Effect of Heat Transfer on the Growing Bubble with the Nanoparticles/Water Nanofluids in Turbulent Flow

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ABSTRACT

This paper is devoted to study the effect of heat transfer on the temperature distribution in a superheated liquid during the growth of vapour bubbles immersed in different types of nanoparticles/water nanofluids between two-phase turbulent flow. The mathematical model is formulated and solved analytically depending on Scriven's theory and using the modification of the method of the similarity parameters between two finite boundaries. The characteristics of vapour bubble growth and temperature distribution are obtained by using the thermo-physical properties of nanoparticles nanofluids. The results indicate that the nanoparticle volume concentration reduces the bubble growth process under the effect of heat transfer. The better agreements are achieved, for bubble dynamics in turbulent nanofluid using the appropriate numerical and theoretical data for the values of concentration rate of nanoparticles $\chi=0,0.2,0.4$. The temperature distribution surrounding the regime of bubble growth in pure water is more intensive than in other cases of Al_2O_3/H_2O, Fe_3O_4/H_2O and CuO/H_2O nanofluids in turbulent flow. A Comparison of the current solution with previous works is carried out and discussed.

1. INTRODUCTION

The field of nanofluids has novel properties of the fluid that make intensively useful in the field of heat transfer and its application including refrigerators, fuel cells, heat exchanger in the systems and conventional colloids [1-6]. It is appeared in the properties of thermal conductivity and convective heat/mass transfer. Measurably, the nanofluids have happened when the scale of measurement is smaller than 100nm functionalized nanoparticles, it is considered that a new class of material. It is known that the efficiency of heat transfer enhancement depends on the number of particles distributed, the type of the substance, the shape of particles, and so on. Bubble dynamics and nanofluids are planned to be used in aircraft, engines, micro-reactors [5, 6]. Prior to the introduction of nanofluids, it was expected that heat transfer could be enhanced by dispersing micron-sized particles. But the fluid of micron-sized particles caused problems due to sedimentation and clogging.

A lot of research has undergone for the last years in the field of boiling systems of nanofluids. However, for most potential applications of nanofluids the situation of interest is flow boiling including bubble dynamics parameters, such as bubble departure diameter, frequency and void fraction during nanofluid flow boiling, that may provide valuable insight in the mechanisms by which the nanoparticles affect the heat transfer [2].

Recently, it has become increasingly necessary to study various methods for increasing heat transfer especially in the field of bubble dynamics and their applications like peristaltic motion [7] and nuclear power system [5], resulting from the transfer of nanoparticles in common liquids called nanofluids. Measurements of nanoparticles are used in nanofluids range from 1:100 nm. These nanoparticles are metal particles such as Cu, Ag or other metal oxides. Typical fluids used in heat transfer usually have a low coefficient of heat transfer, but due to their high coefficient of conductivity, the use of nanoparticles can increase the coefficient of thermal conductivity of mixing fluids. The use of nanofluids is also a suitable approach to improve heat transfer [6, 8-10]. The distribution of temperature around the growth of a vapour bubble is an important process in nanoparticles/water (for example, Al_2O_3/H_2O, Fe_3O_4/H_2O and $CuO//H_2O$) nanofluids between a two-phase turbulent flow. In nanoparticles nanofluid, the growth process is influenced by heat transfer and pressure change between the laminar or turbulent flow of two phases. The problem of heat transfer in a growing bubble with initial conditions in the laminar flow is introduced and discussed in Ref. [11].

The bubble dynamics in nanofluids are studied experimentally and introduced by some efforts of researchers [5, 12-15]. Park et al. [12] attempted to formulate the problem of bubble dynamics in nanofluids and find experimentally the behaviours of boiling bubbles. Moreover single bubble is studied in Al_2O_3/H_2, Fe_3O_4/H_2O and $CuO//H_2O$ nanofluids considering the impact of heating surface. In another side,

some works (as in Refs. [16, 17] showed the model of the bubble dynamics in viscoelastic fluids that took Oldroyd constitutive equation on works [16, 17], which allows for strain relaxation of the fluid and is more suitable to greater deformations. The evaluation of radius under influence of shear stress is theoretically studied in ref. [18, 19]. Recently, Abu-Nab et al. [20] investigated the vapour bubble dynamic in power-law Al_2O_3/H_2 nanofluid, affecting by variable surface tension depending on the method of Plesset and Zwick [21] and Mohammadein et al. [19]. Furthermore, the effects of surface tension behavior with the initial bubble radius, time, and initial rate of bubble radius are investigated within Al_2O_3/H_2 nanofluid. It is found that the growth of the vapour bubble radius is affected by the concentration rate of nanoparticles and other parameters. However, the influence of heat transfer on growth process of bubbles in nanoparticles/water nanofluids between two-phase turbulent flow has not been studied analytically before. Consequently, we will take that into account.

The main aim of the current work is to study the effect of heat transfer on a growing bubble in Al_2O_3/H_2, Fe_3O_4/H_2O and $CuO//H_2O$ nanofluids between two-phase turbulent flow and determine the bubble radius under the influence of thermo-physical parameters of superheating liquid, nanoparticles volume fraction and density in nanoparticles/water nanofluids. Besides, we illustrate the temperature distribution in the regime of bubble growth in turbulent flow, during nanoparticles/water nanofluids.

