

Mathematical modeling and predicting the current trends of human population growth in Bangladesh

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ABSTRACT

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Bangladesh is an overpopulated and the most densely populated country. It is the world's eighth-most populous country in south Asia with over 160 million people. Population problem in Bangladesh is one of the most burning issues in the recent years. So the increasing trend in population is a great threat to the nation and for this reason, the projection of the population of Bangladesh is essential. The purpose of this paper is to model and design the population growth in Bangladesh to predict the future population size. The exponential and the logistic growth models are applied to predict the population of Bangladesh during 1980 to 2080 using the actual data from 1980 to 2016. By using the exponential growth model, the predicted growth rate has been estimated approximately 2.67% and the population of Bangladesh has been predicted to be 1191 million in 2080. We have determined the carrying capacity (K) and vital coefficients a and b for the population prediction in vein of logistic growth model. Thus, the population growth rate of Bangladesh according to the logistic model has been estimated approximately 4.03% and the total population of Bangladesh has been predicted to be 245 million in 2080.

1. INTRODUCTION

Population is the most vital element of world but population projection has become one of the most serious problems in the world. Population sizes and growth in a country directly influence the situation of the economy, policy, culture, education and environment of that country and determine exploring the cost of natural resources [24]. Every government and collective sectors always require proper idea about the future size of various subsistence like population, resources, demands and consumptions for their future activities [11, 23]. To obtain this information, the behavior of the connected variables is analyzed based on the previous data by the statisticians and mathematicians and using the conclusions drawn from the analysis, they make future projections of the aimed at variable [2, 11, 4]. There are enormous concerns about the consequences of human population growth for social, environmental and economic development which intensify all these problems in population growth. Mathematical modeling is a broad interdisciplinary science that uses mathematical and computational techniques to model and elucidate the phenomena arising in real life problems [25]. Thus, it is a process of mimicking reality by using the language of mathematics in terms of differential equations which describe the changing phenomena of the underlying systems. The population models determine the present state in terms of the past and the future state in terms of its present state which describes a typical human way of coping with the reality. The main reason for solving many differential equations is to learn the behavior about an underlying physical process that the equation is believed to model [3, 5]. Many people examine

population growth through observation, experimentation or through mathematical modeling. The population models are used in forecasting of population growth, population decaying, maximum or minimum production, food preserving, environmental, capacity and many other applications [1]. Mathematical models can take many forms including dynamical systems, statistical models and differential equations [12]. These and other types of models can overlap, with a given model involving a variety of abstract structures. A population model is a type of mathematical model that is applied to the study of population dynamics. Models allow a better understanding of how complex interactions and processes work [6]. Modeling of dynamic interactions in nature can provide a manageable way of understanding how numbers change over time or in relation to each other. We refer readers to [7, 19-21] for some recent studies on population models in analyzing biological systems.

In this paper, we discuss an overview of population growth models in terms of nonlinear differential equations in the form of mathematical modeling which have been applied to study the future prediction of human population in Bangladesh. The first order differential equations have been used to govern the growth of the human species. Two simple deterministic population models, namely, Malthusian growth model and logistic growth model have been studied and analyzed to discuss the dynamical behavior of the population viability analysis for both short-term and long-term prediction in Bangladesh. Our study shows that Malthusian model is more accurate for short-term prediction, but for long-term prediction, population growth is unrealistic. On the other hand, logistic model is more realistic than Malthusian model for long term

prediction. Both the exponential and the logistic growth models have been applied to predict the population of Bangladesh during 1980 to 2080 using the actual data from 1980 to 2016 collected from the World Bank [23]. We have shown by the exponential growth model that the predicted growth rate is approximately 2.67% and the population of Bangladesh has been predicted to be 1191 million by 2080. In similar fashion, we have determined the carrying capacity (K) and vital coefficients a and b for the population prediction in vein of logistic growth model. Our analysis shows that the population growth rate of Bangladesh according to the logistic model has been estimated approximately 4.03% and the total population of Bangladesh has been predicted to be 245 million in 2080. Our results show that the predicted populations of Bangladesh are very similar to the present trends of the population size.

2. METHODOLOGY

'Methodology' is more effective than the simple methods in which one intends to use for collecting data. It is often necessary to include a consideration of the concepts and theories which underlies the methods. For instance, if one intends to highlight a specific feature of a mathematical theory or test an algorithm for some aspects of information or test the validity of a particular system, we have to show that we understand the underlying concepts of the methodology.

2.1 Malthusian model

A Malthusian growth model is an essentially exponential growth based on a constant rate. The model is named after Thomas Robert Malthus. The Malthusian growth model is also known as a simple exponential growth model [7]. Malthusian model is single species population model. Let t denotes the time and $y(t)$ denotes the number of individuals presents at a time t .

