Coordination of vendor managed inventory supply chain with price-sensitive demand under consumer balking behaviour

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ABSTRACT. *The vendor managed inventory (VMI) problem has not been widely studied under consumer balking behaviour (CBB), a ubiquitous phenomenon in economic life. To make up for this gap, the purpose of this paper innovatively introduces the CBB to the VMI supply chain whose demand depends on the retail price, explores the effects of the CBB on the coordination of the VMI supply chain with price-sensitive demand, and discusses the impacts on the CBB on the optimal decision and expected revenue of the supply chain. The research results show that it is impossible to coordinate the VMI supply chain with price-sensitive demand under the CBB through the wholesale price contract; however, the wholesale price contract can perfectly coordinate this supply chain after allocating the unmarketable cost to the retailer. Next, it is proved that the optimal inventory factor, the optimal retail price and the expected revenue of the VMI supply chain all decreases with the growth in the balking threshold under certain conditions. Finally, the optimal supply volume of the system was proved to be positively correlated with the balking threshold through numerical experiments. The conclusions of this paper enrich the theory of the supply chain coordination under CBB and facilitate its application in real life.*

RÉSUMÉ. *Le problème de l'inventaire géré par le fournisseur (VMI) n'a pas été largement étudié dans le cadre du comportement de balking du consommateur (CBB), un phénomène omniprésent dans la vie économique. Pour combler cet écart, le présent article introduit de manière innovante le CBB dans la chaîne d'approvisionnement de VMI dont la demande dépend du prix de vente au détail, explore les effets du CBB sur la coordination de la chaîne d'approvisionnement de VMI avec une demande sensible au prix, et discute des impacts sur la CBB sur la décision optimale et les revenus attendus de la chaîne d'approvisionnement. Les résultats de la recherche montrent qu'il est impossible de coordonner la chaîne d'approvisionnement de VMI avec une demande sensible aux prix dans le cadre du CBB via le contrat de prix de gros; Cependant, le contrat de prix de gros peut parfaitement coordonner cette chaîne d'approvisionnement après avoir imputé le coût non commercialisable au* *détaillant. Ensuite, il est prouvé que le facteur d'inventaire optimal, le prix de vente optimal et les revenus attendus de la chaîne d'approvisionnement de VMI diminuent tous avec la croissance du seuil d' hésitation dans certaines conditions. Enfin, il a été démontré que le volume optimal d'alimentation du système était corrélé positivement au seuil d' hésitation par des expériences numériques. Les conclusions de cet article enrichissent la théorie de la coordination de la chaîne d'approvisionnement dans le cadre du CBB et facilitent son application dans la vie réelle.*

KEYWORDS: *VMI, CBB, supply chain, retail price, coordination.*

MOTS-CLÉS: *VMI, CBB, chaîne d'approvisionnement, prix de détail, coordination.*

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1. Introduction

This paper probes into a vendor managed inventory (VMI) supply chain with price-sensitive demand in an environment of CBB. We address the problem of VMI supply chain coordination and analyse the influences of CBB on the optimal decisions and profits of supply chain. VMI is a collective term for the vendor's efforts to achieve the targets of inventory turnover and consumer service level and to improve the efficiency of supply chain efficiency, including monitoring of retailer's inventory level and regular decision-making on order quantity, shipment and replenishment of relevant operations (Waller *et al.*, 1999). It is essentially an inventory management model based on the cooperation between upstream and downstream companies in the supply chain. By contrast, there is virtually no collaboration between these companies in inventory management of traditional supply chain. As a new, representative idea of inventory management, the VMI reflects the management philosophy of system integration and adapts well to changing market demand. The implementation of the VMI model can effectively improve the supply chain's ability to cope with market changes, promote information sharing, and mitigate the bullwhip effect, thereby enhancing the cooperation between upstream and downstream companies in the supply chain (Li *et al.*, 2012). However, it is impossible to coordinate the supply chain using the VMI model alone, as the latter cannot fundamentally eliminate the impact of "dual marginalization" on the supply chain. The main way to coordinate the supply chain lies in supply chain contract. Rational contract design is needed to coordinate VMI supply chain, such that the advanced VMI management model can be adopted to improve the overall benefit of the supply chain.