The introduction presented in section one. The theoretical study, mathematical model and solution of the model are introduced in section two and three. Results and discussions, and conclusion are addressed in section four and five respectively.

2. THEORETICAL MODEL AND MATHEMATICAL FORMULATION

As is known, a single vapor bubble grows between two boundaries of a finite radius R_0 and R_m inside a superheated incompressible liquid (as shown in Figure 1) with different patterns of nanoparticles/water nanofluids like Al_2O_3/H_2, Fe_3O_4/H_2O and $CuO//H_2O$. The single bubble has a spherical geometry. The growth is influenced by the pressure difference between the internal pressure of the bubble $(P(v), (R(t), t)$ and heat transfer in different types of nanoparticles nanofluids between two-phase turbulent flow. Steam and superheated liquid are considered incompressible flows. The pressure inside the bubble is uniform and also the distribution of vapor density inside the bubble is uniform, with the exception of a thin boundary layer at the bubble wall. The thermo-physical properties of particles nanofluids are considered and presented into account in the presented model.

We suppose that the bubble velocity in a laminar flow is greater than the fluctuation velocity for turbulent flow. The equation between them, had been assumed in ref. [22] as

$$\dot{R}_{Lam} = n\dot{R}_{Tur} \qquad (1)$$

here, n is the arbitrary constant; $\dot{R}_{Lam}$, $\dot{R}_{Tur}$ are bubble velocity in laminar and turbulent flow respectively. In the case of study bubble dynamics in turbulent flow, the condition constraint, n≥1 is satisfied.

The primary mathematical model describing the present problem of effect of heat transfer on bubble dynamics with different types of nanofluids between a two-phase turbulent flow, consists of mass, energy equations and the relation of the thermo-physical properties of particles nanofluids.
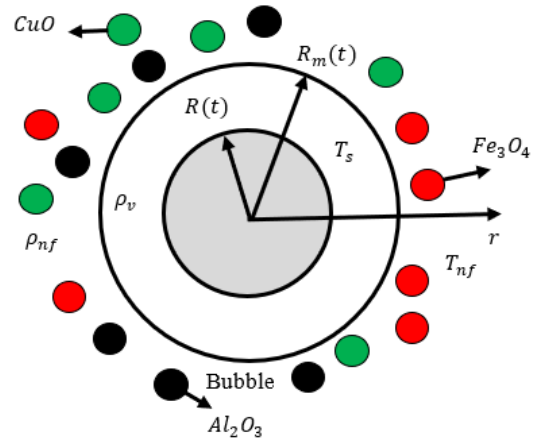

Figure 1. Schematic of a growing vapour bubble immersed in some different types of nanoparticles/water nanofluids between two-phase temperatures

2.1 Mass equation

The continuity equation in an incompressible liquid [23] can be expressed as:

$$\text{div } \mathbf{V} = 0 \qquad (2)$$

where, $\mathbf{V} = \mathbf{V}_{Lam} + \mathbf{V}_{Tur}$; $\mathbf{V}_{Lam}$, $\mathbf{V}_{Tur}$ are the velocity of mixture in laminar flow and turbulent flow respectively.

Assuming the initial and boundary conditions: ($t = 0, r = R, (V_{Lam})_{r=R} = \varepsilon_{nf}a\dot{R}_{Lam}, (V_{Tur})_{r=R} = \varepsilon_{nf}b\dot{R}_{Tur}$); where $\varepsilon_{nf} = 1 - \frac{\rho_v}{(\rho_l)_{nf}}$ is the density ratio in nanofluids; ρ_v and $(\rho_l)_{nf}$ are the density of vapour and nanofluid respectively. a, b are arbitrary constants. Using the above assumptions and Eq. (1) to solve the continuity Eq. (2), then the mixture velocity can be solved as:

$$V = \frac{\varepsilon_{nf}C_kR^2}{r^2}\dot{R} \qquad (3)$$

Here C_k=a+bn. The arbitrary parameter C_k plays a dominant parameter to determine the kind of the flow on bubble dynamic process. That means, if C_k<1, the flow is in turbulent flow, and if C_k≥1, the flow is in laminar flow.

2.2 Energy equation

The energy equation is extended for spherical symmetry to establish the effect of radial conduction resulting from unequal phase temperature. The energy equation in turbulent flow (see for example [24]) can be rewritten in the form

$$\frac{\partial T}{\partial t} + V\frac{\partial T}{\partial t} = c_0^2 a_{Tur}r^2\frac{\partial}{\partial r}(r^2\frac{\partial T}{\partial r}) \qquad (4)$$

Here a_{Tur} is the thermal diffusivity in turbulent flow and c_0 can be defined in ref. [25] as $c_0 = (2.2 \times 10^{-3}P_e)^{-1/2}$; P_e is the relation among the thermal energy related to the fluid and the thermal energy conducted within the fluid, is called the

Péclet number,

The Initial conditions and boundary conditions can be expressed as

$$T(R, 0) = T_s, \qquad T(R_m, t) = T_0,$$

$$\left.\frac{\partial T}{\partial r}\right|_{r=R} = \frac{\varepsilon_{nf} C_k \rho_v (L + ((C_P)_{nf} - C_{Pv})\Delta\Omega)\dot{R}}{k_{nf}} \quad (5)$$

Here, T_s is saturation temperature of bubble, T_0 is initial temperature of liquid. $(C_p)_{nf}$ and C_{Pv} are specific heat of nanofluids and vapour respectively, L is a latent heat vaporization, K_{nf} is liquid thermal conduction in nanofluid and $\Delta\Omega$ is superheating temperature of bulk liquid.