In practice $y(t)$ is a non-negative integer. We assume that $y(t)$ continuously differentiable [1]. The growth rate of population is the rate of at which population changes [5]. If the population $y(t)$ at time t changes to $y(t + \delta t)$ the average per capital growth rate at the time δt is $\frac{y(t+\delta t)-y(t)}{y(t)\delta t}$.

Taking limit $\delta t \rightarrow 0$, we get the instantaneous growth rate at the time δt

$$\lim_{\delta t \rightarrow 0} \frac{y(t + \delta t) - y(t)}{y(t)\delta t} = \frac{y'(t)}{y(t)}$$

Now let,

b = Intrinsic birth rate.

= The average number of off spring born per individuals per time.

d = Intrinsic death rate.

= The fraction of individuals of the population dies per unit time.

$r = b - d$

= Intrinsic growth rate of the population.

= Excess of birth over death per unit time individuals.

Now, we consider a single species of population, the growth model is described by

$$\begin{aligned} \frac{y'(t)}{y(t)} &= r \\ \Rightarrow y'(t) &= ry(t) \end{aligned} \tag{1}$$

with the initial population, $y(t_0) = y_0 > 0$. Thus we have the mathematical model described the growth of single species population as

$$y'(t) = ry(t), \quad y(t_0) = y_0 > 0$$

General solution of this equation (1) is $y(t) = ce^{rt}$. For the initial condition $y(t_0) = y_0$.

We have,

$$y(t_0) = ce^{rt_0} = y_0$$

$$\Rightarrow c = y_0 e^{-rt_0}$$

$$\therefore y(t) = y_0 e^{-rt_0} e^{rt} = y_0 e^{r(t-t_0)}$$

Therefore,

$$y(t) = y_0 e^{r(t-t_0)} \tag{2}$$

which is known as Malthusian law of growth [5].

2.2 Logistic model

Logistic model is a modification of Malthusian model. The Logistic model is a model of population growth first published by a Belgian mathematical biologist Pierre Verhulst [6]. He observed that the population growth not only depends on the population size but also on how far this size is from its carrying capacity [10]. He modified Malthusian model to make a population size proportional to both the previous population and a new term [13].

$$\frac{a - bp(t)}{a} \tag{3}$$

where a and b are the vital coefficients of the population.

Now as the population value gets closer to $\frac{a}{b}$ this new term will become very small and tend to zero, which gives the right feedback to limit the population growth [15]. Thus the second term models the competition for available resources, which tends to unite the population growth [18]. So the modified equation using this new term is

$$\frac{dp}{dt} = \frac{ap(t)[a - bp(t)]}{a} \tag{4}$$

This equation is known as the logistic law of population growth model.

Solving (4) and applying the initial condition $p(t_0) = p_0$ then (4) become

$$\frac{dp}{dt} = ap - bp^2$$

$$\therefore \frac{dp}{p(a-bp)} = dt \quad (5)$$

By the application of separation of variables

$$\frac{1}{a} \left(\frac{1}{p} + \frac{b}{a-bp} \right) dp = dt \quad (6)$$

Now integrating (6), we obtain

$$\Rightarrow \frac{1}{a} [\ln p - \ln(a-bp)] = t + c \quad (7)$$

At $t=0$, we get $p(t_0) = p_0$

$$\therefore c = \frac{1}{a} [\ln p_0 - \ln(a-bp_0)] \quad (8)$$

Equation (7) becomes

$$\frac{1}{a} [\ln p - \ln(a-bp)] = t + \frac{1}{a} [\ln p_0 - \ln(a-bp_0)]$$

$$\therefore p = \frac{\frac{a}{b}}{1 + \left(\frac{b}{p_0} - 1 \right) e^{-at}} \quad (9)$$

If we take the limit of equation (9) as $t \rightarrow \infty$, we get

$$p_{\max} = \lim_{t \rightarrow \infty} p = \frac{a}{b} \quad (10)$$

Taking $t=0$, $t=1$ and $t=2$ the values of p_0 , p_1 and p_2 respectively then we get equation (9)

$$\left. \begin{aligned} \frac{b}{a} (1 - e^{-a}) &= \frac{1}{p_1} - \frac{e^{-a}}{p_0}; \\ \frac{b}{a} (1 - e^{-2a}) &= \frac{1}{p_2} - \frac{e^{-2a}}{p_0} \end{aligned} \right\} \quad (11)$$

