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A priority-slot based continuous-time formulation for crude-oil scheduling problems with oil residency time constraint

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ABSTRACT

The optimal scheduling of crude-oil operation in refineries has been studied by various groups during the past decade leading to different mixed integer linear programming or mixed nonlinear programming formulations. This paper presents a new formulation with oil residency time constraint based on single-operation sequencing (SOS). At the same time, the bilinear constraints in the formulation are replaced by its necessary conditions, which are linear. A simple MILP-NLP procedure has been used to solve this model and leads to a satisfactory optimal result.

Keywords: Oil Refinery, Scheduling, Continuous-Time Formulation, Residency Time Constraint.

1. INTRODUCTION

It is a great challenge to operate an oil refinery [1]. Generally, there are three levels in operating a plant of refinery: production planning, production scheduling, and process control. It is known that when a plant is well operated, it can increase profit by $10 per ton of product or more [2]. Thus, great attention has been paid to the development of effective techniques for the operations of a refinery. Up to now, at the process control level, advanced control systems have been installed for unit control to optimize some production objectives in most oil refinery, resulting in significant productivity gains in plant units. However, without the optimal scheduling, it fails to achieve the global economic optimization of a plant.

At the planning level, oil refineries are increasingly concerned with the better planning of their operations. With the availability of linear programming-based commercial software for refinery production planning, such as process industry modeling system (PIMS) [3], general production plans of a whole refinery can be found. As pointed out by Pelham and Pharris [4], the planning technology can be considered well developed and relevant progress should not be expected. The major advances in the area will be based on model refinement through the use of nonlinear programming.

Because of the NP-hard nature for general scheduling problem [5], usually heuristics and search algorithms, such as simulated annealing algorithms, generic algorithms, and tabu algorithms are applied to solve the scheduling problem in discrete manufacturing operations [6-10]. In recent years, great effort has been made in crude oil scheduling, by using rule-based algorithms [11], search algorithms [12], petri net-based algorithms [13-22], and mixed integer programming [23-27].

Short-term scheduling is at the middle level. It is among the most challenging optimization problems, both in terms of modeling and solution algorithms. As pointed by Shobrys and White [28], to effectively operate a process plant, the three levels should work together. Thus, with the well-developed techniques for planning and process control, it is crucial to develop effective techniques for short-term scheduling [29]. In recent years, mostly mixed integer linear programming (MILP) [30], constraint programming (CP) [31], and genetic algorithm (GA) techniques have been used to tackle these problems. CP has proved to be very efficient for solving scheduling problems but it is rarely used to solve problems arising in the chemical engineering field. One of the reasons is that CP is very efficient at sequencing tasks or jobs that are defined a priori. However, the scheduling of chemical processes usually involves both defining and sequencing the tasks that should be performed. As a consequence, linear programming-based techniques have been preferred with formulations essentially based on time grids as it easily allows modeling tank or unit capacity at the end of each time of interval [32-33].

Uniform time discretization (usually referred to as discrete-time) formulations have first been successfully used to solve batch processes based on an STN or RTN representation of the process [34-35]. The formulation has two advantages: it can easily be applied to many different problems and it has a very tight linear programming (LP) relaxation. However, when a large number of time intervals are needed in order to obtain acceptable accuracy, the problem size may become intractable even for efficient commercial solvers because of a huge number of binary variables.

To reduce the number of discrete variables, non-uniform time discretization (usually referred to as continuous-time) formulations have been introduced based on RTN representation or STN representation [36-38]. The main difference with the discrete-time approach is that the duration of the time intervals is not fixed and has to be determined by the solver. This approach is also easy to implement and may be used for a larger scheduling horizon as it leads to more compact model. However, despite having fewer binary variables, it includes nonlinear constraints and its LP relaxation is in general less tight, which makes the problem difficult to solve.

Recently, Mouret et al. [38] propose a novel priority-slot based continuous time formulation for the scheduling problem crude oil operations. One particular benefit of their model is that the only parameter that needs to be postulated a priori is the total number of operations executed in the schedule. However, in their MINLP model, to make the problem solvable, they ignore oil residency time constraint that is necessary in a real-life refinery. Hence, by the model in [38] which is very difficult to solve, an infeasible solution may be obtained. Because a tank cannot be charged and discharged at the same time and after being charged, the crude oil should stay in the charge tank for a certain amount of time, so we only need consider the concentration of the final process of charging without considering the concentration of all the process of charging. This paper proposes a model based on Sylvain Mouret's formulation, which considers oil residency time constraint. This model is a MINLP model, so it can be solved using MILP-NLP procedure and gets a satisfactory feasible solution with small gap between upper bound and lower bound.

The paper is organized as follows. In the next section, we briefly introduce the operation process of crude oil operations and its short-term schedule. A continuous-time single-operation sequencing (SOS) formulation with residency time constraint is proposed in section III. In section IV, a simple solution method to solve the SOS model is presented. Examples are used to show the application of the proposed model in section V. Finally, section VI gives the conclusions.

