A BETTER RIGID-PLASTIC ESTIMATE FOR EARTHQUAKE-INDUCED PLASTIC DISPLACEMENTS

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ABSTRACT
The earthquake ductility demand on structures may be predicted by means of a rigid-plastic method, which derives the maximum plastic response of elastic-plastic oscillators from that of a simpler rigid-plastic model. The maximum response of the latter is a purely plastic one and may be obtained from the earthquake rigid-plastic pseudo-spectrum, as a function of the oscillator yield acceleration. The results of a wide investigation presented in this paper show that such a method generally leads to a conservative and reliable enough estimate of the maximum plastic displacements. Small mean errors are in fact found for both comparatively short-period and long-period oscillators. In the medium period range, however, the rigid-plastic prediction is found to be less satisfactory. This is due to the appliance in that range of an empirical formula, which estimates the discrepancy between the elastic-plastic and the rigid-plastic peak response. To improve the rigid-plastic prediction in the medium period range, a new semi-empirical formula is derived in the paper which is shown to halve, on average, the error in estimating the earthquake ductility demand on medium period oscillators. Due to the new formula, the mean relative errors are always kept below 15%, whatever the earthquake and the oscillator. This makes the rigid-plastic method competitive with respect to other approximate methods, as discussed in the paper.
Keywords: Earthquake ductility demand, rigid-plastic method, seismic inelastic displacements prediction.

1 INTRODUCTION

Most structures will experience inelastic deformations when subjected to strong earthquakes. The assessment of their maximum plastic displacements is of the utmost importance to ensure that global and local ductility demands are below acceptable limits. If the ground motion is assigned, a time-history analysis can be performed to calculate the plastic displacements. However, the design of earthquake-resistant structures is seldom carried out through a time-history analysis, since it is lengthy, especially when multi-degree of freedom (MDOF) systems are considered. Approximate methods are usually adopted instead, most of which refer to single-degree of freedom (SDOF) models.

The assessment of the seismic displacement demand on SDOF inelastic systems is often based on the theory of linear elastic oscillators. In some approaches, the latter are taken as equivalent elastic oscillators with a lower lateral stiffness and a higher damping ratio than the inelastic oscillators [1–5]. In other approaches, they possess the same stiffness and the same damping ratio as the inelastic oscillators and the peak inelastic displacement is estimated by multiplying the peak elastic displacement for an appropriate modification factor [6,7]. First proposed by Newmark and Hall [6], the modification factor approach has become so popular as to be adopted by the vast majority of current building codes.

An alternative way of estimating the earthquake displacement demand is the rigid-plastic method proposed by Paglietti and Porcu [8] and subsequently improved by Porcu and Carta [9,10]. It predicts the maximum plastic displacement of any elastic-plastic oscillator possessing a given ratio between yield strength and mass (namely a given *yield acceleration*) from that of the corresponding rigid-plastic model. As a function of the yield acceleration only, the peak displacement of a rigid-plastic oscillator can be obtained from

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the earthquake rigid-plastic pseudo-spectrum, which is a single-curve response diagram easy to construct and to use [8,11–14]. Section 2 recalls how the rigid-plastic method can be applied in practice.

By referring to a large variety of earthquakes and oscillators, an investigation is carried out in the present paper which extends the results provided in Ref. [15] and shows that the rigid-plastic method provides, on average, a conservative estimate of the earthquake ductility demand. Mean relative errors below 15% are in fact found in both the short ($T < 0.2$s) and long ($T > 0.75$s) period ranges, whereas, in the medium period range, the error is found to reach up to even 30%. This can be actually attributed to the appliance of an empirical formula provided in Ref. [10], which predicts the discrepancy between the elastic-plastic and the rigid-plastic peak response in the medium period range.

With the aim of improving the rigid-plastic prediction in the medium period range, a semi-empirical formula is derived in Section 3. Based on the well-known Newmark and Hall equal displacement rule [6], such a formula also takes into account the characteristic vibration periods T^* and $\bar{T}$, at which the peak plastic response of an elastic-plastic oscillator coincides with the rigid-plastic one (for $T = T^*$) and vanishes (for $T = \bar{T}$), respectively. The validity of the proposed formula is checked in Section 4, where the mean ratio between the estimated and the calculated values of the displacement ductility ratio is obtained for hundreds of different elastic-plastic oscillators and more than 30 recorded ground motions. Different values of yield acceleration and damping ratio are considered in the investigation. A comparison between the results obtained by considering the 'old' formula provided in Ref. [10] and those obtained by adopting the 'new' formula derived here shows that the latter improves, on average, the estimate of the earthquake ductility demand. In particular, whatever the vibration period, the damping ratio and the yield acceleration of the oscillator, the new formula leads to mean errors always below 15%.

Due to the new formula, the rigid-plastic prediction becomes generally better than –or at least comparable with – that provided by other approximate methods available in current literature, as discussed in Section 5. This result is made stronger by the fact that, unlike other methods, the rigid-plastic method is shown to provide adequate enough estimates even when the plastic displacements are very large. Moreover, based on a direct procedure, the present method may be much faster to apply than other approximate methods, which instead require iteration procedures. In addition, it singles out the range of periods in which the considered elastic-plastic oscillators may plastically yield under a given earthquake – this being a general result which could always be taken into account when predicting inelastic seismic demands.

It can be observed, finally, that other authors [16, 17] adopted a rigid-plastic approximation to predict the response of ductile structures under dynamic loading. Based on equivalent generalized SDOF systems, a rigid-plastic approach was also proposed by Domingues *et al.* [11,13,14], which predicts the maximum plastic displacements of MDOF buildings under strong earthquakes. Due to this, the rigid-plastic method improved in the present paper could also be exploited to assess the seismic ductility demand of MDOF systems. This topic, however, is beyond the scope of the present paper.