Eliminating $\frac{b}{a}$, we have

$$\Rightarrow e^{-a} = \frac{p_0(p_2 - p_1)}{p_2(p_1 - p_0)} \quad (12)$$

Putting this value into the first equation of (11), we get

$$\frac{b}{a} = \frac{p_1^2 - p_0 p_2}{p_1(p_0 p_1 - 2p_0 p_2 + p_1 p_2)} \quad (13)$$

Thus the limiting value of p is given by

$$p_{\max} = \lim_{t \rightarrow \infty} p = \frac{a}{b} = \frac{p_1(p_0 p_1 - 2p_0 p_2 + p_1 p_2)}{p_1^2 - p_0 p_2} \quad (14)$$

3. RESULTS AND DISCUSSIONS

To predict the future population of Bangladesh, we need to determine growth rate of Bangladesh using the exponential growth model in (2).

Using the actual population of Bangladesh in Table 1 below with $t=0$ i.e. t_0 corresponding to the year 1980, we have $P(0) = 82.49$ i.e. $P_0 = 82.49$. We can solve for the growth rate k the fact that $P(5) = 94.28$ i.e. $P_5 = 94.28$ when $t=5$ i.e. t_5 then we get from the equation (2),

$$\begin{aligned} \therefore P_5 &= P_0 e^{k(t_5 - t_0)} \\ \Rightarrow 94.28 &= 82.49 e^{k(5-0)} \\ \therefore k &= 0.03. \end{aligned}$$

Hence, the general solution is

$$P(t) = 82.49 e^{0.0267t} \quad (15)$$

This suggests the prediction rate of population growth is 2.67% in Bangladesh with the exponential growth model. With this we project the population of Bangladesh from 1980 to 2080. Again based on Table 1, let $t=0, 1$ and 2 correspond to the years 1980, 1981 and 1982 respectively. Then P_0 , P_1 and P_2 also correspond 82.49, 84.76 and 87.06 (in million). Substituting the value of P_0 , P_1 and P_2 into (14), we get

$$P_{\max} = \frac{a}{b} = 254.89 \times 10^6 \quad (16)$$

This is the predicted carrying capacity of the population of Bangladesh. From equation (12), we obtain $e^{-a} = 0.96$ hence $a = -\ln(0.96)$. Therefore the value of $a = 0.04$. This also implies that the predicted rate of population growth of Bangladesh is approximately 4% with the logistic growth model [13]. Now from equation (16), we obtain $b = 1.58 \times 10^{-10}$. Substituting the values of P_0 , a and b into equation (9) then we get

$$P(t) = \frac{254893900}{1 + 2.08(0.96)^t} \quad (17)$$

This equation is used to compute the predicted values of the population [17]. The predicted population of Bangladesh using

both models is presented in the Table 1 from 1980 to 2016 with actual data [23].

Table 1. Projected population in Bangladesh from 1980 to 2080 using Exponential and Logistic growth models with help of actual data

Year	Actual population (in millions)	Projected population (in millions)			
		Exponential model	Absolute Percentage Error	Logistic model	Absolute Percentage Error
1980	82	82	0.00%	82	0.0006%
1981	84	84	0.03%	84	0.002%
1982	87	87	0.04%	87	0.003%
1983	89	89	0.02%	89	0.01%
1984	91	91	0.01%	91	0.07%
1985	94	94	0.01%	94	0.18%
1986	96	96	0.02%	96	0.34%
1987	99	99	0.02%	98	0.53%
1988	102	102	0.01%	101	0.71%
1989	104	104	0.12%	103	0.86%
1990	107	107	0.33%	106	0.95%
1991	109	110	0.66%	108	0.97%
1992	112	113	1.08%	111	0.93%
1993	114	116	1.59%	113	0.85%
1994	117	119	2.14%	116	0.77%
1995	119	123	2.72%	119	0.71%
1996	122	126	3.32%	121	0.67%
1997	124	129	3.95%	124	0.64%
1998	127	133	4.64%	126	0.60%
1999	129	137	5.42%	129	0.53%
2000	132	140	6.29%	131	0.40%
2001	134	144	7.27%	134	0.24%
2002	137	148	8.34%	136	0.03%
2003	139	152	9.53%	139	0.23%
2004	141	156	10.86%	142	0.57%
2005	143	160	12.35%	144	1.00%
2006	144	165	14.01%	147	1.53%
2007	146	169	15.82%	149	2.14%
2008	147	174	17.74%	152	2.77%
2009	149	178	19.69%	154	3.36%
2010	151	183	21.61%	156	3.87%
2011	152	188	23.48%	159	4.27%
2012	154	193	25.32%	161	4.58%
2013	156	199	27.15%	164	4.83%
2014	159	204	28.29%	166	4.45%
2015	161	210	30.29%	168	4.72%
2016	162	215	32.38%	171	4.99%
2017		221		173	
2018		227		175	
2019		233		177	
2020		240		179	
2021		246		182	
2022		253		184	
2023		260		186	
2024		267		188	
2025		274		190	
2026		281		192	
2027		289		193	
2028		297		195	
2029		305		197	
2030		313		199	
2031		321		201	
2032		330		202	
2033		339		204	
2034		348		206	
2035		354		207	
2036		367		209	
2037		377		210	

