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Computer-aided analysis of saturation in synchronous machines

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ABSTRACT

There are a number of secondary effects in a synchronous machine which are most often neglected to develop a simplified model. This procedure is simple and straight-forward, but it gives rise to much errors. To avoid the errors, the modelers have developed several techniques. Saturation is one of the secondary effects which become quite dominant at certain load conditions. In the earlier regimes, grapho-analytical techniques were developed to treat the effect of saturation. But such methods were tedious, complex and time-consuming. Now they should be replaced by computer methods. This paper gives a comprehensive discussion on the various methods developed to treat the effect of saturation. It also gives computer-aided methods to replace the older methods.

Keywords: Saturation, Grapho-analytical Technique, Exponential Method, Frolich's Equation, Method of Least Square.

1. INTRODUCTION

There are secondary effects in a synchronous machine which affect the performance variables- they affect the transient as well as steady state performance [1]. The secondary effects are:

i) saturation of the magnetic circuits,
ii) space and time harmonics and
iii) distributed eddy currents in solid metallic parts.

Many authors and modelers have made analysis in this area and bulk of papers has been advanced [2, 3, 4, 5, 6]. Out of these three, saturation also affects the performance of exciters and transformers [7]. Saturation cannot be avoided in machines/ transformers for economic reasons. So, its effect should be accounted for in circuit analysis and design of electrical machines [8, 9, 10]. So only the effect of saturation is being focused in this paper.

The methods improvised in the past were mostly grapho-analytical, as such tedious and time-consuming. They offered various degrees of accuracy. In this paper, the commonly used methods are being discussed and computer methods are being developed to replace the grapho-analytical.

2. THE EXISTING METHODS

There are several techniques to account for saturation in synchronous machine [2, 3, 5] and exciters. These are as given below:

a. Kingsley's method of saturated synchronous reactance.
b. The technique developed by Anderson and Fouad.
c. Frolich's method
d. Modified Frolich's method
e. Exponential method.

Kennelly was also associated with Frolich in his works on saturation. These methods are being discussed one after another along with techniques for computer-aided solutions [11, 12]. For computer-based solutions higher mathematics [13] and numerical methods [14] are adopted.

3. THE KINGSLEY'S METHOD

The Kingsley's method is the simplest one and is commonly used. This is a grapho-analytical method, by which a saturation factor is found out from the OCC/SCC of the machine. The unsaturated synchronous reactance is found out from these curves and the saturated synchronous reactance [15] is found out from:

$$X_{ds} = x_a + (X_{du} - x_a)/k \tag{1}$$

The saturated value of synchronous reactance is used instead of the unsaturated one for computations on stability etc. To find out the saturation factor, the steps are as given below:

a. Plotting the OCC and extend its linear part to get the air-gap line.

b. Find out the voltage behind the armature leakage reactance at full load:

c. $E_r = V + I_a(r_a + jx_a) \simeq V + I_a(jx_a)$ (2)

d. x_a is obtained from manufacturer's data. The effect of r_a is very small and negligible.

e. Finding out excitation required to produce E_r on the air-gap line (OA)

f. Finding out excitation required to produce E_r on the OCC (OB)

g. Finding out the saturation factor $k = \text{OB/OA}$

h. Finding out unsaturated synchronous reactance X_{du} from the air-gap line and SCC.

i. Finding out the base values from the rating of the machine and the p.u. values of reactance.

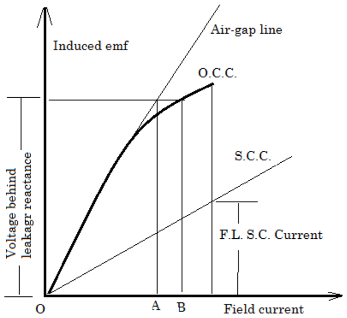

Figure 1. The OCC and SCC of a turbo-generator

3.1 Case-study on Kingsley's method

Saturated synchronous reactance is being found out by KINGSLEY'S method. The data is obtained from reference [16].

The machine in the case study is the 5th unit of BANDEL TPS, manufactured by BHEL.

MW-rating = 210; Rated power factor = 0.85 lag

Rated L-L voltage = 15.75 kV

The corresponding MVA-rating = 247.

The data for OCC is given below:

Table 1. Data for OCC of the synchronous generator

Field current, A	Induced emf, V	Field current, A	Induced emf, V
100	1800	1300	19100
200	3600	1400	19650
300	5400	1500	20100
400	7200	1600	20600
500	9000	1700	21000
600	10800	1800	21400
700	12600	1900	21800
800	14200	2000	22150
900	15600	2100	22450
1000	16700	2200	22700
1100	17700	2300	22900
1200	18500	2400	23100

The OCC is linear up to an excitation of 700 A while it induces a line voltage of 12600 volts (L-L).

