

## **The Inverse Time Depth Migration of Seismic Waves and Interval Velocity for Model Building about Their Method Analyses**

Guangqian Du, Changzhi Zhu, Jing Yuan

Agricultural University of Hebei, Baoding, Hebei, 071001, China

(duguangqian@outlook.com, 1346710159@qq.com)

### **Abstract**

At present, in the process of depth-domain imaging, the inverse time depth migration of seismic wave is a frequently-used and high-precision method, in contrast to the simulation problem of seismic-wave forward number. The technology about inverse-time migration of seismic waves takes advantage of the exact boundary and discrete calculation to solve the two-way. During the progress of calculation, it's no need to give up-and down-going wave decomposition. At the same time, it could perfectly handle the problem of multi-valued traveltimes. This article mainly focused on the basic principles of the inverse time migration of seismic waves and discuss the creation method and fixed way of the primitive-velocity model in depth domain. And we used an empirical model to furtherly analysis it.

### **Key words**

Seismic waves, Inverse time migration, Wave equation, Pre-stack, Reverse-time migration, Velocity of depth domain, Imaging.

### **1. Introduction**

As refer to the current situation, when the researchers studying the topics which are about the earthquake migration, they usually need to adopt some imaging methods. In order to acquire more ideal imaging effect. Some reasonable imaging methods are necessary for researchers to conduct in-depth study. The inverse time depth migration of seismic waves is one of the most popular imaging methods. It could make the inverse-time migration that including complicated dielectric come true through the simple removal of direct-waves and window image processing.

How to achieve the high-precise inverse time migration imaging of complicated dielectric? there are several methods that could be learned by us. First of all, we can utilize a simple cutting way of direct waves or relating windows-imaging to accomplish it. Secondly, from my perspective, using the trace-time imaging to pick up the imagination value is also a good way. But it seems that it overly depends on the speed model of depth domain. In order to ensure that the inverse time migration can achieve the desired effect and perfectly complete imaging. So we need to set up more accurate speed model of depth domain. Therefore, it's necessary to analysis modeling method of the inverse time depth migration of seismic waves and interval velocity.

## 2. The Inverse Time Migration of Seismic Waves

### 2.1 The Inverse Time Migration of Seismic Waves-forward Modeling Algorithm of Seismic Waves

In the studying progress of this article, when we were carrying out foward modeling algorithm of seismic waves ,we have adopted the finite difference staggered grid method to help that. (showing as Figure 1-1). Compared with the regular grid, the staggered grid has better precision, and it is easy to make the convergence, which can get better imaging results. About the specific calculation. Taylor series method can be used to transform the form of differential equation and make it convert into difference equation that could be handled by computer.

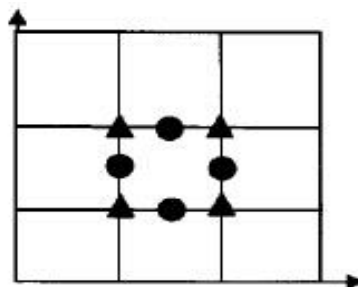


Figure 1-1. Staggered Grid, Finite Difference Scheme

### 2.2 Stability Criteria

According to the frequency of the Fourier transform, the wave number field and the analysis of the characteristic value, we can obtain the staggered grid finite difference stability condition of two dimensional seismic waves.

$$\Delta t V_{\max} \sqrt{\frac{1}{\Delta x^2} + \frac{1}{\Delta z^2}} \leq \frac{1}{\prod_{m=1}^L |a_m|}$$

Among them, the maximum velocity field of seismic wave, in the dielectric, could be represented by  $k=k(x, z)$ , we use  $x$ 、 $z$  to mark the distance of every two continuous grids such as  $x$  axis and  $z$  axis.  $T$  means time step. For 3D seismic wave, the finite difference of the staggered grid can be expressed as:

$$\Delta t V_{\max} \sqrt{\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2}} \leq \frac{1}{\prod_{m=1}^L |a_m|}$$

## 2.3 Boundary Conditions

Within the calculation area, how to stop the fake reflection of boundary is the very important content during the progress of numerical modeling of seismic wave. On the present situation, PML absorbing boundary can accomplish some special tasks perfectly. With the PML absorbing boundary, it can cut off the fake reflection of boundary which is from any angles. In addition, we can also adopt the way of random boundary condition that took advantage of the random boundary, which was produced when the seismic waves was arriving at boundary line to timely eliminate relevant energy caused by fake reflection.

