Modeling of double diffusion with MHD on an inclined flat plate solar captor with non-uniform boundary conditions. Bouyancy ratio, Prandtl, Schmidt and Eckert numbers effects

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ABSTRACT. *The objective of the present work is to develop a valid model to study heat and mass transfer of MHD free convection around a semi-infinite horizontal or inclined plate associated by chemical reaction, radiation heat flux and internal heat generation or absorption using non local similarity transformations. The credibility of this study impose to consider no-uniform conditions at the wall temperature and concentration (Tw(x) =T ∞+axn, Cw(x) =C ∞+bxm). Some plain and relatively simplified differential equations have been gotten, in view of a suitable numerical resolution. The expressions of the local Nusselt number, the skin-friction coefficient, and the local Sherwood number are obtained. The effects of different parameters such as Buoyancy ratio-indicating the relative importance of species and thermal diffusion N, Prandtl, Eckert and Schmidt numbers on velocity, temperature and concentration are carried out in this paper. These results can be useful in the complex design of flat plate solar captor used in renewable energy.*

RÉSUMÉ. *The objective of the present work is to develop a valid model to study heat and mass transfer of MHD free convection around a semi-infinite horizontal or inclined plate associated by chemical reaction, radiation heat flux and internal heat generation or absorption using non local similarity transformations. The credibility of this study impose to consider no-uniform conditions at the wall temperature and concentration (Tw(x) =T ∞+axn, Cw(x) =C ∞+bxm). Some plain and relatively simplified differential equations have been gotten, in view of a suitable numerical resolution. The expressions of the local Nusselt number, the skin-friction coefficient, and the local Sherwood number are obtained. The effects of different parameters such as Buoyancy ratio-indicating the relative importance of species and thermal diffusion N, Prandtl, Eckert and Schmidt numbers on velocity, temperature and concentration are carried out in this paper. These results can be useful in the complex design of flat plate solar captor used in renewable energy.*

KEYWORDS: *MHD, nombres de Schmidt et d'Eckert, reaction chimique, rayonnement, transferts de chaleur et de masse, conditions aux limites.*

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1. Introduction

The free heat and mass transfer intervenes in many natural phenomena and industrial processes. The importance of the study of the heat and mass transfer by free convection on the infinite plates, no uniform conditions on the wall, and inclined is of recognized interest. For example, in renewable energy, some devices are used, like inclined flat plate solar captor, and necessitate an accurate design especially in the complex environment. For this, complex fluids can used to transfer the heat and some phenomenon should be taken into account, as the incident radiative heat flux, the presence of an external magnetic field or a mass transfer with chemical reaction between species.

Classically, several analytical solutions and experimental measurements are advanced for the heat and mass transfer of the vertical plates in the case of the natural convection, few for horizontal or inclined pates.

Gebhart and Pera (1971), Chen *et al*. (1980) were the first of study of heat and mass' transfer along a vertical and inclined plate with uniform wall conditions. Hossain *et al*. (1996) studied the free convection flow from an isothermal plate inclined at a small angle to the horizontal. Anghel *et al*. (2001) presented a numerical solution of free convection flow past an inclined surface.

The effect of electrically conducting fluid was studied in many publications such as Michiyoshi *et al*. (1976) and Gray (1979). Chemical reaction impact of laminar heat and mass transfer over a semi-infinite horizontal plate has been discussed by Anjalidevi and Kandasamy (1999).

Chamka and Khaled (2001) has considered linear variation with space of the temperature and concentration at the wall of the study of MHD heat and mass transfer by free convection around an inclined plate. Singh *et al*. (2007) have presented the technique to obtain similarity solutions in a hydro magnetic flow of the free convection and mass transfer equations past an infinite vertical porous plate. Chen (2004) presented an analysis of the unsteady free heat and mass transfer over a permeable inclined surface with variable wall temperature and concentration. Effect of chemical reaction on magnetic free heat and mass transfer of a viscous, incompressible and electrically conducting fluid around a stretching sheet was done by Afify (2004). Alam *et al*. (2006) published their study concerning the Hall effects on the steady MHD free-convective flow and mass transfer over an inclined stretching sheet in the presence of a uniform magnetic field. The study of heat generation or absorption on the MHD free convection and mass transfer flow past an inclined semi-infinite plat with uniform boundary conditions of temperature and concentration has been developed by Ali *et al*. (2013). In the presence of a magnetic field, thermal radiation and Joule effect heating are investigated for non-Newtonian fluids by Mabood *et al*. (2017) and by Kumar *et al*. (2017). In addition, Srinivasacharya and

Shafeeurrahman (2017) have considered via the concept of entropy generation, the chemical reacting effect for a nanofluid.

