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An Exact Solution for the Propagation of Shock Waves in Self-Gravitating Perfect Gas in the Presence of Magnetic Field and Radiative Heat Flux

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Abstract

Propagation of spherical shock wave with azimuthal magnetic field and radiation heat flux in self-gravitating perfect gas is investigated. The azimuthal magnetic field and the initial density are assumed to vary according to power law. An exact similarity solution is reported when loss of energy due to radiation escape is notable and radiation pressure is non-zero. The entire energy of the shock wave is varying and increases with time. The effects of variation of the radiation pressure number, the initial density variation index, the Alfven-Mach number, the gravitational parameter and the adiabatic exponent are workout in detail. The shock strength increases with an increase in the initial density variation index. On the other hand, presence of magnetic field or an increment in the value of the radiation pressure number or the ratio of specific heats or gravitational parameter the shock strength decreases. It is obtained that increase in the radiation pressure number and gravitational parameter has same behavior on the flow variables. Also, it is observed that an increase in the value of gravitational parameter and the adiabatic exponent have same behavior on the fluid velocity, the material pressure, the radiation pressure, the mass and the radiation flux and azimuthal magnetic field.

Key words

MHD Shock waves, Similarity solution, Self-gravitating perfect gas, Radiation pressure and radiation energy, Radiation heat flux.

1. Introduction

For the first time independently Sedov [1] and Taylor [2, 3] has presented the numerical solutions for shock wave problem. The numerical solutions for self-similar flow in adiabatic case in self-gravitating gas were obtained in [1, 4]. In a self-gravitating gas many authors have investigated the self-similar flows behind a shock wave (see [5-7] and many others). In a variable density medium shock wave has been studied in ([1, 8-14], and others). Their results are more relevant to the shock in the deep interior of stars. Several authors extended the classical self-similar approach of Sedov [1] for blast wave problems by taking radiation into account (see, [15-20] and many others). The majority of research works on radiation gasdynamics are related to the radiative heat flux only and little research has been done under the consideration of radiation energy and radiation pressure in the presence or absence of gravitational field. A detailed study towards gaining a better understanding of the interaction between gasdynamic motion of an electrically conducting medium and magnetic field within the context of hyperbolic system has been carried out by many investigators such as (Shang [21], Lock and Mestel [22]). A detailed review in the field of magnetogasdynamic flows can be seen in the paper (Shang [21]). Lock and Mestel [22] analyzed the annular self-similar solutions in ideal magnetogasdynamics by casting the ideal magnetogasdynamic equations to a three-dimensional autonomous system in which either the magnetic pressure or the fluid pressure vanishes.

In the present study the problem discussed by Vishwakarma et al. [23] (also, see Ashraf and Sachdev [18]) is extended by considering the gravitational effects in spherical geometry. The medium is taken to be inviscid thermally perfect gas and the pressure ahead of the shock is taken into account. The density and the azimuthal magnetic field in the undisturbed medium are assumed to vary as some power of the distance from the point of symmetry.

The exact similarity solutions are derived for isothermal shock with the general shock conditions instead of strong shock conditions. As in [18] we have taken the similarity form for radiation pressure, energy and radiative heat flux, and the ‘Product Solutions’ of Mc. Vittie [24] is used to evaluate them. Radiation flux is obtained from conservation equations.

The effects of variation of Alfven-Mach number, the gravitational parameter, initial density variation index, radiation pressure number and the specific heat ratio of gas on shock strength and the flow variables are discussed in details. The shock strength decreases with the grow in the strength of the surrounding magnetic field strength or the radiation pressure number or the ratio of the specific heat of the gas or the parameter of gravitational effect. On the other hand, initial density variation index has opposite behavior on shock strength.

2. Fundamental Equations of Motions and Boundary Conditions

In Eulerian co-ordinate, the basic equations governing spherically symmetric unsteady motion of an inviscid and perfectly conducting self-gravitating perfect gas under the considerable effects of the radiation heat flux, magnetic field, radiation energy and radiation pressure may be written as (Vishwakarma et al. [23], Whitham [25], Nath et al. [26], Vishwakarma and Singh [27], Nath and Sinha [7])

$$\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial r}+\rho\frac{\partial u}{\partial r}+\frac{2u\rho}{r}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{1}{\rho}\left[\frac{\partial (p+p_R)}{\partial r}+\mu h\frac{\partial h}{\partial r}+\frac{\mu h^2}{r}\right]+\frac{Gm}{r^2}=0, \tag{2}$$

$$\frac{\partial h}{\partial t}+u\frac{\partial h}{\partial r}+h\frac{\partial u}{\partial r}+\frac{hu}{r}=0, \tag{3}$$

$$\frac{\partial (E+E_R)}{\partial t}+u\frac{\partial (E+E_R)}{\partial r}-\frac{(p+p_R)}{\rho^2}\left[\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial r}\right]+\frac{1}{\rho r^2}\frac{\partial (F r^2)}{\partial r}=0, \tag{4}$$

$$\frac{\partial m}{\partial r}=4\pi \rho r^2, \tag{5}$$

where independent space and time coordinates are denoted by r and t, u, ρ, p, p_R, h, μ, E, E_R and F are the fluid velocity, density, material pressure, radiation pressure, azimuthal magnetic field, magnetic permeability, internal energy per unit mass, radiation energy and radiation flux respectively; G is the gravitational constant and m is the mass contained in a sphere of radius r.

We have considered an ideal gas behavior of the medium, so that (Vishwakarma et al. [23], Verma and Vishwakarma [19])

$$p=\Gamma \rho T \ ; \ E=\frac{p}{\rho(\gamma-1)}, \tag{6}$$

where γ is the ratio of specific heats and Γ is the gas constant.

The radiation energy E_R and the radiation pressure p_R are expressed as

$$\rho E_R=3p_R=\sigma T^4, \tag{7}$$

where σ is the Stephen's Boltzmann constant.