## 2.4 Continuation Algorithm

When we analyzing the continuation algorithm of inverse-time migration of seismic waves, we need to calculate that from different condition. Essentially speaking, the inverse-time continuation of seismic waves belongs to the field that named the anti-calculation of seismic forward modeling. Among them, when we continually calculating the two-dimension seismic waves. Intermediate difference scheme could be used to differently discrete the differential equations of seismic waves and then obtaining staggered-grid, finite-difference scheme accompanied by corresponding spatial 2L level accuracy, time 2 level accuracy.

## 2.5 Imaging Condition

In the aspect of inverse time depth migration imagination, Cross correlation imaging is a very popular technical means. In the imaging condition, Essentially, the description of the content

is similar to the correlation among the amplitude, phase seismic wave field and the wave field of detection point. It can be expressed as:  $Images\ c(x,y,z)=\sum_{t=0}^{time} souP(x,y,z) \cdot recP(x,y,z)$

In that, the imagining outcomes of staged points can be expressed with Images  $c(x,y,z)$ . The function of receiving point, when the receiving point of seismic waves capturing inverse-time forward waves filed, that condition could be expressed by  $recP(x,y,z)$ .  $SouP(y, x, z)$  is used to express the wave field value of the forward extension of the blast-point TIME means the maximum time.

## 2.6 Migration Noise Suppression

Now, when we analyzing the imagination of inverse time depth migration, In most cases, it will be affected by the low-frequency noise. However, why this phenomenon happens could be attributed to the up-and-down traveling waves that were generated in the receiving-point waves filed and blast-point waves field. Actually that condition appeared in the process of the extension of the seismic wave field. In the analysis of normalized cross correlation imaging, we only need to analyze the reflected wave field and the wave field of the blast point. but, in fact, The cross-correlation operation is related to the reflection wave field and the detection point penetration. From what has been mentioned above, we can get a conclusion that after the reverse time depth migration, the wave field mentioned above will lead to the emergence of certainly low frequency noise and had an effect on the migration outcomes. finally, it leads to continuous deterioration of experiment result. For that reason, In the analysis of the time, we need to take effective measures to effectively suppress the migration noise. Laplace of Gaussian has a high sensitivity to discrete points and noise. , In the process of edge detection, through the implementation of the image of the differential operation, we can finish that more better. so it is very suitable to suppress the migration noise.

## 2.7 Model Example

In order to analyze different technologies relating to the inverse time migration, in our research content. We decided to combine specific model for the analyzing purpose. That will be convenient for us to study the features and advantages of inverse migration. Marmousi model is a widely applied technology which is used to test the effect of migration imaging . The key section of the imaging is the complex structure including 2400m depth and shallow, middle fault and salt dome below the fault. There is a model, size: m 915 L x 605 m, horizontal and vertical spatial grid are 5 m, maximum speed and minimum speed of 5500 m/s and 1500 m/s, the density is 1 g/cm<sup>3</sup>,

the time step is 0.4 ms, it has good stability. In the process of testing, the earthquake source was ensured as the zero phase ricker wave that maximum frequency won't surpass 80HZ. The way of right shooting was executed in the experiment to multiplely converge the observation system. Waves detector was put on the surface of ground, Channel spacing is 5 m, blast interval is 10 m, total length of spreads is 420 m, the specific parameters is the first blast far away from the model 420m. Marmousi speed model showing as figure 1-2. We calculated the first-arrival times of speed model and regarded it as imaging condition. The prestack reverse-time depth migration was carried out for the numerical simulation records, and the Marmousi model is shown in Figure 1-3 for the prestack reverse time depth migration profile. After compared the figure 1-2 with 1-3. We find the truth that the different geological structure could be clearly and accurately appearance after after imaging processing. Among them, the consolidation interface, the main fault, the anticline can be very clear, for some partly small block, can also be more clearly identified. from what has been mentioned above, we can realize that the smaller power of migration noise could help us get highly precision.

