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PRESENCE OF GOLDEN RATIO RELATIONSHIPS IN Fe–Fe_3C, Cu–Zn AND Cu–Sn ALLOY SYSTEMS

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ABSTRACT
Iron–carbon, copper–zinc and copper–tin are the three most significant alloy systems. Iron–carbon system is commonly known for steels, copper–zinc for brasses and copper–tin for bronzes. This paper shows how eutectoid, peritectic, eutectic, cementite-intermetallic, maximum solubility of carbon in ferrite, maximum solubility of carbon in austenite points in Fe–Fe_3C phase diagram are linked to one another through golden ratio relationships. This paper also shows the ratios between proportions of Cu and Zn at maximum solubility of Zn in Cu at 456 and 0°C and the ratio of proportions of Cu–Sn at ε-Cu_3Sn (IMC) in Cu–Zn and Cu–Sn phase diagrams, respectively, are related to golden ratio. It is predicted that the occurrence of these relationships at vital points in mentioned systems is in accordance with Constructal law of design in nature and golden ratio is the reason for uniqueness and usefulness of these points.
Keywords: cementite, eutectic, eutectoid, ferrite, golden ratio, iron–iron carbide phase diagram, pearlite, peritectic

1 INTRODUCTION

Iron–iron carbide phase diagram is the base of steel metallurgy. Almost every steel grade that has been developed has its roots in Fe–Fe_3C phase diagram. From the simple construction grades of steel, for example, ASTM Grade 40 and Grade 60 [1, 2], which account for more than 65% of the total production of world steel, to a highly advanced M42 High Speed tool steel all are developed after deep understanding of iron–iron carbide phase diagram. It should be noted that eutectoid, peritectic, eutectic, cementite-intermetallic, maximum solubility of carbon in ferrite and maximum solubility of carbon in austenite points are vital in understanding the unique behavior of phases present in iron–carbon system. This paper reveals how the points of concern in Fe–Fe_3C phase diagram are linked with one another through golden ratio.

Similarly, golden ratio relationships are also found at key points of phase diagrams of copper–zinc and copper–tin alloy systems. Cu–Zn and Cu–Sn systems are other two most important systems in material science. This paper also shows that the ratios of proportions of Cu–Zn and Cu–Sn at key points in their respective phase diagrams are associated with golden ratio.

2 GOLDEN RATIO

'If a line segment is divided in such a way that the ratio of its total length a + b to "a" is equals to the ratio of "a" to "b" then this is called golden ratio'. The value of Golden ratio is 1.618 and it is denoted by 'φ' (Wolfram research [3]).

Mathematically,

$$\varphi = \frac{1+\sqrt{5}}{2} = 1.618$$

Golden ratio has some extraordinary mathematical properties (Wolfram research [3]). These properties are,

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1. It satisfies the equation,

$$x^2 - x - 1 = 0$$

   This equation has the following solutions:

$$x = \varphi \text{ and } x = 1 - \varphi$$

2. The golden ratio conjugate is represented by '∅',

$$\varnothing = \varphi - 1$$

3. The ratio also satisfies the following equations:

$$\varphi - 1 = \frac{1}{\varphi} \quad \text{and} \quad \varnothing + 1 = \frac{1}{\varnothing}$$

4. This ratio can be shown as a continued fraction, first used by R. Simson (1753),

$$\varphi = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \dots}}}}$$

   Or,

$$\varphi = 1 + \cfrac{1}{1 + \cfrac{1}{\varphi}}$$

Golden ratio is commonly found in nature. Some scientists have shown it as secret of beauty, strength of a building structure, electronics, properties of materials (such as quasi-crystals), etc. Bejan [4] predicted the golden ratio as a 'single design in nature'. There may be numerous examples of this. Let us take technology as an example, Bejan [4] showed in his paper how a picture with length-to-height ratio nearly equal to 3/2 or the golden ratio has led to the development of a TV screen that appeals to us most. It is an amazing fact that a picture with resolution 1280 × 800 looks more striking as compared with the similar picture with some other resolution say 1024 × 768. The reason behind this is the golden ratio. Apart from number of pixels, the ratio (L/H) of 1280 to 800 is 1.67 while it is 1.33 for 1024 × 768; it means 1.67 is nearer to 1.618 than 1.33. More examples of golden ratio with respect to evolution of technological designs can be found in the book *Design with Constructal Theory* [5].

