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Numerical study of natural convection in an inclined enclosure: application to flat plate solar collectors

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ABSTRACT

In this paper, we present a numerical study of natural convection in an inclined enclosure. This was achieved in order to stimulate the convective heat exchanges that occur over the absorber of solar air flat plate collector. The considered model is an inclined enclosure with adiabatic side walls and aspect ratios $1 \leq AR \leq 12$, and which contain heated air-filled (Pr=0.71). The inclination angle Θ of the enclosure was varied from 00° to 90° with Rayleigh numbers in the range of $10^3 \leq Ra \leq 10^6$. The influences of Θ and Ra on the flow patterns are investigated. The analysis is carried out by a numerical solution of the full governing equations; the resolution of the problem is based on the finite volumes method employing a staggered grid arrangement by the iteratively SIMPLE-C algorithm.The results indicate that there was a strong effect of inclination angle on the flow mode transition.

Keywords: Natural Convection, Solar Air Flat Plate Collector, Inclined Enclosure, Flow Mode Transition, Flow Patterns.

1. INTRODUCTION

Natural convection in confined cavities has received relatively little attention compared to the extensive studies of the other problems, such as: forced convection. This is not because of the greater importance of the latter problems, but rather due to the complexity of the former ones. External Natural convection problems have attracted more attention in recent years for its large applications, such as solar energy systems in inclined flat plate solar collectors, which makes it necessary to gain more understanding on the natural convection in the inclined cavities. Free convection in rectangular enclosures has been experimentally and numerically reviewed in the literature.

Many experimental investigations of heat transfer involving laminar natural convection and temperature distributions are investigated by Eckert and Carlson [1]. The same problem was studied analytically by Batchelor [2]. Ostrach [3] delimited various flow regimes in terms of the range of Grashof numbers and aspect ratio values. Elder's experimental work [4] on laminar natural convection in a vertical slot was perhaps one of the most comprehensive studies. The flow remained two dimensional in cavity with aspect ratio ranged from 1 to 60, Prandtl number equal to 103 and Rayleigh number up to 10^8. In the same year Dropkin [5] conducted an experimental investigation of convective heat transfer in liquids confined by two parallel plates and inclined at various angles with respect to the horizontal. The experiments covered a range of Rayleigh numbers between $5x10^4$and $7.17x10^8$, and Prandtl numbers between 0.02 and 11.56. Hart [6] studied experimentally the stability of flow in a differentially-heated, inclined, shallow box for water and air. After that Ozoe et al.[7] calculated the values of Nusselt number for natural convection heat transfer in an inclined square channel. Further, Ozoe et al. [8] investigated experimentally the flow and heat transfer with aspect ratio varied between 1 to 15.5 and Ra between $3 .10^3$ to 10^6. In most experiments large values of Prandtl number have been used. In 1976, Arnold et al.[9] carried out an experimental investigation of steady natural convection in finite rectangular regions, they studied the effect of angle of inclination on heat transfer across rectangular regions with several aspect ratios was measured for Rayleigh numbers varying between 10^3 and 10^6.Elsherbiny et al. [10] conducted an experimental study on six aspect ratios between 5 and 110 and Rayleigh number in the range of 10^2 to $2x10^7$. Hsieh and Wang [11] studied natural convection heat transfer and flow patterns in cavities.

