Effects of viscosity variation and thermal effects in squeeze films

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ABSTRACT. *Since times immemorial it is known that the application of a material classified as lubricant to two relatively moving surfaces causes motion. The real mechanism of lubrication was explained by Reynolds of the role of lubricants is to support the load between two moving curved surfaces and thus minimizing wear and energy losses reducing friction between them. Thus the proper knowledge and understanding of the process of lubrication becomes a prime necessity to improve standard of design and efficiency of the mechanical system. A study of the behavior of any lubricated system can be best made by developing a mathematical model based on the above factors which depends upon a given a physical situation. The bearing characteristics such as load, flow flux, friction force etc. depends upon the pressure generated in the film and the lubrication process. An attempt has been made to obtain the governing equation for pressure in the lubricant film that are surveyed and summarized in this paper.*

RÉSUMÉ. *Depuis des temps immémoriaux, il est connu que l'application d'un matériau classé comme lubrifiant sur deux surfaces relativement mobiles provoque un mouvement.Le mécanisme réel de lubrification a été expliqué par Reynolds que le rôle de lubrifiants est de supporter la charge entre deux surfaces courbes en mouvement, minimisant ainsi l'usure et les pertes d'énergie, réduisant ainsi les frictions entre elles.Ainsi, la connaissance et la compréhension appropriées du processus de lubrification deviennent une nécessité primordiale pour améliorer les normes de conception et l'efficacité du système mécanique.Une étude du comportement de tout système lubrifié peut être mieux réalisée en développant un modèle mathématique basé sur les facteurs ci-dessus, qui dépend d'une situation physique donnée. Les caractéristiques des roulements, telles que la charge, le flux, la force de friction, etc., dépendent de la pression générée dans le film et du processus de lubrification.Une tentative a été faite pour obtenir l'équation régissant la pression dans le film de lubrifiant qui sont enquêtés et résumées dans le présent document.*

KEYWORDS: *squeeze film, reynolds equation, journal bearing, parallel and circular plates.*

MOTS-CLÉS: *squeeze pressé, équation de Reynolds, palier lisse, plaques parallèles et circulaires.*

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1. Introduction

Most of the lubricated systems may be regarded as combination of moving surfaces (plane/ curved loaded/ unloaded) with a thin film of foreign material known as lubricant between them. The presence of such a thin film is known to promote motion by helping to support load as well as minimizing friction. The characteristics of lubricant like pressure in the film, flow rate of lubricant, frictional force at the surface etc., and this system depend upon the behaviour of the surfaces, lubricant and film boundary conditions etc.

In the case of hydrodynamic lubrication, the equation governing the fluid pressure in a bearing system can be gained by coupling the equation of motion and the equation of continuity. This equation was first originated by Reynolds by considering the following assumptions.

The radius of curvature of the bearing surface is large in comparison with the film thickness.

-The lubricant is an incompressible Newtonian fluid.

-The viscosity of the lubricant is constant.

-Inertia and body force terms are small in comparison to the various pressure terms.

-Due to smallness of the fluid film thickness, the viscosity gradients across the film are large in comparison with the velocity gradients along the film.

-There is no slip at the fluid and solid interface.

Since early years, various attempts have been made to generalize the Reynolds equation applicable to the bearing system functioning under unusual conditions such as high temperatures, high pressures etc. (Reynolds, 1886; Fogg, 1946; Spikes and Cameron, 1974; Tipei, 1982; Ruggiero *et al*., 2011). The first attempt in this direction was made by Fogg (1946) who proposed thermal-wedge concept in the lubricant film. Cope (1949) relaxed the assumption made by Reynolds (1886) to extend the theory for density and variable viscosity along the fluid. But the variations of the fluid pressure and fluid properties across the fluid film remained neglected. In Wannier (1950) showed that the basic Reynolds equation could be obtained from the momentum equations by considering the variations of fluid pressure across the film. Considering the temperature and viscosity variations along as well as across the film, several other workers namely Cameron (1974), Cameron and wood, Hunter and Zienkiewicz (1960) also carried out studies. In early years, Dowson (1962) made a unified approach to study the generalized Reynolds equation by taking into consideration the variation of fluid characteristics across as well as along the film of lubricant.

A different method to study the effect of viscosity variation has also been proposed by Tipei (1982), Das *et al*. (2005). In this approach, we assumed a relation between film thickness and viscosity for convergent.

1.1. Effects of additives in lubrication

A great number of investigations have been carried out in the recent times by tribologists to increase the efficiency of base lubricants. Addition of certain compounds to the lubricant is one such attempt. The observation that adding small amounts of long-chain polymer solutions to Newtonian fluids produces the most effective lubricant due to the flow properties of the lubricants goes a long way in developing suitable lubricant.

The role of the additives is to minimize the sensitivity of the lubricant, to change the shear rate in particular, to improve the characteristics of the base oil and act as rust inhibitors (amine phosphate), corrosion inhibitors (sulphurised olefins), fire resistors (halogenated hydrocarbons), detergents (calcium / barium sulphonates). It is appropriate to point out at this stage that the nature and concentration of additives in the base oil, the resultant lubricants may change its Newtonian character under motion. For example: if the additive is in the form of particles of low concentration, the resulting fluid may be characterized by a micro polar fluid. On the other hand if the additive is in the form of long chain polymeric solution at high concentration the resultant fluid may behave as a non-Newtonian fluid.

