An application of genetic algorithm to improving measurement accuracy of laser interference absolute gravimeter

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ABSTRACT. The purpose of this study was to improve the overall measurement accuracy of laser interference absolute gravimeter. The genetic algorithm approach was adopted to the multiparameter optimization. This paper presents a two-stage vibration isolation system model of laser interference absolute gravimeter, builds a mathematical model of vibration isolation system and searches the optimal solutions of each parameter in the vibration isolation system model based on the genetic algorithm. The results obtained in this study revealed that the overall measurement accuracy of laser interference absolute gravimeter improved by 1.5 times. The findings of this study may serve as a method to improve the overall measurement accuracy of laser interference absolute gravimeter.

RÉSUMÉ. Le but de cette étude était d'améliorer la précision de mesure globale du gravimètre absolu à interférence laser. L'approche par algorithme génétique a été adoptée pour l'optimisation multiparamétrique. Cet article présente un modèle de système d'isolation de vibration en deux étapes du gravimètre absolu à interférence laser, construit un modèle mathématique du système d'isolation de vibration et rechercher des solutions optimales de chaque paramètre du modèle de système d'isolation de vibration basé sur l'algorithme génétique. Les résultats obtenus dans cette étude ont révélé que la précision de mesure globale du gravimètre absolu à interférence laser était améliorée de 1,5 fois. Les résultats de cette étude peuvent servir de méthode pour améliorer la précision de mesure globale du gravimètre absolu à interférence laser

KEYWORDS: vibration isolation, genetic algorithm, absolute gravimeter, gravity measurement.

MOTS-CLÉS: isolation de vibration, Algorithme génétique, gravimètre absolu, mesure de la gravité.

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1. Introduction

Classic laser interference absolute gravimeter was designed based on the principle of Michelson interference; it measures the falling object's time and displacement in high precision, compared with the reference corner cube. There's a premise that the falling object need to be in linear motion or absolute rest during its falling. However, due to the environmental vibration and the self-vibration generated by the control system of the falling object, the reference corner cube presented a complex vibration mode during the object's free falling. It is difficult to correct these deviations, so it is necessary to take measures during the design period of the apparatus to reduce the impact that the system vibration has on the reference corner cube (Zhang, 2015).

For the processing of the vibration, a vibration isolation device using a special designed super spring has been invented in the U.S. (it has a natural period of vibration about 30-60 seconds) after ten years' research, they had successfully isolated the reference corner cube and the systemic vibration, which made absolute gravimeter FG5's precision of measurement close to 1-2 microgram, this is a successful case to implement vibration processing. But using this technology to isolate vibration has some disadvantages: the complexity of its structure and process, the difficulty of its technology and its large size (64cm×56cm×31cm), which is not conductive to the apparatus' miniaturization (Lu, 2015).

China Metrology Institute's high precision absolute gravity of NIM-II uses two kinds of technologies to deal with vibration： increase the falling object's falling tour and adopt that long-period seismometer hanging a reference corner cube, but this method's effect of vibrational isolation is not very good while it has a large volume (Wu *et al*., 2012).

In the research conducted by Institute of Geophysics, China Earthquake Administration, through recast of the falling object's falling tracks to do vibrational compensation, thus to reduce the deviation caused by vibration. Numerical simulation and experimental results show that using the algorithm of vibrational compensation could make the measurement precision improve almost one time. (Long, 2012). But due to the randomness of the vibration signal, inconsistency of measured vibration signal and the actual vibration information of reference corner cube, vibration signal's measurement precision and frequency band and other questions, the gravity measurement precision of the prototype, which was designed based on the algorithm, is not ideal so far.

As the traditional vibrational isolation system, the first level of the vibrational isolation system's frequency band is narrow, while the second level of the vibrational isolation system has a better effect with low frequency interference and it improved vibrational isolation effect to 2~3dB, an 8~10dB improvement to resonant frequency interference. which is a great improvement of vibrational isolation.

This paper proposes the two-stage vibrational isolation system based on the actual research of laser interference absolute gravimeter. Moreover, it adds the genetic algorithm to get parameters' optimal solutions in the two-stage vibrational isolation system's mathematical model quickly.