2038		388		212	
2039		398		213	
2040		409		214	
2041		420		216	
2042		431		217	
2043		443		218	
2044		455		220	
2045		467		221	
2046		480		222	
2047		493		223	
2048		506		224	
2049		520		225	
2050		534		226	
2051		549		227	
2052		564		228	
2053		579		229	
2054		595		230	
2055		611		231	
2056		627		232	
2057		644		233	
2058		662		233	
2059		679		234	
2060		698		235	
2061		717		236	
2062		736		236	
2063		756		237	
2064		777		238	
2065		798		238	
2066		819		239	
2067		841		239	
2068		864		240	
2069		888		240	
2070		912		241	
2071		936		241	
2072		962		241	
2073		988		242	
2074		1014		243	
2075		1042		243	
2076		1070		244	
2077		1099		244	
2078		1129		245	
2079		1159		245	
2080		1191		245	
Mean Absolute Percentage Error			7.22%		1.18%

We have calculated the predicted population of Bangladesh from the equations (15) and (17), In Table 1 we get that the predicted populations of Bangladesh are expected to be 215 million (approximately) and 171 million (approximately) in 2016 by using Exponential model and the Logistic model respectively.

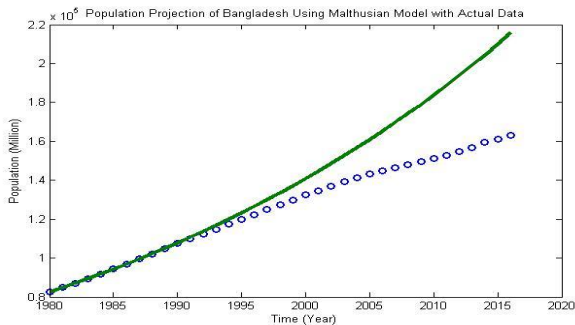


Figure 1. The actual (blue) and predicted (green) populations of Bangladesh using Malthusian model (15) from 1980 to 2016

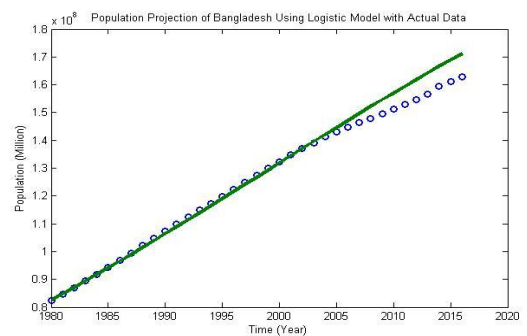


Figure 2. The actual (blue) and predicted (green) populations of Bangladesh using Logistic model (17) from 1980 to 2016

Let us now present the numerical simulation results of the actual and projected populations of Bangladesh from the year 1980 to 2016 using Malthusian model (15) and Logistic model (17) in Figures 1 and 2 respectively.

Now, a combined graph of the Malthusian and Logistic models showing a comparative population size in Bangladesh from 1980 to 2016 is shown in Figure 3.

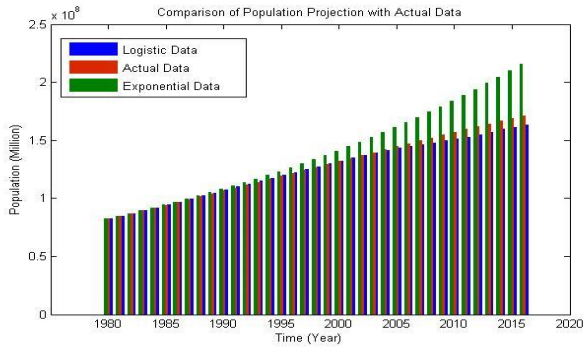


Figure 3. Comparison of population projection between Malthusian and Logistic model with actual data from 1980 to 2016

Again we have calculated the predicted population of Bangladesh from 1980 to 2080 using the equations (15) and (17). As shown in Table 1, we get that the predicted populations of Bangladesh are expected to be 1191 million (approximately) and 245 million (approximately) in 2080 by using Exponential model and the Logistic model respectively. In this situation, population growth is the pressing problem of Bangladesh like every developing country. For our limited resource, it will too difficult to cope with this over population but Bangladesh tries to reduce population growth rate, increase literacy rate and create mass awareness to overcome this problem.