A lot of numerical studies of natural convection inside enclosed cavities were also performed during the last few decades. In 1966 Wilkesand Churchill [12] studied numerically the natural convection of a fluid contained in a long horizontal enclosure of rectangular cross section. Thereafter, Aziz and Heliums [13] presented a finite-difference technique for the numerical solution of three-dimensional natural convection in an enclosure. Ozoe et al. [7] determined numerically, the heat transfer and flow modes of natural convection in an inclined square channel. Further, Ozoe et al, [14] studied the natural convection flow patterns and average Nusselt number in a slightly inclined long box with aspect ratio of 2 and with glycerol as working fluid. Le Quéré and Alziary[15] studied numerically the effect of the thermal boundary conditions of the horizontal walls on the transition characteristics in a differentially heated vertical cavity with aspect ratio ranged from 1 to 10. Lee and Lin [16]investigated numerically the three dimensional natural convection of air flow in an inclined cubic cavity, with the Rayleigh number varied from 10^3 to 10^7 and inclination angles from 0° to 90°.The transition modes of the flow were also studied numerically by Soong et al [17]. Corcione [18] used numerical techniques considering the effect of bidirectional differential heating walls in horizontal cavities with several aspect ratios and Rayleigh numbers between 10^3 and 10^6. Flow mode transition and hysteresis phenomena for Rayleigh numbers greater than 3,000 were demonstrated by Wang and Hamed [19], they conducted a systematic numerical study of Nusselt number variation with different inclination angle and Rayleigh number ranged from 10^3 up to 10^4 and a single aspect ratio of 4. Further, Khezzar et al. [20], studied two-dimensional natural convection in fluid filled cavities heated from below withvaried inclination angle. Two years later, Zhang et al [21] studied numerically thermal equilibrium distribution under different parameters of box-type substation under two kinds of environmental. This was made order to improve the operating efficiency of the heavy oil box-type substation and reduce the construction cost, through the combination of theoretical analysis and simulation calculation method.

In this study, we have used the commercial computational fluid dynamics code Fluent for the simulation and the analysis of the proposed model. This work is part of several theoretical and experimental investigations already carried out in our laboratories [22-27], in the context of the improvement of convective heat exchange and thermal performances of solar collectors.

2. MATHEMATICAL FORMULATION AND NUMERICAL SOLUTION

2.1 Governing equations

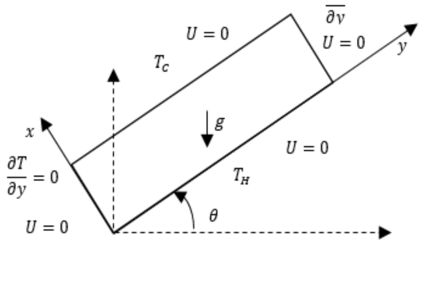

Figure 1. Schematic view of the computational domain with boundaries conditions

For the computational, Figure 1 shows the geometry of the present numerical study. The bottom and the top of the rectangular enclosure are kept at constant temperatures T_H and T_C respectively, and are separated by height H. The other two facing sidewalls are adiabatic as indicated in Figure 1.

Governing equations in dimensionless form for steady two-dimensional laminar flow with constant propertiesand Boussinesq approximation are as follows.[28, 29].

2.1.1 Non-Dimensional formulation

Sometimes for the calculation objectives, it is useful transform the governing equations to non-dimensional equations using dimensional analysis, to extract the numbers non-dimensionalRayleigh number, Grashof and Prandtl..Etc.

Continuity Equation:

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \quad (1)$$

Momentum equations:

x-direction:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\nabla^2 u+\frac{Ra}{Pr}T\sin\theta \quad (2)$$

y-direction:

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\nabla^2 v+\frac{Ra}{Pr}T\cos\theta \quad (3)$$

where: $Ra=\frac{\rho g\beta(T_h-T_c)H^3}{\alpha\mu}$,

$Pr=\frac{\nu}{\alpha}$: *is* the *Prandtl number, set to* 0.71.

Energy equation:

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{Pr}\nabla^2 T \quad (4)$$

The average Nusselt number Nu_H of each horizontal boundary wall and the average Nusselt number Nu_V of each vertical boundary wall are calculated:

$$Nu_H=\int_0^1\frac{\partial\theta}{\partial y}\Big|_{wall}dx \text{ and } Nu_V=\frac{1}{AR}\int_0^{AR}\frac{\partial\theta}{\partial x}\Big|_{wall} \quad (5)$$

In this study we interested by the heat transfer rate from the hot wall, we can use the practical formulation employed the implicit form of exchange coefficient (h) given by:

$$Nu=\frac{hH}{\kappa}=\frac{\left[\frac{q}{\Delta T}\right]H}{\kappa} \quad (6)$$

2.2 Numerical solutions

The system of equations (1)-(4) is solved numerically using a finite-volume method with a pressure-correction method as introduced by using the SIMPLE-C algorithm. [30].

