*M. Rahman, Int. Journal of Design & Nature. Vol. 1, No. 1 (2007) 70–73*

# A TECHNICAL NOTE ON THE RECENT TSUNAMI IN THE INDIAN OCEAN

M. RAHMAN
Department of Engineering Mathematics, Dalhousie University, Halifax, Nova Scotia, Canada.

ABSTRACT
This article presents a brief description of the recent natural disaster in the form of the tsunami that occurred in Southeast Asia on 26 December 2004, killing more than 300,000 people in that region. The name *tsunami* is a Japanese scientific term that means *harbor waves*. In particular, it is defined as a seismic disturbance of the ocean, a great sea wave produced by a submarine, earth movement or volcanic eruption. It can also be described as a tidal wave caused by an earthquake or other disturbances. The effect of a tsunami is caused by the interaction of the solid and the liquid in violent oscillatory movements. Therefore, one must be familiar with both solid and fluid mechanics. In this article I will provide a brief mathematical background of how this devastating natural disaster can be related to the mathematical theories of solids and fluids.
*Keywords: earthquake, hydrodynamics, seismic, soil dynamics, solitary waves, surface waves, tsunami.*

## 1 INTRODUCTION

Tsunamis are the water waves generated by earthquakes. If the ocean floor displacement in the area of the earthquake is known, the water wave problem is a purely hydrodynamic one. Unfortunately, direct measurements near the epicenter are very difficult to make, and a good deal of effort has been centered on using water wave records measured at large distances from the epicenter to infer roughly the nature of tectonic movement. Hence, there have been a considerable number of theoretical studies on water waves due to a variety of ground movements. Among the many features of tsunamis recorded near a coast, two have been frequently reported: one feature is that the arrival of a tsunami is often preceded by the withdrawal of water from the beaches, and the other is that the first crest may not be the largest. I want to emphasize here that the tsunami waves are extremely hostile whereas the surface gravity waves are harmless.

## 2 SEISMIC VIBRATIONS

Many ocean beds on our planet earth consist of seismic zones as many geologists have predicted. There are many volcanic zones on the beds of the Indian Ocean. From time to time, these volcanic zones become active, causing enormous damage when they erupt. The vibration of the earthquake can be mathematically predicted by solving the following idealized linear partial differential equation with the appropriate initial and boundary conditions [1]:

$$u_{tt} + X(t) = \frac{C^2}{h^m}\frac{1}{z}\frac{\partial}{\partial z}\left(z^{1+m}\frac{\partial u}{\partial z}\right). \tag{1}$$

Here $u(z,t)$ is the vibration or oscillation of the ocean bed in the vertical direction due to the earthquake, $u_{tt}$ is the inertia term which is the natural mode of acceleration, $X(t)$ is the general excitation of the earthquake, the last term on the right hand side of eqn (1) is the shear force of the viscoelastic material of the ocean bed, $C$ is the shear velocity of the ocean bed, $h$ is the depth of the ocean and $m$ is the parameter of the model. Note that this partial differential equation is the consequence of Newton's law of motion. It is generally assumed that $m = 0.5$ but the experimental data suggest that $m$ can vary considerably, i.e. $0 \leq m < 1$. If the general excitation is more than


ISSN: 1744-3687 (paper format), ISSN: 1744-3679 (online), http://journals.witpress.com

6.5 on the Richter scale, the earthquake is considered extremely hazardous and the ocean bed or the earth will vibrate with intense magnitude. The ocean bed is usually considered as a sheet of elastic and plastic plates. When there is an excitation, it will start oscillating and this oscillation will cause the water above the bed to oscillate thereby displacing the water mass. The water mass will move up and down just like a heave motion with the intensity of the earthquake. If this phenomenon occurs in the shallow water ocean, its intensity will be great and a tsunami will be produced. In Southeast Asia, the earthquake occurred near the Indonesian coast and the epicenter was a few kilometers away from Banda Aceh. The magnitude of the earthquake was 9.0 on the Richter scale, which is considered a devastating intensity. Once the displaced water moves away from the epicenter, the water mass will travel in the form of a progressive wave with tremendous speed and high amplitude towards the shoreline. The wave moves like a soliton. This is called a solitary wave. The mathematics plays a very important role in predicting this kind of wave phenomenon.