2. PROCESS AND ITS SHORT-TERM SCHEDULING PROBLEM

2.1 Crude oil operations in refinery

Generally, the process of crude oil operations in a refinery can be stated as follows. Crude oil is carried to the port of a plant by crude vessels, where crude oil is unloaded into storage tanks. The crude oil in storage tanks is then transported to charging tanks in the refinery. From the charging tanks, oil is fed into distillers for distillation. The middle products by the distillation are then sent to other production units for further processing, such as fractionation and reaction. This paper is aimed at addressing the short-term scheduling problem for crude oil operations from a crude vessel to distillers.

In a refinery, various types of crude oil are processed which have different components. Sometime, in order to satisfy the requirement of components of distillers, two or more types of crude oil need to be blended. While unloading, crude oil can be unloaded into only an empty storage tank unless the same type of crude oil is in it. After filling a storage tank or a charging tank, crude oil should stay there for a certain amount of time to separate the brine, which is called oil residency time (RT) constraints, and then can be transported to charging tanks and distillers respectively. Besides residency time constraints, there are more constraints in crude oil operations which can be separated into two categories: resource constraints and process constraints. Resource constraints include: 1) the limitation of the number of tanks; 2) the limitation of flow rate of the pipeline; 3) inventory and location (tanks) of all kinds of crude oil. Process constraints include: 1) distillers should operate continuously and cannot be stopped unless maintenance is necessary; 2) when a charging tank feeds into a distiller, this tank must be dedicated to the distiller and cannot be charged at the same time; 3) a tank cannot be charged and discharged at the same time;

2.2 Short-term scheduling

In the process of crude oil operations, resources include docks for crude marine vessels, storage tanks, pipeline, and charging tanks. There are three types of operations: crude-oil unloading from marine vessels to storage tanks, crude oil transportation from storage tanks to charging tanks, and crude oil feeding from charging tanks to distillation units (CDU). The short-term scheduling problem is to arrange production activities in every detail at every time by assigning the resources to the operations. To do so, one knows the initial state information only. The initial state information includes: 1) current inventory of crude oil and types of crude oil in storage and charging tanks; 2) arrival time of marine vessels, types and volume of crude oil in them; and 3) working state of every production device.

Besides the initial state information and the above constraints, a short-term schedule for crude oil operations should be obtained based on: a) a scheduling horizon; b) crude oil property specifications for distillations; and c) demands for different crude oil mixtures. The logistic constraints of the scheduling problem are given as follows.

1) Only one berth is available at the docking station for vessel unloading;

2) Simultaneous charging and discharging of tanks are not permitted;

3) A tank feeds only one CDU at a time;

4) A CDU can be charged by only one charging tank at a time;

5) CDUs must be operated continuously throughout the scheduling horizon without interruption.

Scheduling horizon is different from one refinery to another. Generally, it lasts for a week, 10 days, or even longer. For scheduling of crude oil operations, there are various objectives and it is a typical multi-objective optimization problem [16]. When there is a switch from one charging tank to another in CDU feeding, set point regulation for distillation is necessary. Thus, switches in CDU feeding have a cost. Moreover, when any of such switches occurs, a setup is necessary. Such a setup is not only time consuming but also dangerous in the sense of security. Thus, it is very crucial to minimize the number of switch times, and this is the objective of our model.

A short-term schedule of crude oil operations is composed of a series of crude oil delivering operations. The questions are when an operation should take place, and what and how it should be done. For each operation to take place, a decision should be made to answer them. To describe a short-term schedule in detail, we first define an operation decision (OD).

Definition 2.1: An operation decision is defined as OD = (S, D), where S is the source from which the crude oil comes and D is the destination to which crude oil is to be delivered.

One of the most common constraints appearing in scheduling problem is the non-overlapping requirement for two ODs v and w. This requirement is defined as follows.

Definition 2.2: Assume that ODs v and w are to be executed in $[t_{v1}, t_{v2}]$ and $[t_{w1}, t_{w2}]$, respectively, and it is required that $[t_{v1}, t_{v2}] \cap [t_{w1}, t_{w2}] = \varnothing$. Then, this requirement is called non-overlapping requirement.

A short-term schedule of crude oil operations is composed of a series of ODs. Thus, to describe a schedule by a continuous-time formulation is to present the sequence of these ODs. To do so, a sequence of priority slots is used. A priority slot is a position i on the time coordinate. Priority slot i is said to have a higher scheduling priority than priority slot j with slots i and j being non-overlapping, if i is placed earlier than j on the time coordinate. Such a relation is denoted as $j > i$, or $i < j$. In the formulation for the scheduling problem, each priority slot is assign to exactly one specific OD. In this way, the number of priority slots is equal to the total number of ODs to be executed during the scheduling horizon and the sequence of priority slots corresponds to the sequence of the ODs. One issue is to decide the number of priority slots which should be postulated a priori by the planner.