The flow variables immediately ahead of shock front are as follows

$$u_1 = 0,$$

$$\rho = \rho_1 = \rho_0 R^{w},$$

$$m = m_1 = \frac{4\pi \rho_0 R^{w+3}}{w+3}, \tag{8}$$

$$h = h_1 = h_0 R^{-\alpha},$$

$$p = p_1^* = \frac{\mu h_0^{2}(1-\alpha)}{2\alpha} R^{-2\alpha} - \frac{2\pi G\rho_0^2}{(w+3)(w+1)} R^{2w+2},$$

where ρ_0, w, h_0 and α are constants, $p_1^* = p_1 + p_{R_1}$, $-3 < w < -1$, $\alpha = -(w+1)$, subscript 1 refers the conditions just ahead of the shock front and R is the shock radius given by

$$U^2 = A^2 R^{-\delta}, \tag{9}$$

where A and δ being constants and $U\left(=\frac{dR}{dt}\right)$ denotes the velocity of shock front. The flow configuration is shown in figure - A

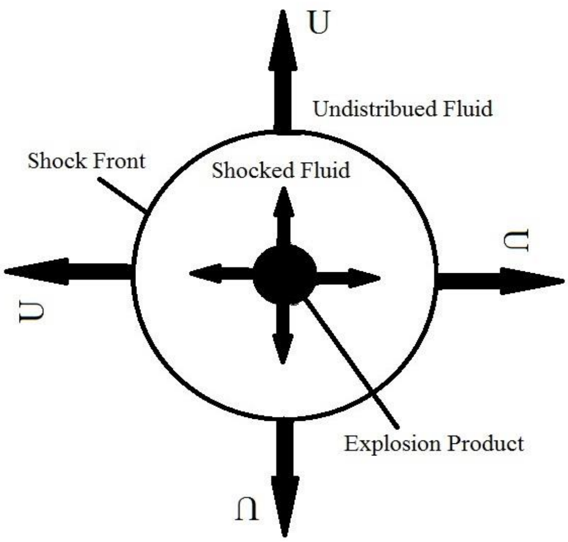

Figure A: Show the flow configutation in spherically symmetric case as a result of point explosion

The Rankine-Hugonite conditions across an isothermal shock wave in an electrically conducting and radiating gas are given by (Vishwakarma et al. [20], Singh [28], Nath and Sinha [7])

$$u_2 = (1-\beta)U, \quad \rho_2 = \frac{\rho_1}{\beta}, \quad p_2 = L\,\rho_1 U^2, \; m_1 = m_2, \quad h_2 = \frac{h_1}{\beta}, \quad p_{R_1} = p_{R_2}, \quad F_2 = F_1, \tag{10}$$

where the subscript 2 refers the condition just behind the shock front, $M = \left(\frac{\rho_1 U^2}{\gamma p_1^*}\right)^{\frac{1}{2}}$ is Mach number related to the frozen speed of sound $\left(\frac{\gamma p_1^*}{\rho_1}\right)^{\frac{1}{2}}$ and $M_A = \left(\frac{\rho_1 U^2}{\mu h_1^2}\right)^{\frac{1}{2}}$ is Alfven-Mach number, $L = \left[(1-\beta) + \frac{1}{\gamma M^2} + \frac{M_A^{-2}}{2}\left(1 - \frac{1}{\beta^2}\right)\right]$, and the density ratio β $(0 < \beta < 1)$ across the shock front is obtained by the quadratic relation

$$\beta^2(\gamma+1) - \beta\left[\frac{2}{M^2} + (\gamma-1) + \frac{\gamma}{M_A^2} + \frac{8(\gamma-1)R_P}{\gamma M^2}\right] + \frac{(\gamma-2)}{M_A^2} = 0, \tag{11}$$

where $R_P = \frac{p_{R_1}}{p_1^*}$ is the radiation pressure number ahead of the shock front.

3. Self-similarity Transformations

To obtain the similarity solutions, the unknown variables may be written in the form (c.f. [7, 23, 29-31])

$$u = UX(\eta), \quad \rho = \rho_1 D(\eta), \quad p = \rho_1 U^2 P(\eta), \quad p_R = \rho_1 U^2 P_R(\eta), \quad \sqrt{\mu}\, h = \sqrt{\rho_1}\, U H(\eta),$$
$$E = U^2 \overline{E}(\eta), \; E_R = U^2\, \overline{E_R}(\eta), \; F = \rho_1 U^3\, \overline{F}(\eta), \; m = m_1 N(\eta), \tag{12}$$

where X, D, P_R, P, H, $\overline{E_R}$, $\overline{E}$, $\overline{F}$ and N are the function of η only, $\eta = \frac{r}{R}$ is the dimensionless quantity.

The shock Mach number M and Alfven-Mach number M_A should be constants for existence of similarity solutions, therefore,

$$w + \delta + 2 = 0, \text{ and } \delta = w + 2\alpha \tag{13}$$

Thus,

$$M^2 = \frac{1}{\gamma\left[\frac{(1-\alpha)}{2\alpha} - \frac{2G_0}{(w+3)(w+1)}\right]} M_A^2 \ . \tag{14}$$

where $G_0 = \frac{G\ \pi\ \rho_0^2}{\mu h_0^2}$ is the gravitational parameter.

By using similarity transformations from equation (12), equations (1)-(5) can be transformed into a system of ODEs

$$(X-\eta)\frac{dD}{d\eta} + D\left(\frac{dX}{d\eta} + w\right) + \frac{2DX}{\eta} = 0, \tag{15}$$

$$(X-\eta)\frac{d\ X}{d\eta} - \frac{\delta\ X}{2} + \frac{1}{D}\left[\left(\frac{d\ P}{d\eta} + \frac{dP_R}{d\eta}\right) + H\frac{d\,H}{d\eta} + \frac{H^2}{\eta}\right] + \frac{4G_0^{'}\ N}{(w+3)\ \eta^2} = 0, \tag{16}$$

$$\left(\frac{w-\delta}{2} + \frac{d\,X}{d\,\eta}\right)H + (X-\eta)\frac{d\ H}{d\ \eta} + \frac{X\ H}{\eta} = 0, \tag{17}$$

$$(X-\eta)\frac{d\ \overline{E}}{d\eta} + (X-\eta)\frac{d\,\overline{E_R}}{d\eta} - \delta\left(\overline{E} + \overline{E}_R\right) - \frac{P}{D}\left(\frac{d\,X}{d\,\eta} + \frac{2X}{\eta}\right) - \frac{P_R}{D}\left(\frac{d\,X}{d\,\eta} + \frac{2X}{\eta}\right) + \frac{1}{D\eta^2}\frac{d}{d\,\eta}\left(\overline{F}\ \eta^2\right) = 0, \tag{18}$$

$$\frac{d\,N}{d\eta} - (w+3)\ D\,\eta^2 = 0, \tag{19}$$

where $G_0^{'} = \left(\frac{G_0\ \mu\, h_0^2}{\rho_0\ A^2}\right)$.