## 3 SURFACE WAVES

The behavior of waves at the surface of water depends on its surface tension and viscosity, on gravity, and, of course, on the boundary conditions, especially the depth of the body of water. For waves of sufficiently long wavelength and with comparatively small amplitude, viscosity and surface tension effects can be neglected, the effect of gravity dominates and these waves have often been called *gravity waves*. To understand this sort of phenomenon [2], we must make some assumptions. First, let us assume that the velocity field $\mathbf{V}$ is irrotational, which implies that there are no eddies or vortices, and therefore it can be derived from a scalar potential. Using vector calculus, we obtain that if $\nabla \times \mathbf{V} = 0$ then there exists a scalar potentials $\phi$ such that $\mathbf{V} = \nabla\phi$. Without fluid sources, the velocity field of an incompressible fluid (water) has no divergence, i.e. mathematically $\nabla \cdot \mathbf{V} = 0$, and therefore the potential must satisfy $\nabla \cdot \nabla\phi = 0$, i.e. $\nabla^2\phi = 0$ which is Laplace's equation.

Essentially, one seeks wave-like solutions to Laplace's equation subject to the boundary conditions that (i) the vertical component of the velocity at the bottom of the body of water be zero and (ii) the pressure at the surface be the atmospheric pressure. The later condition is converted to a condition on $\phi$ through Bernoulli's equation [2]:

$$\frac{P}{\rho} + \frac{\partial\phi}{\partial t} + gz + \frac{1}{2}|\nabla\phi|^2 = 0, \tag{2}$$

where, for waves of small amplitudes, the velocity squared term $|\nabla\phi|^2$ is neglected. Here, $P$ is the pressure, $\rho$ is the density of water, $g$ is the acceleration due to gravity, $gz$ is the hydrostatic term, $\partial\phi/\partial t$ is the inertia term (which is the rate of change of the velocity potential $\phi$), $t$ is the time and $z$ is the vertical coordinate. It is worth noting that eqn (2) is the momentum equation obtained using Newton's law of motion in fluids. In order to solve the differential equation, the method of separation of variables is used and we obtain traveling wave solutions with phase velocity $c$ given by

$$c = \frac{\sigma}{k} = \sqrt{\frac{g}{k}\tanh(kh)}, \tag{3}$$

where $h$ is the depth, $\sigma$ is the wave frequency, $k = 2\pi/L$ is the wave number and $L$ is the wavelength. For waves with wavelength much smaller than the depth, which is the case for deepwater ocean, the hyperbolic tangent tends towards unity, and the wave velocity becomes $c = \sqrt{g/k} = \sqrt{gL/2\pi}$ for $L \ll h$.

For instance, a wave of wavelength 150 m in the kilometers deep ocean will travel approximately 15 m/s, or 55 km/h. But earthquakes in shallow water generate waves with extremely long wavelength, and in this case, tanh($kh$) tends to $kh$ and thus $c = \sqrt{gh}$ for $L \gg h$. The important part here is not necessarily that tsunamis travel fast: they do, of course; a wave with a wavelength of 100 km will travel at 395 m/s or 1422 km/hr. But rather, it turns out that in this limit *there is no dispersion*; the wavelength does not appear in the expression for velocity. So waves created by an earthquake remain in a very small wave packet for thousands of kilometers after their origin; thus whatever they hit receives all the energy essentially at once. The other reason why tsunamis are destructive is that for this sort of shallow water wave, the horizontal component of the water displacement is quite large: the water moves back and forth, mainly [3].

## 4 SOLITARY WAVES

The waves produced by an earthquake in a shallow water ocean can also be described by citing Scott Russell's observation in a channel. Russell's *Report on Waves* in 1844 describes his famous chase on horseback behind a wave in a channel. His interesting and exciting description of the solitary wave that he observed accidentally is reproduced here.

> I was observing the motion of a boat which was rapidly drawn along a narrow channel by a pair of horses, when the boat suddenly stopped—not so the mass of water in the channel which it had put in the motion; it accumulated round the prow of the vessel in a state of violent agitation, then suddenly leaving it behind, rolled forward with great velocity assuming the form of a large solitary elevation, a rounded, smooth and well defined heap of water, which continued its course along the channel apparently without any change of form or diminution of speed, I followed it on horseback, and overtook it still rolling at a rate of some eight to nine miles an hour, preserving its original figure some thirty feet long and a foot to a foot and half in height. Its height gradually diminished and after a chase of one or two miles I lost it in the winding of the channel. Such, in the month of August, 1834, was my first chance interview with that singular and beautiful phenomenon.