Assume that two non-overlapping ODs v and w are assigned to priority slots i and j with $i < j$. Let S_{iv} and S_{jw} be the start time of slots i and j, and D_{iv} and D_{jw} be their durations, respectively. Since OD v has a higher priority than w, w can start only after the completion of v, i.e., we have

$$S_{iv} + D_{iv} \leqslant S_{jw} \qquad (2.1)$$

By using the above precedence rule, given a sequence of ODs, a schedule obtained is feasible if for any pair of non-overlapping ODs (2.1) is satisfied. At the same time, with the above modeling method, given a sequence of ODs, different schedule can be obtained by ordering the ODs with respect to their start time.

A refinery scheduling example from Mouret *et al.* [38] is used to show the modeling method. The Gantt chart of the optimal solution is given in Figure 1. This solution can be represented as a sequence of ODs as **7683513762**. It should be pointed out that it is not trivial to find such a solution.

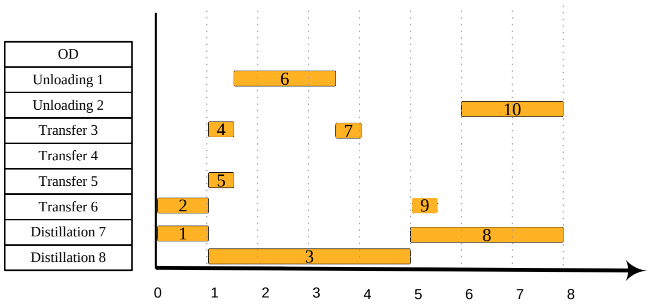

Figure 1. Optimal schedule for the first example presented in Mouret *et al*[38].

3. PROBLEM FORMULATION

This section presents the priority-slot-based continuous-time formulation for the short-term crude oil operations scheduling problem with oil residency time constraint.

3.1 Sets and parameters

$T = \{1, 2, ..., N\}$: Set of priority-slots

W: Set of all operation types for a schedule: $W=W_U \cup W_T \cup W_D$

$W_U \subset W$: Set of oils unloading operation types

$W_T \subset W$: Set of oil transportation operation types

$W_D \subset W$: Set of oil feeding operation types

$R = R_V \cup R_S \cup R_C \cup R_D$: Set of devices

$R_V \subset R$: Set of vessels

$R_S \subset R$: Set of storage tanks

$R_C \subset R$: Set of charging tanks

$R_D \subset R$: Set of CDUs

$I_r \subset W$: Set of operation types that deliver oil into device r

$O_r \subset W$: Set of operation types that deliver oil from device r

C: Set of crude oil types

K: Set of critical components in a type of oil

3.2 Parameters

H: Scheduling horizon

$\underline{S_v}$: Lower bound on the start time of the execution of operation v, $0 \le \underline{S_v} < H$

$\underline{V_v^t}$ and $\overline{V_v^t}$: Lower and upper bounds of the volume delivered by operation v and generally $\underline{V_v^t}$ = 0 for an operation except that an unloading operation is required to unload all the oil of a type in a vessel. In this case, we have $\underline{V_v^t} = \overline{V_v^t}$

$\underline{FR_v}$ and $\overline{FR_v}$: Minimal and maximal flow rates permissive for operation v

S_r: Arrival time of Vessel r

$\underline{x_{vk}}$ and $\overline{x_{vk}}$: Minimal and maximal percentages of Property k permissive for crude oil obtained by oil blending in charging tanks via transportation operation v

x_{ck}: The percentage of Property k of crude oil type $c \in C$

$\underline{L_r^t}$ and $\overline{L_r^t}$: Capacity limit of tank $r \in R_S \cup R_C$

$\underline{D_r}$ and $\overline{D_r}$: Lower and upper bounds of demand on crude oil to be delivered from charging tank $r \in R_C$ during the scheduling horizon

$\underline{N_D}$ and $\overline{N_D}$: Lower and upper bounds of the number of operations for CDU feeding

$NO_{vv'}$: $NO_{vv'}$=1 if operation v and v' must not overlap, and otherwise $NO_{vv'}$=0

L_{0rc}: is the initial accumulated volume of crude oil type c in tank $r \in R_S \cup R_C$

L_{0r}^t: is the initial total accumulated volume of crude oil in tank $r \in R_S \cup R_C$

G_c: Gross margin of crude oil c

RT: Oil residency time in charging tanks and storage ones

3.3 Variables

$Z_{iv} \in \{0, 1\}$, $i \in T$ and $v \in W$: Z_{iv} = 1 if operation v is assigned to priority-slot i; and otherwise Z_{iv} = 0.

$S_{iv} \ge 0$, $D_{iv} \ge 0$, and $E_{iv} \ge 0$, $i \in T$ and $v \in W$: Continuous time variables. S_{iv} is the start time of operation v if it is assigned to priority-slot i, and S_{iv}= 0 otherwise; D_{iv} is the duration of operation v if it is assigned to priority-slot i; and D_{iv}= 0 otherwise. E_{iv} is the end time of operation *v* if it is assigned to priority-slot *i*, $E_{iv} = S_{iv} + D_{iv}$; and E_{iv}=0 otherwise.