2 ESTIMATING THE EARTHQUAKE DUCTILITY DEMAND THROUGH THE RIGID-PLASTIC METHOD

Let us consider an elastic-perfect-plastic oscillator of mass M, natural period T and yield strength F_y. The latter denotes the absolute value of the oscillator strength at yield, which is assumed to be the same for positive and negative forces and to be independent of plastic deformation. The absolute value of the maximum elastic displacement (*yield displacement*) that such an oscillator may undergo is given by

$$u_y = \frac{T^2}{4\pi^2}\frac{F_y}{M} = \frac{T^2}{4\pi^2}\, a_y \tag{1}$$

The quantity $a_y = F_y/M$ appearing in eqn (1) is the absolute value of the maximum acceleration that the oscillator may reach during the motion. It may be referred to as the oscillator *yield acceleration* [10, 18]. When hit by strong enough earthquakes, the elastic-perfect-plastic oscillator will exceed the yield limit and deform plastically. Should $u^P_{\max}$ denote the peak plastic displacement reached by the oscillator during the considered earthquake, the total peak displacement can be expressed as:

$$u_{\max} = u_y + u^P_{\max} \tag{2}$$

All displacements are here intended to be relative to the ground.

The ratio between $u_{\max}$ and u_y is usually denoted by μ and referred to as the *earthquake ductility demand* or *displacement ductility factor* [18]. In view of eqns (1) and (2), μ may be expressed as:

$$\mu = \frac{u_{\max}}{u_y} = 1 + \frac{4\pi^2}{T^2 a_y}\, u^P_{\max} \tag{3}$$

When the elastic-plastic oscillator deforms into the plastic range, $u_{\max}$ is greater than u_y and, consequently, μ becomes greater than unity. Assessing the value of μ is a crucial task in order to assign the system a sufficient ductility capacity to withstand the considered earthquake. Equation (3) shows that assessing the value of μ requires that $u^P_{\max}$ is known. For a given earthquake, the latter depends on the natural period T, on the damping ratio ξ and on the yield acceleration a_y:

$$u^P_{\max} = u^P_{\max}(T, \xi, a_y) \tag{4}$$

As a function of these parameters, $u^P_{\max}$ may be calculated by means of a numerical integration of the non-linear equations of motion of the elastic-perfect-plastic oscillator under the considered earthquake.

A simpler, though less precise, way to predict $u^P_{\max}$ may be the rigid-plastic method [8–10]. This method refers to a rigid-plastic oscillator possessing the same yield acceleration a_y as the actual elastic-perfect-plastic oscillator. For a given earthquake, the maximum displacement of the rigid-plastic oscillator (which is obviously a purely plastic one) only depends on a_y, that is

$$u^{RP}_{\max} = u^{RP}_{\max}(a_y) \tag{5}$$

The displacement $u^{RP}_{\max}$ should be obtained by integrating the equations of motion of the rigid-plastic oscillator, which are simpler than those of the elastic-plastic one [8]. Alternatively, and more quickly, it can be obtained from the rigid-plastic pseudo-spectrum of the earthquake.

It may be recalled that, for a given earthquake, the rigid-plastic pseudo-spectrum is a single-curve diagram, resulting from the integration of the equations of motion of a rigid-plastic oscillator for different values of a_y [8,11,12]. Simpler to construct than the elastic-plastic spectrum, the rigid-plastic spectrum is very easy to use too. For each value of

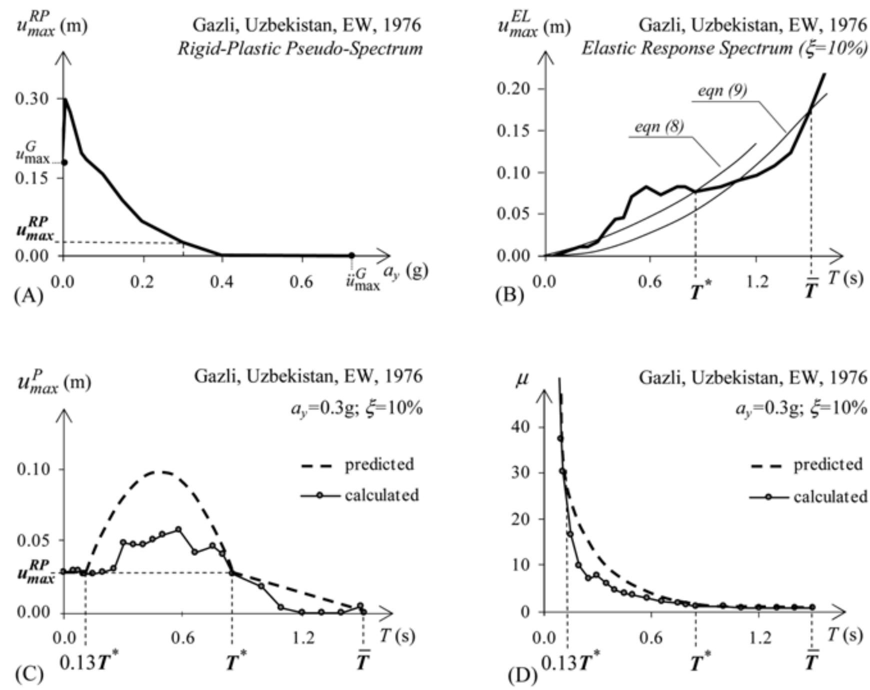

Figure 1: The rigid-plastic method: (**A**) obtaining u_{max}^{RP} from the earthquake rigid-plastic pseudo-spectrum; (**B**) determining T^* and $\bar{T}$ from the displacement response spectrum; (**C**) estimating peak plastic displacements, or otherwise (**D**) estimating the ductility factor.

a_y, it gives the value of the peak plastic displacement $u_{\max}^{RP}$. An instance of such a spectrum is presented in Fig. 1A. It should be noted that any rigid-plastic spectrum starts from the peak ground displacement $u_{\max}^{G}$ and reaches the value $u_{\max}^{RP} = 0$, when a_y is equal to the peak ground acceleration $\ddot{u}_{\max}^{G}$.