The present investigation deals with the study of flow of a fluid on the coupled MHD free heat and mass transfer along a semi-infinite horizontal or inclined plate in the presence of several parameters such chemical reaction, heat flux radiation, and internal heat generation or absorption. The temperature and concentration in the wall are considered a linear or nonlinear variation with space. In the next section, mathematical model is established with the appropriate constitutive equations and more realistic boundaries conditions. In the same section, non-local similarity method is applied to above system and the three level techniques are implemented to obtain a new system easily solved by the classical routines. This section is ended by the report of quantities of interest. In the third section, results obtained by variation of some important dominant numbers are discussed. Some determinants values of the quantities of interest are dressed.

2. Mathematical analysis

The unsteady flow of a viscous incompressible and electrically conducting fluid past an infinite inclined plate from the horizontal with an acute angle γ, in the presence of heat generation/absorption, chemical reaction, and radiative heat flux has been considered. The temperature T_w and the concentration C_w on the wall vary linearly with the distance. Radiative heat flux value following x direction is very small with that in the y direction what explains the negligence of this value, A uniform magnetic field of strength B_0 is imposed along the y axis, Boussinesq approximation is retained. Under these conditions, the flow can be shown to be governed by the following system of coupled linear partial differential equations along with boundary and initial conditions:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta\cos\gamma\frac{\partial}{\partial x}\int_y^\infty(T-T_\infty)dy+\nu\frac{\partial^2 u}{\partial y^2}+$$

$$g\beta(T-T_\infty)\sin\gamma-\frac{\sigma B_0^2}{\rho}u+g\beta^*(C-C_\infty)\sin\gamma \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\propto\frac{\partial^2 T}{\partial y^2}+\frac{\sigma B_0^2}{\rho c_p}u^2+\frac{\nu}{C_P}(\frac{\partial u}{\partial y})^2+\frac{Q_0}{\rho C_P}(T-T_\infty)-\frac{1}{\rho C_P}\left(\frac{\partial q_r}{\partial y}\right) \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-K_r(C-C_\infty)) \tag{4}$$

The boundary conditions are defined as follow:

$$At\ y=0: u=v=0, T=T_w=T_\infty+ax^n, andC=C_w=C_\infty+bx^m \tag{5}$$

$$For\ y \to \infty: T \to T_\infty\ and\ C \to C_\infty$$

Where a and b are constants, n and m are exponents. u, v, T and C are velocity component in x direction, velocity component in y direction, temperature and concentration respectively. g is the acceleration due to gravity, T_w and C_w are the wall temperature and concentration respectively, T_∞ and C_∞ are the temperature and concentration of the uniform flow respectively, α is thermal conductivity, v is the kinematic viscosity, C_p is the specific heat at constant pressure, k is the thermal conductivity of the fluid, ρ is density of the ambient fluid, σ is the electrical conductivity, P is the static pressure difference induced by the buoyancy force, β is the Volumetric coefficient of thermal expansion, β^* is the Volumetric coefficient of thermal expansion with concentration, B_0 is the externally imposed magnetic field in the y direction, Q_0 is heat generation or absorption constant, q_r is the component of radiative heat flux, D is the molecular diffusivity, $K_r is$ chemical reaction parameter.