2.3 Thermo-physical properties of nanoparticles/water nanofluids

In the study of bubble dynamics in nanofluids, the main key thermo-physical properties of nanofluids are to investigate the properties of density, specific heat capacity, thermal conductivity and surface tension during the bubble dynamic process. The thermo-physical relations (see for examples, refs. [5, 9, 26, 27]) describe the problem of heat transfer on a growing vapor bubble between a two-phase turbulent flow has the following form as

$$\rho_{nf} = (1 - \chi)\rho_f + \chi\rho_p \quad (6a)$$

$$\left(c_p\right)_{nf} = (1 - \chi)\left(c_p\right)_f + \chi\left(c_p\right)_p \quad (6b)$$

$$k_{nf} = \frac{\left(k_p + 2k_f\right) - 2\chi\left(k_p - k_f\right)}{\left(k_p + 2k_f\right) + \chi\left(k_p - k_f\right)} k_f \quad (6c)$$

where, ρ_f, ρ_p are the density of fluid and particle respectively. χ is the concentration rate of nanoparticle in nanofluids. The index nf indicates to nanofluid. $(C_p)_f$, $(C_p)_P$ are the specific heat of fluid and particle respectively. k_f, k_p are thermal conductivity of fluid and particle respectively.

Eq. (6a) represents the equation of density in nanofluid. Equation (6b) represents the equation of specific heat in nanofluid. Eq. (6c) is the relation of thermal conductivity in nanofluids, is used to in ref. [27], in our model, we consider that the effects of the Brownian motion are negligible due to the randomly large particles at a low concentration of nanoparticles. The thermo-physical properties of pure water and some nanoparticles are illustrated in Table 1. These thermo-physical properties are used into presented account.

Table 1. Thermo-physical properties of water based copper oxide, iron oxide and alumina nanofluids [26, 28, 29]

Properties	Base Fluid	CuO	Al$_2$O$_3$	Fe$_3$O$_4$
C_p(J/kg K)	4179	531.8	765	670
ρ(kg/m^3)	997.1	6320	3970	5180
k_l(W/m K)	0.613	76.5	40	9.7

3. SOLUTION OF THE MODEL

This section is to present the solution of theoretical model of the temperature distribution surrounding the growing bubbles in some different types of nanoparticles/water nanofluids, based on the reduced system of Eqns. (1-6) and the method of the similarity parameter between two finite boundaries, we will use the following as

$$T(r, t) = T(s), \quad r = \frac{1}{\beta} s \, f(t) \ \text{ and } \ \frac{\partial s}{\partial t} = 0 \quad (7)$$

At the boundary of bubble, we suppose r=R then

$$s = \beta \quad (8)$$

And r=f(t).

The temperature gradient in the mixture of nanofluids at boundary of bubble takes

$$4\pi R^2 k_{nf}(\frac{\partial T}{\partial r})_{r=R} = \frac{d}{dt}\left[\frac{4\pi R^3}{3}(L + ((C_P)_{nf} - C_{Pv})\Delta\Omega)\rho_v\right] \quad (9)$$

Combining (3) and (9), the temperature gradient $(\frac{\partial T}{\partial r})_{r=R}$ becomes

$$(\frac{\partial T}{\partial r})_{r=R} = \frac{\varepsilon_{nf} C_k \rho_v}{k_{nf}} \dot{R}(L + ((C_P)_{nf} - C_{Pv})\Delta\Omega) \quad (10)$$

On the basis of the above assumptions and applying the above assumptions and separating of variables methods on Eq. (4), then the energy Eq. (4) converts to two equations, the obtained equation can be divided to two differential equations as

$$\dot{f}(t)f(t) = \mathcal{A} \quad (11)$$

And

$$\frac{d}{ds}\ln\left(\frac{dT}{ds}\right) = \frac{\mathcal{A}}{a_{Tur}c_0^2}\left[-\frac{s}{\beta^2} + \frac{\varepsilon_{nf}C_k\beta}{s^2}\right] - \frac{2}{s} \quad (12)$$

We integrate Eq. (12), we get

$$\frac{dT}{ds} = \frac{\mathcal{A}}{s^2} e^{\left[\frac{-\mathcal{A}}{a_{Tur}c_0^2}\left(\frac{s^2}{2\beta^2} + \frac{\varepsilon_{nf}C_k\beta}{s}\right)\right]} \quad (13)$$

The boundary condition (5) can be converted to

$$\left(\frac{dT}{dr}\right)_{r=R} = \frac{\mathcal{A}\,\varepsilon_{nf}\,C_k\rho_v(L + ((C_P)_{nf} - C_{Pv})\Delta\Omega)}{k_{nf}\,R} \quad (14)$$