For a given earthquake and for each value of a_y, the rigid-plastic peak displacement $u_{\max}^{RP}$ is a single well-defined quantity, while $u_{\max}^{P}$ depends also on T and ξ; see eqn (4). Once the value $u_{\max}^{RP}$ is obtained from the earthquake rigid-plastic pseudo-spectrum at the considered a_y, the peak plastic displacement $u_{\max}^{P}$ may be estimated by means of the following formulae [10]:

$$u_{\max}^{P} = u_{\max}^{RP} \quad \text{for } T \leq 0.13T^*, \tag{6a}$$

$$u_{\max}^{P} = u_{\max}^{RP} + \overline{\Delta u}^{p} \quad \text{for } 0.13T^* \leq T \leq T^*, \tag{6b}$$

$$u_{\max}^{P} = u_{\max}^{RP} \frac{T - \bar{T}}{T^* - \bar{T}} \quad \text{for } T^* \leq T \leq \bar{T}, \tag{6c}$$

$$u_{\max}^{P} = 0 \quad \text{for } T > \bar{T}, \tag{6d}$$

where

$$\overline{\Delta u}^{P} = -0.039\, a_y \left[\frac{2.5\left(\overline{T} + T^*\right)\left(1 - \sqrt{\frac{a_y}{g}}\right)}{\sqrt[4]{\xi}} - \frac{\left(T^*\right)^2}{\overline{T}} - 1 \right] \left[\left(\frac{T}{T^*}\right)^2 - 1.13 \left(\frac{T}{T^*}\right) + 0.13 \right] \quad (7)$$

Note that eqn (7) holds true in this form provided that time is expressed in seconds [10].

Quantities T^* and $\overline{T}$ appearing in the above equations are two characteristic values of period that depend on the earthquake and on the values of a_y and ξ. In particular, for a given earthquake and for a pair of values of a_y and ξ, the period T^* is the one at which it is $u^P_{\max} = u^{RP}_{\max}$. An approximate value of T^* can be easily evaluated by intercepting the displacement elastic response spectrum with the following curve, derived in Ref. [9]:

$$u^*(T, a_y) = \frac{a_y T^2}{4\pi^2} \sqrt{\frac{8\pi^2 u^{RP}_{\max}}{a_y T^2} + 1} \quad (8)$$

On the other hand, $\overline{T}$ denotes the value of T at which $u^P_{\max} = 0$. In fact, any elastic-plastic oscillator possessing a natural period $T \geq \overline{T}$ will behave as a purely elastic oscillator under the considered earthquake [9]. This means that at $T = \overline{T}$ the yield displacement u_y of the elastic-plastic oscillator coincides with the maximum displacement $u^{EL}_{\max}$ that the corresponding purely elastic oscillator would reach under the given earthquake. The actual value $\overline{T}$ of can then be rigorously obtained by intercepting the displacement elastic response spectrum with the curve:

$$u_y\left(T, a_y\right) = \frac{a_y T^2}{4\pi^2} \quad (9)$$

It should be stressed that the values of T^* and $\overline{T}$ are obviously different for different earthquakes and for different values of a_y and ξ. However, the simple graphical procedure recalled above makes it quite a simple task to determine their value, as Fig. 1B also shows. It should be observed, finally, that when curves (8) and (9) intercept the elastic spectrum more than once, the greatest value of T^* and of $\overline{T}$ must always be chosen (see Fig. 1B).

Once $u^{RP}_{\max}$ is taken from the rigid-plastic pseudo-spectrum (Fig. 1A) and the pair of characteristic periods T^* and $\overline{T}$ is taken from the elastic response spectrum (Fig. 1B), $u^P_{\max}$ can be predicted directly from eqns (6) and (7). An instance of such a prediction is given in Fig. 1C. By introducing the estimated value of $u^P_{\max}$ into eqn (3), the ductility factor μ can also be obtained, see Fig. 1D. Note that in the considered example it is assumed ξ=10%, this being a realistic value for the damping factor when the stress is at yield [18].

The instance given in Fig. 1 shows that the rigid-plastic method is quite simple to apply, provided the rigid-plastic pseudo-spectrum and the elastic response spectrum of the earthquake are both available. Figure 1C,1D also show that the rigid-plastic estimate is rather good both for short-period and long-period oscillators. However, it may be highly conservative in the range $0.13T^* \leq T \leq T^*$, where eqn (7) applies. Actually, the results of Section 4 highlight that the rigid-plastic method generally gives rather high mean errors in this range. A new formula will therefore be derived in the next section, which can be more satisfactorily adopted than eqn (7).

Table 1: Recorded earthquakes considered in the present investigation [19–21].

