An optimal Phasor Measurement Unit placement techniques for achieving complete perceptibility of a network even when PMU failure

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ABSTRACT. *For finding the power system state estimation, first we should know the observability of the system. In order to observe the system, we have to monitor the entire system for each and every second. For monitoring and control of an entire power system, number of techniques is available. Out of all phasor Measurement Unit (PMU) is one of most important technique for monitoring of system. Placement of PMU's is the major trouble because they are little expensive. This problem can be overcome by finding minimum no of PMU's and their optimal locations of PMU's using network connectivity information. In this paper, we found optimal locations of PMU's using two different techniques, they are Binary Integer Programming (BIP) and three stage algorithm. These methods are solved by considering a single PMU outage case along with Zero Injection buses (ZIB's). The results are shown for different IEEE test systems like 14 bus,24 bus, 30 bus,39 bus,57 bus and 118 bus. The results of both techniques are shown along with their computational time.*

RÉSUMÉ. *Pour trouver l'estimation d'état du système électrique, nous devons d'abord connaître l'observabilité du système. Afin d'observer le système, nous devons surveiller le système entier à chaque seconde. Pour surveiller et contrôler le système électrique entier, plusieurs techniques sont disponibles, parmi lesquelles l'Unité de Mesure de Phaseur (PMU) est une des plus importantes. Vu qu'il coûte cher, le placement de PMU est un problème majeur. Nous pourrons résoudre ce problème par atteindre le minimum de PMU et trouver la localisation optimale de PMU à l'aide des informations de connectivité de réseau. Dans cet article, nous avons trouvé la localisation optimale de PMU en adoptant deux techniques différentes. Elles sont la Programmation Binaire en Nombres Entiers (BIP) et l'algorithme à trois étapes. Imaginer un seul cas de panne de PMU avec les bus de Zero Injection (ZIB), est la source d'inspiration de ces deux méthodes. Les résultats sont présentés par les systèmes différents de test d'IEEE tels que les bus 14, 24, 30, 39, 57 et 118. Les résultats des deux techniques sont présentés avec leur temps de calcul.*

KEYWORDS: *state estimation, observability, optimization, phasor measurement unit (PMU), binary integer programming (BIP), PMU outage.*

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MOTS-CLÉS: estimation d'État, observabilité, optimisation, unité de mesure de Phaseur (PMU), programmation binaire en nombres entiers (BIP), panne de PMU.

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1. Introduction

As compared to past, power demand is increased now a day's, crowded the existing network and as a result stability boundaries are decayed. To ensure appropriate and assured operation of power network, an accurate measurement set and network monitoring is essential. By using this measurement set we are finding state estimation of system (Yoon, 2005; Phadke *et al.*, 1986; Jamuna and Swaup, 2011; Xu and Abur, 2005). In order to estimate the state of system we have to observe the network. For checking this observability, we are having mainly two methods (Sodhi *et al.*, 2010).

(1) Numerical observability: In this we are finding rank of matrix by using measurement set, which involves number of combinations. Increases the computational burden of analysis explained in (Koutsoikis *et al.*, 2013).

(2) Topological observability: Here we are considering topological graphs i.e. based on graph theory explained in (Nuqui and Phadke, 2005).

For finding this measurement set we are using SCADA which gives unsynchronized measurements. To get exact measurements we are using Wide Area Monitoring System's (WAMS). This WAMS consists of PMU's which gives synchronized voltages, currents and phase angles through the timing signals received from GPS.One of the advantage of PMU placement is it gives direct measurements of all system states without running any state estimator. As the PMU gives the synchronized phasors it will provide the system conditions like stability margins, maximum loading conditions, disturbance identification and dynamic response analysis (Phadke, 1993).

The techniques BIP and three stage algorithm finds the minimum number of PMU placement buses for complete of system. In three stage algorithm, the first two stages determine the imperative PMU placement bus locations. The third stage audits the more reduction of PMU's from the final set. Binary Integer programming method is also solved using MATLAB. Two techniques are solved with and without ZIB's and also considering the single PMU outage case. The several test systems like IEEE-14 bus, IEEE-24 bus, IEEE-30 bus, IEEE-39 bus, and IEEE-57 bus are tested under normal operating conditions considering ZIB's. The computational time for two techniques under test systems IEEE-14 bus, IEEE-24 bus, IEEE-30 bus is compared. The intended method is examined only PMU measurements for complete perceptibility of the system. Optimal numbers of PMU's obtained are equal but PMU placed bus locations may vary from one technique to another.