It may be pointed out here that addition of additives of long chain molecules to the base lubricant produces a tendency in the former to cling themselves to the surface of the bearing in thin films. More pronounced effects are noticed when the surface is of porous nature. The particles attached to the surface interact with the base fluid and form a extreme viscous layer close to the surface. Thus the viscosity of the film is increased.

1.2. Effect of viscosity variation

Generally, majority of the lubricated systems can be envisaged as consisting of two sliding surfaces with a thin film of foreign material called lubricant between them. The lubricant film helps to support load besides reducing friction to a minimum. The lubrication characteristics of the bearing system like frictional force at the surface, lubricant flow rate, pressure in the film etc. depend on the characteristics of surface, characteristics of lubricant, boundary conditions etc.

The governing mathematical model of the lubricant film pressure in a bearing can be derived by coupling the momentum equation and continuity equation. The equation was first published by Reynolds in 1886 in his classical paper by considering the usual lubrication assumptions for an incompressible lubricant and is known after him like Reynolds equation. This equation does not consider the viscosity variation, slip at the surface, thermal compressibility etc. Cope modified the Reynolds equation by taking into consideration of the changes in density and viscosity along the film. Recently, the performance of viscosity variation across as well as along the hydrodynamically lubricated film was studied by Mohite *et al*. (2005), Hu *et al*. (2009) by assuming a connection between film thickness and the viscosity for the convergent films. It was pointed out that the co-efficient of friction is reduced significantly when the viscosity of the lubricant varies across the thickness of the film.

1.3. Thermal effects

Currently, thermal effects are more significant towards limit design and bearing operating characteristics be accurately predicted due to high dependence of lubricant viscosity on field temperature in the lubricant. The temperature field in the lubricant is the consequence of hydrodynamic lubrication process which consists essentially of two surfaces due to relative motion shearing fine layer of film of viscous fluid. With the exception of bearings running at low speeds, the heat balance (Reddy *et al*., 2010) is an important part of the analysis of bearing operating with fluid film. The operating temperature of the bearing is also of interest because the strength of the bearing material is a function of temperature.

In order to account for thermal effects, the classical isothermal theory of hydrodynamic lubrication has to be extended. Energy balance equation is required and has to be solved simultaneously along with the conservation equations of mass and momentum. Coupling of these governing equations makes the solution very complicated and further simplification (approximation) may be necessary.

Another approach for the consideration of thermal effects in lubrication has also been reported. Viscosity-temperature relation has been replaced by viscosity film thickness relation on the assumption that the highest temperature occurs in the zone where the film thickness is least. This approach does not incorporate energy equation at all, so the complexity of the problem made by energy equation is overcome. But on the other hand, the temperature field in the lubricant is not fully known. It seems therefore that this way of approach has been used sparingly.

A good number of papers touched upon the effects of temperature on hydrodynamic bearing performance and requirements. Tipei and Nica obtained the three dimensional temperature variation of journal bearing oil film. Separate relationship was established for both the convergent and divergent regions of the bearing taking into account viscosity variations and side leakage. The theory was well matched with experimental data. Naduvinamani (2016) studied the thermal effects by obtaining solutions to Micropolar fluid squeeze film lubrication of finite porous bearing. Lu *et al*. (2006) considered the effect of inertia in magneto-hydrodynamic annular squeeze films of rigid cylindrical rollers by a Newtonian incompressible fluid theoretically and experimentally.

Lin *et al*., (2013) studied squeeze film behavior between a sphere and a flat plate thermal EHD problem for some fluid to compare the predicted tractions with those obtained experimentally. Manivasakan and Sumathi (2011) presented a numerical solution to theoretical investigation of couple stress in circular geometry. Pressure and temperature distributions and film shape for fully flooded conjunctions were obtained for a paraffinic lubricant and various dimensionless speed parameters while the dimensionless load and material parameters were held constant. Gordon *et al*. (2005) analyzed the archeological and thermal effects in lubricated elliptical

Hertzian constants with collinear speeds. Stress, temperature and shear strain rate distributions along and across the film were calculated for non-Newtonian vicso-elastoplastic fluid including lubricant convection and conduction effects. Further they calculated traction force in CFD analysis contact using rotating machinery fluid model and Roelands relationship for estimating lubricant viscosity. There are several other notable papers on thermal effects in lubrication for different geometries and it is difficult to refer to all of them.

1.4. Squeeze film lubrication

In phenomenon, in which two lubricated surfaces approach each other with normal velocity, in the film of interposing lubricant within the two surfaces acting as a cushion and preventing the surfaces from making instantaneous contact is known as squeeze film lubrication. The fluid characteristics, design of surface and the load applied govern the time required for squeezing out the lubricant.