$V^t_{iv} \geq 0$ and $V_{ivc} \geq 0$, $i \in T$, $v \in W$, and $c \in C$: Operation variables, where V^t_{iv} is the total volume of crude oil delivered by operation *v* if it is assigned to priority-slot *i*, and V^t_{iv} = 0 otherwise; V_{ivc} is the volume of crude oil of type $c \in C$ delivered by operation *v* if it is assigned to priority-slot *i*, and V_{ivc} = 0 otherwise.

L^t_{ir} and L_{irc}, $i \in T$, $r \in R$, and $c \in C$: Resource variables, where L^t_{ir} is the total accumulated volume of crude oil in tank $r \in R_S \cup R_C$ at the beginning of slot *i*; L_{irc} is the accumulated volume of crude oil type *c* in tank $r \in R_S \cup R_C$ at the beginning of slot *i*.

3.4 Auxiliary continuous variables:

$\rho > 0$, it is large enough, as well as positive infinity.

3.5 Problem formulation

With the notation given above, we can present our formulation for the short-term crude oil operations scheduling problem with oil residency time constraints being taken into account by using the priority-slot-based method. First, we present the constraints as follows.

Constraints of time for unloading oil from a vessel: Constraints (1) - (2) are used to enforce that only after the arrival of crude oil vessels to the dock, a vessel can be unloaded.

$$S_{iv} \geq \underline{S_v} \cdot Z_{iv} \quad i \in T,\ v \in W_U \tag{1}$$

$$\underset{v \in O_r}{S_{iv}} \leq S_r \quad r \in R_V,\ i \in T \tag{2}$$

Time constraints: Constraints (3) and (4) restrict the beginning time, duration, and the ending time of operation v.

$$E_{iv} \leq H \cdot Z_{iv} \quad i \in T,\ v \in W \tag{3}$$

$$E_{iv} = S_{iv} + D_{iv} \quad i \in T,\ v \in W \tag{4}$$

Cardinality constraints for unloading and distillation operations: Constraint (5) ensures that each vessel must be unloaded its cargo exactly once. In order to decrease the changeover cost of CDU switches, the total number of distillation operations is bounded by Constraint (6) using lower bound $\underline{N_D}$ and upper bound $\overline{N_D}$.

$$\sum_{i \in T} \sum_{v \in O_r} Z_{iv} = 1 \quad r \in R_V \tag{5}$$

$$\underline{N_D} \leq \sum_{i \in T} \sum_{v \in W_D} Z_{iv} \leq \overline{N_D} \tag{6}$$

Unloading sequence constraints: Constraints (7) - (8) define the unloading sequence of crude oil vessels that must be unloaded in order of their arrival time to the dock.

$$\sum_{i \in T} \sum_{v \in O_{r_1}} E_{iv} \leq \sum_{i \in T} \sum_{v \in O_{r_2}} S_{iv} \quad r_1, r_2 \in R_V,\ r_1 < r_2 \tag{7}$$

$$\sum_{j \in T, j<i} \sum_{v \in O_{r_1}} Z_{jv} \leq \sum_{j \in T, j \leq i} \sum_{v \in O_{r_2}} S_{iv} \qquad i \in T,\ r_1, r_2 \in R_V,\ r_1 < r_2 \tag{8}$$

Continuous distillation constraint: The continuousness of CDU distillation is ensured by Constraint (9). Since each CDU can be charged by only one charging tank, continuous distillation can be defined by the total distillation time equating the whole scheduling horizon H.

$$\sum_{i \in T} \sum_{v \in I_r} D_{iv} = H \quad r \in R_D \tag{9}$$

Assignment constraint: Constraint (10) enforces that at most one operation must be assigned to each priority-slot.

$$\sum_{v \in W} Z_{iv} \leq 1 \quad i \in T \tag{10}$$

Symmetry breaking constraint: Constraint (11) is used to eliminate non-occupancy of a priority-slot for avoiding slot redundancy.

$$\sum_{v \in W} Z_{iv} \geq 1, \quad i \in T \tag{11}$$

Non-overlapping constraints: Constraints (12) - (14) ensure that two operations v_1 and v_2 must not be simultaneously fulfilled.

$$E_{i_1 v_1} + E_{i_1 v_2} \leq S_{i_2 v_1} + S_{i_2 v_2} + H \cdot (1 - Z_{i_2 v_1} - Z_{i_2 v_2}) \qquad i_1, i_2 \in T, i_1 < i_2, v_1, v_2 \in W, NO_{v_1 v_2} = 1 \tag{12}$$

$$E_{i_1 v_1} \leq S_{i_2 v_2} + H \cdot (1 - Z_{i_2 v_2}) \qquad i_1, i_2 \in T, i_1 < i_2, v_1, v_2 \in W, NO_{v_1 v_2} = 1 \tag{13}$$

$$E_{i_1 v_2} \leq S_{i_2 v_1} + H \cdot (1 - Z_{i_2 v_1}) \qquad i_1, i_2 \in T, i_1 < i_2, v_1, v_2 \in W, NO_{v_1 v_2} = 1 \tag{14}$$