2. Mathematical modeling of two-stage vibration isolation system of reference corner cube

In this system, the two-stage vibrational isolation platform is equivalent to the reference corner cube of the absolute gravimeter. The reference corner cube as the second level of the vibrational isolation platform, is hanging on the first level of the vibrational isolation platform by elastic element. The first level of the vibrational isolation platform is hanging on the outside box by elastic component. This structure is two-stage spring system, its mechanical model is shown in Fig.1, it is a two-degrees-of freedom model. In Figure. 1, m_1 is the mass of the first level of the vibrational isolation platform, k is the rigidity of the spring, G_1 is feedback coil's electromechanical coupling coefficient, m_2 is the mass of the second level of the vibrational isolation platform, μ is the magnetic constant of permanent magnet, y is the displacement of outside box, x_1 is the displacement of the first level of the vibrational isolation platform, x_2 is the displacement of the reference corner cube, β_1 is the damping coefficient of the the first level of the vibrational isolation platform, β_2 is the damping coefficient of the the first level of the vibrational isolation platform, f(t) is feedback force (Hua, 2005).

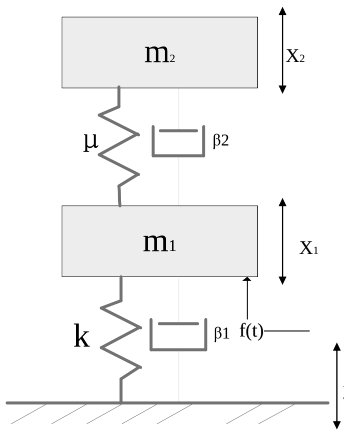

Figure 1. Mechanical model of vibration isolation system

The dynamics equations (Robert, 1983) could be got from the mechanical model shown in Fig.1:

$$m_1\ddot{x}_1(t) = K[y(t) - x_1(t)] + \mu[x_2(t) - x_1(t)] + \beta_1[\dot{y}(t) - \dot{x}_1(t)] + \beta_2[\dot{x}_2(t) - \dot{x}_1(t)] + f(t) \quad (1)$$

$$m_2\ddot{x}_2(t) = \mu[x_1(t) - x_2(t)] + \beta_2[\dot{x}_1(t) - \dot{x}_2(t)] \quad (2)$$

The feedback force F(t) comes from electromagnetic damper and it depends on transfer function of electromagnetic damper. In order to analyze more easily, we set f(t) as:

$$f(t) = G[x_2(t) - x_1(\mathrm{t})] + \beta_s[\dot{x}_2(t) - \dot{x}_1(\mathrm{t})] \tag{3}$$

Use the Laplace transform to combine the three formula above:

$$M_1 S^2 X_1 = (K + \beta_1 \mathrm{S})(\mathrm{Y} - X_1) + (\mu + \beta_2 \mathrm{S})(X_2 - X_1) + F(\mathrm{S}) \tag{4}$$

$$M_2 S^2 X_2 = (\mu + \beta_2 \mathrm{S})(X_1 - X_2) \tag{5}$$

$$F(\mathrm{S}) = (\mathrm{G} + \beta_S)(X_2 - X_1) \tag{6}$$

Combine the three formula:

$$\frac{X_2}{Y} = \frac{(\delta_1 s + \omega_1^2)(\delta_2 s + \omega_2^2)}{s^4 + s^3(\delta_1 + \delta_2 + \delta_s + \delta_{12}) + s^2(\omega_1^2 + \omega_2^2 + \omega_{12}^2 + \omega_s^2 + \delta_1\delta_2) + s(\delta_1\omega_2^2 + \delta_2\omega_1^2) + \omega_1^2\omega_2^2} \tag{7}$$

Among which:

$$\delta_1 = \frac{\beta_1}{m_1}, \delta_2 = \frac{\beta_2}{m_2}, \delta_{12} = \frac{\delta_2}{m_1}, \delta_s = \frac{\beta_s}{m_1}, \omega_1^2 = \frac{K}{m_1}, \omega_2^2 = \frac{\mu}{m_2}, \omega_{12}^2 = \frac{\mu}{m_1}, \omega_s^2 = \frac{G}{m_1}$$