We obtain $\frac{dT}{dr} = \frac{s}{r}\frac{\partial T}{\partial s}$, thus

$$\left(\frac{dT}{ds}\right)_{r=\beta} = \left(\frac{r}{s}\frac{dT}{dr}\right)_{r=R, s=\beta}$$
$$= \frac{\mathcal{A}\,\varepsilon_{nf}\,C_k\rho_v(L + ((C_P)_{nf} - C_{Pv})\Delta\Omega)}{k_{nf}\,\beta} \quad (15)$$

To find the constant $\mathcal{A}_1$, using (15) into (13), then

$$\mathcal{A}_1 = \frac{1}{k_{nf}}\mathcal{A}\,\varepsilon_{nf}\,C_k\rho_v(L + ((C_P)_{nf} - C_{Pv})\Delta\Omega)\beta e^{\left[\frac{\mathcal{A}}{a_{Tur}c_0^2}\left(\frac{1}{2} + \varepsilon_{nf}C_k\right)\right]}$$

Using the value of constant $\mathcal{A}_1$ into Eq. (13), we get

$$\frac{dT}{ds} = \frac{\mathcal{A}_2 \mathcal{A} \, \beta}{s^2} e^{\left[\frac{-\mathcal{A}}{a_{Tur}c_0^2}\left(\frac{s^2}{2\beta^2}+\frac{\varepsilon_{nf}C_k\beta}{s}\right)+\frac{\mathcal{A}}{a_{Tur}c_0^2}\left(\frac{1}{2}+\varepsilon_{nf}C_k\right)\right]} \quad (16)$$

where, $\mathcal{A}_2 = \frac{1}{k_{nf}} \varepsilon_{nf} C_k \rho_v (L + ((C_P)_{nf} - C_{Pv})\Delta\Omega)$.

From Eq. (15), Eq. (16) leads to

$$\frac{\partial T}{\partial r} = \frac{s}{r}\frac{dT}{ds}$$
$$= \frac{\mathcal{A}_2 \mathcal{A} \, \beta}{r \, s} e^{\left[\frac{-\mathcal{A}}{a_{Tur}c_0^2}\left(\frac{s^2}{2\beta^2}+\frac{\varepsilon_{nf}C_k\beta}{s}\right)+\frac{\mathcal{A}}{a_{Tur}c_0^2}\left(\frac{1}{2}+\varepsilon_{nf}C_k\right)\right]} \quad (17)$$

Using the assumptions in (7,8), Eq. (17) becomes

$$\frac{\partial T}{\partial r} = \frac{\mathcal{A}_2 \mathcal{A} \, R}{r^2} e^{\left[\frac{-\mathcal{A}}{a_{Tur}c_0^2}\left(\frac{r^2}{2R^2}+\frac{\varepsilon_{nf}C_kR}{r}\right)+\frac{\mathcal{A}}{a_{Tur}c_0^2}\left(\frac{1}{2}+\varepsilon_{nf}C_k\right)\right]} \quad (18)$$

Eq. (18) can be integrated from any instant t to t_m at r reaches to maximum radius R_m also $T(r, t)=T_0$, that means

$$T(r, t) = T_0 - \mathcal{A}_2\mathcal{A}$$
$$R \int_r^{R_m} \frac{1}{z^2} e^{\left[\frac{-\mathcal{A}}{a_{Tur}c_0^2}\left(\frac{z^2}{2R^2}+\frac{\varepsilon_{nf}C_kR}{z}\right)+\frac{\mathcal{A}}{a_{Tur}c_0^2}\left(\frac{1}{2}+\varepsilon_{nf}C_k\right)\right]} dz \quad (19)$$

In the wall of bubble, we suppose $r=R(t_i)$, Eq. (19) becomes

$$T(R(t_i), t_i) = T_0 - \mathcal{A}_2\mathcal{A} \, R(t_i)$$
$$\int_{R(t_i)}^{R_m} \frac{1}{z^2} e^{\left[\frac{-\mathcal{A}}{a_{Tur}c_0^2}\left(\frac{z^2}{2R^2(t_i)}+\frac{\varepsilon_{nf}C_kR(t_i)}{z}\right)+\frac{\mathcal{A}}{a_{Tur}c_0^2}\left(\frac{1}{2}+\varepsilon_{nf}C_k\right)\right]} dz \quad (20)$$

Supposing that $y=z/R(t_i)$ into Eq. (20), we get the temperature distribution of a growing bubble in nanoparticles nanofluids between two-phase flow in the form

$$T(R(t_i), t_i)$$
$$= T_0$$
$$- \mathcal{A}_2\mathcal{A} \int_1^{\frac{R_m}{R(t_i)}} \frac{1}{y^2} e^{\left[\frac{-\mathcal{A}}{a_{Tur}c_0^2}\left(\frac{y^2}{2}+\frac{\varepsilon_{nf}C_k}{y}\right)+\frac{\mathcal{A}}{a_{Tur}c_0^2}\left(\frac{1}{2}+\varepsilon_{nf}C_k\right)\right]} dy \quad (21)$$