3 IRON–IRON CARBIDE PHASE DIAGRAM

Iron–iron carbide phase diagram is a binary phase diagram of iron and carbon, and it shows the transformation of different phases with respect to change in the percentage of carbon in iron and temperature at a constant pressure of 760 torr. It is a metastable phase diagram in which cementite (Fe_3C) intermetallic compound is a metastable state. Cementite, despite being a metastable state, even at 700°C, takes years to decompose into iron and graphite [6]. The principle of phase diagram states no phase change with respect to time but the time required to decompose cementite is so huge at room temperature that iron–iron carbide phase diagram is considered to practically represent equilibrium conditions [6]. The *x* and *y* axes represent increase in percentage of carbon and temperature, respectively. The useful part of iron–carbon system is up to 6.67% of carbon the only which is termed as iron–iron carbide phase diagram (Fig. 1).

3.1 Description of the important points in iron–iron carbide phase diagram

In Fig. 1, points 1–6 represent the maximum solubility of carbon in ferrite, peritectic, eutectoid, maximum solubility of carbon in austenite (steel limit), eutectic and cementite (intermetallic) points, respectively. The maximum solubility of carbon in ferrite (sometimes called pure iron) is 0.025% at 723°C (Point No. 1) and it decreases with decrease in temperature. The solubility of carbon in ferrite is only 0.008% at room temperature [6]. At 0.18% carbon, at 1493°C, a reaction takes place in which one liquid and one solid (delta iron) convert into one solid called austenite; this reaction is called peritectic reaction (Point No. 2).

$$\text{Liquid} + \delta(\text{ferrite}) \rightarrow \gamma(\text{austenite})$$

This point is of significant importance from continuous casting's perspective. Heat transfer flux between copper mold and liquid steel is lowest for peritectic steels during continuous casting [8]. Point No. 3, eutectoid point, is at 0.8% carbon. At 723°C, a reaction takes place, called eutectoid reaction, in which one solid (austenite) decomposes into two distinctive solids namely ferrite (α) and cementite (Fe_3C) [6].

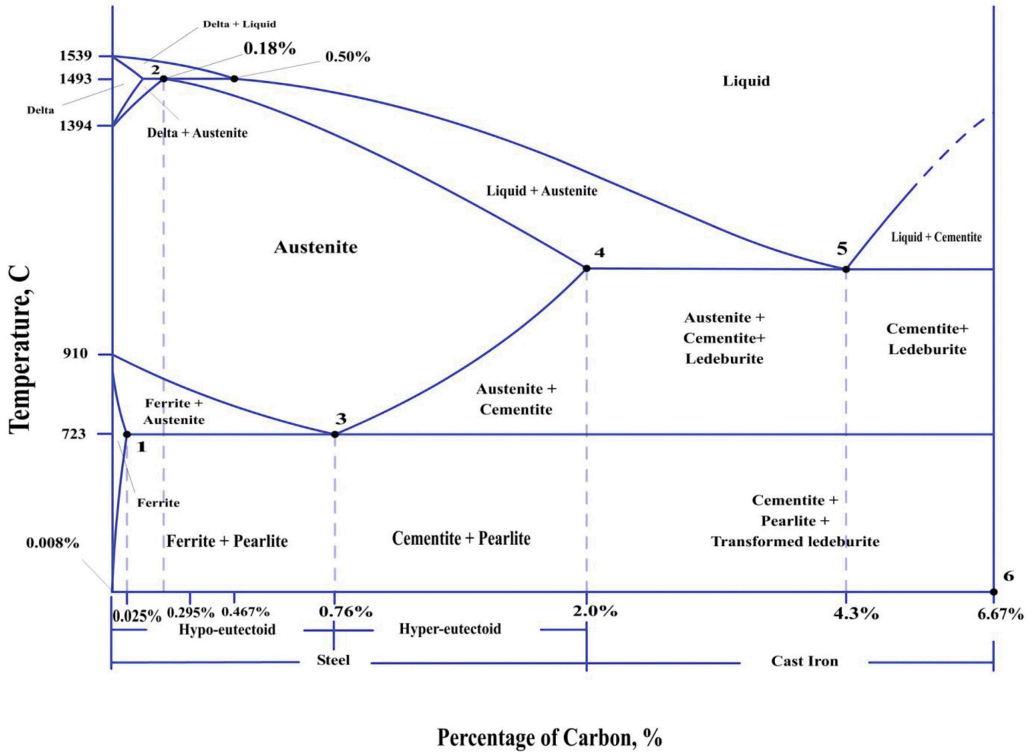

Figure 1: Iron–iron carbide phase diagram [6, 7].

$$\gamma(\text{Austenite}) \rightarrow \alpha(\text{ferrite}) + Fe_3C(\text{cementite})$$

The structure of the steel containing 0.76% carbon contains lamellar structure of ferrite and cementite called pearlite, a naturally existing composite, completely. In pearlite, the percentages of ferrite (α) and cementite (Fe_3C) are 88% and 12%, respectively [7]. Maximum solubility carbon in austenite, represented by Point No. 4 in Fig. 1, is 2%. It is the steel limit or maximum amount of carbon that steel can dissolve, iron containing carbon percentage beyond this limit is not considered as steel. Eutectic point is represented as Point No. 5 in Fig. 1. At this point, at 1147°C, one liquid converts into two solids namely austenite and cementite; the structure formed as a result is called ledeburite [6].

$$\text{Liquid} \rightarrow \gamma(\text{austenite}) + Fe_3C(\text{cementite})$$

The vertical line in Fig. 1 at Point No. 6 shows cementite intermetallic compound. The structure of iron containing 6.67% carbon from this phase diagram will contain cementite only [6].