Traditional supply chain contracts mainly fall into the following categories: wholesale price contract, repurchase contract, revenue sharing contract, and sales rebate contract (Cachon, 2013). In recent years, many scholars have implemented revenue sharing contract in the study on VMI supply chain coordination (Lee *et al.*, 2015; Cai *et al.*, 2006; Li and Hua, 2008; Li *et al.*, 2009). For instance, Tang (2004) proposed a method to coordinate VMI supply chain based on the wholesale price contract. Zhao and Lu (2012) established five different risk-sharing contract models and derived the optimal decision results through analysis. Taleizadeh *et al.* (2015) explored the optimal solution of a VMI model of a two-level supply chain with the features of the Stackelberg game. Liu and Fan (2016) discusses the coordination of

VMI supply chain under unfair aversion and loss avoidance, respectively, revealing that the two-level VMI supply chain can be coordinated by wholesale price contract under certain conditions. To sum up, the above studies either fail to consider the impact of retail price on market demand or ignore the effect of consumer balking behaviour (CBB) on the VMI model. Lan (2017) takes account of the CBB in the VMI model, but considers the exogenous retail price of the commodity, which is different from our research.

In real life, the consumer has less desire to purchase fresh and perishable commodities (e.g. flowers, milk and fruits) if there are only one or a few of them on the shelf, because he/she may think that these commodities are not fresh enough or close to the shelf life (Moon and Choi, 1995). In other words, most consumers may give up buying a commodity, if the inventory of the commodity is at or below a certain threshold. This phenomenon is called the CBB (Moon and Choi, 1995). Pasternack (2001) was the first to build a basic model of the CBB, provided that the demand function is completely defined. Moon and Choi (1995) and Liao *et al*. (2011) created a basic model of the distribution-free newsboy problem under the CBB. Considering the random supply and balking penalty, Lan *et al*. (2005) explored the distribution-free newsboy problem with the CBB. Lee and Jung (2014) disclosed the impact of the CBB on the performance appraisal of the newsboy model, when the CBB parameters are uncertain. Feng *et al*. (2014) tackled the supply chain repurchase contract under the CBB. Nevertheless, none of these scholars has considered the VMI model or the price impacts on the market demand.

This paper contributes to the literatures on VMI supply chain and CBB. In summary, the previous studies on supply chain management have not considered the VMI supply chain coordination under the CBB. Thus, the purpose of this paper innovatively introduces the CBB, a commonplace in real life, to the VMI supply chain whose demand depends on the retail price, and designs a wholesale price contract based on the unmarketable cost allocation to coordinate the supply chain. On this basis, the increasing failure rate (IFR) with demand distributed under the CBB was defined, and adopted for the behavioural research of the VMI model, aiming to disclose the effects of the CBB on the decision-making of supply chain members and the expected revenue of the supply chain.

The rest of this paper is organized as follows. In Section 2 we briefly describe the problem and set parameters. In Section 3 we formulate the integrated and decentralized VMI supply chain models with CBB, the supply chain coordination contract was designed by allocating the unmarketable cost to the retailer. Model Optimization is analyzed in Section 4. In Section 5, numerical experiments are presented. Finally, conclusions and suggestions for future research are detailed in Section 6.

2. Problem description and parameters assumption

This paper probes into the coordination of a two-level VMI supply chain consisting of a vendor and a retailer, under retail price-sensitive market demand and the CBB. It is assumed that the information in the VMI supply chain is absolutely symmetrical, the vendor and the retailer are fully rational and risk-neutral, and the market demand for products is random and affected by the retail price. Within the supply chain, the vendor supplies commodities to the retailer and manages the number of inventory commodities, while the retailer sells the commodities and determines the retail price. The sales amount of a single type of commodities varies by season, and all the unsold commodities are treated by the vendor. All consumers are assumed to have the CBB. In other words, each consumer knows about the commodity inventory throughout the sales period; when the number of inventory commodities falls to or below a critical value (or threshold) t ($0<t\leq q$), the consumer will buy a commodity at the probability of θ ($0<\theta\leq 1$) or not buy the commodity at the probability of 1- θ, and will not buy the commodity again in this sales period.

The other main parameters of our research are listed as follows: c is the vendor's cost to supply a unit of commodities; q is the number of commodities supplied in each cycle; w is the wholesale price per unit of commodities; v is the residual value per unit of the remaining commodities; p is the retail price per unit of commodities. Without loss of generality, it is assumed that $v<c<w<p$. Let $D(p)$ be the market demand for the commodities at the retail price p, and $F(x|p)$ and $f(x|p)$ be the probability distribution function and the probability density function of the demand, with $F(x|p)$ being continuously increasing and differentiable. Since the market demand is a decreasing function of the retail price, we have $\partial F(x|p)/\partial p>0$.