However, the problem can be simplified by using the Rosseland approximation which simplifies the radiative heat flux as:

$$q_r = -\frac{4\sigma^*}{3K^*}\frac{\partial T^4}{\partial y} \tag{6}$$

Where σ^* and K^* are the Stefan-Boltzman constant and the Roseland mean absorption coefficient respectively. To get T^4 we apply Taylor series without taking into consideration upper order terms:

$$T^4 \approx 4T_\infty T - 3T_\infty^4 \tag{7}$$

Using the above approximations (6) and (7), the energy equation (3) becomes:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \propto \frac{\partial^2 T}{\partial y^2} + \frac{\sigma B_0^2}{\rho c_p}u^2 + \frac{v}{C_P}(\frac{\partial u}{\partial y})^2 +$$

$$\frac{Q_0}{\rho C_P}(T - T_\infty) + \frac{1}{\rho C_P}\left(\frac{16T_\infty\sigma^*}{3K^*}\frac{\partial^2 T}{\partial y^2}\right) \tag{8}$$

In order to obtain a similarity solution of the problem we introduce the following non-dimensional variables used by Chen *et al*. (1986) for 0°≤γ <90°:

$$\xi = \xi(x), \eta = \frac{y}{\xi(x)} = \frac{y}{x}(\frac{Gr_x \cos\gamma}{5})^{1/5} \tag{9}$$

The horizontal velocity components are $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$. It can be easily verified that the continuity equation (1) is identically satisfied and introduce the non-dimensional form of velocity, temperature and the concentration as:

$$f(\xi,\eta) = \frac{\psi(x,y)}{5v(Gr_x \cos\gamma/5)^{1/5}}$$

$$\theta(\xi,\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{10}$$

$$\Phi(\xi,\eta)=\frac{C-C_\infty}{C_w-C_\infty}$$

Gr_x is the local Grash of number.

$$f'''' + (3+n)ff''' - (3n-1)f'f'' + \left[\frac{(2-n)}{5}\eta + \xi\right]\theta' -$$

$$n\theta - \frac{(n+3)}{5}\xi\frac{\partial\theta}{\partial\xi} - Mf''\xi^{\frac{4-2n}{3+n}} + \xi N\Phi' - (n+3)\xi\left[f'\frac{\partial^2 f'}{\partial\eta\partial\xi} - f'''\frac{\partial f}{\partial\xi}\right] = 0 \quad (11)$$

$$\frac{1}{Pr}(1+R)\theta'' + (n+3)f\theta' - 5nf'\theta - (n+3)\xi\left(f'\frac{\partial\theta}{\partial\xi} - \theta'\frac{\partial f}{\partial\xi}\right) +$$

$$ME_c f'^2\xi^{(4-2n)/(3+n)} + E_c f''^2 + S\theta\xi^{(4-2n)/(3+n)} = 0 \quad (12)$$

$$\frac{1}{S_C}\Phi'' + (3+n)f\Phi' - 5mf'\Phi + (n+3)\xi\left(\Phi'\frac{\partial f}{\partial\xi} - f'\frac{\partial\Phi}{\partial\xi}\right) - J\Phi = 0 \quad (13)$$

Introduce new variables$F;\ \Theta$ and $\mathbb{C}$ such as:

$F = \frac{\partial f}{\partial\xi}; \Theta = \frac{\partial\theta}{\partial\xi}; \mathbb{C} = \frac{\partial\Phi}{\partial\xi}$. The equations (11), (12), and (13) become:

$$f'''' + (3+n)ff''' - (3n-1)f'f'' + \left[\frac{(2-n)}{5}\eta + \xi\right]\theta' -$$

$$n\theta - \frac{(n+3)}{5}\xi\Theta - Mf''\xi^{\frac{4-2n}{3+n}} + \xi N\Phi' - (n+3)\xi[f'F'' - f'''F] = 0 \quad (14)$$

$$\frac{1}{Pr}(1+R)\theta'' + (n+3)f\theta' - 5nf'\theta -$$

$$(n+3)\xi(f'\Theta - \theta'F) + ME_c f'^2\xi^{(4-2n)/(3+n)} + E_c f''^2 + S\theta\xi^{(4-2n)/(3+n)} = 0 \quad (15)$$

$$\frac{1}{S_C}\Phi'' + (3+n)f\Phi' - 5mf'\Phi + (n+3)\xi(\Phi'F - f'\mathbb{C}) - J\Phi = 0 \quad (16)$$