Constraints (15) - (16) bound crude oil volume transferred by operation v using lower bound $\underline{V^t_v}$ and upper one $\overline{V^t_v}$.

$$V^t_{iv} \leq \overline{V^t_v} \cdot Z_{iv} \quad i \in T,\ v \in W \tag{15}$$

$$V^t_{iv} \geq \underline{V^t_v} \cdot Z_{iv} \quad i \in T,\ v \in W \tag{16}$$

Constraints (17) - (19) enforce material balance for transferring operation.

$$V^t_{iv} = \sum_{c \in C} V_{ivc} \quad i \in T,\ v \in W \tag{17}$$

$$L_{irc} = L_{0rc} + \sum_{j \in T, j<i} \sum_{v \in I_r} V_{ivc} - \sum_{j \in T, j<i} \sum_{v \in O_r} V_{ivc} \qquad i \in T, r \in R, c \in C \tag{18}$$

$$L^t_{ir} = \sum_{c \in C} L_{irc} \quad i \in T,\ v \in W \tag{19}$$

Constraints (20) bound the flowrate by $\underline{FR_v}$ and $\overline{FR_v}$.

$$\underline{FR_v} \cdot D_{iv} \leq V^t_{iv} \leq \overline{FR_v} \cdot D_{iv} \quad i \in T,\ v \in W \tag{20}$$

Property constraint: Constraint (21) bounds property k of the blender transferred by operation v, and calculates property k of the blender from property xck of crude oil c by the assumption that the mixing procedure is linear.

$$\underline{x_{vk}} \cdot V_{iv}^t \leq \sum_{c\in C} x_{ck} V_{ivc} \leq \underline{x_{vk}} \cdot V_{iv}^t \quad i \in T, v \in W, k \in K \tag{21}$$

Constraints (22) - (25) ensure material balance for inventory of tanks.

$$\underline{L_r^t} \leq L_{ir}^t \leq \overline{L_r^t} \quad i \in T,\ r \in R_S \cup R_C \tag{22}$$

$$0 \leq L_{irc} \leq \overline{L_r^t} \quad i \in T, c \in C, r \in R_S \cup R_C \tag{23}$$

$$\underline{L_r^t} = L_{0r}^t + \sum_{i\in T}\sum_{v\in I_r} V_{iv}^t - \sum_{i\in T}\sum_{v\in O_r} V_{iv}^t \leq \overline{L_r^t} \quad r \in R_S \cup R_C \tag{24}$$

$$0 \leq L_{0rc} + \sum_{i\in T}\sum_{v\in I_r} V_{ivc} - \sum_{i\in T}\sum_{v\in O_r} V_{ivc} \leq \overline{L_r^t} \quad r \in R_S \cup R_C, c \in C \tag{25}$$

Constraint (26) is a demand constraint, which define lower and upper bounds, $\underline{D_r}$and $\overline{D_r}$, to restrict the total volume of feedstock charged by each charging tank during the whole scheduling horizon H.

$$\underline{D_r} \leq \sum_{i\in T}\sum_{v\in O_r} V_{iv}^t \leq \overline{D_r} \quad r \in R_C \tag{26}$$

Composition constraint:

$$\frac{L_{irc}}{L_{ir}^t} = \frac{V_{ivc}}{V_{iv}^t}, i \in T, r \in R, v \in O_r, c \in C \tag{27}$$

Oil residency time constraints for storage tanks: A tank must idle for a certain amount of time to separate and remove brine after it receives crude oil and then transport crude oil to charging tank or CDU, which is called oil residency time and an important real-life operational feature. Many existing works do not account for oil residency time, because considering it can increase the difficulty of scheduling and decrease the space of schedulability. As can be seen in [19] that oil residency time is one of the cause of nonschedulability. It is motivation of us to take oil residency time into account.

$$\textstyle\sum_{v\in I_r}(S_{iv} + D_{iv} + RT \times Z_{iv}) \leq \sum_{v\in O_r} S_{jv} + (H + RT)(1 - \sum_{v\in O_r} Z_{jv}),\ i \in T,\ \ j \in T,\ \ i \leq j,\ \ and\ r \in R_S \tag{28}$$

Oil residency time constraints for charging tanks:

$$\textstyle\sum_{v\in O_r} S_{iv} + H \times (1 - Z_{iv}) \geq \sum_{v\in I_r}(S_{jv} + D_{jv}) + RT \times \sum_{v\in I_r} Z_{jv},\ i \in T,\ \ j \in T,\ \ i > j,\ \ and\ r \in R_C \tag{29}$$

Objective function: In scheduling crude oil operations, there are various objectives and it is a typical multi-objective optimization problem. To transport crude oil from storage tanks to charging tanks via a pipeline, it needs to switch from one type of oil to another, leading to unnecessary oil mixing. Also, when there is a switch from one charging tank to another in feeding a CDU, there is a set point regulation process. As a consequence, both switches result in a cost. Moreover, when any of such switches occurs, a setup is necessary. Such a setup is not only time consuming but also hazardous in the sense of security. Thus, it is crucial to minimize the number of switches for both of them, and this is the objectives for our model. Thus, the objective function is as.

$$J = \sum_{i\in T}\sum_{v\in W_D} Z_{iv} \tag{30}$$

Based on the above discussion, the short-term scheduling problem of crude oil operations with oil residency time constraints can be formulated as the following mathematical programming problem.