Applying similarity transformations (12) on shock conditions (10), we get

$$D(1) = \frac{1}{\beta}, \tag{20}$$

$$X(1) = (1-\beta), \tag{21}$$

$$P(1) = L, \tag{22}$$

$$H(1) = \frac{1}{\beta M_A}, \tag{23}$$

$$P_R(1) = \frac{R_P}{\gamma M^2}, \tag{24}$$

$$N(1) = 1. \tag{25}$$

The product solution of the 'progressive wave' is assumed to be (cf. Mc. Vittie [24])

$$u = \frac{a(t)}{t}\ r\,, \tag{26}$$

$$\rho=(\lambda+1)\, t^{-2\varepsilon}\, f(t)\, \xi^{\,\lambda-2}, \tag{27}$$

$$p=\varepsilon^{2}\, t^{-2}\, f(t) b(t) \xi^{\lambda}, \tag{28}$$

$$p_R=\varepsilon^{2}\, t^{-2}\, f(t)\, b(t) \xi^{\lambda}, \tag{29}$$

$$h=\varepsilon\, t^{-1}\, f^{\frac{1}{2}}(t)\, c(t) \xi^{\,\lambda/2}, \tag{30}$$

$$m=4\pi\, f(t) t^{\varepsilon}\, \xi^{\,\lambda+1}, \tag{31}$$

where $\xi=rt^{-\varepsilon}$, λ and ε are constants; and a, f, b and c are functions of t given by

$$a(t)=\frac{\varepsilon\, \lambda-t\dfrac{f'}{f}}{\lambda+1}=\frac{2-2t\dfrac{c'}{c}}{3}, \tag{32}$$

$$2b(t)+\mu c^{2}(t)\frac{(\lambda+2)}{2\lambda}=\frac{(\lambda+1)}{\lambda\varepsilon^{2}}\left[\left(a-a^{2}-ta'\right)-\frac{4\pi\, G\, f(t)\, t^{2-2\varepsilon}\xi^{\lambda}}{\xi^{2}}\right]. \tag{33}$$

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Equations (32) - (33) identically satisfy the equations (1) to (3). On converting this solution to a similarity one, a is obtained as constant given by $a=\left(\frac{2(1-\beta)}{\delta+2}\right)$, applying the boundary conditions (20) - (25) in equations (26) - (31), we obtain

$$X(\eta)=(1-\beta)\eta, \tag{34}$$

$$D(\eta)=\frac{1}{\beta}\eta^{\lambda-2}, \tag{35}$$

$$P(\eta)=L\eta^{\lambda}, \tag{36}$$

$$P_R(\eta)=\frac{R_P}{\gamma M^{2}}\eta^{\lambda}, \tag{37}$$

$$H(\eta)=\frac{1}{\beta M_A}\eta^{\lambda/2}, \tag{38}$$

$$N(\eta)=\eta^{\lambda+1}. \tag{39}$$

The expressions (34) to (39) identically satisfy equations (15)–(17), and hence they represent a solution of equations (15) – (19) in closed form.

Substituting equations (6)–(7), (9) and (34)–(39) in equation (18), we evaluate the value of $F(\eta)$ as given below

$$F(\eta)=\frac{\left(2\beta+\delta\right)\gamma M^{2}L+\left(6\beta+3\delta\right)R_{p}\left(\gamma-1\right)+3\left(\gamma-1\right)\left(1-\beta\right)\left\{\gamma M^{2}L+R_{p}\right\}}{\left(\lambda+3\right)\left(\gamma-1\right)\gamma M^{2}}\eta^{\lambda+1}. \tag{40}$$

Substituting equations (34)–(39) into equations (15)–(16), we obtain

$$\lambda=1+\frac{1}{\beta}\left[1+w+2\left(1-\beta\right)\right], \tag{41}$$

$$\beta(\lambda-1)(1-\beta)-\frac{\delta}{2}(1-\beta)+\frac{4G_{0}^{'}}{(w+3)}\eta^{\lambda-2}+\frac{\beta\lambda}{\gamma M^{2}}(1+R_{P})+\frac{M_{A}^{-2}}{2\beta}(\lambda\beta^{2}+1)=0, \tag{42}$$

Total energy E_T behind the shock front in the flow-field is given as

$$E_{T}=4\pi\int_{0}^{R}\left\{\frac{p}{\left(\gamma-1\right)\rho}+\frac{3P_{R}}{\rho}+\frac{\mu h^{2}}{2\rho}+\frac{u^{2}}{2}-\frac{Gm}{r}\right\}\rho r^{2}dr. \tag{43}$$

Using equations (12) and (34-39), equation (43) becomes

$$E_{T}=4\pi\rho_{0}A^{2}JR^{3+w-\delta}=Bt^{\frac{2(3+w-\delta)}{2+\delta}}, \tag{44}$$

where $J=\int_{0}^{1}\left\{\frac{L}{\left(\gamma-1\right)}+\frac{3R_{P}}{\gamma M^{2}}+\frac{1}{2\beta^{2}M_{A}^{2}}+\frac{\left(1-\beta\right)^{2}}{2\beta}-\frac{4\pi G\rho_{0}\eta^{\lambda-2}}{(3+w)\beta c^{2}}\right\}\eta^{2+\lambda}d\eta$ and B is constant. Equation (44) conveys that the entire energy of the shock front rises with time. Similar rise can be achieved from the time-dependent energy release from an explosive material across the symmetry axis (or point of symmetry).