By integrating the R. H. S of above equation, Eq. (21) becomes

$$T(R(t_i), t_i) = T_0 - \mathcal{A}_2 \left(\frac{\pi}{6} a_{Tur}c_0^2\mathcal{A}\right)^{\frac{1}{2}} e^{\frac{1}{6}\frac{\mathcal{A}(1-\varepsilon_{nf}C_k)^2}{a_{Tur}c_0^2}}$$
$$\left(\text{erf}\left[\left(\frac{3}{2}\frac{\mathcal{A}}{a_{Tur}c_0^2}\right)^{\frac{1}{2}}\left(1-\frac{R(t_i)}{R_m}\right)\right]\right.$$
$$+ (1 - \varepsilon_{nf}C_k)\left(\frac{1}{6}\frac{\mathcal{A}}{a_{Tur}c_0^2}\right)^{\frac{1}{2}}\right] \quad (22)$$
$$\left.- \text{erf}\left[(1 - \varepsilon_{nf}C_k)\left(\frac{1}{6}\frac{\mathcal{A}}{a_{Tur}c_0^2}\right)^{\frac{1}{2}}\right]\right)$$

At $t=t_0$, $R(t_0) = R_0 \Rightarrow \text{erf}(0) = 0$ and $T(R(t_0), t_0)=T_s$, Eq. (22) converts to

$$\Delta\Omega = T_0 - T_s = \mathcal{A}_2\left(\frac{\pi}{6}a_{Tur}c_0^2\mathcal{A}\right)^{\frac{1}{2}} \text{erf}\left[\left(\frac{3}{2}\frac{\mathcal{A}}{a_{Tur}c_0^2}\right)^{\frac{1}{2}}\left(1-\phi_0^{1/3}\right)\right] \quad (23)$$

Here $\phi_0 = (R_0/R_m)^3$, is a void fraction. T_s is the saturation temperature.

Using the value of $\mathcal{A}_2$ into Eq. (23), we get

$$\Delta\Omega$$
$$= \frac{1}{k_{nf}} \varepsilon_{nf} C_k \rho_v (L$$
$$+ ((C_P)_{nf} - C_{Pv})\Delta\Omega)\left(\frac{\pi}{6}a_{Tur}c_0^2\mathcal{A}\right)^{\frac{1}{2}} \text{erf}\left[\begin{array}{c}\left(\frac{3}{2}\frac{\mathcal{A}}{a_{Tur}c_0^2}\right)^{\frac{1}{2}} \\ \left(1-\phi_0^{1/3}\right)\end{array}\right] \quad (24)$$

We note that, $\text{erf}(x) = 1 - \frac{1}{\sqrt{\pi}x} e^{-x^2}$ at any $x \gg 0$, also, for any $a_{Tur} \ll 1$, and for any times, we obtain $0 < \phi_0^{1/3} < 1$. Then Eq. (24) reduces

$$\frac{(J_a)_{nf}(a_{Tur})_{nf}}{\varepsilon_{nf}C_k\left(1+\frac{((C_P)_{nf}-C_{Pv})}{L}\Delta\Omega\right)}$$
$$= \left(\frac{\pi}{6}a_{Tur}c_0^2\mathcal{A}\right)^{\frac{1}{2}}$$
$$- \frac{1}{3\left(1-\phi_0^{\frac{1}{3}}\right)}e^{\left[\left(\frac{-3}{2}\frac{\mathcal{A}}{a_{Tur}c_0^2}\right)^{\frac{1}{2}}\left(1-\phi_0^{1/3}\right)\right]} \quad (25)$$

Here $(J_a)_{nf} = \frac{\rho_{nf}(a_{Tur})_{nf}}{\rho_v L}\Delta\Omega$, is the Jacob number in nanofluids.

For a_{Tur}, we get

$$\frac{\Delta\Omega \, k_{nf}}{\varepsilon_{nf}C_k\rho_v\left(1+\frac{((C_P)_{nf}-C_{Pv})}{L}\Delta\Omega\right)} = \left(\frac{\pi}{6}a_{Tur}c_0^2\mathcal{A}\right)^{\frac{1}{2}}$$

then $\mathcal{A}$ can be written as

$$\mathcal{A} = \frac{6a_{Tur}}{\pi}\left(\frac{(J_a)_{nf}}{c_0\varepsilon_{nf}C_k\left(1+\frac{((C_P)_{nf}-C_{Pv})}{L}\Delta\Omega\right)}\right)^2 \quad (26)$$

Apply the boundary conditions at $t=t_0$, $R=R_0$ in solving Eq. (12a) to get the bubble radius in nanofluids, then we get

$$R = (2\mathcal{A}(t - t_0) + R_0^2)^{\frac{1}{2}} \quad (27)$$

Combining (26, 27), we get the bubble radius in nanofluids between two turbulent flow in a final form

$$R = 2\left(\frac{3a_{Tur}}{\pi}\left(\frac{(J_a)_{nf}}{c_0\varepsilon_{nf}C_k\left(1+\frac{((C_P)_{nf}-C_{Pv})}{L}\Delta\Omega\right)}\right)^2 (t - t_0) + \frac{1}{4}R_0^2\right)^{\frac{1}{2}} \quad (28)$$

The formula (28) denotes the behaviour of vapour bubble growth between two-phase turbulent flow under conditions in (7), that affected by nanoparticles nanofluids.