4 COPPER–ZINC AND COPPER–TIN PHASE DIAGRAMS

Cu–Zn and Cu–Sn systems are commonly known for brasses and bronzes, respectively. Cartridge brass (70Cu–30Zn) is the most widely used brass. The important points in Cu–Zn phase diagram from metallurgical point of view are maximum solubility of Zn in Cu (38% at 456°C) and solubility of Zn in Cu at 0°C (29%) and at room temperature (30% at 20°C) [9], shown in Fig. 2. In 37.2–38% Sn (approximately) in Cu–Sn phase diagram (Fig. 3), there exists microstructure full of ε-Cu_3Sn intermetallic compound [9]. ε-Cu_3Sn is usually found in lead-free solder joints.

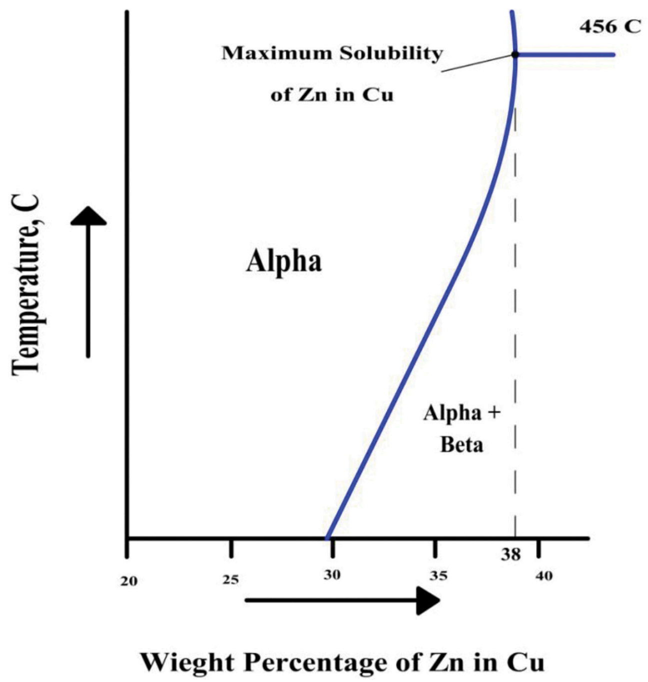

Figure 2: A section of Cu–Zn phase diagram [9].

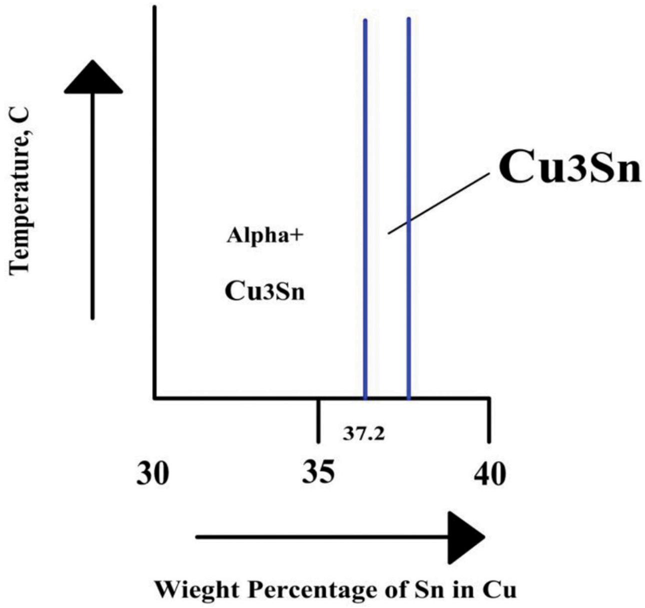

Figure 3: A section of Cu–Sn phase diagram [9].