In general the connection between approach rate and load bearing capacity of the two lubricated surfaces is studied in most squeeze film analysis. The squeeze films that are relevant in engineering applications such as internal combustion engines have initiated several workers to focus their attention on this subject Tichy (1995) studied influences of fluid inertia and visco elasticity on the one-dimensional squeeze film bearings. The role of squeeze films between complaint surfaces in long squeeze film bearings was studied by Blech (1983). Squeeze film problem in elastohydrodynamic lubrication was studied by Sanswade and musafumi, Tsukejshara. Ruggiero developed a mathematical model for human ankle joint in influence of consistency variation of squeeze-film lubrication (Ruggiero *et al.*, 2011). Keeping the above mentioned considerations in view, an attempt has been made through this dissertation to analyze the effects viscosity variations and velocity-slip in squeeze film and spherical bearing systems. The work carried out is presented in summery hereunder.

1.5. Novelty of the manuscript

Manuscript shows the influence of variation in viscosity and effect of thermal in squeeze films. This manuscript will be very helpful in many engineering application like gears, machine tools, disk clutches, aircraft engines, dampers and human joints.

2. Generalized reynolds equation for layer medium

The equation governing fluid pressure in a bearing system can be derived by coupling the momentum equation and the continuity equation and was first originated by Reynolds. He ignored slip at the surface, inertia, roughness, viscosity variation, thermal compressibility for the derivation of equation.

But in deriving this equation viscosity variation, slip at the surface, roughness, inertia, thermal compressibility was ignored. Attempts to revise the equation were made by several authors. Cope modified the Reynold’s equation by considering viscosity variation and density variation along the fluid film. The variation of viscosity across the film thickness was studied by Zienkieicz and Cameron who stressed that the temperature gradient and viscosity variation were also important. A unified approach was made by Dowson to generalize the Reynolds equation by assuming the changes in properties of fluid and together with thickness of lubricant film and ignoring the slip influences at the bearing surfaces.By assuming both energy and Reynolds equations, further studies, in lubricated systems involving the influence of viscosity variation. Additives are added to improve the characteristics of lubricant. It is experimentally proved by Rao and Prasad (2003) that additives form a high viscous layer close to the surface because of their affinity to solid surface. This aspect can be studied theoretically by dividing the lubricant zone in to layered medium, one at the middle and other at the periphery. Keeping this in view, we derive the following the generalized Reynolds equation.

2.1. Fundamental equation

Assuming the streamline flow of a fluid within the two surfaces as shown the Figure 1, Assuming the changes in properties of fluid and together with thickness of film, the momentum and energy equations of continuity for a Newtonian fluid can be as follows:

$$\rho\frac{du}{dt}=\rho x-\frac{\partial p}{\partial x}+\frac{2}{3}\frac{\partial}{\partial x}\left[\eta\left(\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y}\right)\right]+\frac{2}{3}\frac{\partial}{\partial x}\left[\eta\left(\frac{\partial u}{\partial x}-\frac{\partial w}{\partial z}\right)\right]+\frac{\partial}{\partial y}\left[\eta\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)\right]+\frac{\partial}{\partial z}\left[\eta\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right)\right]$$

$$\rho\frac{dv}{dt}=\rho y-\frac{\partial p}{\partial y}+\frac{2}{3}\frac{\partial}{\partial y}\left[\eta\left(\frac{\partial v}{\partial y}-\frac{\partial u}{\partial x}\right)\right]+\frac{2}{3}\frac{\partial}{\partial y}\left[\eta\left(\frac{\partial v}{\partial y}-\frac{\partial w}{\partial z}\right)\right]+\frac{\partial}{\partial z}\left[\eta\left(\frac{\partial w}{\partial w}+\frac{\partial v}{\partial z}\right)\right]+\frac{\partial}{\partial x}\left[\eta\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)\right] \quad (1)$$

$$\rho\frac{dw}{dt}=\rho z-\frac{\partial p}{\partial z}+\frac{2}{3}\frac{\partial}{\partial z}\left[\eta\left(\frac{\partial w}{\partial z}-\frac{\partial u}{\partial x}\right)\right]+\frac{2}{3}\frac{\partial}{\partial z}\left[\eta\left(\frac{\partial w}{\partial z}-\frac{\partial v}{\partial y}\right)\right]+\frac{\partial}{\partial x}\left[\eta\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)\right]+\frac{\partial}{\partial y}\left[\eta\left(\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}\right)\right]$$

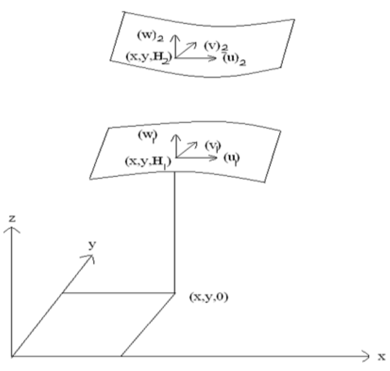

Figure 1. Streamline flow of a fluid within two surfaces

$$\frac{\partial \rho}{dt}+\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)=0 \tag{2}$$

Based on the general consideration of theory of lubrication the above equation (4) may be simplified as:

$$\frac{\partial p}{\partial x}=\frac{\partial}{\partial z}\left[\eta\frac{\partial u}{\partial z}\right] \text{ and } \frac{\partial p}{\partial y}=\frac{\partial}{\partial z}\left[\eta\frac{\partial v}{\partial z}\right] \tag{3}$$

where $P=P(x, y)$ denotes the pressure in film and 'η' is the viscosity.