Equations (34)–(40) give the analytical solution of our considered problem. The solution we obtained is example of exact solution in radiation magnetogasdynamics in presence of gravitational field and similar to ordinary gas dynamics exact solutions obtained by Mc Vittie [24], Ashraf and Sachdev [18] solutions in radiation gas dynamics and Vishwakarma et al. [23] solutions in magnetogasdynamics with radiative heat flux**.**

4. Results and Discussion

For the radiative heat flux to be positive everywhere and the density to be finite at the center, the inequalities obtained from equations (35) and (40) should satisfy:

$$3+w-3\beta>0, \tag{45}$$

$$\left(2\beta+\delta\right)\gamma M^{2}L+\left(6\beta+3\delta\right)R_{p}\left(\gamma-1\right)+3\left(\gamma-1\right)\left(1-\beta\right)\left\{\gamma M^{2}L+R_{p}\right\}>0. \tag{46}$$

In addition to a necessary condition for the density to remain finite at the center inequality (45) must also satisfy (*i.e.* $0<\beta<1$) the condition for the existence of shock wave.

We have calculated the values of the the density $D(\eta)$, the material pressure $P(\eta)$, fluid velocity $X(\eta)$, the radiation pressure $P_R(\eta)$, azimuthal magnetic field $H(\eta)$, the mass $N(\eta)$ and the radiation flux $F(\eta)$ for the values of physical parameters $\gamma=\frac{4}{3},\frac{5}{3}$; $M_A^{-2}=0.06,\ 0.07, 0.1$; $w=-1.6,\ -1.7$; $G_0=0,\ 0.02$; and $R_P=0.5,\ 1$; (Pai [32], Vishwakarma et al. [23], Rosenau and Frankenthal [10]). The value $M_A^{-2}=0$ in non-magnetic case. The value $G_0=0$ in non-gravitating case (the solution obtained in [23]). The present study is the extension of the work of Vishwakarma et al. [23] by taking into account the gravitational effect in both cylindrical and spherical geometry.

Table 1 shows the density ratio β variation across the shock for different values of M_A^{-2}, w, G_0 and R_p with $\gamma=\frac{5}{3}$. Table-2 shows the density ratio β variation across the shock for different values of γ and M_A^{-2} with $w=-1.6,\ G_0=0.02$ and $R_p=1$.

Table 1. Variations of the density ratio β across the shock for different values of M_A^{-2}, w, G_0 and R_p with $\gamma=\frac{5}{3}$

M_A^{-2}	w	G_0	R_p	β
0.06	- 1.6	0.02	0.5	0.3597
			1	0.3814
		0	0.5	0.3537
			1	0.3726
	- 1.7	0.02	0.5	0.3441
			1	0.3587
		0	0.5	0.3385
			1	0.3506
0.07	- 1.6	0.02	0.5	0.3769
			1	0.4021
		0	0.5	0.3699

			1	0.3919
	-1.7	0.02	0.5	0.3588
			1	0.3757
		0	0.5	0.3523
			1	0.3663
0.1	- 1.6	0.02	0.5	0.4274
			1	0.4632
		0	0.5	0.4174
			1	0.4486
	-1.7	0.02	0.5	0.4017
			1	0.4257
		0	0.5	0.3925
			1	0.4124

Table 2. Variation of density ratio across the shock β for different values of γ and M_A^{-2} with w = - 1.6, $G_0 = 0.02$ and $R_p = 1$.

M_A^{-2}	γ	β
0.06	$4/3$	0.2887
	$5/3$	0.3814
0.07	$4/3$	0.3086
	$5/3$	0.4021
0.1	$4/3$	0.3652
	$5/3$	0.4632

In Figs. 1 we plotted the values of the flow variables $X(\eta)$, $D(\eta)$, $P(\eta)$, $P_R(\eta)$, $H(\eta)$, $N(\eta)$ and $F(\eta)$ for $\gamma = \frac{5}{3}$; $M_A^{-2} = 0.06$; $w = -1.6, -1.7$; $G_0 = 0.02$; and $R_p = 1$ as η varies from zero to unity. In Figs. 2 we plotted the values of the flow variables $X(\eta)$, $D(\eta)$, $P(\eta)$, $H(\eta)$, $N(\eta)$ and $F(\eta)$ for $\gamma = \frac{5}{3}$; $M_A^{-2} = 0.07$; $w = -1.6$; $G_0 = 0.02$; and $R_p = 1$ as η varies from zero to unity. In Fig. 3 we plotted the values of $X(\eta)$, $D(\eta)$, $P(\eta)$, $P_R(\eta)$, $H(\eta)$, $N(\eta)$ and $F(\eta)$ for $\gamma = \frac{4}{3}, \frac{5}{3};$ with $M_A^{-2} = 0.06$ $w = -1.6;$ $G_0 = 0.02$; and $R_p = 1$ as η varies from zero to unity.

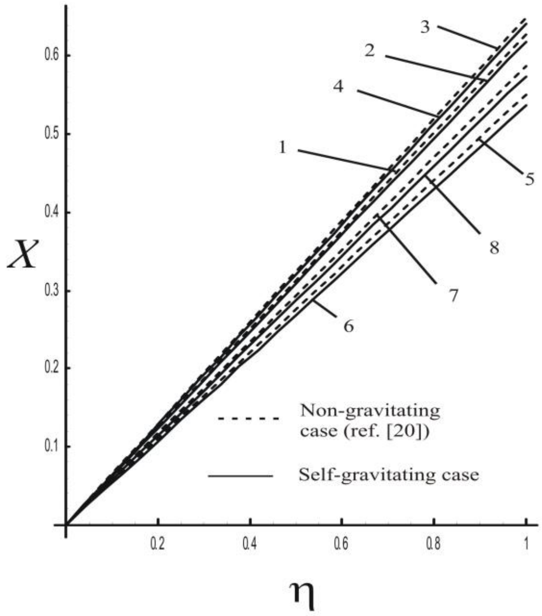
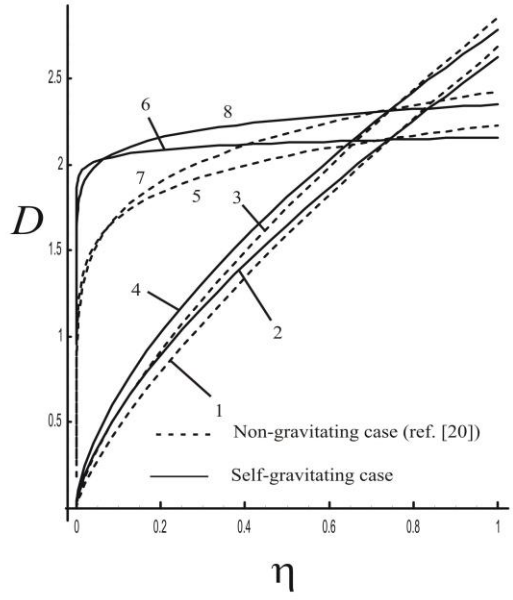