Problem P1: Minimize $J = \sum_{i\in T}\sum_{v\in W_D} Z_{iv}$

Subject to: Constraints (1) - (29).

The splitting of crude oil in tanks makes crude oil scheduling problem nonlinear inherently. In Problem P1, Constraint (27) is non-linear (bilinear), which makes the problem very difficult to solve. To make the problem solvable, the problem is relaxed by neglecting Constraint (27) in [38], leading to an infeasible schedule. Without Constraint (27), the solver is permitted to deliver arbitrary types and amount of any crude oil type to tanks or CDUs rather than in the proportions indicated by the composition of a tank. This results in concentration discrepancies.

By Constraint (27), it states that: 1) the crude oil types that are mixed to form a mixture of oil in a tank are same as that delivered from the tank by ODs that are relative to the tank; and 2) the concentration of every component in the tank is same as that delivered into the tank by ODs and is within the permissive interval [$\underline{x_{vk}}$, $\overline{x_{vk}}$]. Let ρ be a positive real number which is large enough. Then, $\rho \times V_{ivc} \leq Z_{iv} - L_{irc}/\overline{L_r^t}$ assures that if $L_{irc} > 0$ and Z_{iv}=1, V_{ivc} >0 holds, i.e. if *r* contains crude oil *c* before the operation *v* is assigned to priority-slot *i*, it must be partially or totally transferred during operation *v* when operation *v* is assigned to priority-slot *i*. Then, if Statement 1) is true, the following constraints must hold.

$$\begin{cases} V_{iv} = \sum V_{ivc} \\ \rho \times V_{ivc} \leq Z_{iv} - L_{irc}/\overline{L_r^t}\ i \in T, r \in R,\ and\ c \in C \\ L_{ir} = \sum L_{irc} \end{cases} \tag{31}$$

Similarly, if Statement 2) is true, the following constraints must hold.

$$\begin{cases} \underline{x_{vk}} \leq \dfrac{\sum V_{ivc} \times x_{ck}}{V_{iv}} \leq \overline{x_{vk}} \\ \underline{x_{vk}} \leq \dfrac{\sum L_{irc} \times x_{ck}}{L_{ir}} \leq \overline{x_{vk}} \end{cases} \quad i \in T, r \in R,\ k \in K,\ v \in O_r,\ and\ c \in C \tag{32}$$

Although Constraints (31) and (32) are not equivalent to Constraint (27), they present the necessary part of Constraint (27). Thus, if Constraint (27) is approximated by Constraints (31) and (32), a good relaxed formulation can be obtained. Notice that Constraints (31) and (32) are all linear, by replacing Constraint (27) with Constraints (31) and (32), the complexity of solving the problem is greatly reduced. Thus, we have the following formulation for the scheduling problem addressed in this paper.

Problem P2: Minimize $J = \sum_{i \in T} \sum_{v \in W_D} Z_{iv}$

Subject to: Constraints (1) - (26), (28)-(29), and (31) and (32).

With this formulation for the scheduling problem developed above, we discuss how to solve the problem next.

4. SOLUTION METHOD

Since Problem P1 is an MINLP problem, it is very difficult to solve. To solve Problem 1 for the system addressed in [38], the authors use a two-stage method. At Stage 1, they first solve an MILP problem obtained from P1 by removing nonlinear Constraint (27) to obtain z_{iv}'s. Then, at Stage 2, P1 is solved by substituting the value of z_{iv}'s into P1. In this way the computational burden is greatly reduced such that the problem can be solved. However, by doing so, an infeasible schedule may be obtained. Notice that, for the problem addressed in this paper, the oil transportation process from storage tanks to charging tanks via a pipeline is included and the oil residency time constraints are imposed. For such a system, the number of binary variables in P1 must be much greater than that in the model for the system discussed in [38]. Hence, P1 obtained in this paper is more difficult to solve. To make the problem solvable and an obtained schedule feasible, similar to the method used in [38], a two-stage method is proposed as follows. At Stage 1, instead of simply ignoring nonlinear Constraint (27), we solve P2 to obtain z_{iv}'s. Then, as done in [38], P1 is solved with z_{iv}'s being fixed by that obtained at stage 1.

To build priority-slot-based models P1 and P2, it is required that the number of priority slots is known in advance. Unfortunately, in practice, this is not the case and it is very difficult to predict an accurate one. Thus, to build such a model, one has to make a guess to set a value on it. If a small value is given, there might be no solution at all, while a large value could make the problem unsolvable due to the large number of binary variables. Therefore, to obtain a priority-slot-based model, it is crucial to properly determine the number of priority slots. Let S_n be the optimal schedule for the problem when the number of priority slots is n. According to the characteristic of our model we present the following theorem.