We selected a software based on the finite volume method "FLUENT", which is the most widely used in order to provide solutions to all issues flows.

Grid independence tests were performed using four grids for the square cavity for Ra=$10^4$and for the four angles of inclination, 0°, 30°, 60°, 90°.The below Table shows that the grid of 60x60 for AR = 1 suffice because there is not much difference between the "Nusselt number" in the case of 58x58 and case 54x54. The same process is noted for various cavities AR.as shown in Table 1.

Table 1. Results of grid dependence test for Ra = 10^4 at angles of inclination (Θ) = 00°, 30°, 60°, 90°. Values between brackets indicate absolute percent difference to the 60×60 grid.

Grid	Θ	Nu	% Dev	Ψ_{max}%	Dev
50X50	00°	2.1391696	0.241	0.0095310	0.159
50X50	30°	2.4177026	0.211	0.0105569	0.117
50X50	60°	2.4513596	0.207	0.0094445	0.068
50X50	90°	2.2274501	0.225	0.0070324	0.031
54X54	00°	2.1414865	0.132	0.0095380	0.085
54X54	30°	2.4199712	0.117	0.0105626	0.063
54X54	60°	2.4536031	0.116	0.0094474	0.038
54X54	90°	2.2296724	0.126	0.0070333	0.018
58X58	00°	2.1434588	0.041	0.0095437	0.026
58X58	30°	2.4219338	0.036	0.0105673	0.018
58X58	60°	2.4555669	0.036	0.0094499	0.011
58X58	90°	2.2316131	0.039	0.0070342	0.005
60X60	00°	2.1443383	0.000	0.0095462	0.000
60X60	30°	2.4228172	0.000	0.0105693	0.000
60X60	60°	2.4564575	0.000	0.0094510	0.000
60X60	90°	2.2324919	0.000	0.0070346	0.000

3. RESULTS AND DISCUSSION

The present study investigates numerically the effects of cavity aspect ratio, tilt angle and the localized heat sources (presented by Rayleigh number) on the natural convection of air in inclined rectangular cavities.

The numerical simulations are performed for Pr = 0.71 Rayleigh number values in the range $10^3 \leq Ra \leq 10^6$, angle of inclination between 0°≤ Θ ≤ 90° and the aspect ratio of the cavity $1 \leq AR \leq 12$ for 2D model with stationary conditions.

3.1 Heat transfer rates

The figures. 1-3, may provide a general idea about the variation of the heat transfer rates presented by the average Nusselt number 'Nu', this was performed at various Rayleigh numbers 'Ra' and inclination angles (AR=4,8,12). For zero-inclination, the critical Rayleigh number is about 1800 (Nu= 1.0)

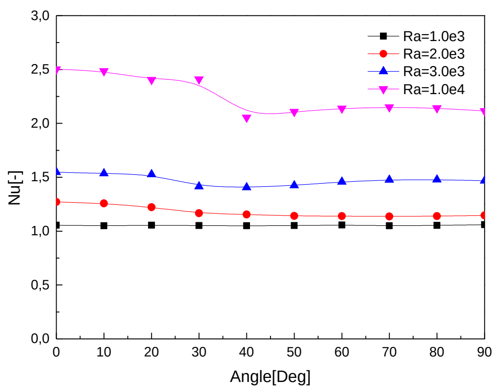

Figure 1. Average Nusselt number Versus the inclination angle Θ for different Ra values (AR =4)