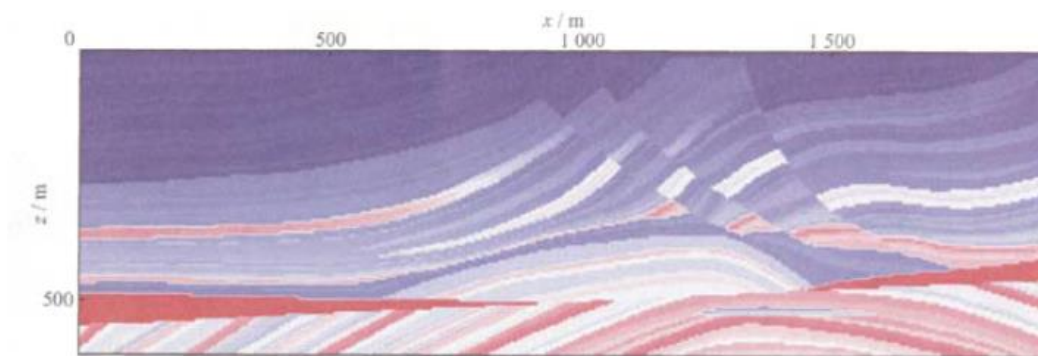


Fig.1-2. Marmousi Velocity Model

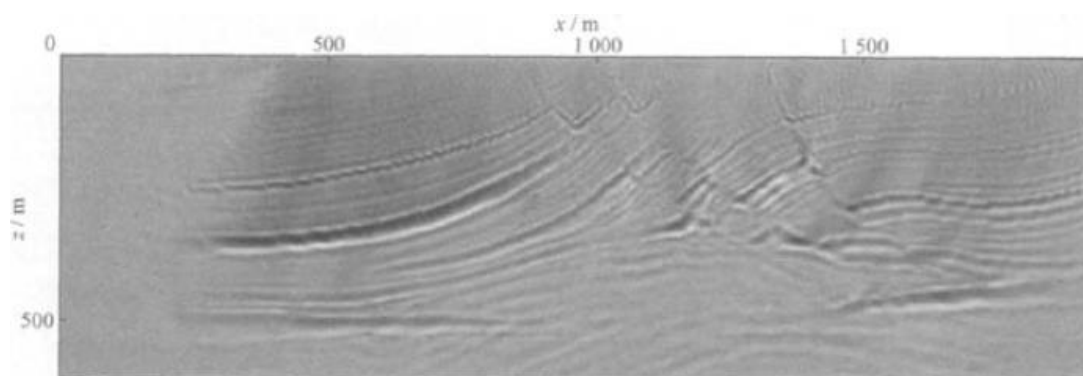


Fig.1-3. Marmousi Forward Recorded during Pre-stack Depth Migration Profile Reverse

### **3. The Depth Domain Velocity of Seismic Waves for Model Building**

#### **3.1 The Establishment of Initial Model**

In the setting process of the velocity model of seismic waves depth domain, Firstly, we need to establish the initial model. The accuracy of the mode bind up with the effect of the final velocity model, if the difference between the real interval velocity and the initial model is greater, The need number of iterations will be more better. Velocity model is also more difficult converging to right model direction. Conversely, if the difference between the two is small, the corresponding number of iterations is more less. Velocity model is also more likely converging to correct model direction.

But actually speaking, in the progress of setting up model. The phenomenon that has so obvious distinction between initial velocity model and real-depth domain velocity model occurs frequently. In this way, velocity model can't correctly converge to right model. The corresponding migration results will be affected concomitantly. Even the deterioration phenomenon starts to appears, the credibility is also significantly decreased. Taking into account the above reasons, when establishing the initial model, we need to be as close as possible to the real speed of the underground, in order to improve the precision of the velocity model, and improve the efficiency of the subsequent migration imaging. In practice, the velocity of seismic wave in underground medium will be affected by different rock lithology. Therefore, when conducting research on the underground macro interface, we need to analysis different geological conditions.