Figure 1(a)



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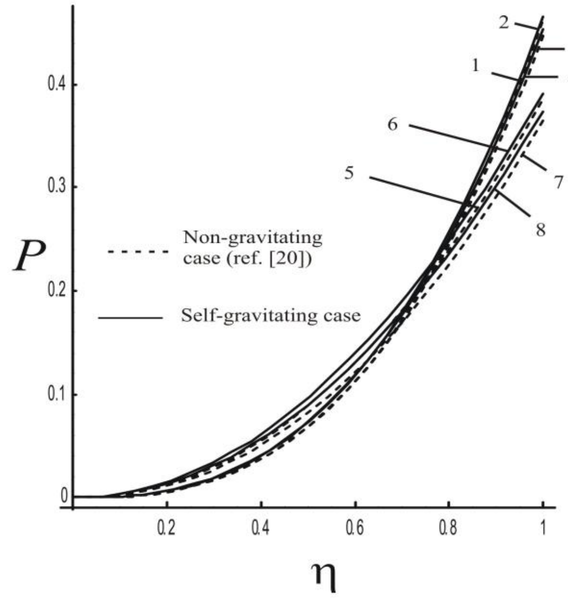
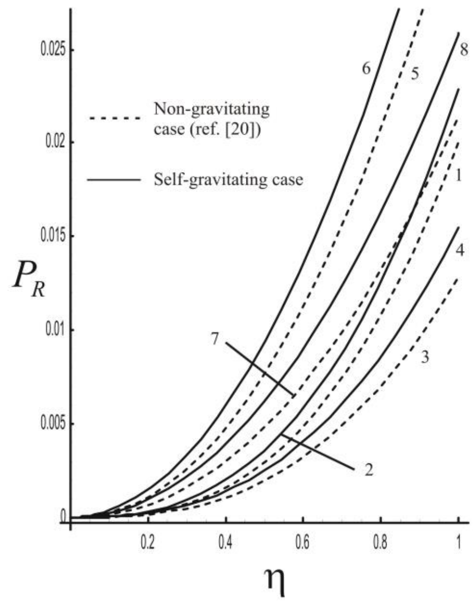


Figure 1(c)



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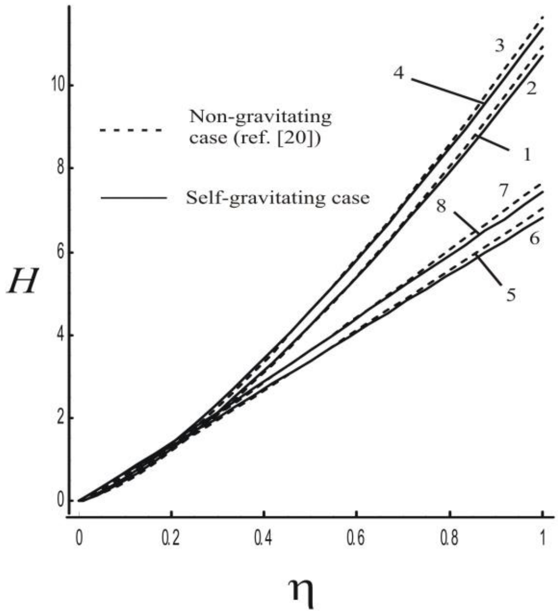

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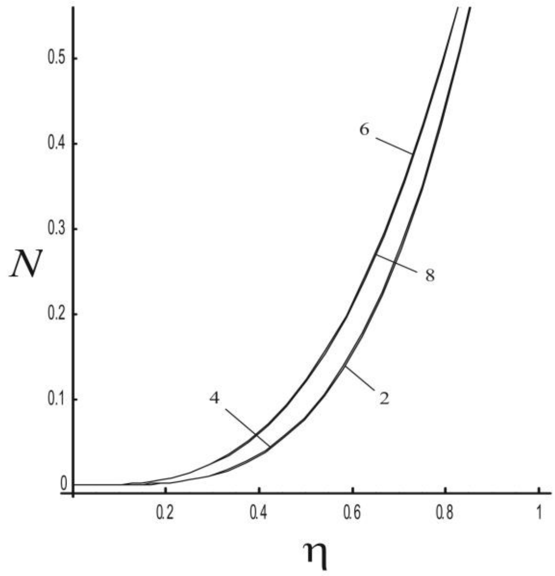


Figure 1(f)