The main issue with PMU's is placement. To find the minimal number of PMU's needed for entire network observability, we are having number of Optimization methods. Some of the important methods are simulated annealing based graph theory approach, the practical development of PMU and its usage is explained by Phadke *et al.,* (1986). Binary integer programming based method for optimal PMU placement (Nuqui and Phadke, 2005; Gou, 2008; Gou, 2008). For getting global optimal results for entire power system observability is explained in (Dua *et al.*, 2008). A three-stage optimal PMU placement technique is analysed (Chakrabarthi and Kyriakides, 2008). The optimal PMU placement by exhaustive search method (Saha *et al.,* 2012) is explained. A two stage PMU placement technique for finding the optimality is discussed (Azizi *et al.,* 2011).

2. PMU placement problem

For ascertaining system observability PMU's are placed in two different ways:

(1) Allocating the PMU's at all system buses and eliminate one by one by checking the observability of network.

(2) Allocating the PMU's at preferred system buses by acquiring the observability of network.

Method (1) is more expensive due to the placement of PMU's in entire network. Method (2) is preferable, but the major problem is finding the peerless set of PMUs' for obtaining complete Observability.

An optimal PMU placement problem is formed for a considering an M-bus system is as follows

Min $\sum_{i=1}^{M} C_i n_i$ Subject to G(N)>A

Where N is the binary selection variable for placement of PMU,

Whose n_i= 1 considering the PMU at ith bus

= 0 no PMU for i=1, 2, 3 …M

A is a vector of unit length M i.e.

A= [1, 1, 1…M]

C_i Is PMU cost installed at ith bus

G (N) is the perceptibility constraint function

= 1 if buses are observable by considering the measurement set

= 0 not observable

The constrained vector function (G(N)) is formed using the bus incidence matrix (B) of network. The elements of B are defined as

$b_{p,q}$ = 1 if p, q are connected each other and are also equal

= 0 p, q are not connected

The constraint function for any (ith) bus in the considered test system is

$G(N) = BN \geq a$

$$f_1 = a_{i,1}n_1 + \cdots + a_{i,i}n_i + \cdots + a_{i,M}n_M$$

ZIB: ZIB's are nothing but the buses which don't any generation and load i.e. no electrical power or electrical current is supplied to the system. This can be used as pseudo measurements to get the system perceptible with minimum number of PMU's.

3. Recommended methods to place PMU's

3.1. BIP

In integer linear programming problem all of the variables or a set of variables are to be restricted to either '1' or '0'. So, this method is considered to be a mathematical optimization problem.

ILP uses following steps to solve the problem

- ✓ From the available bus data form the connection matrix and find the problem which is maximizing or minimizing function.
- ✓ Now form the corresponding linear constraints.
- ✓ Solving the constraints with relating to function, we are having the optimal set of PMUs'.

In mathematical optimization techniques, Binary Integer Programming (BIP) is one which solves the linear objective function by taking linear constraints corresponding to bus impedance matrix. By using the steps described above formation of problem is as follows:

$$\min_{n} G^T n \text{ Such that } \begin{cases} B.n \leq a, \\ Beq.n = aeq, \\ n\ binary. \end{cases}$$

Now the PMU placement problem in the form of linear integer programming as follows

$$\text{Min } \sum_{k=1}^{M} n_k \quad \text{Subject to} \quad G_{pmu}N \geq A_{pmu}$$

Where $N = [N_1, N_2, \ldots \ldots . N_m]^T$ and $A_{PMU} = [1,1,1 \ldots 1]_{Mx1}^T$

In above equation we can see function is minimization and constraints are maximization. So we have to convert the maximization to minimization constraints by using ILP solver in MATLAB.

Binary connectivity matrix (B) and constraints corresponding to bus connectivity matrix are defined for 14 bus system as follows

$$G_{pmu} = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\mathrm{Gpmu}\,.\,\mathrm{N} = \begin{cases} g_1 = n_1 + n_2 + n_5 \geq 1 \\ g_2 = n_1 + n_2 + n_3 + n_4 + n_5 \geq 1 \\ g_{3=} n_2 + n_3 + n_4 \geq 1 \\ g_4 = n_2 + n_3 + n_4 + n_5 + n_7 + n_9 \geq 1 \\ g_5 = n_1 + n_2 + n_4 + n_5 + n_6 \geq 1 \\ g_6 = n_5 + n_6 + n_{11} + n_{12} + n_{13} \geq 1 \\ g_7 = n_4 + n_7 + n_8 + n_9 \geq 1 \\ g_8 = n_7 + n_8 \geq 1 \\ g_9 = n_4 + n_7 + n_9 + n_{10} + n_{14} \geq 1 \\ g_{10} = n_9 + n_{10} + n_{11} \geq 1 \\ g_{11} = n_6 + n_{10} + n_{11} \geq 1 \\ g_{12} = n_6 + n_{12} + n_{13} \geq 1 \\ g_{13} = n_6 + n_{12} + n_{13} + n_{14} \geq 1 \\ g_{14} = n_9 + n_{13} + n_{14} \geq 1 \end{cases}$$

By solving these constraints, we are having the optimal set for placing the PMU's. Due to communication collapse or line interruption, PMU may not be able to give the measurements for total perceptibility of system. In that case, we are moving onto a PMU outage problem. The results are shown for different standard IEEE bus systems.