1	Ardal (Iran), LONG, 1977	17	Landers (California), LCN000, 1992
2	Cape Mendocino (California), PET090, 1992	18	Loma Prieta (Cal), CLS000, 1989
3	Cartago (Costa Rica), LONG, 1991	19	Mammoth Lakes (Cal), LLUL000, 1999
4	Chamoli (India), N20E, 1999	20	Montenegro, N-S, 1979
5	Chi Chi (Taiwan), CHY041N, 1999	21	Morgan Hill (Cal), CYC195, 1984
6	Coalinga (California), D-TSM360, 1983	22	N. Palm Springs (Cal), NPS300, 1986
7	Duzce (Turkey), DZC270, 1999	23	Parkfield (California), C02065, 1966
8	Edgecumbe (New Zealand), N07W, 1987	24	Parkfield (California), 90, 2004
9	El Salvador, LONG, 2001	25	San Fernando (California), S16E, 1971
10	Erzincan (Turkey), N279, 1992	26	South Iceland, LONG, 2000
11	Friuli (Italy), E-W, 1976	27	Spitak (Armenia), GUK000, 1988
12	Gazli (Uzbekistan), E-W, 1976	28	Superstition Hills (Cal), B-SUP135, 1987
13	Imperial Valley (Cal), H-BCR230, 1979	29	Tabas (Iran), N74E, 1978
14	Irpinia (Italy), A-STU270, 1980	30	Tabas (Iran), TAB-LN, 1978
15	Kobe (Japan), N35W, 1995	31	Trinidad, B-RDE000, 1980
16	Kocaeli (Turkey), ATS000, 1999	32	Victoria (Mexico), CPE045, 1980

3 IMPROVING THE RIGID-PLASTIC PREDICTION IN THE MEDIUM PERIOD RANGE

By denoting with Δu^P the discrepancy between $u^P_{\max}$ and $u^{RP}_{\max}$ in the range $0.13T^* \le T \le T^*$, we can set:

$$u^P_{\max} = u^{RP}_{\max} + \Delta u^p \tag{10}$$

For a given earthquake, the discrepancy

$$\Delta u^P = \Delta u^P(T, \xi, a_y) \tag{11}$$

can be rigorously obtained only after $u^P_{\max}$ has been calculated by integrating the non-linear equations of motion of the elastic-plastic oscillator.

However, if an estimate $\overline{\Delta u}^p$ of Δu^P is found, $u^P_{\max}$ might be also predicted as:

$$u^P_{\max} \cong u^{RP}_{\max} + \overline{\Delta u}^p \tag{12}$$

The following conditions should be met by $\overline{\Delta u}^p$:

$$\overline{\Delta u}^p = 0 \qquad for \ \frac{T}{T^*} = 0.13 \tag{13}$$

$$\overline{\Delta u}^p = 0 \qquad for \ \frac{T}{T^*} = 1 \tag{14}$$

Equation (13) takes into account that for $T < 0.13T^*$, typically it is $u^P_{\max} \approx u^{RP}_{max}$. On the other hand, eqn (14) comes from the very definition of T^*, which is the natural period at which $u^P_{\max} = u^{RP}_{max}$ [9]. A simple parabolic curve meeting both eqns (13) and (14) was adopted in Ref. [10] to obtain the estimate $\overline{\Delta u}^p$. This leads to derive eqn (7), which, however, is found not give satisfactory enough estimates of Δu^P, as results provided in Section 4 show.

To improve the rigid-plastic prediction in the range $0.13T^* \le T \le T^*$, an alternative way of obtaining $\overline{\Delta u}^p$ will be explored in what follows. First, we shall assume that the well-known Newmark and Hall *equal displacement rule* [6] applies in that range, that is:

$$u_{\max} \cong u^{EL}_{\max} \tag{15}$$

Here, $u^{EL}_{\max}$ is the peak elastic displacement of a purely elastic oscillator possessing the same natural period T and the same damping ratio ξ as the considered elastic-perfect-plastic oscillator. In view of eqns (2) and (12), we can also write eqn (15) as follows:

$$u^{RP}_{\max} + \overline{\Delta u}^p + u_y \cong u^{EL}_{\max} \tag{16}$$

Taking into account eqn (1), we can also obtain from eqn (16):

$$\overline{\Delta u}^p = \frac{a_y (R_y - 1)}{4\pi^2} T^2 - u^{RP}_{\max} \tag{17}$$

Here, R_y is a *reduction factor* given by Chopra [18]:

$$R_y = \frac{u^{EL}_{\max}}{u_y} \tag{18}$$

It should be noted that the value $\overline{\Delta u}^p$ given by eqn (17) may only give a rather coarse evaluation of the actual discrepancy Δu^P, since it derives from eqn (15) which is roughly satisfied in practice. Above all, as given by eqn (17), $\overline{\Delta u}^p$ does not meet conditions in eqns (13) and (14). To remedy this deficiency, eqn (17) should, more appropriately, be put in this form:

$$\overline{\Delta u}^p = \left(\frac{T}{T^*} - 0.13\right)\left(\frac{T^*}{T} - 1\right)\left[\frac{a_y (R_y - 1)}{4\pi^2} T^2 - u^{RP}_{\max}\right] \tag{19}$$

As given by eqn (19), $\overline{\Delta u}^p$ fulfills both conditions in eqns (13) and (14). Equation (19) could be applied in place of eqn (7) to find the peak plastic displacement in the range $0.13T^* \le T \le T^*$. As it stands, however, it is not actually capable of giving a better estimate of Δu^P than eqn (7). By a trial and error procedure, we found that, eqn (19) may be successfully improved as follows:

$$\overline{\Delta u}^p = 2.5\left(\frac{T}{T^*} - 0.13\right)\left(\frac{T^*}{T} - 1\right)\left[\frac{a_y (R_y - 1)}{4\pi^2} T^2 - u^{RP}_{\max} \frac{T^2}{\overline{T}}\right] \tag{20}$$

