



A Gaussianization-based performance enhancement approach for coded digital PCM/FM

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ABSTRACT

The BER performance of the coded digital PCM/FM telemetry system is dependent on the accuracy of the input likelihood metrics, which are greatly influenced by the click noise. This paper presents a Gaussianization approach to lessen the influence of the click noise. The outputs of the limiter/discriminator are first modeled by a Gaussian mixture model, whose parameters are estimated by the expectation maximization algorithm, and then the amplitudes are adjusted by a proposed Gaussianization filter so that they become more accurate through likelihood metrics. When $(64, 57)^2$ TPC is applied, simulation results show the coding gain is 0.8dB at 10^{-4} BER level.

Keywords: PCM/FM, Limiter/Discriminator, Gaussianization, Turbo Product Codes, LDPC.

1. INTRODUCTION

Pulse coded modulation/frequency modulation (PCM/FM), which has the advantages of being resistant to flame, polarization, multipath fading and phase interference, is a commonly deployed technique in a variety of telemetry areas and other applications. Many forward error-correction (FEC) codes, such as convolution codes, Reed-Solomon (RS) codes, turbo product codes (TPC) and low-density parity check (LDPC) codes [1-4], are employed to enhance the bit error rate (BER) performance of the digital PCM/FM telemetry system. Because of its simplicity, limiter/discriminator (L/D) is often used for the demodulation of a digital FM system. However, it is well known that the noise in the demodulated signal at the output of the L/D becomes impulsive when the carrier-to-noise power ratio decreases below about 10dB, and even the channel is additive Gaussian white noise (AWGN) channel. The most famous description of this kind of noise was proposed in 1963 by S. O. Rice [5] who regarded the noise as the sum of two related components: approximate Gaussian noise and a kind of impulsive noise; namely the so-called click noise. The performance of all soft-input and soft-output (SISO) decoding algorithms, such as the Viterbi decoder, the Chase decoder and the belief propagation decoder, depend on the accuracy of the soft-input likelihood metrics. The accurate likelihood metrics are easily obtained when the amplitude distribution of the noise in the soft-input signal is known, such as Gaussian noise. However, if the amplitude distribution of the noise is unknown or difficult to determine, the accurate likelihood metrics are difficult to obtain. In a practical coded PCM/FM system, the decoders of

the FEC codes adopt Gaussian assumption for the sake of simplicity. However, this is not optimal for the click noise background and decreases the SISO decoding performance. Many efforts which are reported in [6] have been done to improve the SISO decoding performance of the coded digital FM telemetry system with L/D. Most of them are based on Rice's click model and intend to detect and eliminate the click noise. However, it is very difficult to eliminate the click noise completely because of its randomness. Even if the click noise is eliminated completely, the performance of the Gaussian assumption SISO decoder is still not good because the approximate Gaussian noise component in Rice's model is essentially non-Gaussian. In this paper, the noise in the signal at the output of the L/D is not regarded as the sum of the approximate Gaussian noise and the click noise, as described in Rice's model, but is regarded as a kind of non-Gaussian noise as a whole. A Gaussianization approach is proposed to convert the probability distribution of the non-Gaussian noise to be closer to Gaussian, so that the likelihood metrics obtained by the SISO decoders in the digital PCM/FM system with L/D become more accurate and the BER performance of the system is improved.

The remainder of this letter is organized as follows. Section II gives a brief review on the coded digital PCM/FM telemetry systems with L/D. Section III introduces the Gaussian mixture density (GMD) model and the expectation-maximization (EM) algorithm as well as the Gaussianizing filter which are used in the Gaussianization scheme. Section IV presents the proposed Gaussianization approach in the coded digital PCM/FM telemetry systems with L/D. Section

V gives the simulation results of the proposed approach, when the TPC codes and LDPC are employed as FEC codes. Finally, conclusions are drawn in section VI.

2. REVIEW OF CODED DIGITAL PCM/FM SYSTEM

In this section, the coded digital PCM/FM telemetry system with L/D is reviewed. The model of the considered system is shown in Figure 1.

