\frac{k_\beta}{I_b}$, $B_a = \frac{8}{\gamma\Omega^2}\frac{k_\beta}{I_b}$, $A_1 = B_{lat}\delta_{lat}$, $B_1 = A_{lon}\delta_{lon}$, A_1 and B_1 are lateral cyclic pitch input and longitudinal cyclic pitch input, respectively; A_{lon} and B_{lat} represents effective gain coefficient between the main rotor horizontal/longitudinal cyclic pitch input and flapping angle horizontal/longitudinal output.

For a single-rotor helicopter, its stabilized ailerons are connected to the spindle of the rotor system, and the stabilized ailerons can move up and down in a limited amplitude like a seesaw on the longitudinal plane of the spindle. Therefore, when analyzing the stabilized ailerons' dynamics, its rotating plane can be considered as a rotor plane that does not generate the lift force, and has no flapping angle that is generated by the flapping movement. The flapping movement of the stabilized ailerons are similar to the main rotor, expressed as:

$$\beta_s = -c_1 \cos\psi - d_1 \sin\psi \tag{15}$$

Where, d_1 is the longitudinal flapping to the main rotor shaft longitudinal plane, c_1 is the lateral flapping, ψ is the azimuth angle of the stabilized ailerons. As the stabilized ailerons can move freely like a seesaw, so they don't have elastic deformation due to the flapping, that is $k_\beta=0$. The dynamic characteristics of the flapping movement of the stabilized ailerons are expressed as follows:

$$\tau_s \dot{c}_1 = -c_1 - \tau_s q + C_1 \tag{16}$$

$$\tau_s \dot{d}_1 = -d_1 - \tau_s p + D_1 \tag{17}$$

Where: $\tau_s = \frac{16}{\gamma_s \Omega}$ is responding time constant of stabilized ailerons; $C_1=C_{lon}\delta_{lon}$, $D_1=D_{lat}\delta_{lat}$, C_1 and D_1 are the longitudinal cyclic pitch input and the lateral cyclic pitch input of the stabilizing ailerons, respectively; C_{lon} and D_{lat} represents effective gain coefficient between the stabilized ailerons horizontal/longitudinal cyclic pitch input and flapping angle horizontal/longitudinal output.

3.3. Cross-coupling of rotor and helicopter body

There will be a longitudinal flapping a_1 and a lateral flapping b_1, both are relative to the longitudinal plane of the main rotor. The force and torque, generated by the rotor flapping on the center of the propeller hub, will directly affect the change of force and torque of the objective helicopter's centroid. The derivative of the rotor flapping horizontal/longitudinal cyclic pitch to the torque produced by the pulling force T are:

$$M_{a_1} = (k_\beta + hT)/I_{yy} \tag{18}$$

$$L_{b_1} = (k_\beta + hT)/I_{xx} \tag{19}$$

Where h is the longitudinal distance between the rotor head and the centroid of the helicopter body. k_β is elastic deformation of rotor wing root generated by the flapping.

4. Dynamic model parameter identification

4.1. Establishment of parametric model of small unmanned helicopter

Small unmanned helicopter frequency domain identification algorithm means that, by applying specific input excitation signal to the system to obtain the frequency response characteristics of the system, and on this basis, by using spectral analysis methods to get the mathematical model of the system (Leishman, 2000; Adiprawita *et al*., 2007; Liceaga-Castro *et al*., 1995; Tischler, 1987). In this paper, model structure of helicopter attitude angular rate is analyzed, and the parameters are identified when the helicopter is hovering in air or flying in forward direction at a slow speed. As for the transverse passage of the helicopter, we can add the control input of the stabilized ailerons into the flapping movement equation of the main rotor, and get the following transverse passage flapping movement equation of the objective helicopter:

$$\tau_m \dot{b}_1 = -b_1 - \tau_m p + B_d d_1 + B_{lat}\delta_{lat} \tag{20}$$

In a steady state where $\dot{b}_1$=0, equation (20) can be rewritten as:

$$b_1 = -\tau_m p + B_d d_1 + B_{lat}\delta_{lat} \tag{21}$$

Transverse passage flapping movement equation of the helicopter stabilized ailerons is:

$$\tau_s \dot{d}_1 = -d_1 - \tau_s p + D_{lat}\delta_{lat} \tag{22}$$

The Laplace transform of equation (22) is:

$$\tau_s s d_1(s) = -d_1(s) - \tau_s p(s) + D_{lat}\delta_{lat}(s) \tag{23}$$

After reorganizing, the transfer function of the stabilized ailerons of the helicopter is:

$$d_1(s)=\frac{-\tau_s p(s)+D_{lat}\delta_{lat}(s)}{\tau_s s+1} \tag{24}$$

Substituting equation (24) into equation (21) yields:

$$b_1(s)=-\tau_m p(s)+B_{d_1}\left(\frac{-\tau_s p(s)+D_{lat}\delta_{lat}(s)}{\tau_s s+1}\right)+B_{lat}\delta_{lat}(s) \tag{25}$$