Now we present the numerical simulation results of the actual and projected populations of Bangladesh from the year 1980 to 2080 using Malthusian model (15) and Logistic model (17) in Figures 4 and 5 respectively.

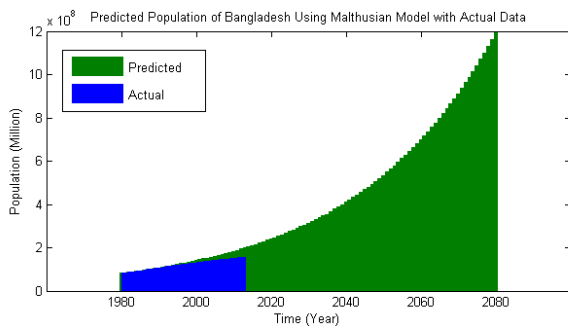


Figure 4. The actual (Blue) and predicted (Green) populations of Bangladesh using Malthusian model (15) from 1980 to 2080

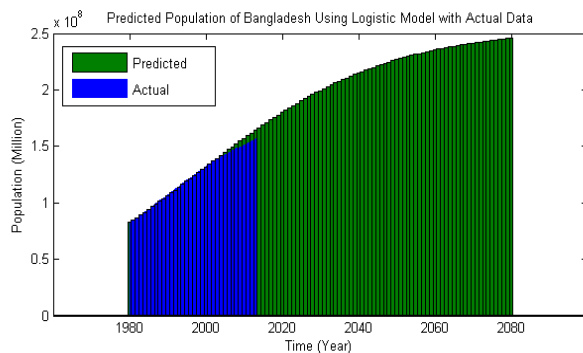


Figure 5. The actual (Blue) and predicted (Green) populations of Bangladesh using Logistic model (17) from 1980 to 2080

Again a combined graph of the Malthusian and Logistic models showing a comparative population size from 1980 to 2080 is shown in Figure 6.

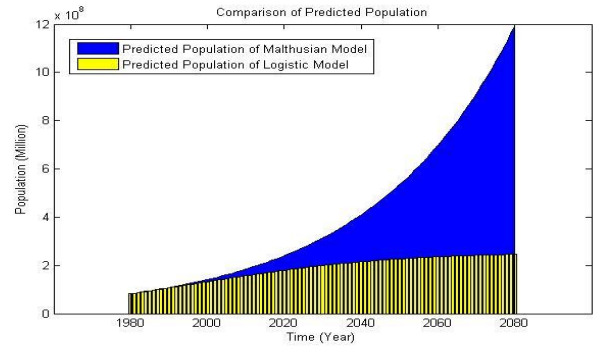


Figure 6. Comparison of population projection between Malthusian and Logistic model from 1980 to 2080

4. CONCLUSIONS

In this paper, we consider two simple deterministic population growth models: the exponential and the logistic and make a comparison to predict the future population of Bangladesh. Our analysis gives good approximation for long term prediction of the population growth trends in comparison to present scenario of human population in Bangladesh. We analyze and investigate to predict the populations for 100 years, which gives the carrying capacity of around 245 million and this leads to an estimated population of around 245.76 million in 2080. If we would predict the population for 50 years, the logistic growth yields a straight line showing a good approximation. According to the World Bank data, the population of Bangladesh was around 172.88 million and 195.07 million in the years 2014 and 2015 respectively which are very close to our predicted population in comparison to the same years. Even our predicted population from 1980 to 2016 is almost similar to that of the World Bank data. So, our study provides a better prediction for the future population size of Bangladesh.

Finally, we find that according to the exponential model the predicted growth rate is approximately 2.67% and predicted population of Bangladesh is 1191 million in 2080 with a Mean Absolute Percentage Error (MAPE) of 7%. On the other hand, the population growth rate of Bangladesh is approximately 4% according to logistic model and the carrying capacity for the population of

Bangladesh is calculated as 254.89×10^6 . By the Logistic model, the population of Bangladesh is calculated to be 245 million in 2080 with a Mean Absolute Percentage Error (MAPE) of 1.2%. As we know that the vital coefficients are responsible for population growth of any country. So we have calculated the vital coefficients a and b are 0.04 and 1.58×10^{-10} respectively. It is also shown that Mean Absolute Percentage Error (MAPE) is very low in logistic model. From this point of view, we can finally conclude that the logistic model gives a good forecasting result as compared to the Malthusian model for a long term prediction.

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