#### **3.2 The Calculation of Bottom Dip Angle and Incidence Angle**

The depth migration paths contain many types, and the common imaging-point paths of angle domain is the important types of them, and there are some differences between the other pre-stack migration paths and common imaging-point paths of angle domain. If the incident angle  $\alpha$ , the formation dip angle  $\beta$  and the reflection point O, all of them were ensured. The corresponding incident and reflected propagation information can be determined. The single path of the incident angle and the reflection angle of the seismic wave is shown in Figure 2-1. It can be found from Figure 2-1 that the seismic wave reflection angle is the same as that of the incident angle, and can be analyzed according to the geometric relation:  $\alpha - \beta = \theta_i = \theta_r$ ,

The above formula shows that if you want to get the incident angle of seismic wave, it is needed to get the angle of the formation and the angle of seismic wave propagation.

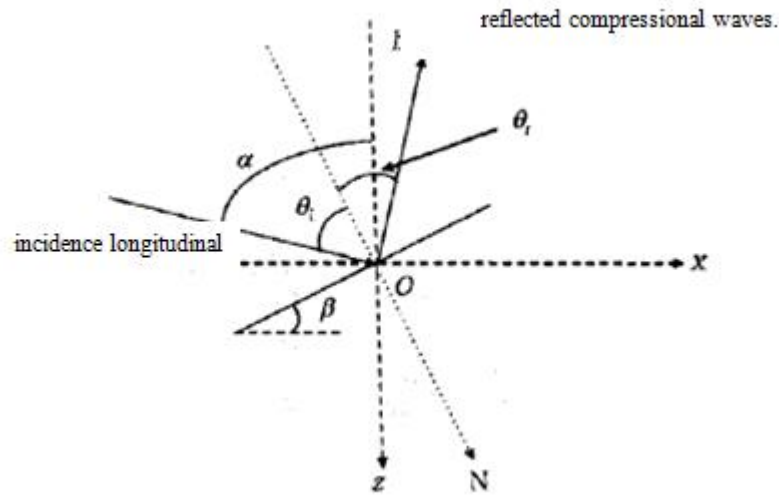


Fig. 2-1. Seismic Wave Incidence Angle, Reflection Angle, Single Path Diagram

### 3.3 Extraction of Corners and Calculation of Residual Curvature

After obtaining the angle of incidence and the angle of the formation, the diagonal gathers can be extracted. In the extraction process, the imaging condition can be fixed according to the Gauss sampling formula, and finally get the corners gather. The corresponding formula as following:

$$\text{Images } c(x, y, z) = \frac{\sum_{T=0}^{\text{TIME}} \text{sou}P(x, y, z) \cdot \text{rec}P(x, y, z) \cdot e^{-\frac{(\beta - \beta_i)^2}{2\sigma^2}}}{\sum_{T=0}^{\text{TIME}} [\text{SOUP}(x, y, z)]^2}$$

In the above formula, TIME represents the maximum moment and  $\sigma$  represents the variance of Gauss's function. The imaging points result of closing grid can be represented with Images  $c(x, y, z)$ , The reflection angle obtained by the imaging point is expressed by the  $\beta$ , and the  $\text{rec}P(x, y, z)$  indicates the wave field value of the seismic wave recorded at the receiving point;  $\text{Sou}P(y, x, z)$  is used to express the wave field value in the forward continuation of the blast-point wave. When calculating the residual curvature of the corner sets, the velocity correction of this study is based on the residual curvature of the corner sets. Therefore, the depth difference of correct velocity imaging position and false velocity imaging position could be identified as corresponding residual curvature. The different velocity migration depth is shown in Figure 3-2, and the residual curvature is  $z_f - z_o$  in figure 2-2.