The boundary conditions can be expressed as follows:

$u=u_1$, $v=v_1$ at $z=H_1$

$u=u_2$, $v=v_2$ at $z=H_2$

Integrating the above equation with the boundary conditions, the following expressions for the velocities of fluid film are achieved.

$$u=u_1+\int_{u_1}^{z}\frac{zdz}{\eta}\frac{\partial p}{\partial x}+\left[\frac{(u_2-u_1)-\frac{\partial p}{\partial x}f_1}{f_0}\right]\int_{z=H_1}^{z}\frac{dz}{\eta} \tag{4}$$

where $f_0=\int_{H_1}^{H_2}\frac{dz}{\eta}$, $f_1=\int_{H_1}^{H_2}\frac{zdz}{\eta}$.

$$v=v_1+\int_{H_1}^{z}\left(\frac{zdz}{\eta}\right)\frac{\partial p}{\partial y}+\left[-\frac{(v_2-v_1)-\frac{\partial p}{\partial y}f_1^1}{f_0^1}\right]\int_{z=H_1}^{z}\frac{dz}{\eta} \tag{5}$$

where $f_0^1=\int_{H_1}^{H_2}\frac{dz}{\eta}$, $f_1^1=\int_{H_1}^{H_2}\frac{zdz}{\eta}$.

Integrating the equation of continuity (2) by taking limits from H_1 to H_2, we get

$$\int_{H_1}^{H_2}\frac{\partial p}{\partial t}dz+\int_{H_1}^{H_2}\frac{\partial(\rho x)}{\partial x}dz+\int_{H_1}^{H_2}\frac{\partial(\rho v)}{\partial y}dz+\left[\rho w\right]_{H_1}^{H_2}=0 \tag{6}$$

Applying the usual integral formula,

$$\frac{\partial}{\partial x}\int_{H_1}^{H_2} f\left(x,y,z\right)dz = \int_{H_1}^{H_2}\frac{\partial}{\partial x} f\left(x,y,z\right)dz + f\left(x,y,H_2\right)\frac{\partial H_2}{\partial x} - f\left(x,y,H_1\right)\frac{\partial H_1}{\partial x} \tag{7}$$

We have the equation,

$$\int_{H_1}^{H_2}\frac{\partial p}{\partial t}dz + \int_{H_1}^{H_2}\frac{\partial(\rho u)}{\partial z}dz + \int_{H_1}^{H_2}\frac{\partial(\rho v)}{\partial x}dz + \left[\rho u_2\right]\frac{\partial H_2}{\partial x} - (\rho v_2)\frac{\partial H_1}{\partial x} + (\rho v_1)\frac{\partial H_1}{\partial y} + \left[\rho w\right]_{H_1}^{H_2} = 0 \tag{8}$$

The integrals of ρv and ρu can be evaluated by integrating by parts which gives the following equation.

$$\int_{H_1}^{H_2}\frac{\partial p}{\partial t}dz + H_2\left[\frac{\partial}{\partial x}(\rho u)_2 + \frac{\partial}{\partial x}(\rho v)_2\right] - H_1\frac{\partial(\rho u)_1}{\partial x} + \frac{\partial(\rho v)_1}{\partial y} - \frac{\partial}{\partial x}\left[\int_{H_1}^{H_2}(\rho z\frac{\partial u}{\partial z} + uz\frac{\partial p}{\partial z})dz\right] -$$

$$\frac{\partial}{\partial y}\left[\int_{H_1}^{H_2}(\rho z\frac{\partial v}{\partial z} + vz\frac{\partial p}{\partial z})dz\right] + [\rho w]_{H_1}^{H_2} = 0 \tag{9}$$

In the above equation, adding the expressions of *v*, *u* for their derivatives, we have

$$\frac{\partial}{\partial x}\left[(f_2 + G_1)\frac{\partial p}{\partial x}\right] + \frac{\partial}{\partial y}\left[(f_2 + G_1)\frac{\partial p}{\partial y}\right] = H_2\left[\frac{\partial}{\partial x}(\rho u)_2 + \frac{\partial}{\partial y}(\rho v)_2\right] - H_1\left[\frac{\partial}{\partial x}(\rho u)_1 + \frac{\partial}{\partial y}(\rho v)_1\right] -$$

$$+\int_{H_1}^{H_2}\frac{\partial \rho}{\partial t}dz + \left[\rho w\right]_{H_1}^{H_2} \tag{10}$$

where $f_0 = \int_{H_1}^{H_2}\frac{dz}{\eta}$, $f_1 = \int_{H_1}^{H_2}\frac{zdz}{\eta}$, $f_2 = \int_{H_1}^{H_2}\frac{\rho z}{\eta}\left(z - F_1/F_2\right)dz$, $f_3 = \int_{H_1}^{H_2}\frac{\rho z dz}{\eta}$

And $G_1 = \int_{H_1}^{H_2}\left\{z\frac{dp}{dz}\left[\int_{H_1}^{z}\frac{zdz}{\eta} - \frac{F_1}{F_0}\int_{H_1}^{z}\frac{dz}{\eta}\right]dz\right\}$, $G_2 = \int_{H_1}^{H_2}\left\{z\frac{dp}{dz}\left[\int_{H_1}^{z}\frac{dz}{\eta}\right]dz\right\}$, $G_3 = \int_{H_1}^{H_2}\left[z\frac{\partial p}{\partial z}\right]dz$.