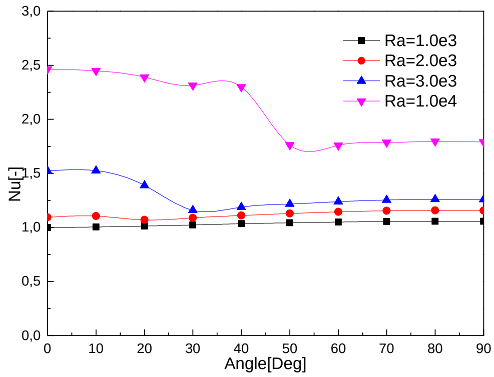

Figure 2. Average Nusselt number Versus the inclination angle Θ for different Ra values (AR =8)

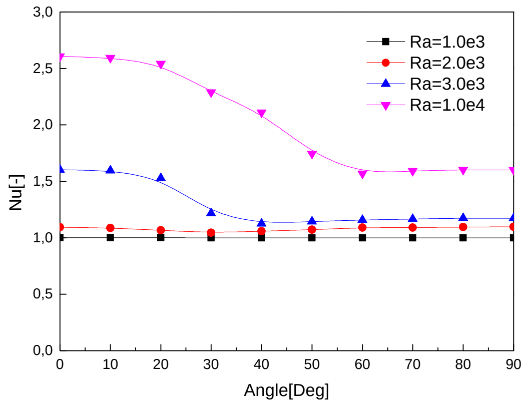

Figure 3. Average Nusselt number Versus the inclination angle Θ for different Ra values (AR =12)

For subcritical state (Ra < Rac), it is observed that there is no dominance between the two modes of heat transfer, convection and conduction.

For a higher Rayleigh number, Ra=2000, which lies at a supercritical state, for Θ= 0° and Nu around 1.1, the convection contributed immediately as evidenced by the value of Nusselt number. As Ra further increased, e.g. Ra = 3000 or 10000, a noticeable drop in Nusselt number appeared when increasing θ. This radical change in the rate of heat transfer implied a mode-transition of the flow pattern.

We clearly distinguish on figure 4 that Nusselt number depends on Rayleigh number values, increasing Rayleigh number leads to increasing Nusselt number.

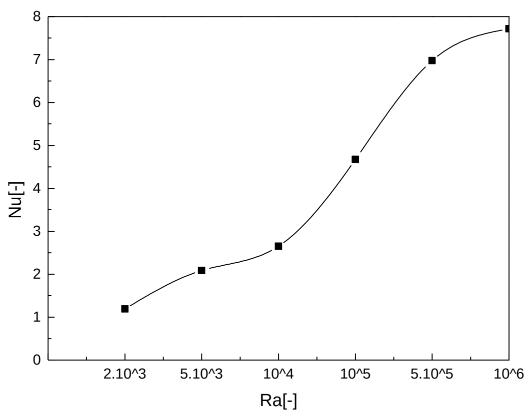

Figure 4. Average Nusselt number VersusRayleigh number (AR =12 /θ = 0°)

3.2 Flow-mode transition

The evolution of flow structure and temperature field illustrated by the contour lines of Stream function ψ and θ for Ra values in the range $10^3 \leq$ Ra $\leq 10^4$ and AR=12, 8 and 4, are represented in figures. 5-13, respectively.

The effect of the inclination angle is also shown in figure 5 for AR=12 andRa=2.10^3, it is found that the initial multi-cell pattern of 12 cell observed at 0° gradually shifts to a lower number of 11 cells at an angle of 2° and decreases with the increase of the inclination angle. Some cells begin to fade at 13°, to become six (06) cells at 14°, they start to overlap later at the angle 20° and become almost only one cell at the angle 30° and completely one cell at the angle 90°.

The same structures are noticed in figure 6 for Ra=3.10^3 and at the angle 0°, they become 11 cells at the angle 15°, and 6 cells at 24°, then 4 cells at 25°, These cells shrink to almost three (03) cells at 26°, and diminished to a one cell at 30°.