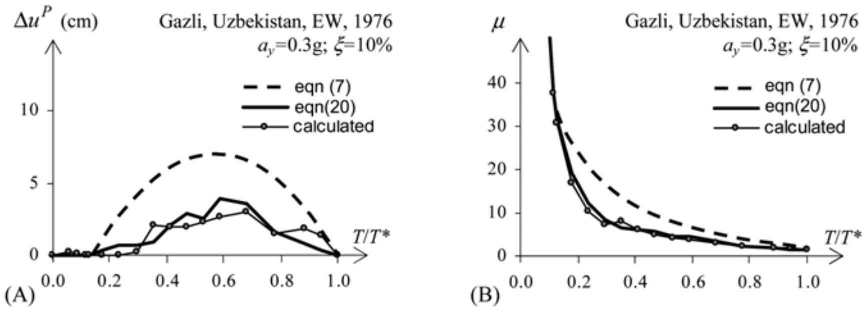

Figure 2: (**A**) Comparing the calculated and the predicted values of Δu^P as obtained by eqns (7) and (20), in the range $0.13 \le T/T^* \le 1$. (**B**) Comparing the ductility predictions. Both figures refer to the same instance considered in Fig. 1.

By referring to the same instance given in Fig. 1, a comparison between the calculated and the estimated values of Δu^P as obtained through eqn (7) and eqn (20) is provided in Fig. 2A. A comparison between the predictions of μ is given in Fig. 2B. Some further instances similar to that given in Fig. 2A, but relevant to different earthquakes and different values of a_y and ξ, are given in Fig. 3. They show that a better – even if not always conservative – estimate of calculated results can be obtained from eqn (20) with respect to eqn (7).

4 COMPARING AVERAGE RESULTS FROM EQUATIONS (7) AND (20)

To assess the effectiveness of eqn (20) herein derived with respect to the 'old' eqn (7) provided in Ref. [10], a numerical investigation is carried out in the present section. The ratio r between the displacement ductility factor $\bar{\mu}$ as estimated by means of the rigid-plastic method, and the 'exact' value μ of the same factor obtained from a non-linear time-history analysis, was computed for elastic-plastic oscillators possessing different realistic values of a_y and different levels of damping ratio ξ. In view of eqns (2) and (3), the ratio r can be expressed as:

$$r = \frac{\bar{\mu}}{\mu} = \frac{u_y + \bar{u}^P_{\max}}{u_y + u^P_{\max}} \tag{21}$$

where $\bar{u}^P_{\max}$ is the plastic displacement estimated through eqns (6) and $u^P_{\max}$ the calculated value. In the investigation, the value $\overline{\Delta u}^P$ to be put into eqn (6b) is taken alternatively from eqns (7) and (20).

It can be noted that the ratio r gives the relative error we introduce when estimating μ with the rigid-plastic method. As the estimated and the calculated values of the plastic displacement tend to coincide, r tends to one, which means that no error is committed in estimating the ductility demand. This obviously happens when T equals zero (rigid-plastic behavior) and when T equals T^*, since in both those cases it is $u^P_{\max} \equiv \bar{u}^P_{\max}$. On the other hand, when T reaches or exceeds the value $\bar{T}$, then $u^P_{\max} \equiv \bar{u}^P_{\max} \equiv 0$, which implies $r = 1$. In all these cases, the rigid-plastic method predicts the earthquake displacement demand exactly. Otherwise, some errors can be produced.

For each earthquake listed in Table 1, for different values of a_y and ξ, and for T ranging from zero to $\bar{T}$, we calculated the value of r, as given by eqn (21). On the whole, more than

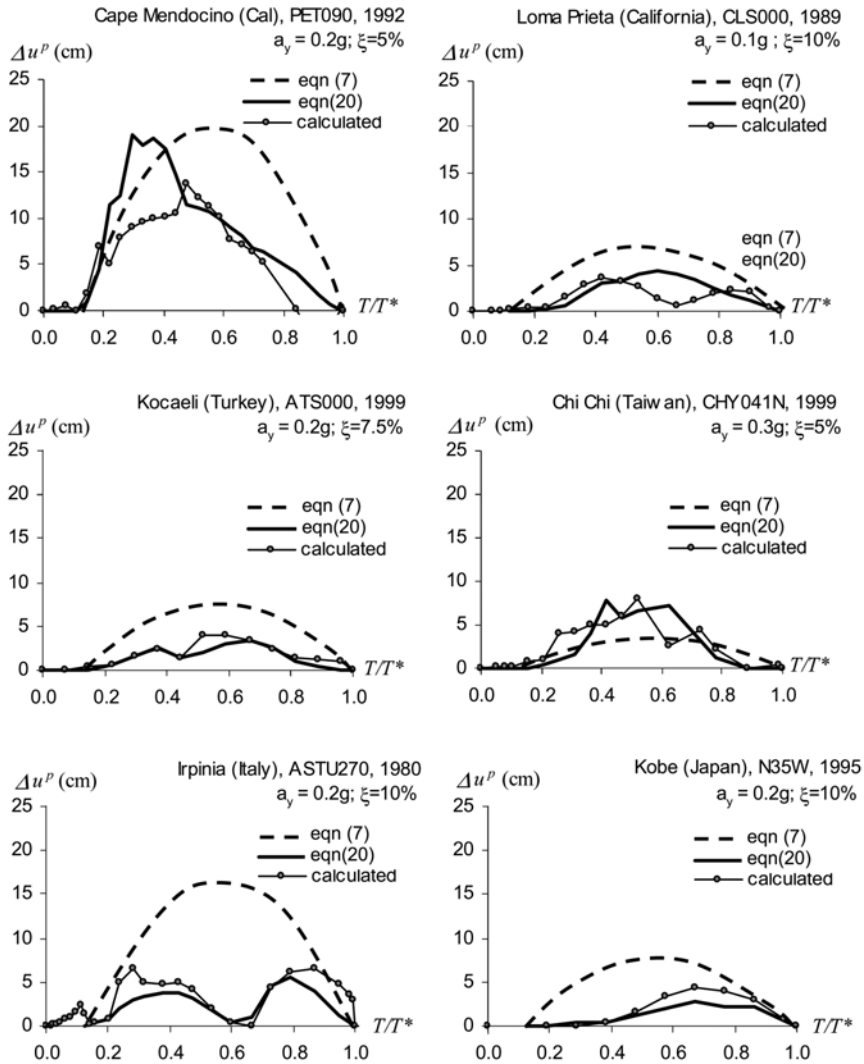

Figure 3: Some instances of comparison between the calculated and the estimated values of the discrepancy Δu^P. The estimated values are obtained from eqns (7) and (20), respectively.