3.1.1. For single PMU outage case

If we consider the single loss case (20), (21) the test system modified as

Min $\sum_{k=1}^{M} n_k$ Subject to $G_{pmu}N \geq A_{pmu}$

Where $N = [N_1, N_2, \ldots\ldots . N_m]^T$ and $A_{PMU} = [2{,}2{,}2 \ldots . . 2]^T_{Mx1}$

3.2. Three stage algorithm

This technique is an iterative procedure. A PMU placement problem in three stage technique is same as the BIP problem i.e.

Min $\sum_{i=1}^{M} C_i n_i$

Subject to G (N)>a

Where N is the decision variable for PMU placement having either 1 or 0,

Whose n_i= 1 considering the PMU at i^{th} bus

0 no PMU for i=1, 2, 3 …M

The Three stages are explained below with considering Zero Injection Buses (ZIB).

Stage (1): It founds the important buses where PMU's are confined among distinct valency buses.

Stage (2): Some of the buses are unobservable due to not having any connection with already placed PMU's or with ZIB's. These unobservable buses are tested here.

Stage (3): This is an eliminating stage. This checks feasible ways for further minimization of PMU's obtained from stages (1), (2).

If we can solve these three steps, a peerless group of PMU's is obtained for complete perceptibility of system.

3.2.1. for single PMU outage case

After getting a peerless group of PMU's, the algorithm for one PMU outage case is modified as below.

(1) Locale PMU at minimum valency buses along with optimal set obtained from above case.

(2) Finding the buses which are perceptible through one PMU with already placed PMU's.

(3) Find highest valency from the buses which are perceptible through one PMU ignoring the placed PMU buses.

(4) Choose any bus from the list of maximum valency buses and remove doubly observable buses.

(5) In order to eliminate the surplus PMU's other than the doubly perceptible, we are performing pruning and final optimal PMU's are obtained.

4. Simulation results

If we can see the simulation results, for IEEE-14bus system the optimal set is 2,6,7,9 without ZIB's and with ZIB's finalized optimal set is 2, 6, and 9. Due to PMU outage the optimal set is 1, 2, 4, 6, 9, 10, and 13.The optimal results for IEEE bus systems are shown in Table 2 without ZIB's under normal operating conditions. The results with ZIB's are also shown in table 3 and also a single PMU failure or outage is shown in table 4. The computational time for IEEE bus systems are also shown in table 1.

The comparison of computational time of different test systems is shown. BIP without ZIB's will take more time to compare three stage algorithms with ZIB's. BIP take 1.16 sec while three stage algorithms take 0.66 sec for IEEE-14 bus system.

Table 1. Computational time of two techniques for obtaining optimal solution

Test System IEEE	BIP	Three Stage Algorithm
14 bus	1.16s	0.66s
24 bus	1.34s	0.76s
30 bus	1.24s	0.83s

Table 2. Optimal no of PMU's and their positions under normal operating condition

IEEE Test system	No of PMU's	BIP	Three Stage Algorithm
		Without ZIB's Optimal PMU Locations	
14 bus	4	2, 6, 7, 9	2, 6, 7, 9
24 bus	7	2, 3, 8, 10, 16, 21, 23	2, 3, 8, 10, 16, 21, 23
30 bus	10	1, 7, 9, 10, 12, 18, 24, 25, 27, 28	2, 3, 6, 9, 10, 12, 15, 19, 25, 27
39 bus	13	2, 6, 9, 10, 12, 14, 17, 19, 20, 22, 23, 25, 29	2, 6, 9, 10, 12, 14, 17, 19, 20, 22, 23, 25, 29
57 bus	17	1, 4, 9, 20, 24, 27, 29, 30, 32, 36, 38, 41, 45, 51, 54	1, 4, 9, 20, 24, 27, 29, 30, 32, 36, 38, 41, 45, 51, 54
118 bus	32	1, 5, 9, 12, 13, 17, 21, 23, 26, 28, 34, 37, 41, 45, 49, 53, 56, 62, 63, 68, 71, 75, 77, 80, 85, 86, 90, 94, 101, 105, 110, 114	1, 5, 9, 12, 13, 17, 21, 23, 26, 28, 34, 37, 41, 45, 49,53, 56, 62, 63, 68, 71, 75, 77, 80, 85, 86, 90, 94,101, 105, 110, 114