In Figure 8. The fluid in this subcritical state (Ra=10^3< Rac) is still considered as stationary. As the enclosure is inclined, θ = 1°, the shear flow along the two longitudinal walls results a large circulation in which there are two weak sub-cells rotating in the same sense as the primary cell. This two cellular structure disappears at an inclination angle between 48° and 49° due to stronger upslope/downslope flows along the x-direction. For $\theta \geq 50°$, the flow field is one-cell mode. The isotherms illustrate a gradual change from a stratification state to a skewsymmetric distortion due to the cellular motion.

Figure 9. Shows a four-cell structure at θ= 0° for a rectangular enclosure with AR = 4, the four- cell structure remained up to θ = 21°, and it changed to a three-cell structure in θ=22°, in the inclination angle θ=38° being to the threecell structure changes to a one cell.

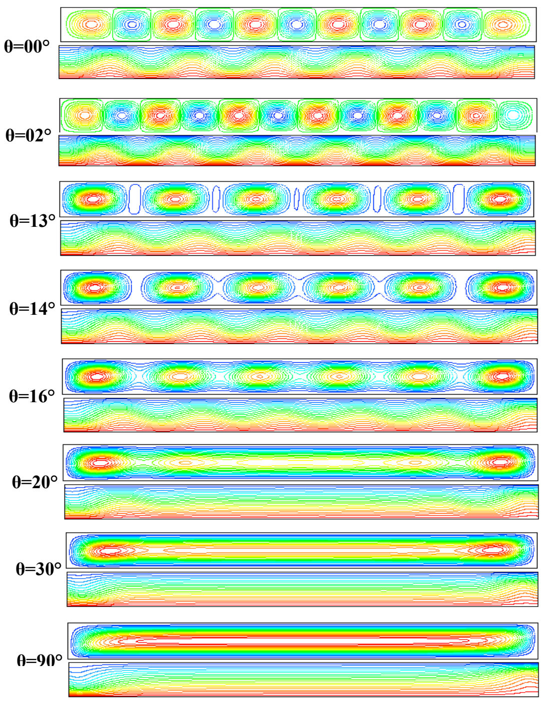

Figure 5. Stream function and Temperature contours at inclination θ for AR = 12 and Ra =2.10^3

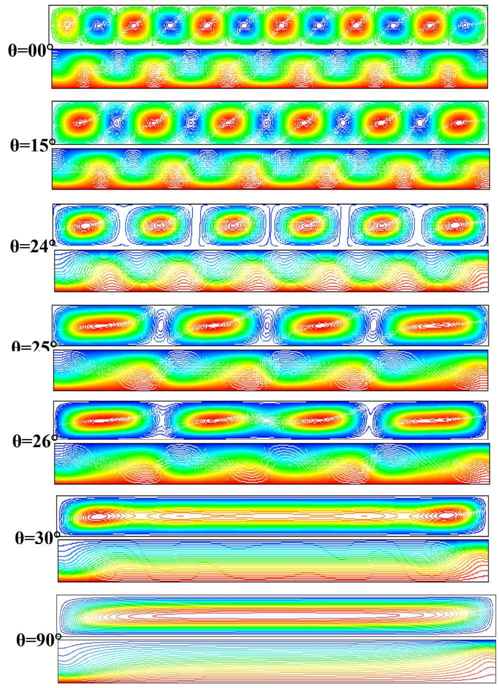

Figure 6. Stream function and Temperature contours at inclination θ for AR = 12 and Ra =3.10^3