In the second step, we derivate the equations (14), (15), and (16) / ξ with neglecting the derivatives terms related to ξ of the new variables $F;\ \Theta$ and $\mathbb{C}$ which are minute, additional equations are then derived and we get:

$$F'''' + (3+n)(Ff''' + fF''') + \theta' - (3n-1)(F'f'' + f'F'') - \frac{(6n+3)}{5}\Theta +$$

$$\left[\frac{(2-n)}{5}\eta + \xi\right]\Theta' + N(\xi\mathbb{C}' + \Phi') - (n+3)[(f'F'' - f'''F) + \xi(F'F'' - F'''F)] -$$

$$MF''\xi^{\frac{(4-2n)}{3+n}} - Mf''\frac{(4-2n)}{(3+n)}\xi^{\frac{1-3n}{3+n}} = 0 \quad (17)$$

$$\frac{1}{Pr}(1+R)\Theta'' + (n+3)(F\theta' + f\Theta') - 5n(F'\theta + f'\Theta) - (n+3)\xi(f'\Theta -$$

$$\theta'F) - (n+3)\xi(F'\Theta - \Theta'F) + 2E_c F''f'' + ME_c f'^2\frac{(4-2n)}{(3+n)}\xi^{\frac{1-3n}{3+n}} +$$

$$2ME_c f' F'^{\xi^{\frac{4-2n}{3+n}}} + S\Theta\xi^{\frac{4-2n}{3+n}} + S\theta\frac{(4-2n)}{(3+n)}\xi^{\frac{1-3n}{3+n}} = 0 \qquad (18)$$

$$\frac{1}{S_C}\mathbb{C}'' + (3+n)(F\Phi' + f\mathbb{C}') - 5m(F'\Phi + f'\mathbb{C}) -$$
$$J\mathbb{C} + (3+n)[(\Phi' F - f'\mathbb{C}) + \xi(\mathbb{C}' F - F'\mathbb{C})] = 0 \qquad (19)$$

Where

$M = \sigma B_0^2(\tan\gamma)^2/\mu\alpha_1^{10/(3+n)}$: Magnetic parameter with $\alpha_1 = \left[\frac{\alpha g\beta}{\nu^2}(\cos\gamma/5)\right]^{1/5}\tan\gamma$.

$E_c = \left[\frac{5\nu}{x}(\mathrm{Gr_x}\cos\gamma/5)^{2/5}\right]^2/\mathrm{C_p}(T_w - T_\infty)$: Eckert number

$S_c = \frac{\nu}{D}$: Schmidt number; $S = Q_0(\tan\gamma)^2/\mu C_p\alpha_1^{10/(3+n)}$: Heat generation or absorption parameter,

$R = 16\sigma^* T_\infty/3K^* k$: Thermal radiation parameter,

$J = K_r\rho(\tan\gamma)^2/\mu\alpha_1^{10/(3+n)}$: Chemical reaction parameter,

$N = \beta^*(C_w - C_\infty)/\beta(T_w - T_\infty)$: Buoyancy ratio-indicating the relative importance of species and thermal diffusion,

$Pr = \frac{\mu C_p}{k}$: Prandtl number,

( )' prime indicates differentiation with respect to η.

Boundary layers become:

$$f(\xi,0) = f'(\xi,0) = f'(\xi,\infty) = 0$$
$$F(\xi,0) = F'(\xi,0) = F'(\xi,\infty) = 0$$
$$\theta(\xi,0) = 1, \theta(\xi,\infty) = 0 \qquad (20)$$
$$\Theta(\xi,0) = \Theta(\xi,\infty) = 0$$
$$\Phi(\xi,0) = 1, \Phi(\xi,\infty) = 0$$
$$\mathbb{C}(\xi,0) = \mathbb{C}(\xi,\infty) = 0$$

The local Nusselt number, the skin-friction coefficient, and the local Sherwood number are important physical parameters. These can be defined and derived as:

$$Nu_x = \frac{hx}{k} = -(Gr_x\cos\gamma/5)^{\frac{1}{5}}\theta'(\xi,0) \qquad (21)$$

$$\tau_w = \mu(\partial u/\partial y)_{y=0} = \frac{5\nu\mu}{x^2}(Gr_x\cos\gamma/5)^{3/5} f''(\xi,0) \qquad (22)$$

$$Sh_x = \frac{m_w}{(C_w - C_\infty)}\left(\frac{x}{D}\right) = -(Gr_x \cos\gamma/5)^{1/5}\,\Phi'(\xi, 0) \tag{23}$$

With $m_w = -D(\partial C/\partial y)_{y=0}$: is the mass flux

3. Results and discussion

The set of coupled ordinary differential Equations (14)-(19), subject to the boundary conditions (20) was solved numerically using finite difference method via Lobatto III approach. The impact of Buoyancy ratio-indicating *N*, Prandtl number *Pr*, Eckert number, and Schmidt number are examined and discussed in detail.