Table 3. Optimal no of PMU's and their positions under normal operating condition considering ZIB's

IEEE Test System	No of PMU's	BIP	Three Stage Algorithm
		With ZIB's Optimal PMU Locations	

14 bus	3	2, 6, 9	2, 6, 9
24 bus	6	2, 8, 10, 15, 18, 20	1, 2, 8, 16, 21, 23
30 bus	7	2, 4, 10, 12, 15, 18, 27	2, 3, 10, 12, 18, 24, 30
39 bus	8	3, 8, 12, 16, 20, 23, 25, 29	3, 8, 12, 16, 20, 23, 25, 29
57 bus	11	1, 6, 9, 19, 25, 27, 32, 37, 38, 46, 51, 53, 56	1, 6, 13, 19, 25, 29, 32, 38, 51, 54, 56
118 bus	28	3, 8, 12, 15, 17, 21, 27, 31, 32, 34, 40, 45, 49, 53, 56, 62, 65, 70, 75, 77, 80, 85, 87, 90, 94, 102 105, 110	1, 6, 8, 12, 15, 17, 21, 25, 29, 34, 40, 45, 49, 53, 56, 62, 72, 75, 77, 80, 85, 86, 90, 94, 101, 105, 110, 114

Table 4. Optimal no of PMU's and their positions under normal operating condition considering ZIB's with single PMU outage

IEEE Test System	No of PMU's	BIP	Three Stage Algorithm
		Optimal PMU Locations FOR Single PMU Outage With ZIB's	
14 bus	7	1, 2, 4, 6, 9, 10, 13	1, 2, 4, 6, 9, 10, 13
24bus	13	1, 2, 7, 8, 9, 10, 11, 15, 16, 17, 20, 21, 23	1, 2, 7, 8, 9, 10, 11, 15, 16, 17, 20, 21, 23
30bus	15	1, 2, 3, 5, 6, 10, 12, 13, 15, 16, 18, 19, 24, 27, 30	1, 2, 3, 5, 6, 10, 12, 13, 15, 16, 18, 19, 24, 27, 30
39 bus	18	2,3,5,6,8,13,16,17,20,22,23,25,26,29,34, 36,37,38	2,3,5,6,8,13,16,17,20,22,23,25,26,29,34, 36,37,38
57bus	26	1, 2, 5, 6, 8, 9, 11, 12, 15, 17, 19, 22, 24, 25, 27, 29, 30, 32, 33, 36, 38, 41, 47, 50, 51, 53, 54, 56	1, 2, 5, 6, 8, 9, 11, 12, 15, 17, 19, 22, 24, 25, 27, 29, 30, 32, 33, 36, 38, 41, 47, 50, 51, 53, 54, 56
118 bus	64	1, 2, 5, 6, 8, 9, 11, 12, 15, 17, 19, 20, 21, 23, 25, 27, 28, 29, 32, 34, 35, 37, 40, 41, 43, 45, 46, 49, 50, 51, 52, 53, 56, 59, 62, 66, 68, 70, 71, 72, 75, 76, 77, 78, 80, 83, 85, 86, 87, 89, 90, 92, 94, 96, 100, 101, 105, 106, 108, 110, 111, 112, 114, 117	1, 2, 5, 6, 8, 9, 11, 12, 15, 17, 19, 20, 21, 23, 25, 27, 28, 29, 32, 34, 35, 37, 40, 41, 43, 45, 46, 49, 50, 51, 52, 53, 56, 59, 62, 66, 68, 70, 71, 72, 75, 76, 77, 78, 80, 83, 85, 86, 87, 89, 90, 92, 94, 96, 100, 101, 105, 106, 108, 110, 111, 112, 114, 117

5. Conclusion

Two optimized techniques are discussed in this paper which gives complete observability of system. BIP is simple technique but we cannot achieve minimization and maximization at the same time. The three step method is quick and smooth to implement by using network connectivity information. The results for IEEE test systems sight the optimal set of PMUs' for complete perceptibility of system. The intended method is examined only PMU measurements for complete perceptibility of the system. Optimal numbers of PMU's obtained are equal but PMU placed bus locations may vary from one technique to another. Further work we have to mainly focus on an optimal PMU placement for incomplete perceptibility of system.

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