The Laplace inverse transform of equation (25) gives the dynamic response equation of the helicopter's transverse angular rate:

$$b_1(s)=\frac{-\left(\tau_m+B_{d_1}\tau_s\right)p(s)}{s+1/\tau_s}+\frac{\left(B_{lat}+B_{d_1}D_{lat}\right)\delta_{lat}(s)}{s+1/\tau_s} \tag{26}$$

Movement equation of transverse rotation of the helicopter body is $\dot{p}=L_{b_1}b_1$, its Laplace transform is:

$$p(s)=L_{b_1}b_1(s) \tag{27}$$

Substituting equation (26) into equation (27), after reorganizing, we can get the model structure transfer function of the transverse angular rate of the helicopter as follows:

$$\frac{p(s)}{\delta_{lat}(s)}=\frac{L_{b_1}\left(B_{lat}+B_{d_1}D_{lat}\right)/\tau_s}{s^2+(1/\tau_s)s+L_{b_1}\left(\tau_m+B_{d_1}\tau_s\right)/\tau_s} \tag{28}$$

Similarly, when the control input of the stabilized ailerons is added, we can get the model structure transfer function of the longitudinal angular rate of the helicopter as follows:

$$\frac{q(s)}{\delta_{lon}(s)}=\frac{M_{a_1}\left(A_{lon}+A_{c_1}C_{lon}\right)/\tau_s}{s^2+(1/\tau_s)s+M_{a_1}\left(\tau_m+A_{c_1}\tau_s\right)/\tau_s} \tag{29}$$

The flight test data collected by the model identification of the helicopter include the dynamic response characteristics of the actuator. Therefore, when conducting structure analysis of model identification, the dynamic characteristics of the actuator should be added to the input-output transfer function of the system. The dynamic characteristics of the helicopter actuator this paper studies can be described by the following second-order system:

$$H_{servo}=\frac{\omega_s^2}{s^2+2\zeta_s\omega_s s+\omega_s^2} \tag{30}$$

Where, ω_s and ξ_s are the actuator's natural frequency and the damping ratio.

When ignoring cross-coupling of the rotor flapping and other factors, the transfer function of the transverse and longitudinal control actuator input and attitude angular rate output of the helicopter are:

$$\frac{p(s)}{\delta_{lat}(s)} = \frac{L_{b_1}\left(B_{lat} + B_{d_1} D_{lat}\right)/\tau_{sp}}{s^2 + \left(1/\tau_{sp}\right)s + L_{b_1}\left(\tau_{mp} + B_{d_1}\tau_{sp}\right)/\tau_{sp}} \bullet \frac{\omega_s^2}{s^2 + 2\zeta_s \omega_s s + \omega_s^2} \tag{31}$$

$$\frac{q(s)}{\delta_{lon}(s)} = \frac{M_{a_1}\left(A_{lon} + A_{c_1} C_{lon}\right)/\tau_{sq}}{s^2 + \left(1/\tau_{sq}\right)s + M_{a_1}\left(\tau_{mq} + A_{c_1}\tau_{sq}\right)/\tau_{sq}} \bullet \frac{\omega_s^2}{s^2 + 2\zeta_s \omega_s s + \omega_s^2} \tag{32}$$

4.2. System parameter identification of the small unmanned helicopter

According to the helicopter's input auto-power spectrum $G_{xx}(f)$, and input/output cross-power spectrum $G_{xy}(f)$, we can get the system frequency response of the to-be-identified passage $H(f)$ as follows (Bendat and Piersol, 1990):

$$H(f) = \frac{G_{xy}(f)}{G_{xx}(f)} \tag{33}$$

Due to the serious cross-coupling between different passages of the helicopter, the vibration of the system is also quite large. The interference of these external factors will make the measurement signal vulnerable to the noise signals. In order to detect the linear correlation between the output $y(t)$ and the input $x(t)$ in the frequency domain, a coherent function is introduced. The expression is:

$$\gamma_{xy}^2 = \frac{\left|G_{xy}(f)\right|^2}{\left|G_{xx}(f)\right|\left|G_{yy}(f)\right|} \tag{34}$$

Where, γ_{xy}^2 is the coherence function, and $0 \le \gamma_{xy}^2 \le 1$. If $y(t)$ is a linear response of $x(t)$, then $\gamma_{xy}^2 = 1$; if $y(t)$ and $x(t)$ are completely unrelated, then $\gamma_{xy}^2 =0$. Under normal circumstances, the coherence function of the system is in the range of $0 < \gamma_{xy}^2 < 1$. When identifying the frequency domain of the helicopter, the identified result of the coherent function $\gamma_{xy}^2 > 0.6$ can be considered as satisfactory; if it is smaller than the limit, the system will have a large random error in the system. If $\gamma_{xy}^2 > 0.8$ or closer to 1, the identification result will be more accurate.