Theorem 1: If S_n is the optimal schedule obtained for the problem with n being the number of priority slots, an optimal schedule S_k for the problem with $k > n$ is not better than S_n.

Proof is omitted.

Let |T| be the cardinality of T and N = |T|. By Theorem 1, the scheduling problem for crude oil operations can be solved as follows. We can make a guess on N. Then, with N as the number of slots, the problem is solved. If a solution is found, let N ← N-1 and the problem is solved again. This process is repeated until no solution can be found for an N. In this case, an optimal solution is found with N+1 being the number of slots. If a solution is not found for the guessed N, let N ← N+1 and the problem is solved again. We continue this process until a solution is found for an N. The problem is how to guess an appropriate N such that the problem can be solved with less computation.

By the definition of an OD, an operation may be executed more than one time, depending on the demands of CDUs. However, generally an operation should be executed at least once. Thus, although it is very difficult to guess the number of slots, it is reasonable to let the number of operations be the number of slots to start the solution process. Let the guessed number of slots be N = Π, we can solve P2 iteratively as follows.

Algorithm 1: Given N = Π, find the optimal solution of P2

1) N ← Π, Ω ← Π, Θ ← 0, and Γ ← M;
2) Solve P2 with the number of slots being N by using a commercial solver;

2.1) If a solution is found and N > 2 and N ≤ Ω, then Θ ← 1, N ← N-1, and go to 2);

2.2) If a solution is found and N = 1, then Θ ← 1 and go to 5);

2.3) If a solution is not found and Θ = 0, then N ← N + 1 and go to 3);

2.4) If a solution is not found and Θ = 1, then N ← N + 1 and go to 5);

3) Solve P2 with the number of slots being N by using a commercial solver;

3.1) If a solution is not found, then N ← N + 1; if N ≤ Γ, go to 2), otherwise go to 3);

3.2) If a solution is found, then Θ ← 1 and go to 5);

4) No solution can be found, then go to 6);
5) Return N and the solution, then go to 6);
6) Stop.

In solving P2, when the number of slots is set to be large and a solution is not found, we think that a solution cannot be found and the algorithm should stop. In the algorithm, by Step 2), we find a solution with the smallest N (≤ Π) if it exists. If, for N ≤ Π, no solution can be found, by Step 3) we then find a solution with N > Π. When N > M and a solution is not found yet, the algorithm stops without finding a solution. In this way, according to Theorem 1, for P2, an optimal solution with the smallest priority-slot number is found if it exists.

When a solution obtained from P2 by using Algorithm 1, the number of slots N = |T| and z_{iv}'s, $i \in T$, are determined. Let J_{P2} be the value of objective obtained for the optimal solution. Then, by substituting the value of z_{iv}'s obtained into P1, P1 can be solved by a commercial solver just as done in [38] and let J_{P1} be the value of objective for the obtained solution. Notice that P2 is a relaxed problem of P1 and J_{P2} must be a lower bound of P1. Also, with z_{iv}'s being fixed as constant for solving P1, J_{P1} must be an upper bound of P1. Let G = (J_{P1}-J_{P2})/ J_{P1} be gap of them. Then, we can use G to evaluate the performance of the proposed method. Besides, the optimality of the solution got with our heuristic approach closely depends on the tightness of P2.

5. NUMERICAL EXAMPLE

In this section, numerical experiments are carried to show the performance of the proposed method by using an example adapted from [38].

The refinery for the example consists of two charging tanks, two storage tanks, and one CDU. In [38], it is assumed that there is an independent pipeline from any storage tank to any charging tank.

For crude oil to be processed by the CDU, there is particular specification on a certain component, such as sulfur. If this requirement is not satisfied for a type of crude oil, it should be mixed with other types of oil through oil transportation. This is done by transporting the types of oil to be mixed one type by one type into a charging tank. Since oil residency time constraints are a critical requirement, we take them into account though they are neglected in [38]. The data presenting the initial state information are given in Table I.

We use our proposed heuristic method to solve two problems with priority-slot number 10 and objective function $\sum_{i\in T}\sum_{v\in W_D} Z_{iv}$, from above example, one is problem P1' which is problem P1 in our paper neglecting Constraint (31) and Constraint (31), the other is problem P1'', i.e, problem P1 in our paper, to illustrate that P1'' can get much less concentration discrepancies. The gap between P1 and P2 in each problem above is also defined as (J_{P1}- J_{P2})/ J_{P1}. Two situations are assumed in order to compare concentration discrepancies of two problems. Situation A has the same data in Table I; situation B changes the demand of the CDU for Crude mixes X and Y to 500. The system stores superfluous crude oil in the storage tanks as inventory which will be processed in the next scheduling horizon. Situation B keeps all other data constant as those in Table I. The results of two problems are shown in Table II, and Table III, respectively. Figure 2 is the schedule gant chart of situation A. Because the initial conditions are different in two situations, the smallest priority-slot number obtaining optimal solution is different. Situation A and B have the smallest priority-slot number 11, and 9, respectively.