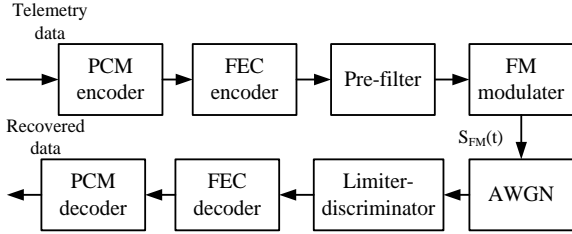


Figure 1. Model of PCM/FM telemetry system

In the coded digital PCM/FM telemetry system, the telemetry data is first encoded by a PCM encoder and then is encoded by a FEC encoder. The FEC encoder can be an encoder of RS codes, TPC codes or LDPC codes. The pre-filter in this system is used to eliminate the inter-symbol interference and improve the band efficiency. L/D is adopted to be the demodulator, which is followed by a FEC SISO decoder. The modulated FM signal $S_{FM}(t)$ is as Equation (1):

$$S_{FM}(t) = A \cos \left[\omega_c t + K_{FM} \int f(t) dt \right] \quad (1)$$

where $f(t)$ is the modulate signal, which is the output of the pre-filter, ω_c is the carrier frequency, A is the carrier amplitude, and K_{FM} is the frequency deviation constant which is decided by the hardware circuit. The structure of the L/D is as depicted in Figure 2:

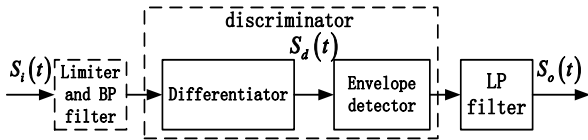


Figure 2. Discriminator

The L/D is composed of a differentiator and an envelope detector. The input signal of the differentiator is $S_{FM}(t)$ in equation [1], and the output of the differentiator $S_d(t)$ is as Equation (2):

$$S_d(t) = -A \left[\omega_c t + K_{FM} \int f(t) dt \right] \sin \left[\omega_c t + K_{FM} \int f(t) dt \right] \quad (2)$$

From equation [2] it can be determined that $S_d(t)$ is the differentiation of $S_{FM}(t)$. The envelope of $S_d(t)$ is proportional to $f(t)$. The output signal of the envelope detector excluding the direct current is as Equation (3):

$$S_o(t) = K_d K_{FM} f(t) \quad (3)$$

where K_d is a constant decided by the L/D circuit. From Equation [3] it is known that $f(t)$ can be recovered.

3. GAUSSIANIZATION APPROACH

The Gaussianization approach is an important technique in many signal processing areas and detection areas, such as the non-Gaussian autoregressive process [7] and speech processing. In these areas, the noise background is usually assumed to be Gaussian for the sake of simplicity. However, a wide variety of signal probability distributions are non-Gaussian. The mismatch between the assumption and the actual distribution results in poor performance of the match filter, correlation test, maximum likelihood decoding, etc. However, utilizing the statistics characteristic of the non-Gaussian background, the probability distribution of them can be converted to be more ‘‘Gaussian-like’’. In other words, the amplitudes of the background noise can be adjusted so that the probability distribution becomes more similar to Gaussian distribution, and thus can improve performance.

The procedure of a typical Gaussianization approach is as follows. First, the probability distribution function of the non-Gaussian background should be fitted by a non-Gaussian probability model [7]. Then, the parameters of the non-Gaussian probability model should be determined by the parameter estimation algorithms [8]. Finally, according to the estimated parameters, the amplitudes of the non-Gaussian background are adjusted by a Gaussianization processing module so that the probability distribution is made more Gaussian.

There are many non-Gaussian probability models, such as the Gaussian mixture density (GMD) model, the class-A model, the K-distribution model, and so on [7]. The GMD model adopted in this paper, which is an effective model for fitting varieties of non-Gaussian probability distributions, is as Equation (4):

$$f(s) = \sum_{i=1}^M \varepsilon_i f_i(s | \mu_i, \sigma_i), \quad \sum_{i=1}^M \varepsilon_i = 1 \quad (4)$$

where s is the signal with non-Gaussian background noise, $f(s)$ is the GMD model of s , $f_i(s)$ are the Gaussian probability distribution functions with different mean μ_i and variance σ_i , M is the order of the GMD model, namely the number of $f_i(s)$, and ε_i is the mixture parameter which denotes the relative weighting of each $f_i(s)$. M corresponds to the statistics characteristic of s . The larger the M is, the more accurate the GMD model is. However, a large M means high computational cost. Therefore, the value of M is usually a trade-off between the computational cost and the accuracy in practice. In this paper, M is set to be 2, which is the simplest case. Therefore, the GMD model is a 2-order one, as Equation (5):

$$f(s) = \varepsilon f_B(s | \mu_B, \sigma_B) + (1 - \varepsilon) f_I(s | \mu_I, \sigma_I) \quad (5)$$

where $f_B(s)$ is a Gaussian probability distribution function with mean μ_B and variance σ_B , and $f_I(s)$ is a Gaussian probability distribution function with mean μ_I and variance σ_I . Compared with Rice’s click model, $f_B(s)$ describes the random property of the non-Gaussian background noise, while $f_I(s)$ describes the impulsive property of the non-Gaussian background noise.

