AMSE JOURNALS-AMSE IIETA publication-2017-Series: Modelling B; Vol. 86; N°4; pp 885-906
Submitted Jan. 2017; Revised Feb. 15, 2017, Accepted March 15, 2017
https://doi.org/10.18280/mmc_b.860405

Numerical Modeling of Two Phase Jet Flow and Heat Transfer with Charged Suspended Particulate Matter (SPM)

*Tumnanath Samantara, **S.K. Mishra, ***T.C. Panda

[*] Orissa Engineering College, Bhubaneswar, Odisha, India-752050
(tnsamantara@gmail.com)
[**] R.C.M.Science College, Khallikote, Odisha, India-761030
(s1_mishra@yahoo.com)
[***] Berhampur University, Odisha, India-760007
(tc_panda@yahoo.co.in)

Abstract

The problem of flow of fluid with charged suspended particulate matter through a jet is studied. The particles are allowed to diffuse through the carrier fluid and the random motion of the particles has been taken into consideration, as the size of the particles is very small. The terms related to the heat added to the system, to slip-energy flux in the energy equation of particle phase is considered. The governing systems of nonlinear partial differential equations are solved by perturbation methods followed by similarity transformation and finite difference technique using non-uniform grid. The effects of volume fraction on skin friction, heat transfer and other boundary layer characteristics have been studied. The effects of electrification on the velocity and temperature are analyzed and presented graphically. Variation of Nusselt number and Skin-friction coefficient for various values of physical parameter are presented through tables. It is observed that the electrification of particles reduces the velocity and temperature gradient, leading to reduction of skin friction and heat transfer.

Key words

Two phase flows, Electrification of particle, Skin friction, Volume fraction.

1. Introduction

Jets are a common configuration used in many mixing and thrusts producing devices. The enhancement of jet flow mixing is frequently desirable in a broad range of engineering applications. Considerable literature has been published on the study of jet behavior. Bansal [8], Bansal and Tak(7) had derived approximate solutions of heat and momentum transfer in laminar plane wall jet for clear fluid. The topic of two-phase flows has, in a wide variety of engineering systems, become increasingly important for optimal design and safe operations. Gas-particle flows, dusty fluid flows and the flow of suspensions have received considerable attention due to the importance of these types of flow in various engineering applications. It is well known that many organic or metallic powders like cornstarch, coal, aluminum and magnesium are suspended in air form explosive mixtures due to huge specific surface area of fine dispersed particles. However, the particles may be aerodynamically entrained into the air flow, which can be induced by a primary gaseous explosion, propagates over the deposit layer. This leads in the formation of dust cloud and increase the air temperature. In all these applications, a basic understanding of how particles interact with the fluid flows is necessary to allow the use of computational fluid dynamics (CFD) models in the optimization and performance improvement of existing equipment and processes, the identification and solution of operating problems, the evolution of retrofit options, and the design of new equipment systems and plant, including process scale-up. Soo [10] had developed the mathematical approach to multiphase flows. A detailed derivation of the momentum equations for disperse two-phase systems was studied by Rietema and van der Akker [6]. Palani and Ganesan [5] have used the implicit finite difference method to the study the heat transfer effects and velocity profiles on the dusty-gas flow past on a semi-infinite isothermal inclined plate. Matsusake et.al (2) have studied the measurement and control of particle charging. Xie et.al. (1) have studied the electrification of homogeneous particles in contact and collision. Mishra S.K. et.al. (3) have studied two phase flow problem using non uniform grid. Panda J.P et.al (4) have studied heat transfer through viscous fluid. To date there is insufficient published information about the two-phase jet flow with charged suspended particulate matter. In the Present analysis, the particles will be allowed to diffuse through the carrier fluid i.e. the random motion of the particles shall be taken into account because of the small size of the particles. This can be done by applying the kinetic theory of gases and pressure diffusion.

As a general statement, any volume element of charge species, with charge “e” experiences an instantaneous force given by the Lorentz force law given by $\vec{f} = e\vec{E} + \vec{J} \times \vec{B}$ where $\vec{B}$ is the magnetic flux density. The current densities in corona discharge are so low that the magnetic force term $\vec{J} \times \vec{B}$ can be omitted, as this term is many orders of magnitude smaller than the coulomb term $e\vec{E}$. The ion drift motion arises from the interaction+ of ions, constantly subject to the Lorentz force with the dense neutral fluid medium. This interaction produces an effective drag force on the ions. The drag force is in equilibrium with the Lorentz force so that the ion velocity in a field $\vec{E}$ is limited to $k_m\vec{E}$, where k_m is the mobility of the ion species. The drag force on the ions has an equal and opposite reaction force acting on the neutral fluid molecules via this ion-neutral molecules interaction, the force on the ions is transmitted directly to the fluid medium, so the force on the fluid particles is also given by $\vec{f} = e\vec{E}$, Soo [12]