When CBB occurs at a given price and supply volume, the expected sales volume $S(q,p)$ can be expressed as:

$$S(q,p)=\int_0^{q-t} xf(x|p)dx+\int_{q-t}^{q-t+t/\theta}\left[(q-t)+\theta(x-q+t)\right]f(x|p)dx+\int_{q-t+t/\theta}^{\infty} qf(x|p)dx. \quad (1)$$

On the right side of equation (1), the first, second and third terms are respectively the expected sales volume when the market demand falls between 0 and q-t, between q-t and q-t+t/θ, and above q-t+t/θ.

Equation (1) can be simplified as:

$$s(q,p)=q-\int_0^{q-t} F\left(x\,|\,p\right)dx-\theta\int_{q-t}^{q-t+t/\theta} F\left(x\,|\,p\right)dx \quad (2)$$

Thus, we have $\partial S(q,p)/\partial p<0$. Then, the retailer's expected remaining volume $I(q,p)$ can be expressed as:

$$I(q,p)=q-S(q,p)=\int_0^{q-t} F(x|p)dx+\theta\int_{q-t}^{q-t+t/\theta} F(x|p)dx. \quad (3)$$

3. Modelling and analysis

3.1. Integrated VMI supply chain

In an integrated VMI supply chain, the expected revenue of the supply chain system can be expressed as:

$$\begin{aligned}\pi_{rs}(q,p) &= pS(q,p) + vI(q,p) - cq \\ &= (p-v)S(q,p) - (c-v)q \\ &= (p-c)q - (p-v)\left[\int_0^{q-t} F(x|p)dx + \theta \int_{q-t}^{q-t+t/\theta} F(x|p)dx\right].\end{aligned} \tag{4}$$

Under the given retail price *p*, the optimal volume of commodities q^0 supplied by the vendor must satisfy the following first-order condition:

$$\begin{aligned}\frac{\partial \pi_{rs}(q^0,p)}{\partial q} &= (p-v)\frac{\partial S(q^0,p)}{\partial q} - (c-v) \\ &= (p-c) - (p-v)\left[\theta F(q^0 - t + t/\theta | p) + (1-\theta)F(q^0 - t | p)\right] = 0,\end{aligned}$$

That is

$$\theta F(q^0 - t + t/\theta | p) + (1-\theta)F(q^0 - t | p) = \frac{p-c}{p-v}. \tag{5}$$

Under the given volume of commodities *q* supplied by the vendor, the optimal retail price p^0 of the retailer must satisfy the following first-order condition:

$$\frac{\partial \pi_{rs}(q,p^0)}{\partial p} = (p-v)\frac{\partial S(q,p^0)}{\partial p} + S(q,p^0) = 0. \tag{6}$$

Since equation (4) is not necessarily a concave function or a unimodal function, there may be more than one optimal solution (q^0, p^0) to πrs(q,p). Hence, q^0 and p^0 should satisfy equations (5) and (6), respectively, to coordinate supply chain contract.

3.2. Decentralized VMI supply chain

Under a decentralized VMI supply chain, the expected revenue functions of the retailer and the vendor can be expressed as:

$$\begin{aligned}\pi_r(p) &= (p-w)S(q,p)\\ &= (p-w)q-(p-w)\left[\int_0^{q-t}F(x|p)dx+\theta\int_{q-t}^{q-t+t/\theta}F(x|p)dx\right],\end{aligned} \tag{7}$$

$$\begin{aligned}\pi_s(q) &= wS(q,p)+vI(q,p)-cq\\ &= (w-v)S(q,p)+(v-c)q\\ &= (w-c)q-(w-v)\left[\int_0^{q-t}F(x|p)dx+\theta\int_{q-t}^{q-t+t/\theta}F(x|p)dx\right].\end{aligned} \tag{8}$$

The optimal retail price p^* of the retailer in the decentralized VMI supply chain should satisfy:

$$\frac{\partial \pi_r(p^*)}{\partial p} = (p-w)\frac{\partial S(q,p^*)}{\partial p} + S(q,p^*) = 0. \tag{9}$$

Comparing equations (9) and (6), it can be seen that $\frac{\partial \pi_r(p)}{\partial p} < \frac{\partial \pi_{rs}(p)}{\partial p}$. Therefore, it is impossible to coordinate the decentralized VMI supply chain with price-sensitive demand under the CBB relying on wholesale price contract alone.

Let q^* be the optimal volume of commodities supplied in the decentralized VMI supply chain. Then, the Property 1 listed below is valid.