The slope of the air-gap line = 18 V/A

Field current to produce rated voltage on a-g line = 875 A

Base current of the machine = 9056.5 A

Field current required to produce f.l. short-circuit current = 1850 A

S.C. current at field current corresponding to rated a.g. voltage = 4283.5 A

The value of d-axis unsaturated synchronous reactance = 2.1229 Ω

The base impedance = 1.0041 Ω

The value of d-axis unsaturated synchronous reactance = 2.1143 p.u.

The p.f. angle at rated loading = 31.791°

The voltage behind leakage reactance = 1.0878 p.u.

The reqd. field current at E_r is obtained by linear approximation.

Field current required to produce E_r = 1043.2 A

Field current required to produce E_r on a.g. line = 951.8 A

$$S_{G1.2} = A_G e^{B_G 0.4} \; S_{G1.2} = A_G e^{B_G 0.4}$$

The saturation factor = 1.09607

The saturated synchronous reactance = 1.9423 p.u. (Compared to unsaturated = 2.1143 p.u.)

The method of synchronous reactance is simple and straight-forward but it produces errors- its accuracy is limited. Moreover, the value of the saturated synchronous reactance depends on the load.

4. THE METHOD SUGGESTED BY ANDERSON AND FOUAD

Anderson and Fouad [3] in their famous treatise on power system have suggested a rather simple method to account for saturation of the generator. They have defined a p.u. saturation factor S_G. It is defined in terms of O.C. terminal voltage and field current. As an example, with reference to fig. 1, S_G at

$$E_r = \text{(OB-OA)/OA} \quad (3)$$

With this definition, we specify S_G for two values of terminal voltage 1.0 and 1.2 p.u. as S_{G1} & $S_{G1.2}$ following eqn. 3. P.U. S_G is given as: $S_G = A_G e^{B_G V_\Delta}$, where

$$V_\Delta = V_t - V_{th} = V_t - 0.8 \quad (4)$$

From these equations, we get: $S_{G1} = A_G e^{B_G 0.2}$;

$$1.2 S_{G1.2} = A_G e^{B_G 0.4} \quad (5)$$

which yield the following expressions:

$$A_G = S_{G1}^2 / (1.2 S_{G1.2}) \; ; \; B_G = 5 \ln(1.2 S_{G1.2} / S_{G1}) \quad (6)$$

The saturation factor can be estimated from these expressions for any operating voltage.

4.1.1 Case study on Anderson and Fouad’ method

Slope of the air-gap line = 18 V/A S_G

There is no saturation up to field current, I_f = 700 A

The Saturation starts at a field current, I_f = 800 A

Threshold voltage = 0.8 p.u.

Field current required for developing rated voltage on a.g. line = 875 A

Field current required to produce rated voltage at no load = 913.64 A

Field current required to produce 20% overvoltage on air-gap line = 1050 A

Field current required to produce 20% overvoltage at no load = 1266.7 A

Normalized saturation function: SG_1= 4.4156E-02: SG_2 = 0.20635

Saturation as an exponential function:

$$SG = 7.8739E\text{-}03 \times e^{8.6208 \times V_\Delta} \text{ ; where } V_\Delta = V - V_t$$

5. THE EXPONENTIAL METHOD

There is another way to account for saturation. That is the exponential method. In this method, the describing eqn. is given as:

$$E = aI_f^b \tag{7}$$

The equation can be re-written as:

$$\log E = \log a + b.\log I_f \tag{8}$$

In this form, it is the equation of a straight line and the method of linear regression can be applied to it [13, 14]. The case-study is given below.

5.1 Fitting exponential equation to OCC

Table 2. Actual and calculated values by the exponential method

I_f, A	E_{actual}, V	$E_{calcu.}$, V	% deviation
800	14200	14294	0.6637
900	15600	15120	-3.0749
1000	16700	15900	-4.7926
1100	17700	16639	-5.9934
1200	18500	17344	-6.2470
1300	19100	18019	-5.6581
1400	19650	18668	-4.9991
1500	20100	19292	-4.0187
1600	20600	19895	-3.4205
1700	21000	20479	-2.4804
1800	21400	21045	-1.6582
1900	21800	21595	-0.9405
2000	22150	22130	-0.0910
2100	22450	22651	0.8949
2200	22700	23159	2.0227
2300	22900	23655	3.2989
2400	23100	24141	4.5047
2500	23250	24615	5.8721
2600	23400	25080	7.1799

The sum of squares of % deviation (in the saturated zone for I_f>=800A) = 324.6

The least square line is being fitted to saturation curve:

The fitting eqn. for the saturation curve is given below:

$$E = 589.37 \times I_f^{0.477}$$

Checking the fitting:

The actual values are being compared with the calculated values using the exponential method and are given in table 2.