Table 1. Initial state information for the example

	Scheduling horizon		8 days
Vessel	Arrival time	Composition	Amount of crude oil (Mbbl)
Vessel 1	0	100% A	1000
Vessel 2	4	100% B	1000
Storage tank	Capacity (Mbbl)	Initial composition	Initial amount of crude oil (Mbbl)
Tank s1	[0,1000]	100% A	250
Tank s2	[0,1000]	100% B	750
charging tank	Capacity (Mbbl)	Initial composition	Initial amount of crude oil (Mbbl)
Tank c1	[0,1000]	100% C	500
Tank c2	[0,1000]	100% D	500
Crude oil type	Sulfur concentration	Gross margin($/bbl)	
Crude oil A	0.01	9	
Crude oil B	0.06	4	
Crude oil C	0.02	8	
Crude oil D	0.05	5	
Crude mixture	Sulfur concentration	Demand (Mbbl)	
Crude oil mix X	[0.015,0.025]	[1000,1000]	
Crude oil mix Y	[0.045,0.055]	[1000,1000]	
Unloading flow rate	[0,500]	transportation flow rate	[0,500]
Distillation flow rate	[50,500]		

As can be seen from Table II and Table III that because of arbitrary transportation of types and amounts of crude oil to tanks or CDUs by solver, there are two situations where there are extreme concentration discrepancies in result of P1', i.e. the oil types in resource are not identical to ones delivered from it, among three situations. Crude oil types discrepancies happen in V7 of situation B and V7 of situation C, respectively. At the same time, due to adding constraints which are used to restrain the crude oil types that are mixed to form the oil in a tank are identical to ones delivered from the tank by ODs, Table II, Table III show that there is no phenomena happening in result of P1'' where the types of crude oil in a tank are not identical to the ones which are transferred from it among three situations. The significant thing should be noticed is that in two situations all optimality gaps are 0.0%, which is interpreted by the actuality that the composition constraints always hold because of adding Constraint (31) and (32). In other words, in our model we need not solve NLP problem and get optimal solution. So, our model is better than the one in [38].

Table 2. Results of situation A

Situation A	Priority-slot	1	2	3	4	5	6	Gap
	operation	V8	V3	V1	V3	V7	V6	
	resource	R6	R3	R1	R3	R5	R4	
	Oil types in resource	D	A	A	A	C	B	
MILP result of	Oil types delivered from	D	A	A	A	C	B	0%
MILP result of	Oil types delivered from	D	A	A	A	C	B	0%
Situation A	Priority-slot	7	8	9	10	11		Gap
	operation	V1	V4	V8	V5	V2		
	resource	R1	R3	R6	R4	R2		
	Oil types in resource	A	A	AB	B	B		
MILP result of	Oil types delivered from	A	A	AB	B	B	B	0%
MILP result of	Oil types delivered from	A	A	AB	B	B	B	0%

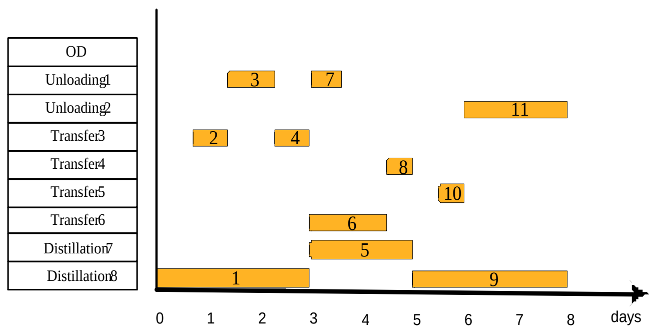

Figure 2. The schedule gant chart of situation A

Table 3. Results of situation B

Situation B	Priority-slot	1	2	3	4	5	Gap
	operation	V8	V5	V7	V5	V3	
	resource	R6	R4	R5	R4	R3	
	Oil types in resource	D	B	BC	B	A	
MILP result of P1’	Oil types delivered from resource	D	B	C	B	A	2.4%
MILP result of P1’’	Oil types delivered from resource	D	B	BC	B	A	0.0%
Situation B	Priority-slot	6	7	8	9		Gap
	operation	V1	V6	V8	V2		
	resource	R1	R4	R6	R2		
	Oil types in resource	A	B	BD	B		
MILP result of P1’	Oil types delivered from resource	A	B	BD	B		2.4%
MILP result of P1’’	Oil types delivered from resource	A	B	BD	B		0.0%

6. CONCLUSION

A new continuous-time scheduling formulation with a residency time constraint is proposed to address crude oil scheduling problems with the objective of minimize the number of switches between transportation ODs and the number of switches between distillation ODs, which is a linear function. We introduce a new MILP relaxation formulation of original problem, which is tighter than the one in [38], makes us to find the solution of original problem easier, as well assures that the crude oil types that are mixed to form the oil in a tank are same as that delivered from the tank by ODs that are relative to the tank.

According to the characteristics of the model, we propose Theorem 1, based on which we come up with a method which can get a satisfactory optimal solution with the smallest priority-slot number.

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