The non-linear part of the small unmanned helicopter system and the input and output noise are the main factors affecting the linear dependence of the system. Because the helicopter is a typical multiple-input multiple-output system, and the cross-coupling between axes can also affect the consistency of the coherence function, therefore, for the objective helicopter's multiple-input and multiple-output system, its linear correlation can be simplified as a multi-input and single-output question. The longitudinal and lateral passages of the helicopter will produce cross-coupling when controlling input by the sweep control, in order to detect the linear correlation between the output *y(t)* and the input *x(t)* in the frequency domain and to obtain more accurate identification results; the method of partial coherence analysis is used to analyze whether the cross-coupling of the control input will affect the system identification result, and then remove the control input which affects identification result of the passage to be identified. The double-input single-output system structure is shown in Figure 3.

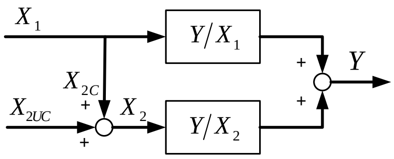

Figure 3. Double-input Single-input system structure

Where, Y/X_2 passage's control input X_2 is consisted of X_{2c}, which is relevant to X_1, and X_{2UC}, which is irrelevant to X_1, namely:

$$X_2 = X_{2C} + X_{2UC} \tag{35}$$

The partial coherence functions $\gamma^2_{2y,1}$ for input X_2 and output Y and remove linear effects X_{2c} are:

$$\gamma^2_{2y,1} = \frac{\left|G_{2y}G_{11} - G_{1y}G_{21}\right|^2}{G_{11}^2 G_{22} G_{yy}\left(1-\gamma_{12}^2\right)\left(1-\gamma_{1y}^2\right)} \tag{36}$$

Frequency response of passage Y/X_2, which is to be identified, is:

$$H(f) = \frac{Y}{X_2} = \frac{G_{2y,1}}{G_{22,1}} = \frac{G_{11}G_{2y} - G_{1y}G_{21}}{G_{11}G_{22}\left(1-\gamma_{12}^2\right)} \tag{37}$$

In this formula, G_{21} is the cross-spectrum of X_2 and X_1, G_{1y} and G_{2y} are cross-spectrum of X_1 and Y; X_2 and Y, respectively; G_{11}, G_{22} and G_{yy} are self-spectrum of X_1, X_2 and Y, respectively; γ^2_{12} is the coherent function of X_1 and X_2, γ^2_{1y} is the coherent function of input X_1 and output Y .

4.3. Flight test data identification results

Frequency domain identification of small unmanned helicopter system allows parametric identification of their entire system or subsystem (actuator model or engine model). In this paper, a kind of sinusoidal swept frequency input is used as the input excitation signal to the objective helicopter. According to the flight characteristics and working bandwidth of the objective helicopter, this paper selects frequency band of the swept input as *0.6~18rad/sec*. Through the analysis and extraction of the identification data of the flight data acquired from the flight test of the helicopter, several useful samples of the input and output are obtained to ensure the validity and accuracy of the model identification results.

According to the model structure of the helicopter, the frequency response data obtained from the spectrum analysis are fitted by the above parameter identification algorithm, and the angular velocity transfer functions of the transversal and longitudinal passages of the single-rotor helicopter are finally obtained. The expressions are as follows:

$$\frac{p(s)}{\delta_{lat}(s)}=\frac{-10107000}{s^4+18.35s^3+1674.43s^2+17691.15s+277535.18} \tag{38}$$

$$\frac{q(s)}{\delta_{lon}(s)}=\frac{10391000}{s^4+18.69s^3+1276.94s^2+11828.72s+294843.01} \tag{39}$$

5. Identification parameter time-domain verification

The non-recognized sample's sweep input is used as the input excitation signal of the identified small unmanned helicopter dynamics model. By comparing the model output data obtained in frequency domain identification with the output data collected in flight test, the model adaptability of the objective helicopter by the frequency domain identification method, and the time domain verification results under the longitudinal and lateral sweep excitation are shown in Figure 4 and Figure 5. From the figures we can see, the lateral and longitudinal passage output response and the actual flight test data collected have fewer output errors, and can reflect the flight dynamics of the helicopter more accurately, so the identified dynamic model has a strong adaptability.