Property 1 The optimal volume of commodities q^*supplied by the vendor in a decentralized VMI supply chain is smaller than that q^0 in an integrated VMI supply chain.

Proof: The first-order condition of equation (8) relative to q can be expressed as:

$$\frac{\partial \pi_s(q^*)}{\partial q} = (w-c)-(w-v)\left[\theta F(q^*-t+t/\theta|p)+(1-\theta)F(q^*-t|p)\right]=0,$$

That is

$$\theta F(q^*-t+t/\theta|p)+(1-\theta)F(q^*-t|p) = \frac{w-c}{w-v}. \tag{10}$$

Whereas

$$\partial\left[\theta F(q-t+t/\theta|p)+(1-\theta)F(q-t|p)\right]\big/\partial q = \theta f(q-t+t/\theta|p)+(1-\theta)f(q-t|p)>0,$$

the function $\theta F(q-t+t/\theta|p)+(1-\theta)F(q-t|p)$ is an increasing function of q. Comparing equations (10) and (5), we have $\frac{w-c}{w-v}<\frac{p-c}{p-v}$. Thus, $q^*<q^0$. Q.E.D.

According to Property 1, the optimal volume of commodities supplied by the vendor in the decentralized VMI supply chain tends to be conservative and lower than the optimal level in the integrated VMI supply chain, even if the retail price of the retailer remains the same. To solve the incentive incompatibility between the vendor and the retailer and improve the coordination between VMI supply chain members, the coordination strategy for the VMI supply chain with price-sensitive demand under the CBB is further discussed based on wholesale price contract.

3.3. Coordination of VMI supply chain

Let $m(0<m<c-v)$ be the retailer's subsidized price to the vendor per unit of unsalable commodities. Then, the expected transfer payment becomes $T=wS(q,p)+mI(q,p)$. In this case, the expected revenues of the retailer and the vendor can be expressed as:

$$\begin{aligned}\pi_r'(p) &= (p-w)S(q,p)-mI(q,p)\\ &= (p-w+m)S(q,p)-mq,\end{aligned} \tag{11}$$

$$\begin{aligned}\pi_s'(q) &= wS(q,p)+vI(q,p)-cq+mI(q,p)\\ &= (w-v-m)S(q,p)+(v+m-c)q.\end{aligned} \tag{12}$$

Next, the author derived the condition of the contract parameter m that enables the above wholesale price contract, which is based on unmarketable cost allocation, to coordinate the VMI supply chain with price-sensitive demand under the CBB.

Comparing equations (11) and (4), let

$$\begin{cases} p-w+m=\lambda(p-v), \\ \qquad\quad m=\lambda(c-v). \end{cases} \tag{13}$$

for any $0<\lambda<1$. Thus, $\pi'_r(p)=\lambda\pi_{rs}(q,p)$, indicating that $\pi'_r(q)=(1-\lambda)\pi_{rs}(q,p)$. In other words, the expected revenue of the vendor and that of the retailer are both affine functions of the expected revenue of the supply chain. Therefore, when the contract parameter m satisfies equation (13), the wholesale price contract, which is based on unmarketable cost allocation, can perfectly coordinate the supply chain, allowing the supply chain revenue to be distributed randomly between the retailer and the vendor.

4. Model optimization

This section explores the optimal supply volume q^0 and the optimal retail price p^0 of the VMI supply chain through model optimization.

The market demand $D(p)$ is assumed to be a function of retail price p and a random factor ε. Note that ε is a random variable in the interval [A,B] and independent of retail price. Let $G(y)$ and $g(y)$ be the probability distribution function and probability density function of ε, respectively, with $G(\cdot)$ being strictly increasing and differentiable and $g(\cdot)$ be continuous and greater than zero. It is further assumed that the market demand is the sum of the retail price and the random factor, i.e. $D(p)=d(p)+\varepsilon$, where $d(p)=a\text{-}bp(a>0, b>0)$.

Since $D(p)=d(p)+\varepsilon$, the probability density function and probability distribution function of the market demand $D(p)$ can be expressed as:

$F(x|p)=G(x\text{-}d(p)), f(x|p)=g(x\text{-}d(p))$.