Because this mechanical system has high quality factor and high gain servo loop, so:

$$\delta_1 \ll \delta_s, \delta_2 \ll \delta_s, \delta_{12} \ll \delta_s, \omega_1 \ll \omega_s, \omega_2 \ll \omega_s, \omega_{12} \ll \omega_s$$

Thus, the formula could be reduced as:

$$\frac{X_2}{Y} = \frac{(\delta_1 s + \omega_1^2)(\delta_2 s + \omega_2^2)}{s^4 + s^3\delta_s + s^2\omega_s^2 + s(\delta_1\omega_2^2 + \delta_2\omega_1^2) + \omega_1^2\omega_2^2} \tag{8}$$

In order to anm'alyze the systes time-domain response, break the formula above as:

$$\frac{X_2}{Y} = \frac{(\delta_1 s + \omega_1^2)(\delta_2 s + \omega_2^2)}{(s^2 + As + B)(s^2 + Cs + D)} \tag{9}$$

Among which:

$$BD = \omega_1^2\omega_2^2, AD + BC = \delta_1\omega_2^2 + \delta_2\omega_1^2, AC + B + D = \omega_S^2, A + C = \delta_S$$

From above formula we could get the transfer function of the vibration isolation system:

$$\frac{X_2}{Y}=\frac{(\delta_1 s+\omega_1^2)(\delta_2 s+\omega_2^2)}{(s^2+s\delta_s+\omega_s^2)[s^2+(\frac{\delta_1\omega_2^2+\delta_2\omega_1^2}{\omega_s^2}-\delta_s\frac{\omega_1^2\omega_2^2}{\omega_s^4})s+\frac{\omega_1^2\omega_2^2}{\omega_s^2}]} \tag{10}$$

3. Use genetic algorithm to calculate the optimal solution of the model

Genetic algorithm is a calculation model which draws lessons from evolution laws and genetics mechanism, searches the optimal solution through simulating the process of natural evolution. It provides a universal framework to solve those complicated system optimization questions. It does not depend on the specific area of the questions so is robust to the categories of questions. Thus, it is now applied widely on many disciplines (Li *et al*., 2015).

In order to use genetic algorithm to calculate the optimal solution of the model of vibration isolation system, the method of partial fractions is used to calculate formula (10) 's inverse Laplace transform, then the expression of vibration isolation system on the time domain could be draw as:

$$z(t)=\zeta^{-1}(\frac{X_2}{Y})=k_1e^{h_1t}+k_2e^{h_2t}+k_3e^{h_3t}+k_4e^{h_4t} \tag{11}$$

Among which:

$$k_1=\frac{[2\omega_1^2+\delta_1(\mathrm{a}-\delta_s)][2\omega_2^2+\delta_2(\mathrm{a}-\delta_s)]}{\mathrm{a}[(\mathrm{a}+\mathrm{c}-\delta_s)^2-(\mathrm{c}^2-4\mathrm{b})]}$$

$$k_2=-\frac{[2\omega_1^2-\delta_1(\mathrm{a}+\delta_s)][2\omega_2^2-\delta_2(\mathrm{a}+\delta_s)]}{\mathrm{a}[(\mathrm{c}-\mathrm{a}-\delta_s)^2-(\mathrm{c}^2-4\mathrm{b})]}$$

$$k_3=\frac{[2\omega_1^2+\delta_1(d-c)][2\omega_2^2+\delta_2(d-c)]}{\mathrm{d}[(\delta_s+\mathrm{d}-\mathrm{c})^2-\mathrm{a}^2]}$$

$$k_4=-\frac{[2\omega_1^2-\delta_1(d+c)][2\omega_2^2-\delta_2(d+c)]}{\mathrm{d}[(\delta_s-\mathrm{d}-\mathrm{c})^2-\mathrm{a}^2]}$$

$$h_1=\frac{\mathrm{a}-\delta_s}{2},h_2=-\frac{\mathrm{a}+\delta_s}{2},h_3=\frac{d-c}{2},h_4=-\frac{d+c}{2}$$

$$\mathrm{a}=\sqrt{\delta_s^2-4\omega_s^2},\mathrm{b}=\frac{\omega_1^2\omega_2^2}{\omega_s^2}$$