The effect of Buoyancy ratio-indicating *N* on the velocity, temperature, and concentration is shown on the Figure 1. The reduction of both the temperature and concentration appears whenever *N* moves up. At the proximity of the wall, the velocity and *N* are proportional, but inversely proportional far away from the wall. The effect of mass buoyancy on the velocity is clear when N=10. Differences between the velocities are noted near the wall. The temperature and concentration of species profiles are not affected by the variation of N, result expected because the buoyancy is strictly a flow parameter.

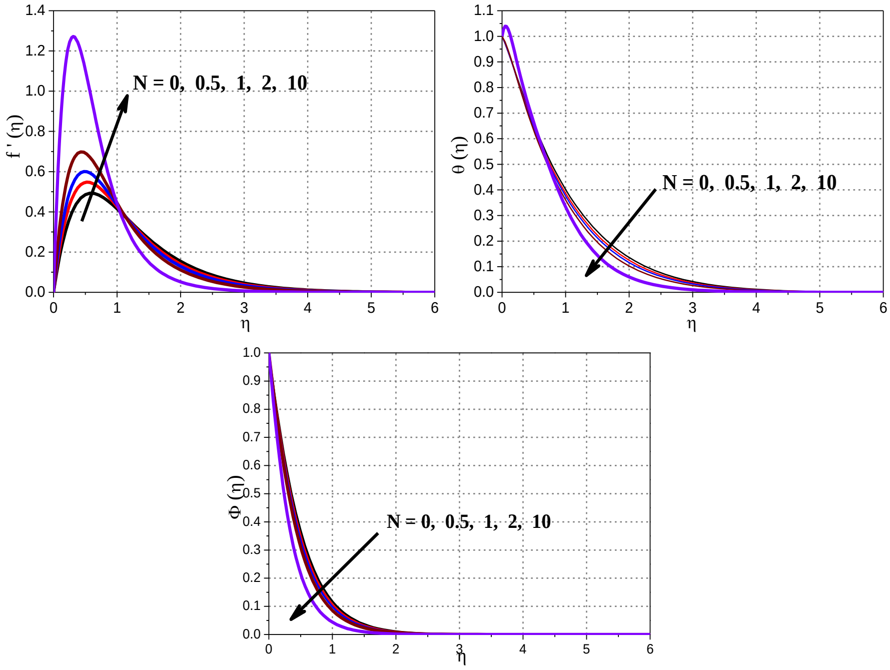


Figure 1. Velocity a), temperature b), and concentration c) profiles for various values of N with n=1; Pr=0.72; ξ =5; M=1; Ec=1; S=0.2; R=1; m=2; Sc=0.5; J=2