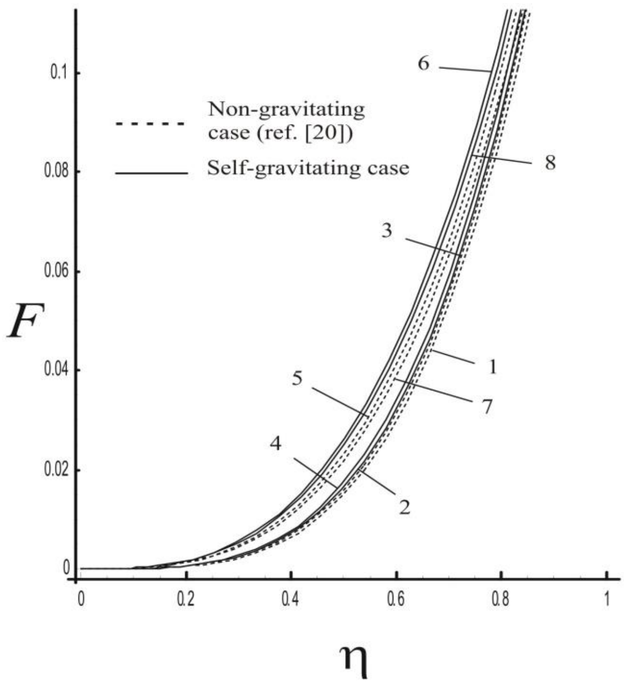


Figure 1(g)

Fig. 1. Variation of the flow variables with the distance in the region behind the shock front at $\gamma = \frac{5}{3}$ and $R_p = 1$ (a) fluid velocity $X(\eta)$, (b) the density $D(\eta)$, (c) the material pressure $P(\eta)$, (d) the radiation pressure $P_R(\eta)$, (e) the azimuthal magnetic field $H(\eta)$, (f) the mass $N(\eta)$, (g) the radiation flux $F(\eta)$:

1. $M_A^{-2} = 0.06$, $w =$ - 1.6, $G_0 = 0$; 2. $M_A^{-2} = 0.06$, $w =$ - 1.6, $G_0 = 0.02$; 3. $M_A^{-2} = 0.06$, $w =$ - 1.7, $G_0 = 0$; 4. $M_A^{-2} = 0.06$, $w =$ - 1.7, $G_0 = 0.02$; 5. $M_A^{-2} = 0.1$, $w =$ - 1.6, $G_0 = 0$; 6. $M_A^{-2} = 0.1$, $w =$ - 1.6, $G_0 = 0.02$; 7. $M_A^{-2} = 0.1$, $w =$ - 1.7, $G_0 = 0$; 8. $M_A^{-2} = 0.1$, $w =$ - 1.7, $G_0 = 0.02$.

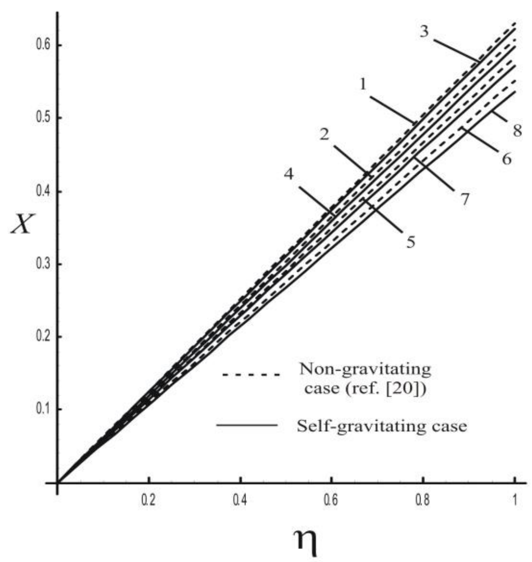
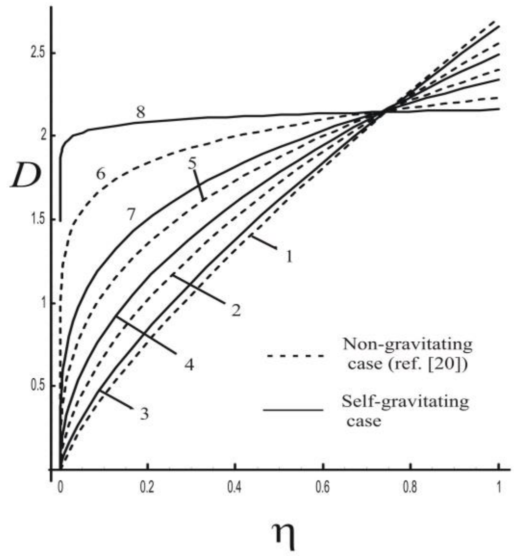


Figure 2(a)



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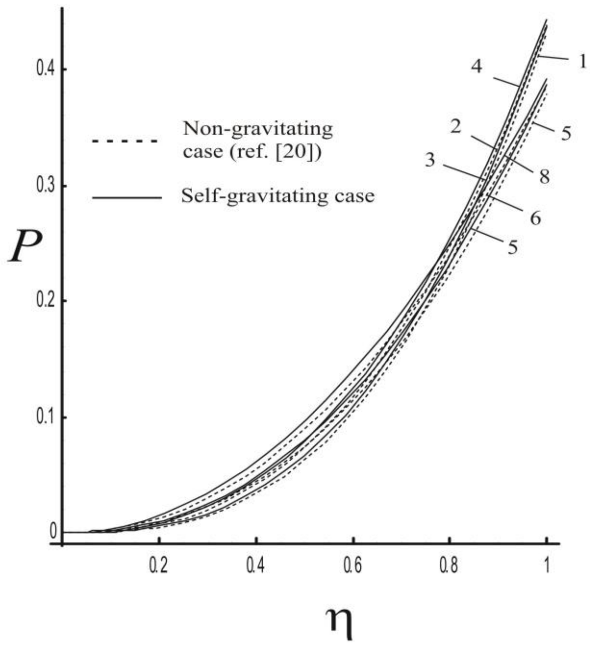

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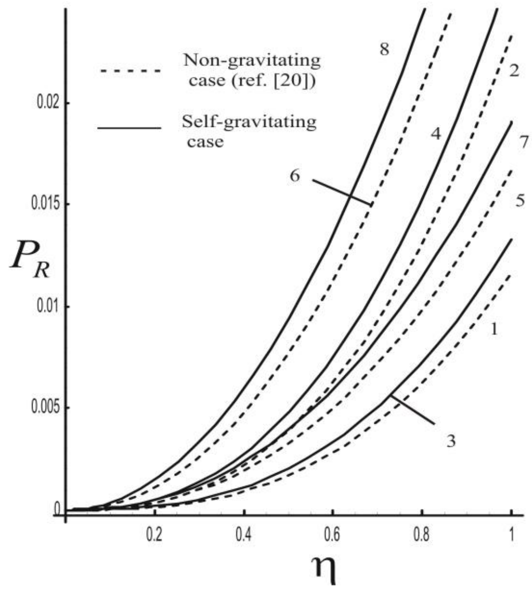