From the above equation, deduction of general form of Reynolds equation which is obtained by Dowson with the consideration of $H_1=0$ and $H_2=h$. The various forms of equation of fluid film lubrication derived by Reynolds, Cope and Zienkewicz can also be deduced from the generalized equation.

2.1.2. Various special cases

case-1

In this case we consider ρ as a constant. Then all the G–functions vanish and generalized equation simplifies to the following form:

$$\frac{\partial}{\partial x}\left[f_2\frac{\partial p}{\partial x}\right]+\frac{\partial}{\partial y}\left[f_2\frac{\partial p}{\partial y}\right]=H_2\left[\frac{\partial}{\partial x}(u)_2+\frac{\partial}{\partial y}(v)_2\right]-H_1\left[\frac{\partial}{\partial x}(u)_1+\frac{\partial}{\partial y}(v)_1\right]-\frac{\partial}{\partial x}\left[\frac{(u_2-u_1)f_3}{f_0}\right]-$$

$$\frac{\partial}{\partial y}\left[\frac{(v_2-v_1)f_3}{f_0}\right]+\left[w\right]_{H_1}^{H_2} \tag{11}$$

where $f_0=\int_{H_1}^{H_2}\frac{dz}{\eta}$, $f_1=\int_{H_1}^{H_2}\frac{zdz}{\eta}$, $f_2=\int_{H_1}^{H_2}\frac{z}{\eta}\left(z-f_1/f_2\right)dz$, $f_3=\int_{H_1}^{H_2}\frac{zdz}{\eta}$

Case-2

The viscosity of the lubricant may changes throughout the film and can be differ from close to the bearing surfaces owing to reaction of surfactants and additives.

Consider the logical cases in which the lubricant viscosity close to the bearing surface may be differ from that of the central region, we get:

$$\eta=\eta_1(x,y),\ \eta=\eta_2(x,y),\ \eta=\eta_3(x,y) \tag{12}$$

The generalized equation becomes,

$$\frac{\partial}{\partial x}\left[f_2\frac{\partial p}{\partial x}\right]+\frac{\partial}{\partial y}\left[f_2\frac{\partial p}{\partial y}\right]=H_2\left[\frac{\partial}{\partial x}(u)_2+\frac{\partial}{\partial y}(v)_2\right]$$

$$-H_1\left[\frac{\partial}{\partial x}(u)_1+\frac{\partial}{\partial y}(v)_1\right]-\frac{\partial}{\partial x}\left[\frac{(u_2-u_1)f_3}{F_0}\right]-\frac{\partial}{\partial y}\left[\frac{(v_2-v_1)f_3}{F_0}\right]+\left[w\right]_{H_1}^{H_2} \tag{13}$$

where $f_0=\frac{h_1}{\eta_1}+\frac{h_2}{\eta_2}+\frac{h_3}{\eta_3}$, $f_1=\frac{h_1(2H_1+h_1)}{2\eta_1}+\frac{h_2(2H_1+2h_1+h_2)}{2\eta_2}+\frac{h_3(2H_1+2h_1+2h_2+h_3)}{2\eta_3}$,

$$f_2=\frac{1}{3\eta_1}\{(H_1+h_1)^3-H_1\}^3+\frac{1}{3\eta_2}\{(H_1+h_1+h_2)^3-(H_1+h_1)\}^3+\frac{1}{3\eta_3}\{H_2{}^3-(H_1+h_1+h_2)\}^3-\frac{f_1f_3}{f_0},$$

$$f_3=\frac{h_1}{2\eta_1}(2H_1+h_1)+\frac{h_2}{2\eta_2}(2H_1+2h_1+h_2)+\frac{h_3}{2\eta_3}(2H_1+2h_1+2h_2+h_3)$$

where $u_1=U$ *and* $[w]_{H_1}^{H_2}=u_1\frac{\partial H_1}{\partial x}-v_s$, v_s the resultant velocity towards the film.

Case-3

$u_1=U$ and $v_2=u_2=v_1=0$.