Obviously, the parameter group $g = [\mu, \sigma, \varepsilon]$ of the GDM model should be decided by the statistics characteristic of s . There are many estimating approaches that can be used to obtain the parameter group g , such as the expectation

maximization (EM) algorithm [8], the penalized maximum likelihood estimation algorithm, the indirect least squares estimation algorithm for cumulant generating function, and so on [9]. As a widely used and highly efficient algorithm, the EM algorithm is adopted in this paper to be the parameter estimation algorithm. The EM algorithm is an iterative algorithm, which requires initial values of parameter group g . The initial value of g can be set by experience or according to the statistic characteristic of s .

The Gaussianization processing module in this paper is the so-called Gaussianizing filter, which was proposed in reference [9]. The function of a Gaussianizing filter is to adjust the amplitudes of the input signal according to the estimated parameters obtained by the estimation algorithm by strengthening the smaller and weakening the larger, so that the amplitudes distribution of the output signal becomes closer to Gaussian. Two Gaussianizing filters, U-filter and G-filter, are mentioned in reference [9]. The Gaussianizing filter proposed in this paper is a revised version of the G-filter, which is discussed in detail in next section. It should be noted that after the Gaussianization approach, the information in the original signal should be kept, else the performance is still poor. In this paper, the similarity between the signals before and after the Gaussianization approach is evaluated by the traditional concept of correlation coefficient, defined as Equation (6):

$$r_{xy} = \frac{\sum_{i=1}^N (x_i - x_m)(y_i - y_m)}{\sqrt{\sum_{i=1}^N (x_i - x_m)^2} \sqrt{\sum_{i=1}^N (y_i - y_m)^2}} \quad (6)$$

where x and y denote the signals before and after the Gaussianization approach respectively, and x_m and y_m are the mean values of them respectively. The value of r_{xy} is within the range of $[-1, +1]$. The larger the r_{xy} is, the more similar to

each other the two signals are. When r_{xy} is 1, the two signals are exactly the same. When r_{xy} is 0, the two signals are independent from each other. From this perspective, if the r_{xy} is high, it can be considered that most information is kept after the Gaussianization approach.

4. THE PROPOSED GAUSSIANIZATION SCHEME

The traditional Gaussianization approaches mentioned in the previous section is also far applied in signal processing areas and detection areas. To the best of our knowledge, the idea of the Gaussianization approach has not been applied in the SISO decoding algorithm. In this paper, a Gaussianization approach is proposed to improve the BER performance of the SISO decoder in the coded digital PCM/FM telemetry system with L/D. The idea is based on the following fact. Because of the discriminator in the L/D, the noise in the demodulated signal, which is the output of the L/D, is non-Gaussian, even if the noise in the channel is additive white Gaussian noise. However, the likelihood metrics in the traditional SISO decoding algorithm, such as the Chase decoding algorithm, the belief propagation decoding algorithm, etc., all adopt the Gaussian background assumption. The mismatch between the actual distribution of likelihood metrics and the Gaussian assumption causes the inaccurate likelihood metrics, which results in a poor performance in decoding BER. For the sake of clarity, in the following the demodulated signals refer to the output signals of the L/D which are the sum of the useful signal and the non-Gaussian noise. Since it can adjust the amplitudes of the demodulated signals, the Gaussianization approach is adopted so that the distribution of the demodulated signals becomes closer to Gaussian, resulting in more accurate likelihood metrics and better performance in SISO decoding. Figure 3 shows the proposed Gaussianization module in the receiver of the coded digital PCM/FM telemetry system.