2. Mathematical Modeling

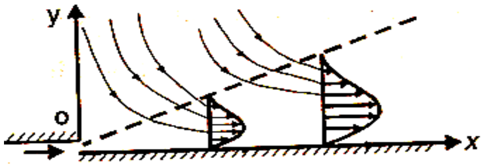


Fig. 1. Plane wall jet

Let an incompressible fluid with SPM be discharged through a narrow slit in the half space along a plane wall and mixed with the same surrounding fluid being initially at rest having temperature T_∞. The wall is also maintained at the same constant temperature T_∞.Taking the origin in the slit and the co-ordinate axis x and y along and normal to the plane wall respectively, the boundary layer equations for the continuity, momentum and energy are given by considering electrification of particles.

Introducing the non-dimensional quantities like

$$x^* = \frac{x}{L} \;,\; y^* = \frac{y}{L}\sqrt{Re} \;,\quad u^* = \frac{u}{U},\; v^* = \frac{v}{U}\sqrt{Re},\quad u_p^* = \frac{u_p}{U},\; v_p^* = \frac{v_p}{U}\sqrt{Re}\,,$$

$$\rho_p^* = \frac{\rho_p}{\rho_{p_0}} \;,\; T^* = \frac{T - T_\infty}{T_\infty} \;,\; {T_p}^* = \frac{T_p - T_\infty}{T_\infty} \tag{1}$$

And dropping stars the governing equations can be given by,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$\frac{\partial}{\partial x}\left(\rho_p u_p\right) + \frac{\partial}{\partial y}\left(\rho_p v_p\right) = 0 \tag{3}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}-\alpha\frac{1}{1-\varphi}\frac{FL}{U}\rho_p\left(u-u_p\right)+\frac{1}{1-\varphi}\alpha M\rho_p \tag{4}$$

$$u_p\frac{\partial u_p}{\partial x}+v_p\frac{\partial u_p}{\partial y}=\epsilon\frac{\partial^2 u_p}{\partial y^2}+\frac{FL}{U}\left(u-u_p\right)+M \tag{5}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{Pr}\frac{\partial^2 T}{\partial y^2}+\frac{2\,\alpha}{3\,Pr}\frac{1}{1-\varphi}\frac{FL}{U}\rho_p\left(T_p-T\right)+Ec\left(\frac{\partial u}{\partial y}\right)^2$$
$$+\frac{1}{1-\varphi}\frac{FL}{U}\alpha Ec\,\rho_p\left(u-u_p\right)^2+\frac{1}{1-\varphi}\alpha\,EcM\rho_p u_p \tag{6}$$

$$u_p\frac{\partial T_p}{\partial x}+v_p\frac{\partial T_p}{\partial y}=\frac{\epsilon}{Pr}\frac{\partial^2 T_p}{\partial y^2}-\frac{FL}{U}\left(T_p-T\right)+\frac{3}{2}Pr.\epsilon Ec\left[\left(\frac{\partial u_p}{\partial y}\right)^2+u_p\frac{\partial^2 u_p}{\partial y^2}\right]$$
$$-\frac{3}{2}\mathrm{Pr}\,Ec\frac{FL}{U}\left(u-u_p\right)^2+\frac{3}{2}\mathrm{Pr}\,Ec\,Mu_p \tag{7}$$

$$u_p\frac{\partial \rho_p}{\partial x}+v_p\frac{\partial \rho_p}{\partial y}=\epsilon\frac{\partial^2 \rho_p}{\partial y^2} \tag{8}$$

Subjected to the boundary conditions

$$y=0:\ u=0,\quad u_p=u_{p_w},\ \rho_p=\rho_{p_w},\ T=0,\ T_p=T_{p_w},$$
$$y=\infty:\ u=u_p=0,\ \rho_p=0,\ T=0,\ T_p=0 \tag{9}$$

and the integral conditions $\frac{\partial}{\partial x}\int_0^\infty\{u^2(\int_0^y udy)\}\,dy+\frac{1}{1-\varphi}\frac{FL}{U}\ \alpha\int_0^\infty\left\{u\int_y^\infty\rho_p\ (u-u_p)dy\right\}dy-$

$$\frac{1}{1-\varphi}\alpha M\int_0^\infty u(\int_y^\infty\ \rho_p dy)dy=0 \tag{10}$$

$$\frac{\partial}{\partial x}\int_0^\infty uT(\int_0^y\ u\,dy)\,dy=\frac{1}{1-\varphi}\frac{2\,\alpha}{3\,Pr}\frac{FL}{U}\int_0^\infty\left\{u\int_y^\infty\rho_p\left(T_p-T\right)dy\right\}dy$$
$$-\frac{1}{1-\varphi}\frac{FL}{U}\alpha\,Ec\int_0^\infty\left\{u\int_y^\infty\rho_p\left(u-u_p\right)^2dy\right\}dy+Ec\int_0^\infty\left\{u\int_y^\infty\left(\frac{\partial u}{\partial y}\right)^2dy\right\}dy$$
$$+\frac{1}{1-\varphi}\alpha MEc\int_0^\infty u\left(\int_y^\infty\rho_p \mathrm{u_p}\,dy\right)dy \tag{11}$$