3,000 different instances were actually examined in the present investigation. For each value of T, the mean value of the ratio r, say M_r, was finally obtained. The resulting diagrams are presented in Figs. 4A, 4B and 5A, 5B. Figures 4A, 4B plot M_r for a given value of ξ and three different values of a_y. Similarly, Fig. 5A, 5B refer to a given value of a_y and to three different values of ξ. Results provided by Figs. 4A and 5A are obtained by means of eqn (7), whereas those given by Figs. 4B and 5B are obtained by adopting eqn (20).

Diagrams in Figs. 4,5 show that a rather good prediction (mean errors less than 15%) is provided, on average, by the rigid-plastic method for short-period oscillators, say for $T < 0.2s$, as well as for long-period oscillators, say for $T > 0.75s$. Yet, a less good prediction

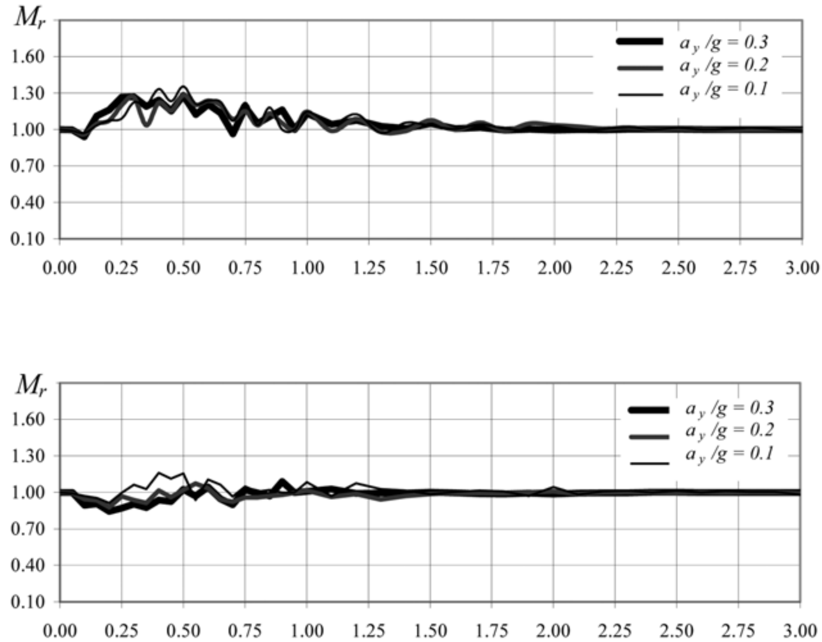

Figure 4: Mean ratio of predicted to calculated maximum displacements for different values of a_y (ξ=*10%*). (A) Results obtained from the 'old' eqn (7); (B) results obtained through the 'new' eqn (20).

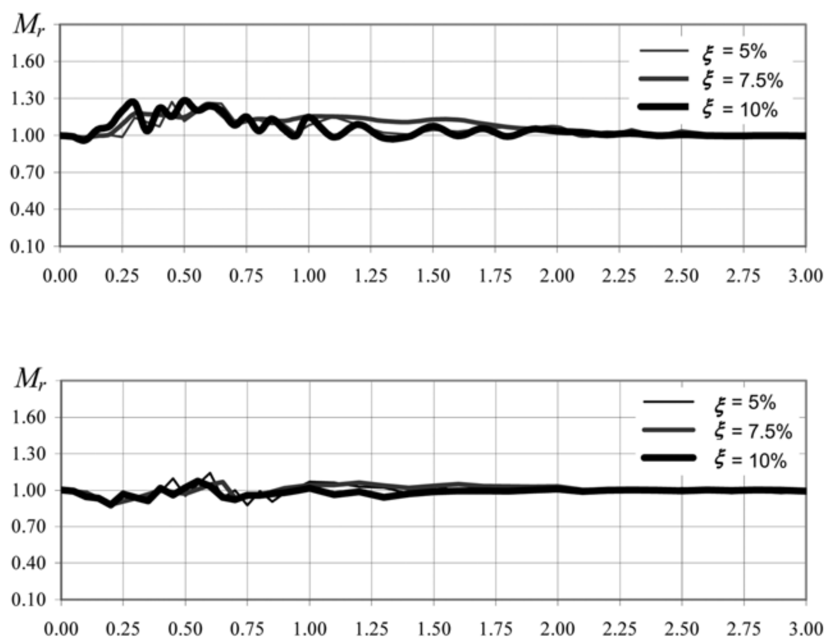

Figure 5: Diagrams analogous to those in Fig. 4, but derived for different values of ξ (a_y*=0.2g*).

may be found in the range $0.2s \le T \le 0.75s$. In fact, Figs. 4A and 5A show that mean errors of up to even 30% may be actually found when reference to eqn (7) is made, both for different values of a_y (see also results provided in Ref. [10]) or for different values of ξ. On the contrary, a quite good prediction is obtained in the medium period range when eqn (20) is adopted in place of eqn (7). In particular, for all the considered values of damping ratio and yield acceleration, eqn (20) always gives mean relative errors below 15%, whatever the natural period T of the elastic-plastic oscillator; see Figs. 4B and 5B. It should be noted, however, that, on average, results obtained via eqn (7) are always conservative (values of M_r larger than one). On the contrary, when applied through eqn (20) the rigid-plastic prediction may give either positive or negative mean errors.