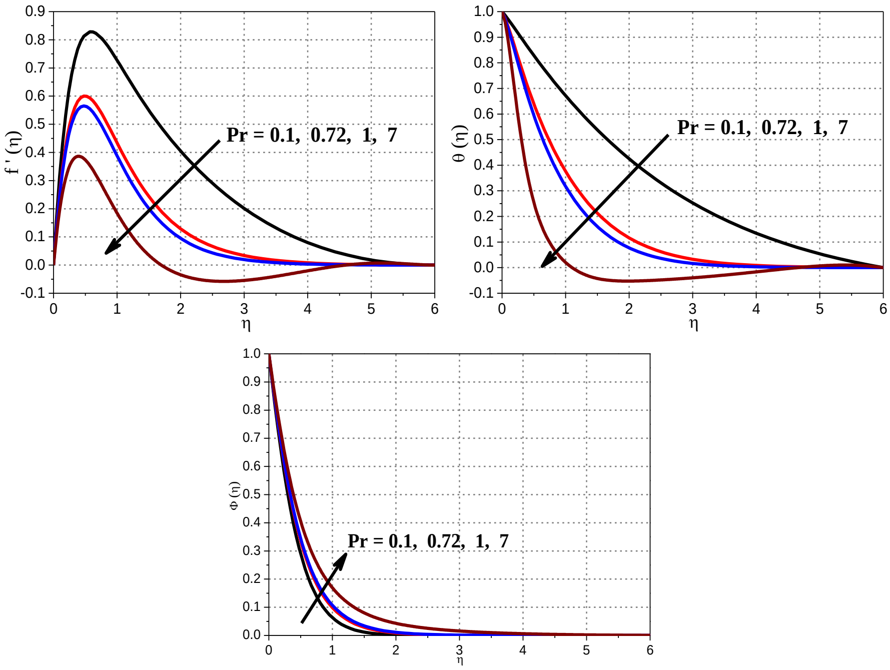


Figure 2. Velocity a), temperature b), and concentration c) profiles for various values of Pr with n=1; ξ =5; M=1; Ec=1; S=0.2; R=1; m=2; Sc=0.5; J=2, N=1

The velocity, temperature and concentration are affected by Prandtl number *Pr*, that's what Figure 2 clarifies. When the Prandtl number increases, the velocity and temperature of the fluid go down considerably. As the fluid becomes more viscous or less conductive, the velocity and the temperatures profiles are reduced in the respective boundary layers. This is not the case for the concentration profile which is not affected by the Prandtl number.

The influence of Schmidt number *Sc* on velocity, temperature and concentration of the fluid boundary layer is shown on Figure 3. A slight rise of the thermal boundary layer is observed when Schmidt number *Sc* increases, but the velocity and concentration decrease, caused by a relatively more viscous fluid than mass diffusive. The thickness of the concentration boundary layer is reduced, because a big diffusivity is imposed by the specific fluid. Sc does not act on the temperature profile.

Figure 4 presents typical profiles for the velocity, temperature and concentration for various values of Eckert number *Ec*. As shown, the velocity and the temperature are increasing with increasing *Ec*, the natural convection is dominant over the thermal capacity of the fluid. The concentration of species profile decreases slightly as *Ec* increase.

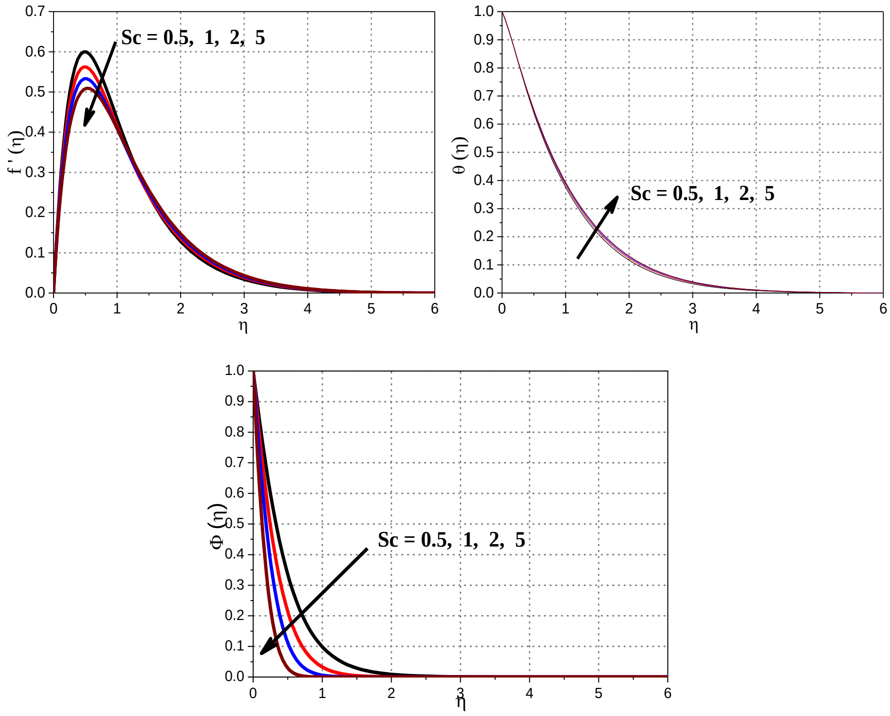
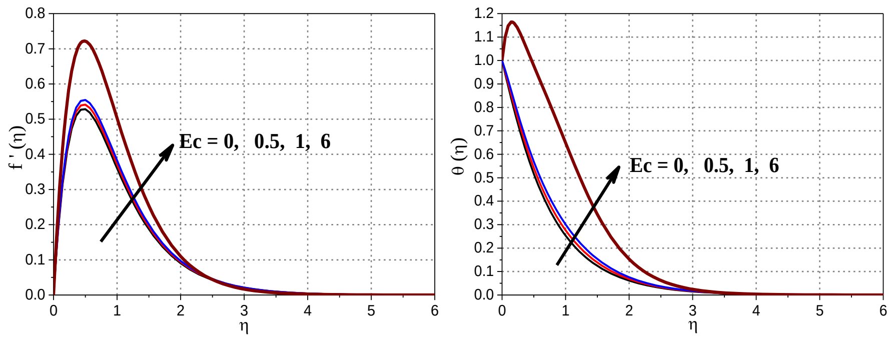