The study of solitary waves was first initiated by Korteweg and de Vries in 1895 after Scott Russell's excellent field observation about the phenomenon of this beautiful wave in shallow water ocean. A tsunami is the nonlinear shallow water wave produced by a natural disaster such as an earthquake on the ocean bed. The wave elevation can be predicted by solving a nonlinear partial differential equation subject to given initial and boundary conditions. As mentioned above, it is not a dispersive wave. The mathematical equation in one-dimensional form is:

$$\eta_t + \eta\eta_x + \eta_{xxx} = 0, \tag{4}$$

where $\eta(x, t)$ is the wave elevation in dimensionless form. Equation (4) is the nonlinear dimensionless wave equation and is obtained using Newton's law of motion in shallow water ocean.

The solution of this equation can be found as

$$\eta\,(x, t) = (3b)\,\mathrm{sech}^2 \left\{ \frac{\sqrt{b}}{2}\,(x - bt) \right\}. \tag{5}$$

Here the amplitude of the wave is $(3b)$ and $b$ is the dimensionless wave speed. Thus, from this simulation it is clear that with the increase of wave speed the wave amplitude will be increased. It can be easily seen that the solution $\eta$ always lies above the $x$-axis with a hump progressing forward with respect to time without diminishing its original shape and height. However, with the amplified speed, the hump of the wave will be extremely steep behaving like a Dirac delta function. The energy contained in the hump is proportional to the square of the wave amplitude, i.e. $(3b)^2$, in this simulation.

[Note: The energy $E = \left(\frac{1}{2}\rho g\right)\int_{-\infty}^{\infty}|\eta(X)|^2\,\mathrm{d}X = \left(\frac{1}{2}\rho g\right)\left(18b^2\right)\int_0^{\infty}\mathrm{sech}^4 X\,\mathrm{d}X = \left(\frac{1}{2}\rho g\right)\left(12b^2\right)$, where $X = \frac{\sqrt{b}}{2}(x - bt)$. Here the speed of the tsunami seems to be equal to the speed of sound.] Thus, the wave will travel forward carrying enormous energy until it hits some objects on the way (recall Einstein's energy formula $E = mv^2$, where $m$ is the mass of the particle and $v$ is the speed of light). This is exactly what happened during the recent tsunami that occurred in the Indian Ocean on 26 December 2004.

## 5 CONCLUDING REMARK: HOW A TSUNAMI FORMS

Tsunamis are usually caused by underwater earthquakes. These often occur offshore at subduction zones (places where a tectonic plate that carries an ocean is gradually slipping under a continental plate). Part of the sea floor can snap upward abruptly, while other areas sink downward, when sections of the plates that have been locked together for a while move suddenly under the strain. In the instant after such an underwater earthquake, the shape of the sea surface mirrors the new contours of the sea floor—some areas of water are pushed upwards and others sink.

This starts a series of waves that rush outwards—the beginning of a tsunami. These waves travel very far and very fast (more than 800 km/h, or the speed of a jet airplane). At first, out at sea in deep water, the waves are very far apart—sometimes hundreds of kilometers—and their crests are not very high, perhaps only a few meters above the rest of the surface (although these crests are only the tips of the vast masses of water in motion).

To an observer on a passing ship, or a low-flying plane, these waves would probably not even be noticeable. However, when a tsunami leaves deep water and approaches the shore, it slows down and its height grows. The wave crests also squeeze closer together. Depending on the shapes of the sea floor off the coast and of the coastline, a tsunami hitting the coast may appear as a series of towering walls of water that can level buildings.

## ACKNOWLEDGMENT

Thanks are extended to the Natural Sciences and Engineering Research Council (NSERC) of Canada for its financial support.

## REFERENCES

[1] Dakpulas, P. & Gazetas, G., A class of inhomogeneous shear models for seismic response of dams and embankments, *Journal of Soil Dynamics and Earthquake Engineering*, **4(4)**, pp. 166–182, 1985.

[2] Rahman, M., *Water Waves: Relating Modern Theory to Advanced Engineering Applications*, Oxford University Press: Oxford, 1995.

[3] Elmore, W.C. & Heald, M.A., *Physics of Waves*, McGraw-Hill: New York, 1969.