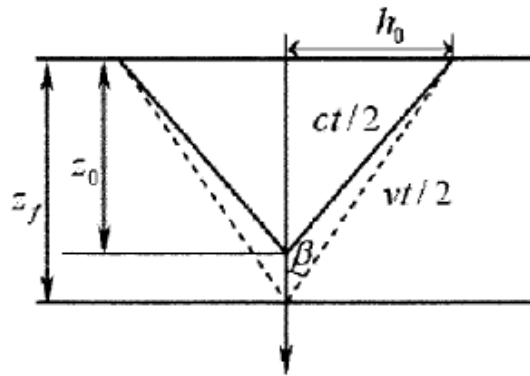


Fig.2-2. Schematic Diagram of Different Velocity Migration Depth

### 3.4 The Calculation of Travel Time Residuals

After calculating the value of the incident angle and the residual curvature of the corner sets, we can calculate the travel time residuals of seismic waves by the mutual transformation among the different indexes. The specific calculation method as following:

$$\frac{1}{2} \Delta t = s \cdot \Delta z \cdot \cos \alpha \cos \beta$$

The formation dip angle is expressed by the incident angle of seismic wave used  $\alpha$  to represent itself, the current residual curvature of Imaging point angle sets could be represented with  $\Delta z$ . At the present time, the slow degree of imaging point is expressed by  $s$ , and the travel time residuals of angular channel sets, we usually mark it with  $\Delta t$ .

### 3.5 Matrix Computation about Sensitivity

In the calculation of the sensitivity matrix of the inversion equations, it is needed to refer to the travel route of the ray in the velocity model grid. Different methods can be used in the process of ray tracing. This study applied the constant velocity gradient that is based on the eikonal equation of radiation, and the velocity model was divided into a number of rectangular elements. To establish the coordinate axis, the velocity gradient is assumed to be constant in the same rectangular element. Using this method, it can be more convenient and fast to calculate the sensitivity matrix. In the calculation of ray travel time, starting from the eikonal equation of the radiation, the corresponding equations are expressed as follows:

$$\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2 + \left(\frac{\partial T}{\partial z}\right)^2 = \frac{1}{c^2}$$

Among them, the seismic wave velocity is expressed by  $c=c$ , and  $C(x, y, Z)$ .



### 3.6 The Construction and Calculation of the Tomographic Inversion Equation Groups

When we constructing tomographic inversion equation groups, the spatial model will be divided into a grid, and the mesh parameters are speed. By using the ray tracing model to calculate the sum of the travel time, the corresponding calculation method is:

$$u(\rho, \theta) = \int_L f(x, y) ds$$

Among them, the projection function of space function  $f(x, y)$  is  $u(\rho, \theta)$ . Space coordinates  $(x, y)$  function using  $f(x, y)$  to express. .

The least square orthogonal decomposition is used to solve the problem of the tomographic inversion equation groups. The specific steps of the solution mainly include the initialization and the diagonal transformation, as well as the orthogonal transformation and iterative solution, the final solution of the equations of the output  $X$ .

### 3.7 Velocity Model Optimization

By solving the inverse equations, we can get the slow speed vector, that is, the update of the speed. According to this result, the initial model of the layer velocity can be optimized. When modifying the velocity model in depth domain, using the way of striping layers is a good choice. Through the repeated iteration method revised the model constantly. At the same time, attention should be paid to the analysis of the direction of the angular channel sets extracted after the reverse time migration, and whether the observation period is in a horizontal state. Finally, the modified layer velocity model was obtained, which is the final depth domain layer velocity model which is suitable for reverse time migration.

### 3.8 Modelexample

Do the artificial synthetic seismic experiment, the source frequency is Ricker wave with 35Hz. We recorded the composition of the blast sets, which include a total of 50 blasts. Each blast contains 300 data units in all of 15000, we basically ensured the blast point as middle. A total of 300 channels reception exist in the both sides of blast-point. The shot interval has been set at 30m, track interval was set to 5 m, a total record of 1.25S.