The generalized equation reduces to,

$$\frac{\partial}{\partial x}\left[f_2\frac{\partial p}{\partial x}\right]+\frac{\partial}{\partial y}\left[f_2\frac{\partial p}{\partial y}\right]=U\frac{\partial}{\partial x}\left(\frac{f_3}{f_0}\right)+[w]_{H_1}^{H_2} \tag{14}$$

$$f_0 = \frac{h_1}{\eta_1} + \frac{h_2}{\eta_2} + \frac{h_3}{\eta_3}, \; f_1 = \frac{h_1(2H_1 + h_1)}{2\eta_1} + \frac{h_2(2H_1 + 2h_1 + h_2)}{2\eta_2} + \frac{h_3(2H_1 + 2h_1 + 2h_2 + h_3)}{2\eta_3}$$

$$f_2 = \frac{1}{3\eta_1}\{(H_1 + h_1)^3 - H_1\}^3 + \frac{1}{3\eta_2}\{(H_1 + h_1 + h_2)^3 - (H_1 + h_1)\}^3 + \frac{1}{3\eta_3}\{H_2{}^3 - (H_1 + h_1 + h_2)\}^3 - \frac{f_1 f_3}{f_0}$$

$$f_3 = \frac{h_1}{2\eta_1}(2H_1 + h_1) + \frac{h_2}{2\eta_2}(2H_1 + 2h_1 + h_2) + \frac{h_3}{2\eta_3}(2H_1 + 2h_1 + 2h_2 + h_3) \tag{15}$$

Case-4

Let $\eta_1 = \eta_2; \eta_3 = \mu$ & $H_1=0$, $H_2=a$ and $h_1=h_3=H/2$ $h_2=h$.

Then we obtain Reynolds equation in one - dimensional for two layer lubricant is,

$$\frac{\partial}{\partial x}\left(f_1 \frac{\partial p}{\partial x}\right) + U\frac{\partial}{\partial x}\left(h + 2a\right) = V \tag{16}$$

$$f_1 = \frac{1}{3}\frac{a^3}{2\eta} + \frac{1}{4}(h+a)^2\frac{2a}{\eta} + \frac{h^3}{12\mu} \tag{17}$$

3. Effects of viscosity variation in squeeze films

The equation, governing the fluid pressure produced in lubricant film can be derived by paring the momentum and continuity equations was first derived by Reynolds and is called as Reynolds equation. But in deriving this equation, the thermal and compressibility effects as well as viscosity variation in the film are ignored, in practice; there will be viscosity variation in the pressure and the presence of additives etc. A considerable amount of work is done using this aspect of viscosity variation by different workers.

In the previous section, we derived the Reynolds equation for two layered lubricant and various special cases have been obtained. Now in this chapter we apply the Reynolds equation for squeeze film bearings to study the viscosity variation across the film and the various characteristics of bearings.

3.1. Parallel plates

In this section, we consider the flow of lubricant between two parallel plates of length 2d, approaching each other normally with velocity V as given in the Figure 2. The plates are separated by a film thickness 2h. The viscosity varies across the film. Taking this into account, we divide the lubricant zone into layer medium. The Reynolds equation for such a flow is given by equation (16) for one dimensional equation by taking $U=0$, we get

$$\frac{\partial}{\partial x}\left(f_1 \frac{\partial p}{\partial x}\right)=V \tag{19}$$

$$f_1=\frac{1}{3}\frac{a^3}{2\eta}+\frac{1}{4}(h+a)^2\frac{2a}{\eta}+\frac{h^3}{12\mu} \tag{20}$$

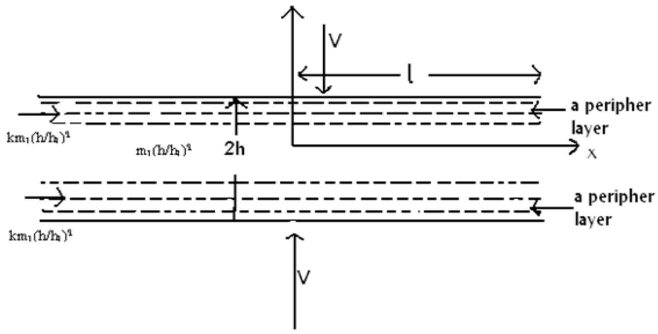

Figure 2. Squeeze film between parallel plates

Now taking $\eta=k\mu$ and $\mu=m_1(h/h_1)^q$ and then integrating with respect to x, since pressure attains its maximum at x = 0, we have

$$\frac{\partial p}{\partial x}=-\frac{m_1}{h_1^3}\left(\frac{3vx}{f}\right)\left(\frac{h_1}{h}\right)^{3-q} \tag{21}$$

where $f=1-\left(1-{}^{1}\!/_{k}\right)\left\{1-\left(1-{}^{a}\!/_{h}\right)^3\right\}$, Integrating equation (21) using condition $p=0$ at $x=d$, denoting pressure p with $p_{k,q}$ is

$$p_{k,q}=\frac{3m_1V}{2h_1^3 f}\left(\frac{h_1}{h}\right)^{3-q}\left[d^2-x^2\right] \tag{22}$$

The load capacity $w_{k,q}$ per unit width is given by

$$w_{k,q}=2\int_0^d p_{k,q}(x)dx \tag{23}$$

Integrating equation (23) with equation (22), we get

$$w_{k,q}=2\int_0^d \frac{3m_1V}{2h_1^3 f}\left(\frac{h_1}{h}\right)^{3-q}\left[d^2-x^2\right]dx,\ w_{k,q}=\frac{2m_1V}{h_1^3 f}\left(\frac{h_1}{h}\right)^{3-q}d^3 \tag{24}$$

$$\overline{w} = \frac{1}{f}\left(\frac{h_1}{h}\right)^{3-q} \tag{25}$$

where $\overline{w} = \dfrac{w_{k,q} h_1^3}{2m_1 V d^3}$

The squeezing time $t_{k,q}$ from initial film thickness $2h_1$ to a subsequent film thickness $2h_2$, say, is obtained by putting $v = -\partial h/\partial t$ in equation (24) and integrating, we have

$$w_{k,q} = \frac{2m_1 d^3}{h_1^q h^{3-q}}\left(\frac{-\partial h}{\partial t}\right), \; \frac{\partial t}{\partial h} = \frac{-2m_1 d^3}{h_1^q w_{k,q}} \frac{1}{h^{3-q}}$$