5 SOME ADVANTAGES OF THE RIGID-PLASTIC METHOD

Several approximate methods have been proposed to estimate the earthquake ductility demand on structures. Some of them refer to *equivalent linear elastic systems* (with lower lateral stiffness and higher damping ratio than the elastic-plastic systems) (cf. e.g. [1–5]). Other methods multiply the response of the corresponding linear elastic systems (with the same stiffness and damping ratio as the inelastic systems) by some *modification factors* (cf. e.g. [6, 7].). Both *equivalent linear methods* and *modification factor methods* usually evaluate the earthquake ductility demand μ by means of parameters that are a function of μ itself. This implies iteration procedures and, often, convergence problems [22, 23].

This criticism does not affect the rigid-plastic method, which predicts the inelastic displacement demand on any elastic-plastic oscillator by means of the direct procedure recalled in Section 2. The latter is relatively quick to apply, once the rigid-plastic pseudo-spectrum and the elastic response spectrum of the considered earthquake are both available. The elastic response spectrum is usually available in practice, whereas the rigid-plastic pseudo-spectrum is less used. Depending only upon the yield acceleration, however, this spectrum is much simpler to construct than an elastoplastic response spectrum and could be profitably introduced even by standard codes.

The elastic response spectrum provides a first crucial item: the characteristic period $\bar{T}$, which spots the range of periods ($0 \le T \le \bar{T}$) in which the inelastic demand prediction actually needs to be obtained under a given earthquake (see Fig. 1). For $T > \bar{T}$, no plastic deformation is in fact demanded by the earthquake to the considered elastic-plastic oscillators. This is a general result, which could be adopted by any approximate method that aims at estimating the earthquake ductility demand on elastic-plastic oscillators. It could be observed, for instance, that the well-known Newmark's *equal displacement rule* [6], assuming that the maximum displacement of an elastic-plastic oscillator coincides with that of the corresponding purely-elastic oscillator, becomes more and more accurate as T approaches $\bar{T}$. Of course, for $T \ge \bar{T}$ that rule gives exact results.

On the other hand, the rigid-plastic pseudo-spectrum provides, for each given value of yield acceleration a_y, the peak rigid-plastic displacement $u_{\max}^{RP}$, which is a reference value for obtaining the peak inelastic displacement prediction through the rigid-plastic method. As it stands, the value $u_{\max}^{RP}$ gives a very good prediction of the peak plastic displacement of comparatively short-period elastic-plastic oscillators, as Figs. 4,5 also show. It can be noted that in the short-period range the rigid-plastic prediction is generally better than that of other methods, as can be inferred from the diagrams plotting the mean errors relevant to some approximate methods provided by Miranda and Ruiz Garcia [24] and Akkar and Miranda [25]. The same value $u_{\max}^{RP}$ may be exploited to obtain, through eqn (6c), a fairly good prediction even in the comparatively long-period range, say for $T > 0.75s$; see Figs. 4,5. In this range the rigid-plastic method prediction is, on average, comparable with that of other approximate methods (cf. e.g. the results presented in [24, 25]).

As to the reliability of the rigid-plastic prediction in the medium period range, it should be noted that adopting eqn (20) rather than eqn (7) produces a perceptible improvement on the average results. When the rigid-plastic method is applied by means of eqn (20), in fact, the mean relative errors are always below 15%; see Figs. 4B and 5B. This is a rather good result, since other methods, as for instance those evaluated in Refs. [24, 25], may give errors ranging from 25 to 80% in the medium period range.

This result is made even stronger by the fact that the rigid-plastic method is capable of estimating very high values of μ with reasonably narrow mean errors. In fact, the mean ratio M_r plotted in Figs. 4 and 5 is relevant to assigned values of a_y, which may entail very high values of μ in the short-medium period range (μ >10)), as Fig. 1D shows. On the contrary, rather low values of μ are usually assigned when evaluating the mean errors relevant to other approximate methods (e.g. μ ranging from 1.5 to 6 is considered by Miranda and Ruiz Garzia [24]). Moreover, the errors relevant to other methods generally increase as μ increases, this being especially so in the medium period range [24, 25].

6 CONCLUSIONS

A rigid-plastic method to estimate the inelastic displacement demand under strong earthquakes is evaluated in the present paper. By computing the mean ratio between predicted and calculated ductility factors relevant to thousands of different instances, the paper shows that the method gives reliable enough results for both relatively short and relatively long-period oscillators. Less reliable results – although generally conservative – are found instead for medium period oscillators. To improve the rigid-plastic prediction in the medium period range, a semi-empirical formula is derived in the paper, which estimates the discrepancy between the elastic-plastic and the rigid-plastic peak response of medium period oscillators. The validity of such a formula is checked over a wide variety of recorded earthquakes and oscillators. It is found that, due to this formula, the mean errors are always kept below 15%, whatever the natural period, damping ratio or yield acceleration of the elastic-plastic oscillator. A comparison with other approximate methods available in the literature shows that the rigid-plastic procedure presents some advantages and may give, on average, a better estimate on the whole, especially for high levels of ductility demand.

REFERENCES

[1] Rosemblueth, E. & Herrera, I., On a kind of hysteretic damping. *Journal of Engineering Mechanics Division ASCE*, **90**, pp. 37–48, 1964.