Thus

$$\begin{aligned} S(q,p) &= q - \int_0^{q-t} G(x-d(p))dx - \theta \int_{q-t}^{q-t+t/\theta} G(x-d(p))dx \\ &= q - \int_A^{q-t-d(p)} G(y)dy - \theta \int_{q-t-d(p)}^{q-t+t/\theta-d(p)} G(y)dy. \end{aligned} \tag{14}$$

If the inventory factor $z=q\text{-}d(p)$, then $q=z+d(p)$. Thus, the retailer's inventory q can be obtained from the value of z. Hence, equation (14) can be transformed into:

$$S(q,p) = z + d(p) - \int_A^{z-t} G(y)dy - \theta \int_{z-t}^{z-t+t/\theta} G(y)dy. \tag{15}$$

Substituting the above equation into equation (4), the supply chain revenue function can be established as:

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$$\begin{aligned} \pi_{rs}(q,p) &= (p-v)S(q,p) - (c-v)q \\ &= (p-c)(z+d(p)) - (p-v)\Lambda(z) \\ &\triangleq \pi_{rs}(z,p), \end{aligned} \tag{16}$$

where $\Lambda(z) = \int_A^{z-t} G(y)dy + \theta \int_{z-t}^{z-t+t/\theta} G(y)dy$. The determination of the optimal supply volume q^0 and the optimal retail price p^0 can be converted into the determination of (p^0,z^0) by changing the inventory factor.

Substituting equation (15) into equations (7) and (8), the revenue functions of the retailer and the vendor in the decentralized VMI supply chain can be obtained as:

$$\begin{aligned} \pi_r(p) &= (p-w)S(q,p) \\ &= (p-w)(z+d(p)) - (p-w)\Lambda(z), \end{aligned} \tag{17}$$

$$\begin{aligned} \pi_s(q) &= (w-v)S(q,p) + (v-c)q \\ &= (w-c)(z+d(p)) - (w-v)\Lambda(z) \\ &\triangleq \pi_s(z). \end{aligned} \tag{18}$$

The studies on price-sensitive demand often assume that the random factor ε of the market demand has an IFR [3], that is, $dh(y)/dy$ must be greater than zero if the failure rate of the demand distribution is denoted as $h(y)=g(y)/[1-G(y)]$. Here, the above assumption is extended. The failure rate function of the random factor ε under the CBB can be defined as:

$$\eta(z) = \frac{\theta g(z-t+t/\theta)+(1-\theta)g(z-t)}{1-\theta G(z-t+t/\theta)-(1-\theta)G(z-t)}. \tag{19}$$

In equation (19), if t=0 or θ=1, then $\eta(z)=h(z)$ is the failure rate function of the random factor ε of the usual market demand. As with the distribution assumption of the general demand function, the failure rate function of the random factor ε under the CBB should also be assumed as incremental, i.e. $d\eta(z)/dz>0$. In fact, this property is found in many common distributions (e.g. normal distribution, uniform distribution and exponential distribution).

Theorem 1 When the market demand satisfies the additive model, $Dp=a-bp+\varepsilon$, for any $z \in [A+t-t/\theta, B+t]$, the integrated VMI supply chain has a unique optimal retail price:

$$p^0 = \frac{z+a+bc-\Lambda(z)}{2b}, \tag{20}$$

Besides, when $A+t-t/\theta+bc+a-2bv>0$ and $d\eta(z)/dz>0$, the integrated VMI supply chain also has a unique optimal inventory factor z^0, which satisfies:

$$\theta G(z^0-t+t/\theta)+(1-\theta)G(z^0-t) = \frac{z^0+a-bc-\Lambda(z^0)}{z^0+a+bc-2bv-\Lambda(z^0)}. \tag{21}$$

Proof: Whereas

$$\frac{\partial \pi_{rs}(z,p)}{\partial p} = z+a-2bp+bc-\Lambda(z), \tag{22}$$

$$\frac{\partial^2 \pi_{rs}(z,p)}{\partial p^2} = -2b < 0. \tag{23}$$

For any given $z \in [A+t-t/\theta, B+t]$, $\pi_{rs}(z,p)$ is a strict concave function of the retail price p. Let $\partial \pi_{rs}(z,p)/\partial p = z+a-2bp+bc-\Lambda(z)=0$. Then, the optimal retail price p^0 of the retailer satisfies equation (20).