$$c=\frac{\delta_1\omega_2^2+\delta_2\omega_1^2}{\omega_s^2}-\delta_s\frac{\omega_1^2\omega_2^2}{\omega_s^4}$$

$$d=\sqrt{c^2-4b}$$

Set vibration input function as $\mathrm{y(t)} = y_0 sin\omega t$, using convolution operation to calculate inverse Laplace transform and input function could draw the output function of vibration isolation system on the time domain (Wan *et al*., 2012; Zhang, 2015):

$$x_2(t) = \int_0^t z(\tau) y(t-\tau) d\tau \tag{12}$$

The cube's displacement, x2, was produced by the vibration displacement y, in order to achieve vibration isolation, displacement need to be close to zero, so set x2 as the objective function of the genetic algorithm. It could be known from formula (12) that x2's value mainly depends on $\omega_1, \omega_2, \omega_s, \delta_1, \delta_2, \delta_3$, So those parameters' value could be transferred as an optimization question of multi-objective on given region, which is a multi-objective optimization problem.

Among most intelligent search algorithm to objective optimization problems, genetic algorithm's overall optimization has the maximum probability. Genetic algorithm gives strong support to solve multi-objective questions. Use genetic algorithm to search for the optimization solutions of the six parameters above while x2 close to zero infinitely. The specific coding methods are as follows (Lian *et al*., 2015):

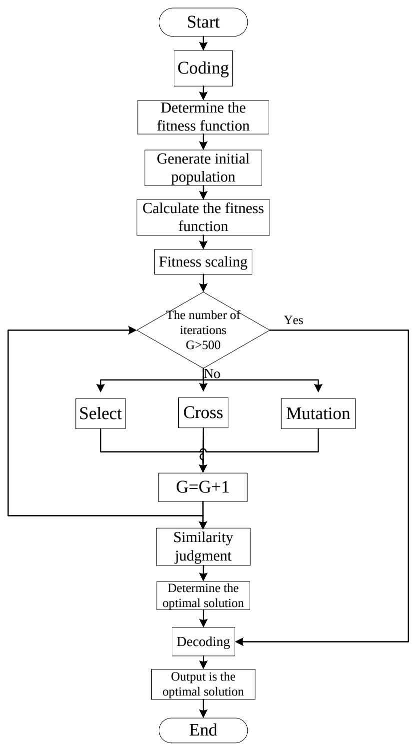

Figure 2. Genetic algorithm program flow chart

(1) According to the actual situation of the cube, determine the six parameters' range of value firstly.

(2) According to the range of value, use 25-bit binary number to represent all the parameters, the gene length is 128, the actual parameter's values accurate to 15 decimal digits, this precision level completely meets the need of absolute gravimeter's requirement to vibration isolation system.

The program flow of the genetic algorithm is shown as Fig.2.

y0's value's magnitude is micrometer, its value is 1e-6m, the generation gap is 0.9, search the optimization solution of the six parameters when the hereditary algebra is 500. The population objective function's main value variation and optimization solution' variation is shown as Fig.3 and Fig.4. When the objective function reaches its minimization, the values of the six parameters are: $\omega_1 = 0.7, \omega_2 = 0.5, \omega_s = 5, \delta_1 = 0.8, \delta_2 = 29.6$.

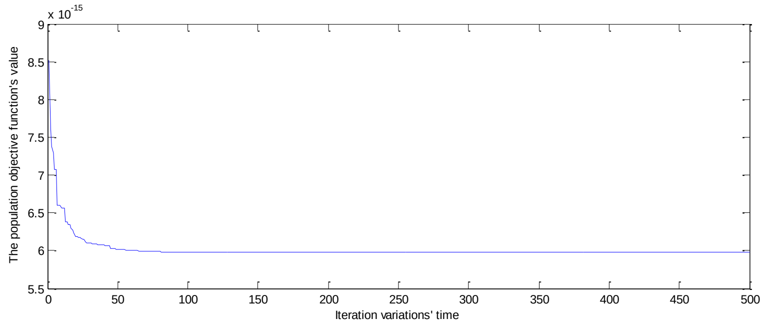

Figure 3. The population objective function's value after 500 times' iteration variations