Figure 3. Velocity a), temperature b), and concentration c) profile for various values of Sc with n=1; Pr=0.72; ξ =5; M=1; Ec=1; S=0.2; R=1; m=2; J=2; N=1



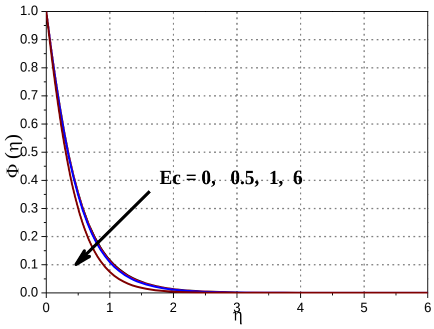


Figure 4. Velocity a), temperature b), and concentration c) profile for various values of Ec with n=1; Pr=0.72; ξ=5; Ec=1; S= - 0.5; R=1; Sc=0.5; m=2; J=2; N=1

Table 1. The effect of parameters Ec, Sc, N, and Pr on $-\theta'(\xi,0), f''^{(\xi,0)}$, *and* $-\Phi'(\xi,0)$ *with* $n = 1;\ \xi = 5;\ S = 0.2;\ R = 1;\ m = 2;\ J = 2;\ M = 1$

E_c	S_c	N	P_r	$-\theta'(\xi,0)$	$f''(\xi,0)$	$-\Phi'(\xi,0)$
0.5	0.5	1	0.72	0.7206	3.0824	1.8049
1				0.4712	3.1498	1.8206
1.5				0.1969	3.2220	1.8371
Slp				-0.5486	0.1444	0.033
1	**0.5**	1	0.72	0.4712	3.1498	1.8206
	1			0.4910	3.0164	2.4259
	5			0.5290	2.7252	4.6601
	Slp			0.01156	-0.086	0.6032
1	0.5	**0.5**	0.72	0.5235	2.6531	1.7543
		1		0.4712	3.1498	1.8206
		3		0.1570	4.9834	2.0295
		Slp		-0.1496	0.9277	0.1084
1	0.5	1	**0.1**	0.2827	3.8344	1.9741
			0.72	0.4712	3.1498	1.8206
			3	0.5746	2.6477	1.6663
			Slp	0.0856	-0.3580	-0.0957

Table 1 represents values of the local Nusselt number, the skin-friction coefficient, and the local Sherwood number. According to more demonstration, it is useful to use the slope of the linear regression through the data points method to show the influence of one parameter on the quantities of interest. As we can see in this table, Strong influence of the buoyancy parameter and the Prandtl number are observed on the friction coefficient, while the Schmidt number have no effect on the Nusselt number and the friction coefficient. In the same way, the Eckert number does not affected the Sherwood number. Finally, it is clear that in others cases, the quantities of interest are moderately influenced by the parameters.

4. Conclusion

In this paper, a numerical analysis is presented to investigate the influence of Eckert, Schmidt, Prandt numbers, and Buoyancy ratio-indicating N on the MHD free heat and mass transfer around horizontal or inclined plate in the presence of chemical reaction, radiation heat flux and internal heat generation or absorption.

The outcomes of this present analysis are listed below:

The flow conducted with a strong buoyancy parameter and with a fluid that has a great Prandtl number can lead to an undesirable friction coefficient.

The variation of the Eckert and Prandtl numbers, the buoyancy parameter affects moderately the Nusselt number.

These preliminary results can help the designer to adjust the parameters and numbers in order to produce more efficiency of the flat plate solar captor.

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