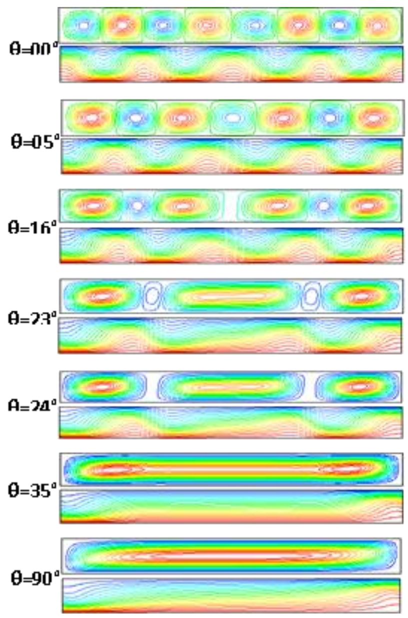

Figure 7. Stream function and Temperature contours at inclination θ for AR = 8 and Ra =3.10^3

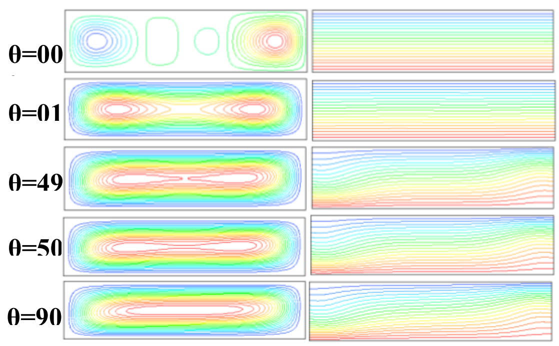

Figure 8. Stream function and Temperature contours at inclination θ for AR =4 and Ra =10^3

θ=0

θ=2

θ=

θ=3

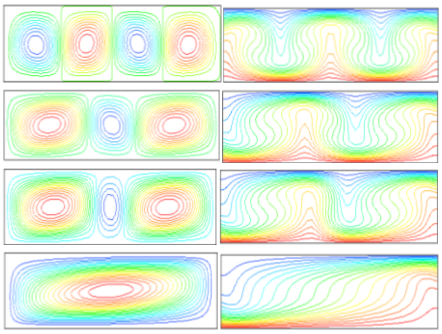

Figure 9. Stream function and Temperature contours at inclination θ for AR =4 and Ra =10^4

7. CONCLUSIONS

In this work we investigate the natural convection flow in rectangular cavities by using many controlled parameters such as: the aspect ratio (AR) , inclination angle (Θ) and Rayleigh number (Ra), where the Prandtl number (Pr) is fixed in all the study. We propose a phenomenological study about the influence of some parameters on the flow patterns and heat transfer rate (average Nusselt number). The analysis is carried out by a numerical solution of the full governing equations, on the basis of the finite volumes approach employing a staggered grid arrangement by the iterative SIMPLE-C algorithm. The results indicate that there is a sensible effect of the inclination angle on the flow mode transition and multiplicity of solutions at various Rayleigh numbers. The existence of such multi-steady solutions strongly depends on the value of Rayleigh number, it is also found, that the inclination angle whene the minimum heat transfer occurs, has a close relationship with the flow structure transition

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NOMENCLATURE

AR	aspect ratio
H	height of the side wall, m
L	length of the side wall, m
q	Heat flux, Wm^{-2}
h	Heat transfer coefficient, $Wm^{-2}K^{-1}$
Nu	mean Nusselt number
Ra	Rayleigh number
Pa	Prandtl number
g	gravitational acceleration, ms^{-2}
T	temperature, K
P	pressure, Nm^{-2}
U	velocity, ms^{-1}
t	time, s
x, y	coordinates

Greek symbols

α	thermal diffusivity, m^2s^{-1}
β	thermal expansion, K^{-1}
K	Thermal conductivity, $Wm^{-1}K^{-1}$
μ	dynamic viscosity, kg $m^{-1}s^{-1}$
ν	kinematic viscosity, m^2s^{-1}
ρ	fluid density, kg m^{-3}
Ψ_{max}	dimensionless stream function
Θ	inclination angle, deg

Subscripts

c	cold wall
h	hot wall
H	horizontal wall
V	vertical wall