4. RESULTS AND DISCUSSIONS

The results of the present model and its solution are described in Figures 2-8 depending on the physical parameters in Table 1 and using the data of parameters that can be expressed as following as: $\rho_v = 0.579 \frac{kg}{m^3}$ [30], $C_{pv} = 2160 \frac{J}{kg\,K}$ [30], $\Delta\Omega = 2.5\,K, T_0 = 373\,K, p_e = 0.03$ [16], $L = 533000\,J/kg$ [30], $R_0 = 5 \times 10^{-4}m$, $R_m = 5 \times 10^{-3}m$, $\dot{R}_0 = 10^{-3}m/s$, $a = 0.1, b = 0.1, t_0 = 0.1s$, $n = 1.1$.

The radius of bubble grows in the terms of time in Figure 2 where Figure 2 shows the bubble growth in different types of nanoparticles nanofluids (like $CuO//H_2O$, Fe_3O_4/H_2O and Al_2O_3/H_2O) and pure fluid (water) between two-phase turbulent flow. We observed that the bubble growth in water fluid is more than in the case of nanoparticles nanofluids. Moreover, the bubble growth in the type of Fe_3O_4/H_2O nanofluids is bigger than in other types of nanoparticles nanofluids. The effect of nanoparticles nanofluids reduces the bubble growth.
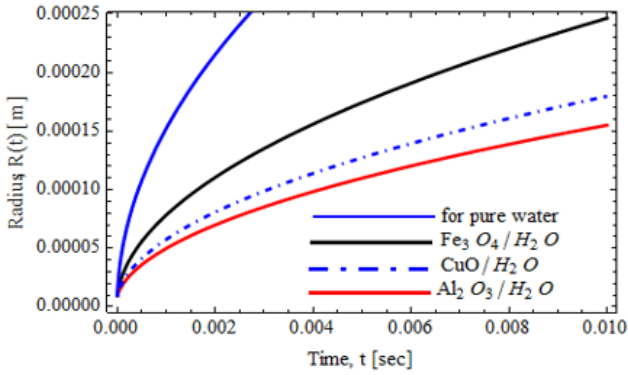

Figure 2. Bubble radius growth R(t) with time t at different values of the presented types of nanoparticles/water nanofluids and pure water
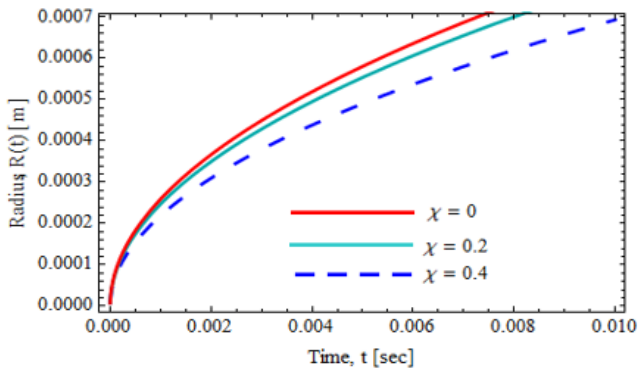

Figure 3. Bubble radius growth R(t) with time t at different values of nanoparticles volume concentration of Al_2O_3/H_2O nanofluids

Figure 3 shows the effect of nanoparticles volume concentration on a growing vapour bubble between two-phase turbulent flow. The growth of bubble in the case no-nanofluids is higher than in the case of nanofluids. We conclude that the nanoparticles volume concentration in nanofluids weakness the growth process.
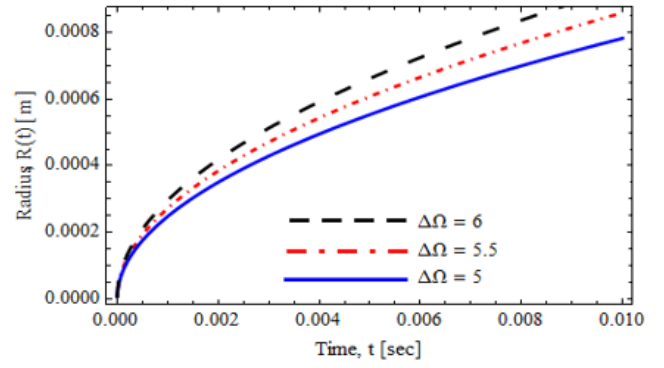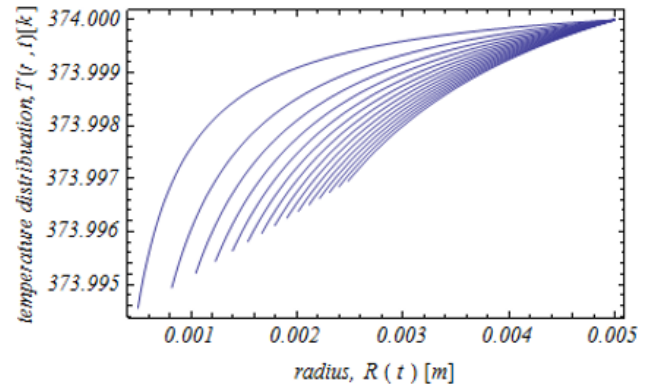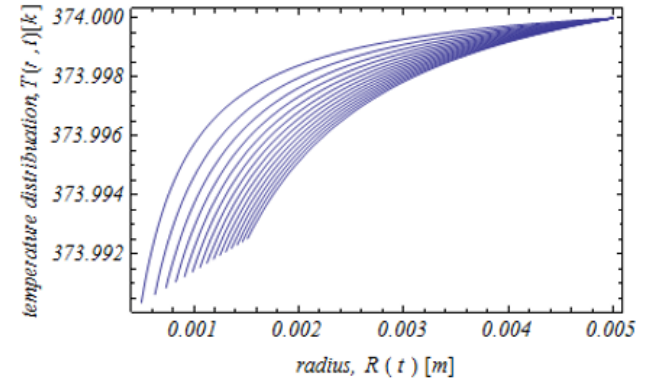