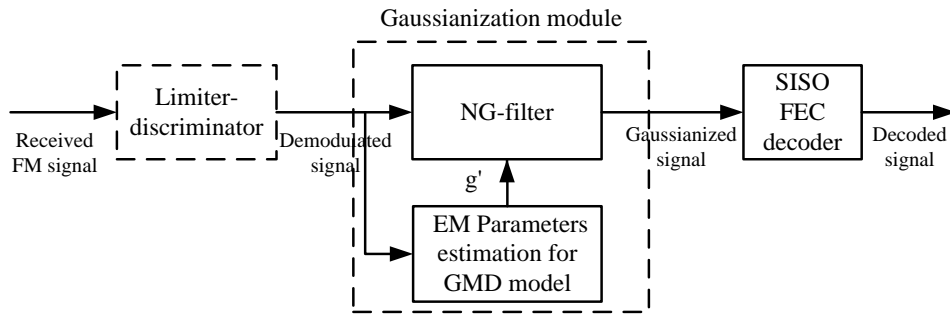


Figure 3. Gaussianization module in the coded digital PCM/FM receiver

In the proposed Gaussianization module, the probability distribution of the demodulated signals is approximated by the 2-order GDM model in reference [5]. The EM algorithm is adopted to estimate the parameter group $g = [\mu, \sigma, \varepsilon]$ of the 2-order GDM model. The initial setting of g is crucial to the final results of the EM algorithm. The algorithm will converge to the local maximum value if the initial setting is inappropriate. In experiments, the following initial setting is appropriate: the initial mean μ_1 and μ_2 are set to be the mean of the input signals of the Gaussianization module and the amplitude of the baseband data; the initial variance σ_1 and σ_2 are set to be the variance of the input signals of the

Gaussianization module and I ; the initial mixture parameter ε is initialized by 0.5 and 0.5. After a fixed number of iterations (50) or being terminated by the stopping criteria, the EM algorithm gives the estimated parameter group g . The Gaussianizing filter proposed in this paper is the normalized G-filter (NG-filter), as Equation (7):

$$f_{NG}(s | g') = \frac{\Phi^{-1} \left\{ \sum_{i=1}^M \varepsilon'_i \Phi \left(\frac{s - \mu'_i}{\sigma'_i} \right) \right\}}{\Phi^{-1} \left\{ \sum_{i=1}^M \varepsilon'_i \Phi \left(\frac{m - \mu'_i}{\sigma'_i} \right) \right\}} \quad (7)$$

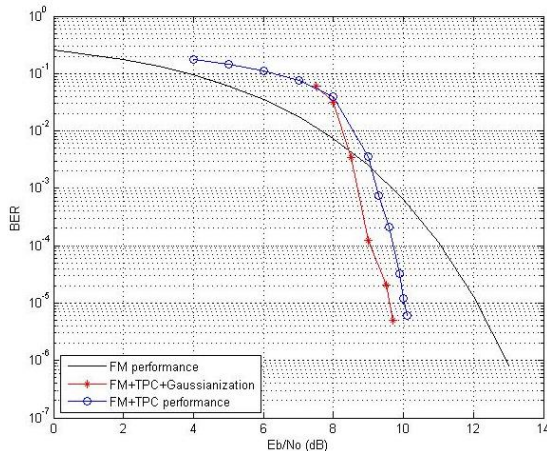
where $f_{NG}(s|g')$ are the outputs of the NG-filter, $\Phi(x)$ is the standard Gaussian cumulative distribution function, $\Phi^{-1}(x)$ is its inverse function, $g'=[\mu', \sigma', \varepsilon']$ are the estimated 2-order GDM model parameters obtained by the EM algorithm, s are the input signals of the Gaussianization module, and m is the amplitude of the baseband data. A normalization term is added as the denominator compared with the G-filter in reference [9]. Because of the normalization term, the outputs of the NG-filter become the representations of the relative magnitude to the amplitude of the baseband data. Therefore, these outputs can be fed into the SISO decoder as more accurate likelihood metrics.

Almost all the computational cost of the proposed Gaussianization approach concentrates on the EM algorithm. The larger the number of iterations, the larger the computational cost is. In fact, not a very large number of iterations, such as 50 iterations as adopted in this paper, can produce exact estimations. In a practical telemetry system, the Gaussianization module can be realized by hardware or software. It can be an optional module for the performance enhancement in the telemetry system, which is in the place between the L/D and the SISO decoder.