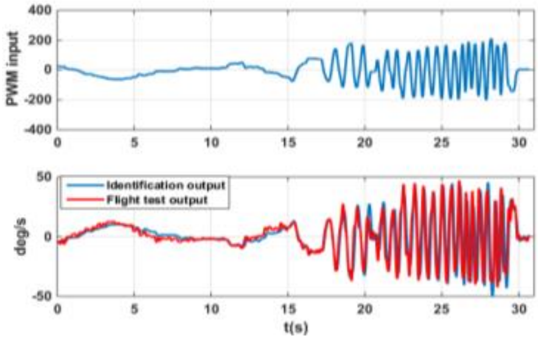

Figure 4. Longitudinal passage identification model sweep time domain verification

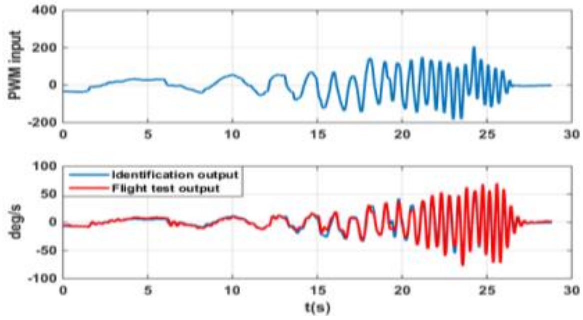

Figure 5. lateral passage identification model sweep time domain verification

6. Conclusion

In this paper, a dynamic model frequency domain identification method of the small unmanned helicopter is proposed. The dynamic model structure of the helicopter is determined by the mechanism modeling method. The model parameters which are hard to determine by mechanism modeling, are obtained by frequency domain identification method which based on partial coherence analysis, and finally we can get the longitudinal dynamic model of the helicopter. In the time domain, the cross-validation method is used to verify the flight test data of the objective helicopter. The results show that the dynamic model of the helicopter can well reflect the dynamic characteristics of the angular velocity of the objective helicopter. It provides a more accurate dynamic model for the design of the attitude controller for the next step.

Acknowledgment

The authors would like to thank the whole automation team for their invaluable assistance and support. In particular, Wang Min and Hong Liang. The Scientific Research Program Funded by Shaanxi Provincial Education Department under Grant No.17JK0345, Doctoral startup fund of Xi'an Polytechnic University under No.BS1415.

References

Adiprawita W., Ahmad A.S., Semibiring J. (2007). Automated flight test and system identification for rotary wing small aerial platform using frequency responses analysis. *ICIUS*, Vol. 4, No. 4, pp. 50-56. https://doi.org/10.1016/s1672-6529(07)60037-7

Bendat J. S., Piersol A. G. (1990). Random, data: Analysis and measurement procedures. *Wiley*.

Budiyono A., Yoon K. J., Daniel F. D. (2009). Integrated identification modeling of rotorcraft-based unmanned aerial vehicle. *17th Mediterranean Conference on Control & Automation*, Vol. 10, pp. 898-903. https://doi.org/10.1109/MED.2009.5164659

Castillo P., Lozano R., Dzul A. E. (2005). Modeling and control of mini-flying machines. *New York Springer,* Vol. 26, No. 3, pp. 122-124. https://doi.org/10.1007/1-84628-179-2

Leishman J. G. (2000). Principles of helicopter aerodynamics. *Cambridge University Press*, Vol. 5, pp. 78.

Liceaga-Castro J., Verde C., O'Reilly J., Leithead W. E. (1995). Helicopter flight control using individual channel design. *IEE Proceedings of Control Theory and Application*, Vol. 142, No. 1, pp. 58-72. https://doi.org/10.1049/ip-cta:19951575

Ljung L. (1993). Some results on identifying linear system using frequency domain data. *32nd IEEE Conference on Decision and Control*, Vol. 4, pp. 3534-3538. https://doi.org/10.1109/CDC.1993.325876

Tischler M. B. (1987). Frequency-response identification of XV-15 tilt-rotor aircraft dynamics. *NASA Technical Report.*

Tischler M. B., Remple R. K. (2006). Aircraft and rotorcraft system identification: engineering methods with flight-test example. *American Institute of Aeronautics and Astronautics.*

Zhou J., Hong L., Li W. C. (2014). Design and implementation of small-scale single rotor. *Journal of Xi'an Polytechnic University*, Vol. 28, No. 5, pp. 626-630.