Figure 4. Bubble radius growth R(t) with time t for different values of superheating $\Delta\Omega$ of Al_2O_3/H_2O nanofluids

(5A)) $\chi = 0.1$

(5B)) $\chi = 0.5$

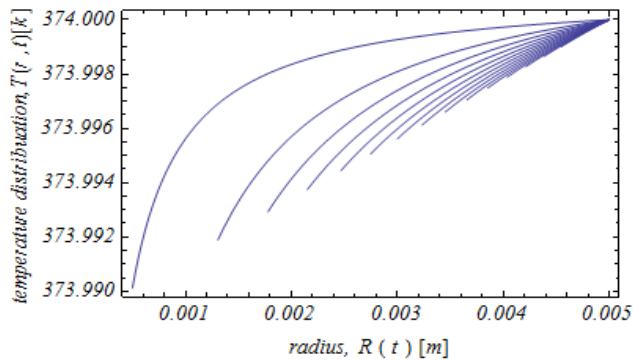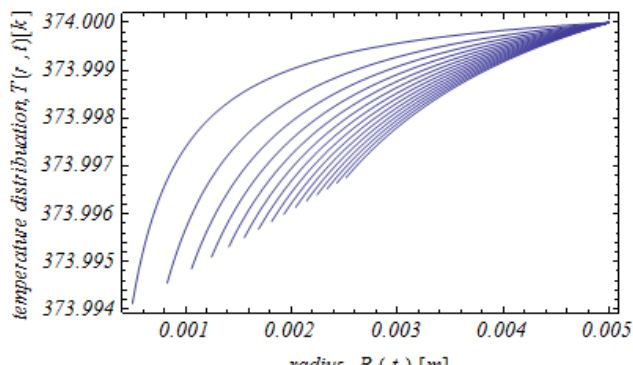**Figure 5.** Temperature distribution around a growing bubble in Fe_3O_4/H_2O nanofluids for some different values of nanoparticles volume concentration χ

Figure 4 shows the bubble growth for some different values of superheating of nanofluids. We observed that the bubble radius is increasing when the superheating of nanofluids increases.

Figure 5 reveals the influence of nanoparticle volume concentration χ on the temperature distribution around the growing of bubble in Al_2O_3/H_2O nanofluids where we note that the temperature distribution is proportional with χ. Figure 6 illustrates the temperature distribution around a growing bubble in Fe_3O_4/H_2O, $CuO//H_2O$, Al_2O_3/H_2O nanofluids and water fluid in the turbulent flow. We observed the temperature distribution surrounding the bubbly flow in water is more intensive than in other cases of nanoparticles nanofluids.
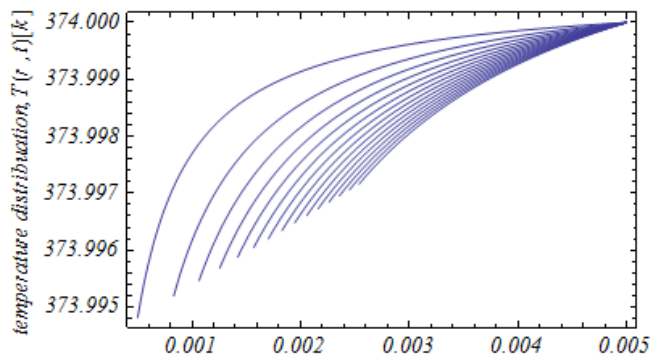
(6A) Al_2O_4/H_2O nanofluids

(6B) $CuO//H_2O$ nanofluids
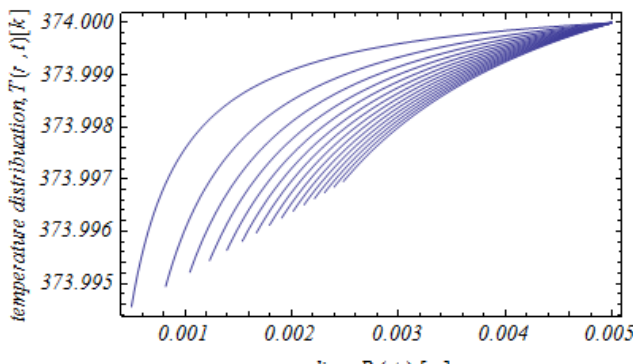
(6C) Fe_3O_4/H_2O nanofluids

(6D) pure water

Figure 6. Temperature distribution around a growing bubble wthin some different nanoparticles/ water nanofluid and pure water