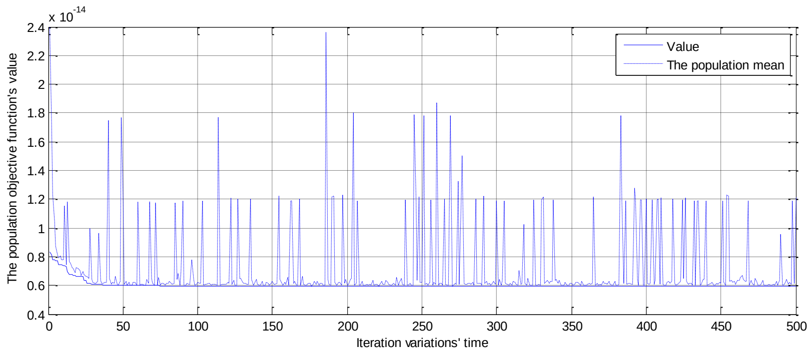

Figure 4. The population objective function's value variation and optimization solution' variation after 500 times' iteration

4. The vibration isolation's simulation analysis

The transfer function of the vibration isolation of the key parameters is shown by formula 13.

$$\frac{X_2}{Y}=\frac{1.08s^2+0.86s+0.12}{s^4+29.6s^3+25s^2+0.86s+0.12} \tag{13}$$

The amplitude-frequency characteristic curve and the phase-frequency characteristic curve of the transfer function could be drew from formula 13, shown as Figure 5 (Holterman, 2002).

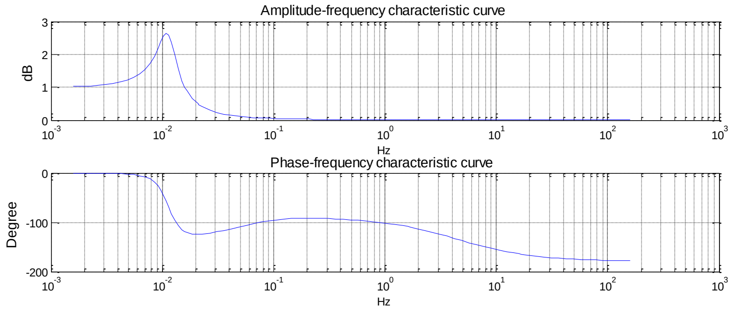

Figure 5. Genetic algorithm's optimization solution's amplitude-frequency characteristic curve and the phase-frequency characteristic curve

From Figure 5 we could know, there is a clear resonant peak at around 0.01Hz in the vibration isolation system under the genetic algorithm's optimization solution, meanwhile the system's stability is poor. So, based on the optimization solution above, combined with actual applications, revise those parameters:

$$\omega_1=0.7,\omega_2=0.5,\omega_s=5,\delta_1=5,\delta_2=5,\delta_s=29.6$$

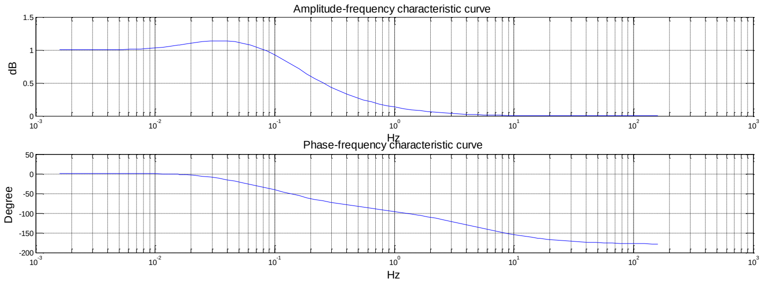

Figure 6. The vibration isolation system's amplitude-frequency characteristic curve and the phase-frequency characteristic curve

Use those revised parameters to do simulation analysis on frequency domain, the ceiling's cutoff frequency is at around 0.15Hz, and has a good performance at low frequency. The analyze result is shown as Figure 6.