6. FROLICH’S EQUATION

One of the methods is fitting Frolich’s equation to the saturation curve data. Frolich has proposed the following equation for the saturation curve:

$$E_f = \frac{a I_f}{b + I_f} \tag{9}$$

To fit this eqn. to saturation curve, by the method of linear regression, the following modification has to be made:

$$\frac{1}{E_f} = \frac{1}{I_f}\frac{b}{a} + a \tag{10}$$

This is the eqn. of a straight line. Hence the method linear regression can be used [13, 14].

6.1 Fitting Frolich’s equation to saturation curve by least square line

The fitting eqn. for the saturation curve using the method of least squares is given below:

$$E_f = \frac{35067\, I_f}{1154 + I_f}$$

Checking the fitting:

The actual values obtained by Frolich’s method are being compared to the calculated values and is given in table 3.

Table 3. Actual and calculated values by the Frolich’s method

Field current, A	E_{actual}	$E_{calcu.}$	% deviation
800	14200	14356	1.1032
900	15600	15365	-1.5069
1000	16700	16280	-2.5175
1100	17700	17113	-3.3160
1200	18500	17876	-3.3743
1300	19100	18576	-2.7421
1400	19650	19222	-2.1785
1500	20100	19819	-1.3983
1600	20600	20373	-1.1039
1700	21000	20887	-0.5359
1800	21400	21367	-0.1521
1900	21800	21816	0.0735
2000	22150	22236	0.3889
2100	22450	22630	0.8038
2200	22700	23001	1.3270
2300	22900	23351	1.9675
2400	23100	23680	2.5118
2500	23250	23992	3.1907
2600	23400	24287	3.7899

The sum of squares of % deviation (in the saturated zone for $I_f >= 800A$) = 85.24

7. MODIFIED FROLICH'S EQUATION

Frolich proposed a modified form which gives less error. It is given by equation

$$E_f = \frac{aI_f}{b + I_f} + cI_f \tag{11}$$

Fitting modified Frolich's equation to saturation curve is a little more difficult as it cannot be reduced to the linear regression form. The coefficients a, b are found out by linear regression using eqn. 12, by varying values of c in a loop (a recursive procedure) [14]:

$$E_f' = \frac{aI_f}{b + I_f} = E_f - cI_f \tag{12}$$

The fitting eqn. (for modified Frolich) is given below:

$$E_f = \frac{52853 I_f}{1735 + I_f} - 3.0 I_f$$

Checking the fitting:

The actual values are being compared to the calculated values, using modified Frolich's method and are given in table 4.

Table 4. Actual and calculated values by the modified Frolich's method

Field Current, A	Eactual	Ecalcu.	% deviation
800	14200	14277	0.5429
900	15600	15350	-1.6038
1000	16700	16322	-2.2625
1100	17700	17205	-2.7980
1200	18500	18007	-2.6660
1300	19100	18736	-1.9047
1400	19650	19400	-1.2725
1500	20100	20004	-0.4770
1600	20600	20554	-0.2229
1700	21000	21055	0.2598
1800	21400	21510	0.5128
1900	21800	21923	0.5660
2000	22150	22299	0.6719
2100	22450	22639	0.8420
2200	22700	22947	1.0868
2300	22900	23224	1.4160
2400	23100	23474	1.6186
2500	23250	23698	1.9250
2600	23400	23897	2.1242

The sum of squares of % deviation (in the saturated zone for If >= 800A) = 44.28

Another method is that of piece-wise linearization on the basis of straight line segments between data points. The computation is lengthier but the accuracy is more.

8. CONCLUSIONS

Secondary effects are neglected in simplified modeling using generalized machine theory. Saturation is one of the secondary effects which affect the performance variables and it should be accounted for. There has been many modeling approach to account for saturation. The first one is the use of saturated synchronous reactance in place of the unsaturated one. This is covered by Kingsley's method. A better method has been suggested by Anderson and Fouad. They have suggested an exponential form. The coefficients have been found out by using data for p.u. voltages of 1.0 and 1.2. A still better method is the exponential method, in which curve-fitting has been obtained by linear regression. Due to the use of curve-fitting procedure, the error is relatively less. Another fitting equation was proposed by Frolich. This equation has also been fitted using linear regression. The error is less than that for fitting exponential equation. Later on, Frolich suggested a modified equation for better matching. That could also be fitted using a combination of recursive methods and linear regression. The error in this case is much smaller compared to other methods, but the procedure is a little more complex. Piecewise linearization is another popularly used method.

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NOMENCLATURE

r_a, x_a : Armature resistance/ leakage reactance
K: Saturation factor
X_{du} : Unsaturated synchronous reactance
X_{ds} : Saturated synchronous reactance
V_t, V_{th} : Terminal, threshold voltage
V_Δ : Difference voltage from threshold.
S_G : P.U. saturation factor