The comparison between the present model in nanofluids and other published models [22, 24] in the cases turbulent flow are shown in Figure 7. Moreover, Figure 8 shows also the comparison between current model and previous studies [11, 20, 21, 31] in laminar flow and pure water. Our results agree with the results of the model [20] in the case nanoparticle/water nanofluids. In final, we conclude the behavior of bubble growth in nanoparticles/water nanofluids is lower than in the case of pure water in turbulent flow under the influence of heat transfer. Besides, the given results demonstrate that the growth of vapour bubble in laminar flow is bigger than in turbulent flow. The presented results are the agreement with the theoretical study in ref. [22] in turbulent flow.
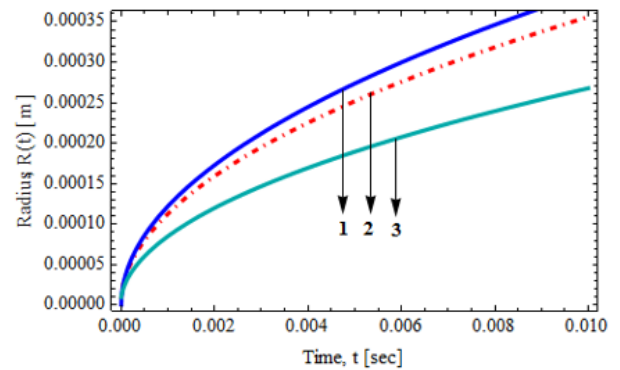

Figure 7. Comparison the present model in Fe_3O_4/H_2O nanofluids with the previous studies in refs. [22, 24] in turbulent flow. Curve 1- results in ref. [22], curve 2- results in ref. [24] and curve 3- presented results in equation (28)
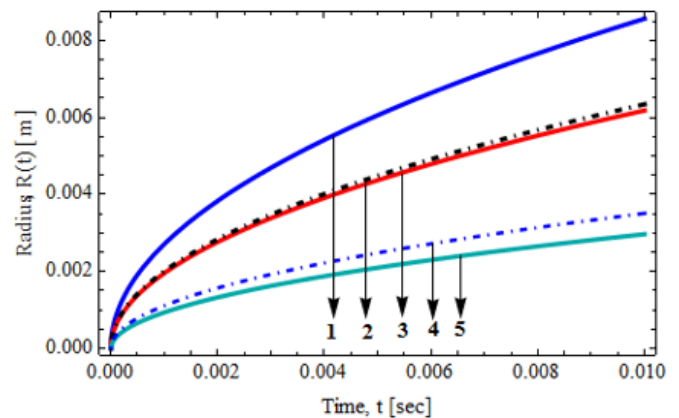

Figure 8. Comparison between the current model and some of previous works in the case of laminar flow. Curve 1- results by Scriven model [11], curve 2- results by Plesset and Zwick model [21], curve 3- results by Mohammadein and Gouda model [31], curve 4-results by Abu-Nab et al. [20], and curve 5- given solution in equation (28); for $C_k=1$

5. CONCLUSIONS

We present the results of the influence of heat transfer on bubble dynamics with some different types of nanofluids like Fe_3O_4/H_2O, CuO/H_2O, Al_2O_3/H_2O and water fluid between a two-phase turbulent flow. The results show that the effect of nanoparticles volume concentration in nanofluids reduces the growth process under the effect of heat transfer. The radius of vapour bubble in nanoparticles/water nanofluids is less than in pure water. The temperature distribution surrounding the regime of bubble growth in water is more intensive than in other cases of nanoparticles/water (Al_2O_3/H_2O, Fe_3O_4/H_2O,

and CuO/H_2O) nanofluids. Besides, the given results demonstrate that the bubble radius in turbulent flow is less than in turbulent flow. The obtained results of bubble growth in the nanofluids are agreement with the results of previous studies [11, 21, 22, 24, 31] at special cases of turbulent flow and laminar flow. This conclusion must be taken into account in the field of the nuclear power system and other applications of bubbles field.

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NOMENCLATURE

a, b	arbitrary constants in Eq. (2)
a_l	thermal diffusivity
C_k	constant defined by Eq. (3)
C_p	specific heat, $(J\,kg^{-1}\,K^{-1})$
c_0	constant defined by Eq. (4)
L	latent heat, (Jkg^{-1})
n	constant in Eq. (1)
V	velocity vector in mixture $(msec^{-1})$

p_e	Péclet number
r	radial coordinate
R	radius of bubble (m)
$\dot{R}$	velocity of bubble $((msec^{-1})$
T	temperature (K)
s	parameter defined by in (7)
t	time (sec)

Greek symbols

k_l	thermal conductivity, $(Wm^{-1}K^{-1}))$
$\mathcal{A}$	arbitrary constants in Eq. (26)
σ	surface tension $(Kg\,sec^{-2})$
ρ	density $(Kg\,m^{-3})$
β	parameter defined in (7)
$\Delta\Omega$	superheating temperature (K)
J_a	Jacob number
χ	nanoparticles volume concentration
ϕ_0	void fraction
ϵ	ratio of density

Subscripts

0	initial value
m	maximum value
Lam	laminar flow
Tur	turbulent flow
v	vapour
nf	nanofluid
l	Liquid
f	fluid
p	particle