Figure 2(d)

It is shown that the pressure, fluid velocity, azimuthal magnetic field, density, radiation pressure, radiation flux and mass approaches to zero at the point of symmetry. The values of all physical variables increase from the zero at the point of symmetry to the highest at the shock. The shock strength increases with an increase in M_A^{-2} or $\bar{b}$ or G_0 or R_P or γ; whereas the initial density variation index w **(**which ultimately decreases the value of α **)** has reverse effect on shock strength (see Tables 1& 2)**.** The flow variables the fluid velocity $X(\eta)$, the azimuthal magnetic field $H(\eta)$ decreases; whereas the radiation flux $F(\xi)$, the radiation pressure $P_R(\xi)$, the mass $N(\eta)$ increases with an increases in M_A^{-2} or G_0 or R_P (see Figure 1 (a, d-g) and 2 (a, d-g)). Also, the density $D(\eta)$ decreases near shock; whereas it increases near inner boundary with increase in M_A^{-2} or G_0 or R_P (see Fig. 1 (b) and Fig. 2 (b)), and the material pressure $P(\xi)$ increases anywhere in the flow field with increase in G_0 or R_P; but it decreases near shock and increases near inner boundary surface with increase in M_A^{-2} (see Figures 1(c) and 2(c)). The flow variables $D(\eta)$, $P(\eta)$, $P_R(\eta)$, $H(\eta)$, $N(\eta)$ and $F(\eta)$ increases at any point in the flow field behind the shock (see curve 2,3, 5-7 respectively in Fig. 3), but the fluid velocity $X(\eta)$ decreases everywhere in the flow-field behind the shock (see curve 1 in Fig. 3) with an increase in the value of adiabatic exponent γ **.** Also, the radiation pressure $P_R(\xi)$ is almost unaffected with an increase in $\gamma\gamma$ (see, curve 4 in Fig. 3). The flow variables $X(\eta)$**,** $D(\eta)$, $H(\eta)$ increases; whereas $P(\xi)$, $P_R(\xi)$ decreases with increase in the density variation index w (Figs 1(a, b, c, d)). Also, In the flow-field behind the shock the mass $N(\eta)$ and the radiation flux $F(\eta)$ increases for $M_A^{-2} = 0.06$ whereas these flow variables decrease for $M_A^{-2} = 0.01$ with an increase in w (see Figs.1 (f, g)).

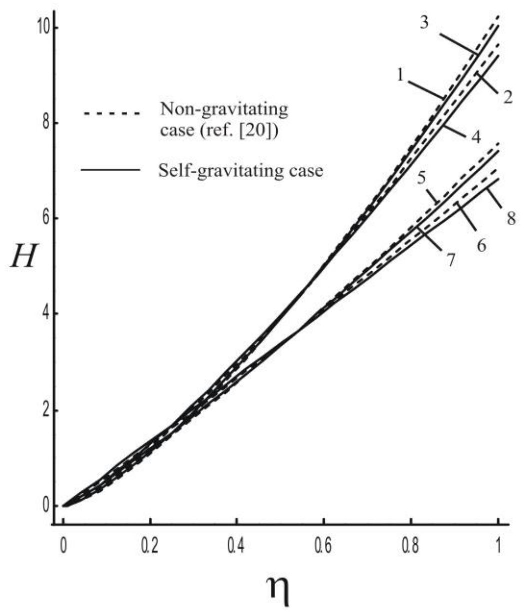

Figure 2(e)

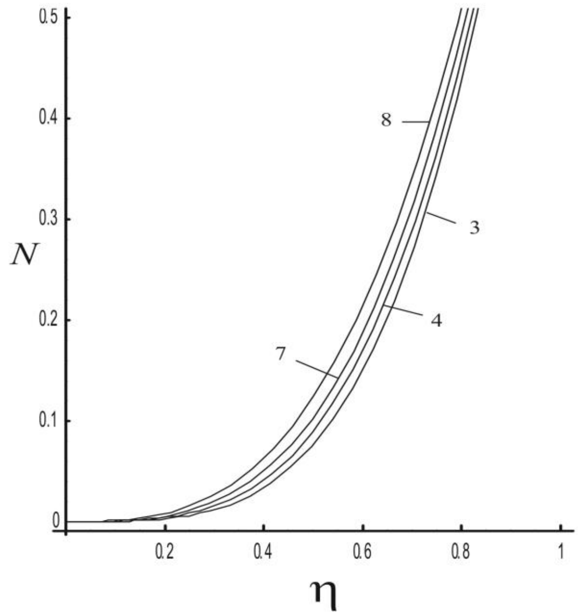

Figure 2(f)

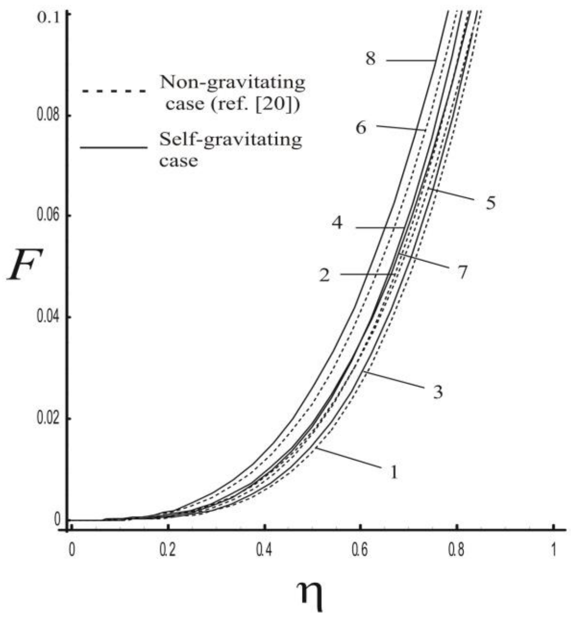

Figure 2(g)