We tried to do the moving experiment of seismic source with the method of bilingual comparisons and the same time kept the receiving site stable. And then we can start to construct an observation system. Something needing to be done we should remember. Select CMP path

groups and set the interval at 5m. the range of CMP should within 2-598. The number of maximum coverage won't surpass 50 times. There were several important points we should continuously execute. Pick up the speed spectrum of superposition and maximum energy. Then construct vertical function and adjust corresponding CMP path groups to horizontal status. Finally, after we finished the collective work of vertical function of stacking velocity. We can begin to transfer vertical function to stack velocity profile and carry out the flexible correction stack. The more details as following 2-3、 2-4:

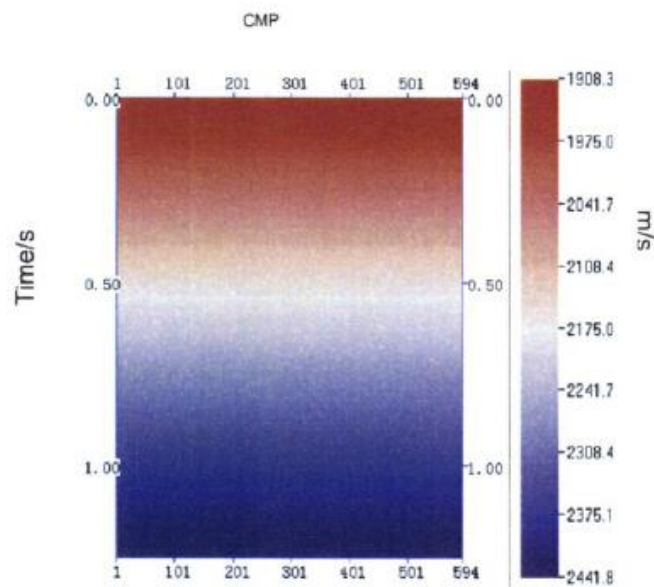


Fig. Stack Velocity Profile of 2-3

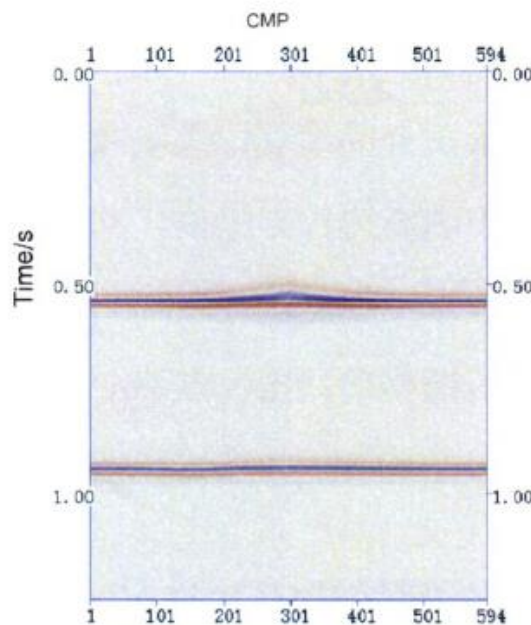


Fig. 2-4 Stack Profile

Converted the stack speed, start to carry out time migration after the quiet converter was converted to the root mean square speed. In accordance with the Dix formula ,i was transformed it into time domain layer velocity. When the conversion was completed, we should execute the bending ray time migration. After the implementation of a depth conversion and then we can begin to construct the initial depth domain layer velocity model . After the model was constructed, according to the formula, we can calculate the residual delay of the first layer and the second layer and respectively obtain the corresponding residual delay spectrum. (details as 2-5) .We can separately pick up the remaining delay of the first layer and the second layer for finding the largest energy position, The reason to do so is to ensure that the common imaging point of first layer and second layer always in a horizontal level.