Integrating the above equation and denoting time t by $t_{k,q}$

$$t_{k,q} = \frac{2m_1 d^3}{h^q{}_1 w_{k,q}} \int_{h_2}^{h_1} \frac{dh}{f(h)^{3-q}} \tag{26}$$

Let $H=h/h_1$, then $dH=dh/h_1$ and $h=Hh_1$

From equation (26) by taking $\overline{h_2} = h_2/h_1$

$$t_{k,q} = \frac{2m_1 d^3}{h_1^2 w_{k,q}} \int_{\overline{h_2}}^{1} \frac{dH}{\overline{\overline{f}}(h_1 H)^{3-q}}, \text{ where } \overline{f} = 1 = (1-1/k)\left\{1-\left(1-\overline{a}/H\right)^3\right\}$$

Simplifying the above equation becomes

$$\bar{t} = \int_{\overline{h_2}}^{1} \frac{dH}{f(h_1 H)^{3-q}}, \text{ where } \bar{t} = \frac{t_{k,q} h_1^2 w_{k,q}}{2m_1 d^3} \tag{27}$$

3.1.1. Parallel circular plates

We consider squeezing between two circular parallel plates each of radius R separated by a film thickness 2h as in Figure 3. With normal velocity v symmetrically, plates approach each others. The Reynolds equation for the flow of power law lubricant in the radial direction r can be obtained as

$$\frac{1}{r}\frac{\partial}{\partial r}\left[\frac{1}{3} h_1^q f h^{3-q} r\left(\frac{-1}{m_1}\frac{\partial p}{\partial r}\right)\right] = v \tag{28}$$

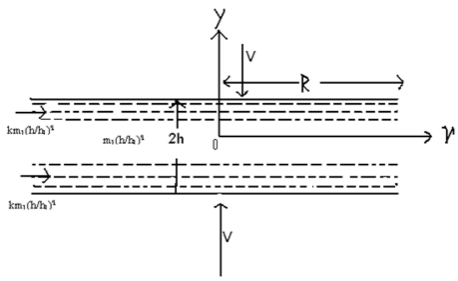

Figure 3. Squeeze film between parallel circular plates

Where f is defined in equation (20). Now using the boundary conditions

$$\frac{\partial p}{\partial r} = 0 \text{ at } r=0 \text{ \&- } p=0 \text{ at } r=R \tag{29}$$

We can obtain an expression for pressure by integrating twice equation (28). Denoting it by $p_{k,q}$, we have

$$p_{k,q} = \frac{3m_1}{4h_1^q}\frac{v}{f}\frac{1}{h^{3-q}}\left[R^2 - r^2\right] \tag{30}$$

The load capacity $w_{k,q}$ is given by

$$w_{k,q} = \int_0^R 2\pi rpdr \tag{31}$$

Which on using, equation (30) yields

$$w_{k,q} = \frac{3\pi m_1}{8h_1^3}\frac{v}{f}R^4\left(\frac{h_1}{h}\right)^{3-q} \tag{32}$$

The time of squeezing $t_{k,q}$, to reduce the initial film thickness $2h_1$ to a subsequent film thickness $2h_2$ is given by

$$t_{k,q} = \frac{3\pi m_1}{8w_{k,q}}\frac{R^4}{h_1^q}\int_{h_2}^{h_1}\frac{dh}{f(h)^{3-q}} \tag{33}$$

4. Results

In order to highlight the effects of various physical parameters on the load capacity w, the numerical computations are performed and numerical results for the load capacity are evaluated with the help of graphical illustration from figures 4 to 11. It is observed that due to increase in thermal parameter *q*, load capacity and response time *t* decrease. Figure 4 demonstrates that load capacity gradually decreases with increase in film thickness. This happened because of the compliant surface becomes more elastic due to increase in film thickness. Figure 5 illustrates that due to enhancement in consistency ratio *k*, load capacity also increases. It is noticed from Figure 6 that load capacity decreases due to increase in peripheral layer thickness *a* when (k<1). Figure 7 illustrates that load capacity is getting enhanced due to enhancement in peripheral layer thickness when ($k>1$). It is obvious from figures 8 and 9 that response time *t* is inverse proportional to oil film thickness. Figure 10 and 11 illustrate that due to increase in peripheral layer thickness, there is decrement in response time when k<1 and opposite behaviour of response time when k>1.

5. Conclusion

In this manuscript a general form of Reynolds equation is studied and applied to observe the effects of thermal factors at viscous layer near the peripheral in squeeze film between parallel plates and parallel circular plates. It is observed that the presence of viscous layer near the peripheral is increased the load capacity and time of squeezing, but due to thermal factor load capacity and squeezing time decreases.