Substituting equation (20) into equation (16), we have:

$$\pi_{rs}(z,p(z)) = (p^0-c)[z+a-bp^0]-(p^0-v)\Lambda(z). \tag{24}$$

According to the chain rule and the optimality of p^0, we have:

$$\begin{aligned}\frac{d\pi_{rs}(z,p(z))}{dz} &= \frac{\partial\pi(z,p(z))}{\partial p^0}\frac{\partial p^0}{\partial z}+\frac{\partial\pi_{rs}(z,p(z))}{\partial z}\\ &= \frac{\partial\pi_{rs}(z,p(z))}{\partial z}\\ &= (p^0-c)-(p^0-v)\left[\theta G(z-t+t/\theta)+(1-\theta)G(z-t)\right]\\ &= \frac{1}{2b}(z+a-bc-\Lambda(z))\\ &\quad -\frac{1}{2b}(z+a+bc-2bv-\Lambda(z))\left[\theta G(z-t+t/\theta)+(1-\theta)G(z-t)\right].\end{aligned} \tag{25}$$

Let $R(z)=d\pi_{rs}(z,p(z))/dz$. Then, we only need to prove the existence of a unique z^0 such that $R(z^0)$=0, that is, to derive equation (21)/

Since $R(z)$ is continuous in $[A+t-t/\theta, B+t]$, and

$$R(A+t-t/\theta)=\frac{A+t-t/\theta+bc+a-2bv}{2b}>0,\ R(B+t)=v-c<0,$$

There is at least one zero point of $R(z)$ in $[A+t-t/\theta, B+t]$.

The next step is to prove that $R(z)$ is a single-peak function in $[A+t-t/\theta, B+t]$.

Whereas

$$\frac{dR(z)}{dz}=-\frac{\rho(z)}{2b}\delta(z), \tag{26}$$

where $\rho(z)=\left[\theta g(z-t+t/\theta)+(1-\theta)g(z-t)\right]>0$,

$$\delta(z)=(z+a+bc-2bv-\Lambda(z))-\frac{\left[1-\theta G(z-t+t/\theta)-(1-\theta)G(z-t)\right]}{\eta(z)},$$

$$\frac{d^2R(z)}{dz^2}=\frac{dR(z)/dz}{\rho(z)}\frac{d\rho(z)}{dz}-\frac{\rho(z)}{2b}\frac{d\delta(z)}{dz}.$$

Since

$$\begin{aligned}\frac{d\delta(z)}{dz} &= \frac{1-\theta G(z-t+t/\theta)-(1-\theta)G(z-t)}{\eta^2(z)}\left[2\eta^2(z)+\frac{d\eta(z)}{dz}\right]\\ &\geq \frac{1-\theta G(z-t+t/\theta)-(1-\theta)G(z-t+t/\theta)}{\eta^2(z)}\left[2\eta^2(z)+\frac{d\eta(z)}{dz}\right]\\ &= \frac{1-G(z-t+t/\theta)}{\eta^2(z)}\left[2\eta^2(z)+\frac{d\eta(z)}{dz}\right]>0,\end{aligned} \tag{27}$$

We have:

$$\left.\frac{d^2R(z)}{dz^2}\right|_{dR(z)/dz=0} = -\frac{\rho(z)}{2b}\frac{d\delta(z)}{dz} < 0. \tag{28}$$

This means R(z) is a single-peak function in [A+t-t/θ,B+t]. Hence, there is a unique inventory factor z^0 satisfies equation (21). Q.E.D.

Property 2 When the market demand satisfies the additive model, for any balking probability θ and any balking threshold $t\in[0,q]$, the optimal inventory factor z^0 and the optimal retail price p^0 are decreasing functions of t, when

$$A+t-t/\theta+bc+a-2bv-\left[\theta g(A)\right]^{-1}>0,\ d\eta(z)/dz>0,\ g(z^0-t+t/\theta)\ge g(z^0-t).$$

Proof: Let

$$H(z,t)=R(z)=\frac{1}{2b}(z+a-bc-\Lambda(z))$$
$$-\frac{1}{2b}(z+a+bc-2bv-\Lambda(z))\left[\theta G(z-t+t/\theta)+(1-\theta)G(z-t)\right],$$

Then, H(z^0,t)=0. According to the derivation rule of implicit functions, we have

$$\frac{dz^0}{dt}=-\frac{\partial H(z^0,t)}{\partial t}\bigg/\frac{\partial H(z^0,t)}{\partial z^0}.$$

It can be seen from equation (27) that $d\delta(z)/dz>0,$ and

$$\delta(A+t-t/\theta)=A+t-t/\theta+bc+a-2bs-\left[\theta g(A)\right]^{-1}>0.$$

Thus, for any $z\in[A+t-t/\theta,B+t]$, there exists $\delta(z)$>0. Hence, it can be seen from equation (26) that:

$$\frac{\partial H(z^0,t)}{\partial z^0}=\frac{dR(z^0)}{dz^0}=-\frac{\rho(z^0)}{2b}\delta(z^0)<0. \tag{29}$$