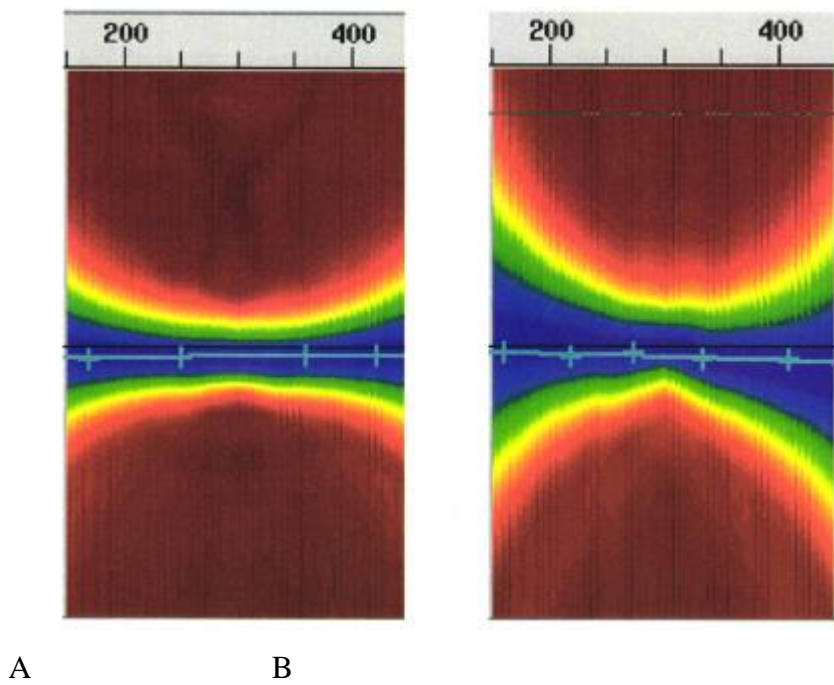


Figure note: A is the residual delay spectrum of first layer , B is the residual delay spectrum of second layer

Fig. 2-5 residual delay spectrum

#### 4. Case Analysis

The is a depression model with size of 500 m \* 500 m, the vertical and horizontal length of spatial grid are 5 m, the density is 1 g/cm<sup>3</sup>. We can find the appearencing details of depression model. In the progress of experiment, We set Ricker waves which involves 60Hz as seismic source. As we can see, the time interval is 0.5ms. it has very excellent stability. The ground surface was placed witha waves detector, and the adjacent spacing has been set to 10 m, a total of 51 common blast point groups. in the aspect of numerical simulation, the corresponding accuracy

simulation is 2-level-time and 10-level in space. Exert the superposition of multi-shot pre-stack reverse time migration on the common shot gather through the way that stimulating time to become the pre-stack inverse migration technology of imagining condition. Pre-stack reverse-time migration stack section of the depression model is shown in Figure 3-2

According to the comparison of figure 3-1 and figure 3-2, we can find that after the imagining the breakpoint produced in stack profile of pre-stack inverse time migration could be accurately and clearly represented and correctly converge surrounding around the center. Aim at the interference producing in the experiment progress, such as multiple waves, we have many ways that can suppress it and then obtain better imagining outcomes. In addition, the reflection wave was not observed at the boundary, the effective wave field does not cause the phenomenon of frequency dispersion, however the signal noise is so high.