Use the optimization parameters drew from genetic algorithm to do simulation analysis of vibration isolation system on time domain. While doing the simulation, set a ground vibration signal with single frequency point, its frequency is 150Hz and amplitude is 1um, sampling rate is 3333Hz. After the vibration signal goes through the system, it causes the second suspended-mass body vibration's x2's value, as shown in Figure 7. The result of simulation analysis shows that the vibration isolation system's isolation efficiency could reach 100dB, and it completely meets the requirement of the research and development of the reference cube (Hua, 2005).

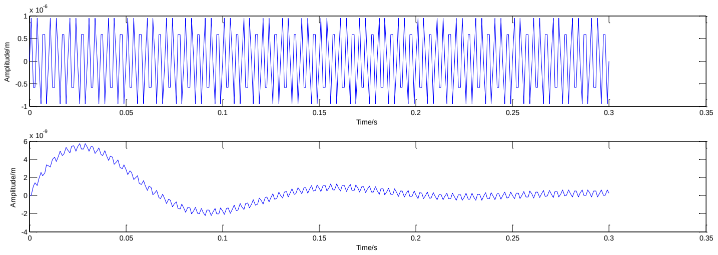

Figure 7. The time domain analysis of the single frequency point under the optimization solution

5. Test with the Age-110 laser interference absolute gravimeter

The reference corner cube vibration isolation system which based on absolute velocity feedback was tested collaboratively with the Age-110 laser interference absolute gravimeter which was self-developed by Institute of Geophysics, China Earthquake Administration. The collaborative test was carried out at National Bureau of Surveying and Mapping's absolutely gravity routine measuring point in Pingliang, Gansu province (Zhang, 2017).

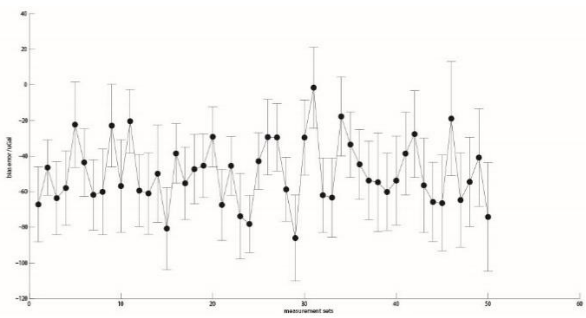

Figure 8. The measurement results without vibration isolation system

Using Age-110 laser interfere absolute gravimeter with the reference cube (later in this paper, ‘no vibration isolation system results’ means the measurement results are acquired without using the two levels of magnetic suspension vibration isolation system designed in this paper.) to test for 40 sets and 50 data for each set. The results are shown as Figure 8.

Without the vibration isolation system, the measurement accuracy is less than 50 uGal for every set of the measurement results, the overall measurement accuracy is 6.42 uGal.

Carry out the test with Age-110 laser interference absolute gravimeter’s reference corner cube in the absolute velocity feedback vibration isolation system for 50 sets, 50 data for each set. The results are shown as Figure 9.

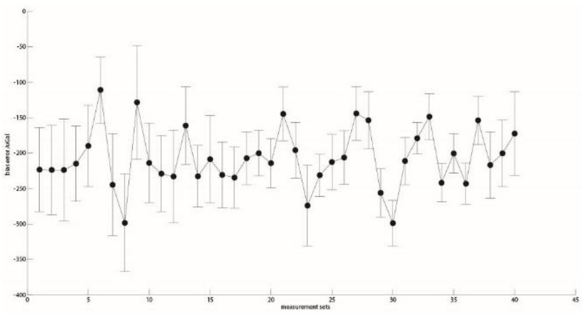

Figure 9. The measurement results with vibration isolation system

With the vibration isolation system, the measurement accuracy is within 20 uGal for every set of the measurement results, the overall measurement accuracy is 2.5 uGal.

6. Conclusion

For the vibration isolation system’s key parameters’ setting, using traditional research method that based on analyzing and repeated debugging would be difficult, cumbersome, a long period and too much work, it also has many casual factors.

The parameters obtained through genetic algorithm performs well, its isolation efficiency could reach 100dB while low frequency could reach 0.15Hz. The experimental data indicated that the overall measurement accuracy of laser interference absolute gravimeter improved by 1.5 times. Therefore, it completely meets the high measurement precision laser interference absolute gravimeter’ requirement of vibration isolation.

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