[2] Gulkan, P. & Sozen, M., Inelastic response of reinforced concrete structures to earthquakes motion. *ACI Journal*, **71**, pp. 604–610, 1974.

[3] Iwan, W.D., Estimating inelastic response spectra from elastic spectra. *Earthquake Engineering and Structural Dynamics*, **8**, pp. 375–388, 1980. doi: http://dx.doi.org/10.1002/eqe.4290080407

[4] Hadjian, A.H., A re-evaluation of equivalent linear models for simple yielding systems. *Earthquake Engineering and Structural Dynamics*, **10**, pp. 759–767, 1982. doi: http://dx.doi.org/10.1002/eqe.4290100602

[5] Kowalsky, M., Priestley, M.J.N. & McRae, G.A., Displacement-based design of RC bridge columns in seismic regions. *Earthquake Engineering and Structural Dynamics*, **24**, pp. 1623–1643, 1995. doi: http://dx.doi.org/10.1002/eqe.4290241206

[6] Newmark, N.M. & Hall, W.J., *Earthquake Spectra and Design*, Earthquake Engineering Research Institute: Berkeley, 1982.

[7] Miranda, E., Inelastic displacements ratios for structures on firm sites. *Journal of Structural Engineering*, **126**, pp. 1150–1159, 2000. doi: http://dx.doi.org/10.1061/(ASCE)0733-9445(2000)126:10(1150)

[8] Paglietti, A. & Porcu, M.C., Rigid-plastic approximation to predict motion under strong earthquakes. *Earthquake Engineering and Structural Dynamics*, **30**, pp. 115–126, 2001. doi: http://dx.doi.org/10.1002/1096-9845(200101)30:1<115::AID-EQE999>3.0.CO;2-V

[9] Porcu, M.C. & Carta, G., Rigid-plastic bound to the seismic inelastic response of flexible elastic-plastic oscillators. *European Earthquake Engineering*, **3**, pp. 3–9, 2007.

[10] Porcu, M.C. & Carta, G., Rigid-plastic seismic analysis to predict the structural ductility demand. *International Journal of Applied Engineering Research*, **4**, pp. 309–325, 2009.

[11] Domingues Costa, J.L., Bento, R., Levtchitch, V. & Nielsen, M.P., Simplified non-linear time-history analysis based on the theory of plasticity. *Proc. of 5th World Conf. on Earthquake Resistant Engineering Structures*, eds C.A. Brebbia, D.E. Beskos, G.D. Manolis & C.C. Spyrakos, WIT Press: Southampton, pp. 375–386, 2005.

[12] Porcu, M.C. & Mascia, M., Rigid-plastic pseudo-spectra: peak response charts for seismic design. *European Earthquake Engineering*, **3**, pp. 37–47, 2006.

[13] Domingues Costa, J.L., Bento, R., Levtchitch, V. & Nielsen, M.P., Rigid-plastic seismic design of reinforced concrete structures. *Earthquake Engineering and Structural Dynamics*, **36**, pp. 55–76, 2007. doi: http://dx.doi.org/10.1002/eqe.617

[14] Hibino, Y., Toshikatsu, I., Domingues Costa, J.L. & Nielsen, M.P., Procedure to predict the storey where plastic drift dominates in two-storey building under strong ground motion. *Earthquake Engineering and Structural Dynamics*, **38**, pp. 929–939, 2008. doi: http://dx.doi.org/10.1002/eqe.874

[15] Porcu, M.C. & Carta, G., Evaluating a rigid-plastic method to estimate the earthquake ductility demand on structures. *Proc. of 8th World Conf. on Earthquake Resistant Engineering Structures*, eds C.A. Brebbia & M. Maugeri, WIT Press: Southampton, pp. 261–271, 2011. doi: http://dx.doi.org/10.2495/ERES110221

[16] Fatt, M.S.H., Wierzbicki, T., Moussouros, M. & Koenig, J., Rigid-plastic approximation for predicting plastic deformation of cylindrical shell subject to dynamic loading. *Shock & vibration*, **3**, pp. 169–181, 1996.

[17] Makris, N. & Black, C.J., Dimensional analysis of rigid-plastic structures under pulse-type excitations. *Journal of Engineering Mechanics*, **130**, pp. 1006–1018, 2004. doi: http://dx.doi.org/10.1061/(ASCE)0733-9399(2004)130:9(1006)

[18] Chopra, A.K., *Dynamics of Structures. Theory and Application to Earthquake Engineering*, Prentice Hall: New Jersey, 2001.

[19] The European Strong-Motion Database (ESD), available at http://www.isesd.cv.ic.ac.uk

[20] PEER Strong Motion Database, available at http://peer.berkeley.edu/smcat

[21] COSMOS Virtual Data Center, available at http://db.cosmos-eq.org

[22] Chopra, A.K. & Goel, R.K., Evaluation of NSP to estimate seismic deformation: SDF systems. *Journal of Structural Engineering*, **126**, pp. 482–490, 2000. doi: http://dx.doi.org/10.1061/(ASCE)0733-9445(2000)126:4(482)

[23] Fajfar, P., A nonlinear analysis method for performance-based seismic design. *Earthquake Spectra*, **16**, pp. 573–592, 2000. doi: http://dx.doi.org/10.1193/1.1586128

[24] Miranda, E. & Ruiz Garcia, J., Evaluation of approximate methods to estimate maximum inelastic displacement demands. *Earthquake Engineering and Structural Dynamics*, **31**, pp. 539–560, 2002. doi: http://dx.doi.org/10.1002/eqe.143

[25] Akkar, S.D. & Miranda, E., Statistical evaluation of approximate methods for estimating maximum deformation demands on existing structures. *Journal of Structural Engineering*, **131**, pp, 160–172, 2005.