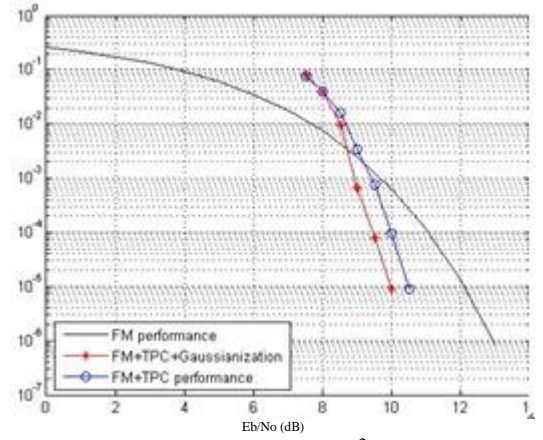
5. SIMULATION RESULTS

As mentioned in section III, the correlation coefficient is used to characterize the similarity of the signals before and after the Gaussianization module. In simulation, the average value of the correlation coefficient is 0.9879, indicating that almost all information is kept while the amplitudes have been adjusted.

Simulation results are provided in this section to show the improved performance in decoding BER of the proposed Gaussianization approach. The FEC codes adopted in this paper are TPC codes and LDPC codes. The adopted TPC codes are TPC $(64, 57)^2$ and TPC $(32, 26)^2$, which has extended Hamming codes as their component codes [10]. Both kinds of TPC codes have been chosen as FEC codes in the PCM/FM telemetry system [3]. The adopted LDPC code is $(8160, 7136)$, which is the suggested FEC code by the Consultative Committee for Space Data Systems (CCSDS) [11]. The simulation system is built by MATLAB. The carrier frequency of FM is 80 MHz, and the baseband data rate is 10 Mbps. The maximum frequency deviation coefficient is 0.35. The BER performance comparison in the TPC coded PCM/FM system with L/D, as well as the performance of the digital PCM/FM with L/D is presented in Figure 4.



(a) TPC $(64, 57)^2$



(b) TPC $(32, 26)^2$

Figure 4. Comparison of BER performance with and without Gaussianization approach in TPC coded PCM/FM with limiter-discriminator

The SISO decoding algorithm of both TPC codes in Figure 4 is the Chase II algorithm with 8 iterations [10]. For the BER of 10^{-4} level, the needed E_b/N_0 of the TPC without the Gaussianization approach are about 9.8dB (TPC $(64, 57)^2$) and 10dB (TPC $(32, 26)^2$), while that of the TPC with the Gaussianization approach is 9dB (TPC $(64, 57)^2$) and 9.5dB (TPC $(32, 26)^2$), which yields 0.8dB and 0.5 dB coding gain respectively.

The BER performance comparison of the LDPC coded PCM/FM system with and without the Gaussianization approach is presented in Figure 5. The SISO decoding algorithm of the LDPC code is the minimum-sum algorithm, with α being 1.25 and β being 0. The number of iterations is 50, and the quantization mode is 1-3-4 [11]. For the BER of 10^{-4} level, the needed E_b/N_0 of the LDPC without the Gaussianization approach is about 9.6dB, while that of the LDPC with the Gaussianization approach is 9.2dB, which yields 0.4dB coding gain.

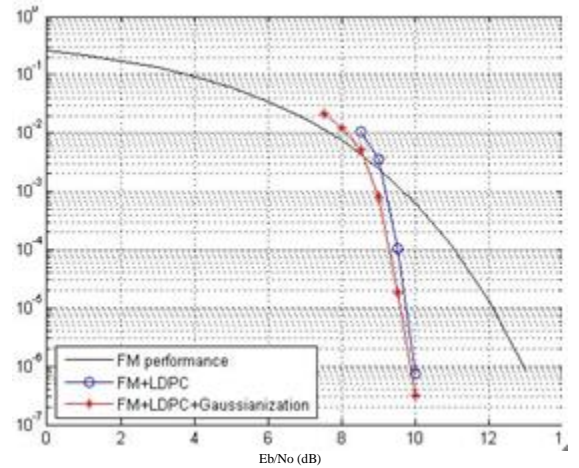


Figure 5. Comparison of BER performance with and without Gaussianization approach in LDPC coded PCM/FM with limiter-discriminator

6. CONCLUSIONS

In this work, a novel Gaussianization approach is proposed to improve the BER performance of the SISO decoder in the

digital PCM/FM telemetry system with L/D. The simulation results show that a coding gain of about 0.8dB at 10⁻⁴ BER level has been achieved when the employed FEC code is TPC. The proposed approach can easily be applied to a digital PCM/FM telemetry system with other kinds of FEC codes that employ SISO decoding algorithms, such as convolution codes and Turbo codes, which is the subject of future study.

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