Whereas

$$\frac{\partial H(z^0,t)}{\partial t}=\frac{1}{2b}(\theta-1)[G(z^0-t+t/\theta)-G(z^0-t)]\left[1-\theta G(z^0-t+t/\theta)-(1-\theta)G(z^0-t)\right]$$
$$+(p^0-s)(\theta-1)\left[g(z^0-t+t/\theta)-g(z^0-t)\right].$$

Since

$$1-\theta G(z^0-t+t/\theta)-(1-\theta)G(z^0-t)>0,\ g(z^0-t+t/\theta)-g(z^0-t)\ge 0,$$

$$G(z^0-t+t/\theta)-G(z^0-t)>0,\ (p^0-s)(\theta-1)<0,$$

we have $\partial H(z^0,t)/\partial t<0$. According to equation (29), we have:

$$\frac{dz^0}{dt}=-\frac{\partial H(z^0,t)}{\partial t}\bigg/\frac{\partial H(z^0,t)}{\partial z^0}<0. \tag{30}$$

Finding the first-order derivation of equation (20) relative to *t*, the following can be derived from the implicit function theorem:

$$\begin{aligned}2b\frac{dp^0}{dt}&=\left[1-\theta G(z^0-t+t/\theta)-(1-\theta)G(z^0-t)\right]\frac{dz^0}{dt}\\&+(\theta-1)[G(z^0-t+t/\theta)-G(z^0-t)].\end{aligned}$$

According to equation (30), we have:

$$dp^0/dt<0 \tag{31}$$

From equations (30) and (31), Property 2 is proved valid. Q.E.D.

Property 3 When the market demand satisfies the additive model, the maximum expected revenue $\pi_{rs}(z^0,p^0)$ of the integrated VMI supply chain will decrease under the CBB.

Proof: Finding the first-order derivation of $\pi_{rs}(z^0,p^0)$ in equation (4) relative to *t*, the following can be derived from the chain rule and the optimality of p^0 and q^0:

$$\begin{aligned}\frac{d\pi_{rs}(q^0,p^0)}{dt}&=\frac{\partial\pi_{rs}(q^0,p^0)}{\partial t}+\frac{\partial\pi_{rs}(q^0,p^0)}{\partial p^0}\frac{\partial p^0}{\partial t}+\frac{\partial\pi_{rs}(q^0,p^0)}{\partial q^0}\frac{\partial q^0}{\partial t}\\&=(p^0-v)(\theta-1)\left[F(q^0-t+t/\theta\,|\,p^0)-F(q^0-t\,|\,p^0)\right]<0,\end{aligned}$$

This means the retailer's expected revenue $\pi_{rs}(z^0,p^0)$ decreases with the increase of t.

When there is no CBB, i.e. t=0, the value of $\pi_{rs}(z^0,p^0)$ reaches the maximum; when there is CBB, i.e. $0<t\le q^0$, the value of $\pi_{rs}(z^0,p^0)$ is smaller than that at $t=0$. Therefore, the maximum expected revenue $\pi_{rs}(z^0,p^0)$ of the supply chain will decrease under the CBB. Q.E.D.

According to Property 3, the VMI supply chain with no CBB (t=0) has a lower maximum expected revenue than that with CBB, and the difference is positively correlated with the balking threshold.

Theorem 2 When the market demand in the decentralized VMI supply chain satisfies the additive mode, the retailer's expected revenue function $\pi_r(p)$ and the vendor's expected revenue function $\pi_s(z^*)$ are respectively the strict concave functions of the retail price p and the inventory factor z, and the values of $\pi_r(p^*)$ and $\pi_s(z^*)$ reach the maximum when the optimal retail price and the optimal inventory factor (p^*,z^*) satisfy equations (32) and (33), respectively:

$$p^*=\frac{z^*+a+bw-\Lambda(z^*)}{2b}, \tag{32}$$

$$\theta G(z^* - t + t/\theta) + (1-\theta)G(z^* - t) = \frac{w-c}{w-v}. \tag{33}$$

Proof: Finding the first-order derivative of equation (17) relative to p and the second-order derivation of equation (18) relative to z, Theorem 2 can be proved according to the first-order condition. Q.E.D.

5. Numerical experiments

It is assumed that the market demand satisfies the additive model $D(p)$=100-2p+ε, where the random factor ε is uniformly distributed in [0,10]. Then, we have $g(y)$=0.1 and $G(y)$=0.1y. The other parameters are listed in Table 1 below.