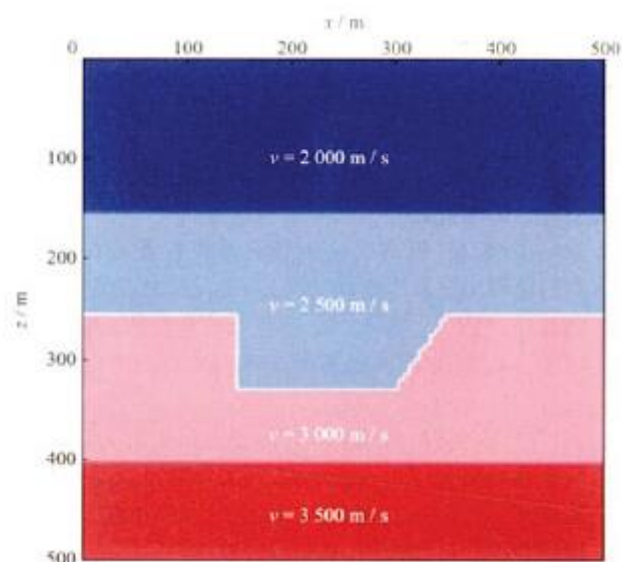


Fig3-1 Depression Velocity Model

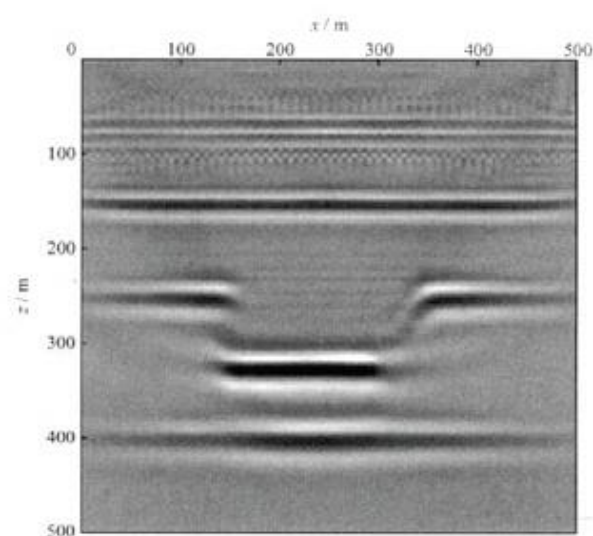


Fig. 3-2. Stack Profile of the Depression Model in the Pre-stack Inverse Time Migration

## Conclusion

Based on the analysis of this paper, the analysis of seismic wave reverse time migration technique was finished, and some key techniques in the construction of the model of inverse time migration depth domain layer velocity were carefully analyzed by us. Through the analysis, we can find that in the process of applying reverse time migration imaging the large storage space is needed. According to the characteristics of reverse time migration, the storage condition can be set to random boundary when realizing the effective inverse time delay of seismic wave. In this way, when the blast-point being in the realization of the forward extension, the seismic wave field of the final moment can be preserved, and the other values are not retained. And then realize the reverse extrapolation, and finally reconstruct the wave field. In this way, that can be a very good way to reduce the large number of migration data storage and resolve the problem of migration data storage. At the same time, it is found that the key factor for the success of reverse time migration lies in whether the high precision processing of the depth domain layer velocity modeling is realized. In the study, the velocity model was modified according to the migration channel set, so as to obtain a higher reliability. In addition, depending on the depth domain layer velocity model, it is necessary for us to construct the tomographic inversion equations. The residual curvature of angular trace gather can be constructed by using the method of least square orthogonal decomposition. In order to update the initial layer velocity model, it is necessary to meet the actual needs of the depth domain layer velocity model.

## References

1. W.J. Li, S.L. Qu, X.C. Li, Discussion on the technology of non- zero offset VSP elastic wave pre-stack reverse time depth migration, 2012, Chinese Journal of Geophysics, vol. 55, no. 1, pp. 238-251.
2. X. Xiao, W.S. Leaney, Local vertical seismic profiling (VSP) elastic reverse-time migration and migration resolution: Salt-flank imaging with transmitted P-to-S waves, 2010, Geophysics: Journal of the Society of Exploration Geophysicists, vol. 75, no. 2, pp. S35-S49.
3. S.L. Liu, X.F. Li, Pre-stack reverse time depth migration research on high precision seismic wave, Proceedings of the twenty-eighth annual conference of the Chinese Geophysical Society, 2012, pp. 289-289.
4. J.L. Wu, P. Wu, Y. Gao, The application and research of inverse time pre-stack depth migration in the exploration of depressed buried-hill locating at the west area of LIAOHE, Proceedings of the Symposium on geophysical prospecting technology in 2012.

5. Y.F. Wang, Comparison of seismic wave interference migration and pre-conditioned regularized least square migration imaging, 2013, Chinese Journal of Geophysics, vol. 56, no. 1, pp. 230-238.
6. K.Y. Chen, X.C. Fan, Analysis on the migration noise of the common imaging point gathers in the different regions of the seismic wave inverse time migration, 2014, vol. 26, no. 2, pp. 118-124.
7. X.S. Bao, J.M. Zhang, C. Yin, The difference of average energy and the relationship between the change of reservoir velocity, 2008, Geophysical Prospecting for Petroleum, vol. 47, no. 1, pp. 24-29.
8. T. Chen, L.J. Huang, Imaging steeply-dipping fault zones using elastic reverse-time migration with a combined wave field-separation and pointing-vector imaging condition, 39th Workshop on Geothermal Reservoir Engineering 2014, Vol. 2, Stanford Geothermal Program Workshop Report SGP-TR-202.Stanford, California(US).24-26 February 2014.2014:891-901.