Figure 2. Variation of the flow variables with the distance in the region behind the shock front at $\gamma = \frac{5}{3}$ and $w = -\ 1.6$(a) fluid velocity $X(\eta)$, (b) the density $D(\eta)$, (c) the material pressure $P(\eta)$, (d) the radiation pressure $P_R(\eta)$, (e) the azimuthal magnetic field $H(\eta)$, (f) the mass $N(\eta)$, (g) the radiation flux $F(\eta)$:

1. $M_A^{-2} = 0.07$, $G_0 = 0, R_p = 0.5$; 2. $M_A^{-2} = 0.07$, $G_0 = 0, R_p = 1$; 3. $M_A^{-2} = 0.07$, $G_0 = 0.02, R_p = 0.5$; 4. $M_A^{-2} = 0.07$, $G_0 = 0.02, R_p = 1$; 5. $M_A^{-2} = 0.1$, $G_0 = 0, R_p = 0.5$; 6. $M_A^{-2} = 0.1$ $G_0 = 0$, $R_p = 1$; 7. $M_A^{-2} = 0.1$, $G_0 = 0.02$, $R_p = 0.5$; 8. $M_A^{-2} = 0.1$, $G_0 = 0.02, R_p = 1$.

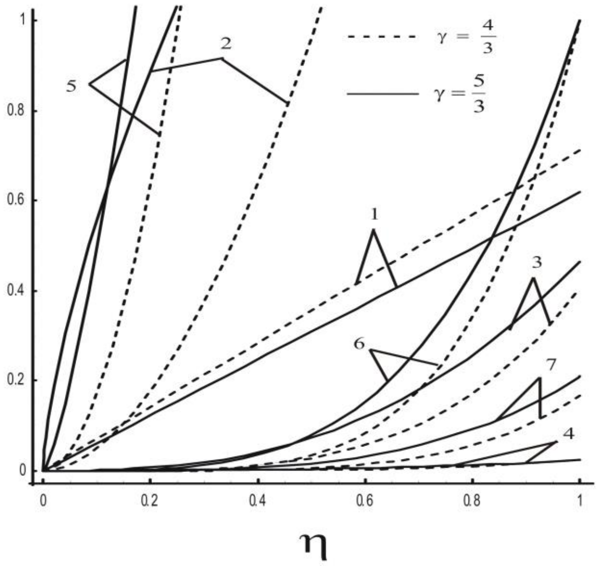

Figure 3

Figure 3. Variation of the flow variables with the distance in the region behind the shock front *for* $\gamma = \frac{4}{3}, \frac{5}{3}$; $M_A^{-2} = 0.06; w =$- 1.6, $G_0 = 0.02; R_p = 1$: 1. the fluid velocity $X(\eta)$, 2. the density $D(\eta)$, 3. the material pressure $P(\eta)$, 4. the radiation pressure $P_R(\eta)$, *5.* the azimuthal magnetic field $H(\eta)$, *6.* the mass $N(\eta)$, 7. the radiation flux $F(\eta)$.

Conclusion

In the present problem the flow behind the magnetogasdynamics shock waves with or without self-gravitating effects and radiation heat flux in a non-uniform perfect gas have been discussed. On the basis of result obtained in the present study, we may conclude the following:

(i) An increase in the strength of the surrounding magnetic field or the adiabatic exponent or the radiation pressure number or the parameter of gravitation the shock strength decreases; whereas it increases with increase in the variation index of initial density.

(ii) An increase in the parameter of the gravitational effect G_0, fluid velocity and the magnetic field decreases; whereas the material pressure, the radiation pressure, the mass and the radiation flux increases. The density increases near inner boundary surface but it decreases near shock with increase in the gravitational parameter G_0.

(iii) There is a same effect on the fluid velocity, the material pressure, the density, the radiation pressure, the mass, the magnetic field and the radiation flux with an increase in the gravitational parameter G_0 and the radiation pressure number R_p .

(iv) An increase in the value of the gravitational parameter G_0 and initial density variation index w have opposite behavior on the material pressure, the fluid velocity, the radiation pressure, the mass, the magnetic field, and the radiation flux for $M_A^{-2} = 0.01$.

(v) same effect on the density, radiation pressure, fluid velocity, magnetic field, the mass and radiation flux with an increase in the gravitational parameter G_0 and M_A^{-2}.

(vi) an increase in the value of the gravitational parameter G_0 and the ratio of specific heats γ have same behavior on the fluid velocity, the material pressure, the mass and the radiation flux.

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Nomenclature

A constant

a function of t

B constant

b function of t

c function of t

D non-dimensional density

E internal energy per unit mass

$\overline{E}$ non-dimensional internal energy per unit mass

E_R radiation energy

$\overline{E}_R$ non-dimensional radiation energy

E_T total energy of the flow-field behind shock front

F radiation flux

$\overline{F}$ non-dimensional radiation flux

f function of t

G the gravitational constant

G_0 the gravitational parameter

H non-dimensional azimuthal magnetic field

h azimuthal magnetic field

h_0 constant

J abbreviation

L abbreviation

M shock Mach number

M_A Alfven- Mach number

m mass contained in a unit cylinder of radius r or in a sphere of radius r

N non dimensional mass

P non-dimensional fluid pressure

P_R non-dimensional radiation pressure

p material pressure

p_R radiation pressure

p_a^* Sum of partial pressure and radiation pressure ahead of shock front

R Shock radius

R_p radiation pressure number

r independent space coordinate

T temperature of the gas

t independent time coordinate

U shock velocity

u fluid velocity

X non-dimensional fluid velocity

w density variation index

Greek Letters

ρ the fluid density

ρ_0 constant

δ shock radius variation index

α magnetic field variation index

Γ gas constant

γ ratio of specific heats

σ Stephen's Boltzmann constant.

β ratio of density across the shock front

μ magnetic permeability

ξ arbitrary function of r and t

λ constant

ε constant

η similarity variable

Subscripts

1 immediately ahead the shock

2 immediately behind the shock

Superscript

$'$ derivative with respect to t