Table 1. Contract parameters of the VMI supply chain

Parameter	t	θ	v	c	w	m
Value	5	0.5	3	6	12	2.3

Table 2. Calculated values of the parameters of the VMI supply chain in the decentralized mode and the integrated mode

Supply chain contract	Retail price	Supply amount	Vendor's revenue	Retailer's revenue	Supply chain revenue
Wholesale price contract based on unmarketable cost allocation	p^0=28.92	q^0=51	241.5	793.7	1035.2
Pure wholesale price contract	p^*=31.80	q^*=43	227.2	784.0	1011.2

Table 2 provides the calculated values of the parameters of the VMI supply chain in the decentralized mode and the integrated mode. It can be seen that, for the VMI supply chain with price-sensitive demand under the CBB, the optimal inventory level of the vendor in the decentralized mode was lower than that in the integrated mode, while the optimal retail price of the retailer in the former mode was higher than that in the latter mode. Therefore, it is impossible to coordinate the supply chain under the wholesale price contract only. By contrast, the supply chain revenue reached the optimal value of the integrated supply chain, while the retailer's revenue and vendor's revenue realized Pareto improvement, encouraging the two parties to join the contract, after the introduction of the unmarketable cost allocation to the retailer and proper adjustment of the contract parameter m. These results indicate that the wholesale price contract based on unmarketable cost allocation can coordinate the supply chain in a perfect manner.

Table 3. Effects of t on the optimal decision and revenue of the integrated and decentralized VMI supply chains

t	z^0	p^0	q^0	$\pi_{rs}(z^0,p^0)$	$\pi_r(p^*)$	$\pi_s(z^*)$
1	8.856	29.22	50.41	1066.5	807.9	234.4
2	8.854	29.18	50.49	1062.6	804.9	233.5
3	8.851	29.12	50.61	1056.1	799.9	232.0
4	8.848	29.03	50.78	1046.9	792.9	229.9
5	8.843	28.92	51.00	1035.2	784.0	227.2
6	8.836	28.78	51.27	1021.0	773.1	223.9
7	8.829	28.62	51.59	1004.3	760.4	220.0
8	8.820	28.43	51.95	985.2	745.8	215.5
9	8.810	28.22	52.37	963.7	729.5	210.4
10	8.799	27.98	52.84	939.8	711.5	204.7

Table 3 illustrates the effects of the balking threshold t on the optimal decision and revenue of the integrated and decentralized VMI supply chains when θ=0.5. It is clear that the value of the balking threshold t is negatively correlated with the optimal inventory factor and optimal retail price of the integrated VMI supply chain, but positively correlated with the supply volume. This is because the CBB suppresses the actual market demand; in response, the decision-maker will bolster the market demand by lowering the price, and increase the supply volume so that the number of inventory commodities is not below the stalking threshold, thus preventing the CBB. It can also be seen from Table 3 that, under the CBB, the maximum revenue in the integrated VMI supply chain continued to decline with the increase of the balking threshold t; the same trend was observed for the maximum revenues of the retailer and the vendor in the decentralized VMI supply chain.

6. Conclusions and future research

This paper introduces the CBB, a commonplace in real life, to the VMI supply chain with price-sensitive demand, and investigates the effects of the CBB on the decision-making of supply chain members and the expected revenue of the supply chain. Since the traditional wholesale price contract cannot coordinate the VMI supply chain with price-sensitive demand under the CBB, the author put forward the wholesale price contract, in which the unmarketable cost is allocated to the retailer.

With different parameter settings, this contract ensures that the supply chain revenue reaches the optimal value of the integrated supply chain, while the retailer's revenue and vendor's revenue realize Pareto improvement. Furthermore, the IFR with demand distribution under the CBB was defined to obtain the sufficient conditions for the supply chain to have a unique optimal inventory factor and a unique optimal retail price, when the random market demand equals to sum of the retail price and the random factor. Next, it is proved that the optimal inventory factor, the optimal retail price and the expected revenue of the VMI supply chain all decreases with the growth in the balking threshold, under certain conditions. Finally, the optimal supply volume of the system was proved to be positively correlated with the balking threshold through a case analysis.

In this paper, the coordination of the VMI supply chain with price-sensitive demand under the CBB is mainly explored based on the wholesale price contract. The future research will investigate the coordination of this supply chain based on other contracts (e.g. repurchase contract, revenue sharing contract, and sales rebate contract). In addition, the supply chain is assumed to be information symmetrical and its members risk-neutral in this research. More complicated conditions will be studied in the future research on the VMI supply chain.

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