5 AUTHOR'S FINDINGS

Before going into the details of findings, some calculations related to it are presented that will later help in better understanding of the finding of this paper. In Fe–Fe_3C alloys system, let us suppose that 'W_α' and 'W_p' are the weight fractions of ferrite and pearlite, respectively, for the microstructure containing ferrite and pearlite phases,

$$Wp = 1 - W\alpha \tag{1}$$

Now, consider the ratio of the weight fraction of ferrite phase to the weight fraction of pearlite phase is equal to golden ratio 1.618

$$\frac{W\alpha}{Wp} = \frac{\varnothing}{1-\varnothing} = 1.618 \tag{2}$$

This equation can also be written as

$$\frac{W\alpha}{1-W\alpha} = \frac{\varnothing}{1-\varnothing} = 1.618 \tag{3}$$

From this equation, 'W_α' is equal to 0.618.

This ratio is now applied to the iron–iron carbide phase diagram to find out at what percentage of carbon this ratio will be obtained. Since it is known that at room temperature in equilibrium conditions, ferrite and pearlite contain 0.008% and 0.76% carbon, respectively, so, for 0.618 weight fraction ferrite, carbon percentage is given by

$$\%C = 0.008W\alpha + 0.76(1 - W\alpha) \tag{4}$$

When the value of 'W_α' is substituted in eqn (4), we obtain the carbon percentage 0.295.

Similarly, if the ratio of 'W_p' to 'W_α' is considered to be 1.618, then the percentage of carbon will be 0.467.

Now, the actual findings of this paper will be discussed. The numbers 0.295, 0.467 and 0.76 nearly obey the definition of golden ratio

$$0.295 + 0.467 = 0.762 \text{ (eutectoid point)} \tag{5}$$

$$\frac{0.295 + 0.467}{0.467} \approx \frac{0.467}{0.295} \approx 1.618 \tag{6}$$

Equations that relate key points of iron–carbon alloy system with golden ratio are shown in Table 1. In each equation, '2' is divided; the reason behind it is that 2% carbon is the maximum solubility of carbon in austenite thereby each key point is linked to this point (2%) with golden ratio.

When the temperatures are normalized with the temperature of points 4 and 5, the results obtained are shown in Table 2.

In copper–zinc alloy system, the approximate equations with golden ratio for key points are shown in Table 3.

Table 1: Mass % carbon approximate equations with golden ratio.

Carbon (%)	Approximate equations with golden ratio
0.008	$\frac{2}{2\varphi^{10}} = 0.00813$
0.025	$\frac{2}{\varphi^{9}} = 0.0263$
0.18	$\frac{2}{\varphi^{5}} = 0.18$
0.295	$\frac{2}{\varphi^{4}} = 0.292$
0.467	$\frac{2}{\varphi^{3}} = 0.472$
0.760	$\frac{2}{\varphi^{2}} = 0.764$
4.30	$\frac{2}{2\varnothing^{3}} = 4.24$
6.67	$\frac{2}{2\varnothing^{4}} = 6.85$

Table 2: Temperature (°C)/1147°C approximate equations with golden ratio.

Key temperatures (°C)	Approximate equations with golden ratio
1493	$\frac{1493}{1147} = 1.30 \approx \frac{\varphi^2}{2} = 1.31$
1394	$\frac{1394}{1147} = 1.21 \approx 2\varnothing = 1.24$
910	$\frac{910}{1147} = 0.79 \approx \frac{\varphi}{2} = 0.81$
723	$\frac{723}{1147} = 0.63 \approx \varnothing = 0.618$

Table 3: Mass % Cu and Zn, approximate equations with golden ratio.

Cu (%) and Zn (%)	Approximate equations with golden ratio
Cu= 62 Zn= 38	$\frac{62}{38} = 1.63 \approx \varphi = 1.618$
Cu= 71 Zn= 29	$\frac{71}{29} = 2.45 \approx \varphi^2 = 2.62$

In Cu–Sn alloy system, the percentage range of Sn at which the microstructure is completely of ε-Cu_3Sn is 37.2–38 (approximately). Approximate equation with golden ratio for this point is given by

$$\frac{62.8}{37.2} = 1.69 \approx \varphi = 1.618 \tag{7}$$

6 CONCLUSION

For a given alloy system, we could obtain infinite number of alloys by slightly varying the proportions of individual constituents. Phase diagrams show equilibrium conditions, that is, at each point of phase diagram, alloy must solidify in such a way that it takes the easiest route of solidification to attain the lowest possible free energy state and to gain maximum thermodynamical and chemical stability, which is also in accordance with universal Constructal law design in nature. A combination of Constructal law with qausi-symmetric properties of golden number, present between two entities of similar dimensions, has led to some striking phenomena occurring at some points in the above-mentioned phase diagrams in the form of maximum solubility, formation of natural composite structure (eutectoid reaction) in steels, intermetallic compound formation, metallurgical reactions, etc. This combination of Constructal law and golden ratio has been the pattern of nature found in most balanced, beautiful and strong things on earth. However, the better understanding of this phenomenon in material science requires further work. Other alloys systems, binary, ternary or multi-components could be explored in the perspective of present work.

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