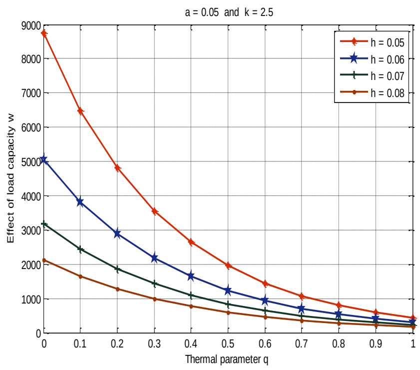

Figure 4. Variation of load ratio with changes of thermal factor for various oil film thickness

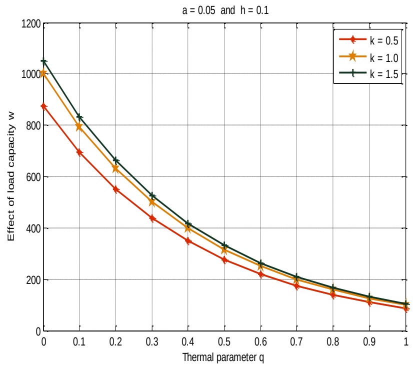

Figure 5. Variation of load ratio with increases of thermal factor for various consistency ratio

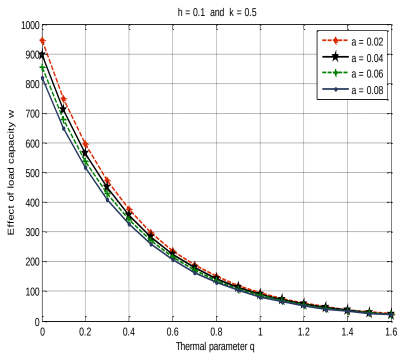

Figure 6. Variation of load ratio with increases of thermal factor for various peripheral layer thickness

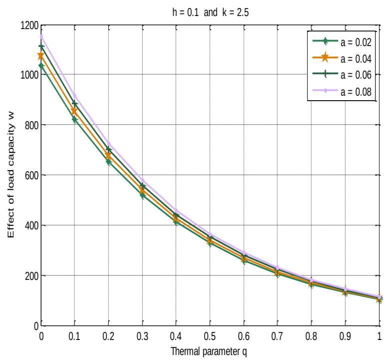

Figure7. Variation of load ratio with increases of thermal factor h=0.1 and k=2.5

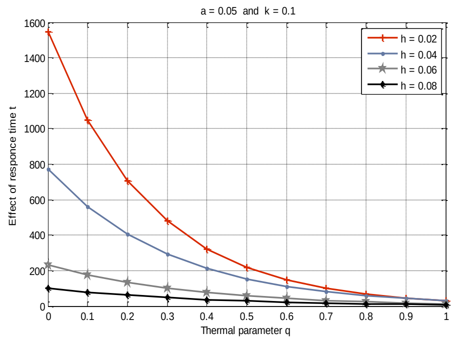

Figure 8.Variation of response time ratio with increases of thermal factor for a 0.05 and k=0.1

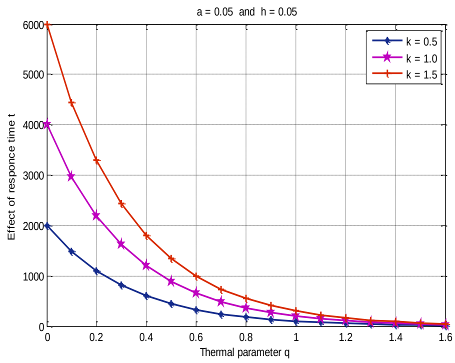

Figure 9. Variation of response time ratio with increases of thermal factor h=0.05 and k=0.05

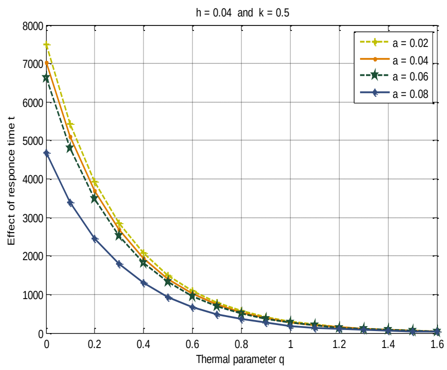

Figure 10. Variation of response time ratio with increases of thermal factor h=0.04 and k=0.5

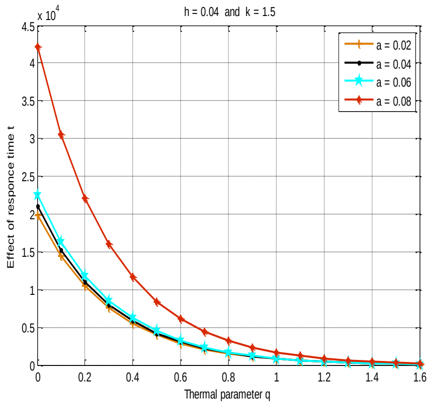

Figure 11. Variation of response time ratio with increases of thermal factor h=0.04 and k=1.5

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