Where u and v are velocity components of fluid phase and u_p ,v_p are velocity components of particle phase along the x and y co-ordinates respectively, ρ and ρ_p are the mass density of fluid and particle phase respectively, φ is the volume fraction of suspended particulate matter(SPM),F is the friction parameter between fluid and particle, L is the reference length , U is free stream velocity, α is the loading ratio, ϵ is the diffusion parameter , M is the electrification parameter , T and T_p are the temperature of fluid and particle phase.

3. Method of Solution

Solution for the velocity distribution

By taking a perturbation on the Schlichting [9] model by writing

$$u = u_0 + u_1,\ u_p = u_{p_0} + u_{p_1}, \quad \rho_p = \rho_{p_0} + \rho_{p_1}, \tag{12}$$

Where $u_1, u_{p_1}, \rho_{p_1}, T_1$ and T_{p_1} are perturbation quantities and substituting in equations (2) to (5), we get two sets of equations as follows.

1st set and its solution:

$$\frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial y} = 0 \tag{13}$$

$$\frac{\partial}{\partial x}\left(\rho_{p_0} u_{p_0}\right) + \frac{\partial}{\partial y}\left(\rho_{p_0} v_{p_0}\right) = 0 \tag{14}$$

$$u_0 \frac{\partial u_0}{\partial x} + v_0 \frac{\partial u_0}{\partial y} = \frac{\partial^2 u_0}{\partial y^2} - \frac{1}{1-\varphi}\frac{FL}{U}\alpha\, \rho_{p_0}\left(u_0 - u_{p_0}\right) + \frac{1}{1-\varphi}\alpha M \rho_{p_0} \tag{15}$$

$$u_{p_0} \frac{\partial u_{p_0}}{\partial x} + v_{p_0} \frac{\partial u_{p_0}}{\partial y} = \epsilon \frac{\partial^2 u_{p_0}}{\partial y^2} + \frac{FL}{U}\left(u_0 - u_{p_0}\right) + M \tag{16}$$

$$u_{p_0} \frac{\partial \rho_{p_0}}{\partial x} + v_{p_0} \frac{\partial \rho_{p_0}}{\partial y} = \epsilon \frac{\partial^2 \rho_{p_0}}{\partial y^2} \tag{17}$$

Subjected to the boundary conditions

$$y = 0:\ u_0 = 0, \quad u_{p_0} = u_{p_{wo}},\ \rho_{p_0} = \rho_{p_{wo}}$$

$$y = \infty:\ u_0 = u_{p_0} = 0,\ \rho_{p_0} = 0 \tag{18}$$

Together with the integral condition

$$\frac{\partial}{\partial x}\int_0^\infty \left\{u_0^2\left(\int_0^y (u_0)dy\right)\right\}dy + \frac{1}{1-\varphi}\frac{FL}{U}\ \alpha \int_0^\infty \left\{u_0 \int_y^\infty \rho_{p_0}\left(u_0 - u_{p_0}\right)dy\right\}dy$$

$$-\frac{1}{1-\varphi}\alpha M \int_0^\infty u_0\left(\int_y^\infty\ \rho_{p_0} dy\right)dy = 0 \tag{19}$$

Since we are considering the case of a dilute suspension of particles, following Soo [11], the velocity distribution in the fluid is not significantly affected by the presence of the particles. Therefore, the drag force term [i.e. 2nd term in the R.H.S. of equation (15)] in equation (15) is dropped. But for the submicron particles, Brownian motion can be significant, the concentration distribution equation (14) above will then be modified by Brownian diffusion equation (17). With the above consideration the equations (15) and (19) become

$$u_0 \frac{\partial u_0}{\partial x} + v_0 \frac{\partial u_0}{\partial y} = \frac{\partial^2 u_0}{\partial y^2} \tag{20}$$

$$\frac{\partial}{\partial x}\int_0^\infty \left\{u_0^2\left(\int_0^y (u_0)dy\right)\right\}dy = 0$$

$$\text{Or, } \int_0^\infty \left\{u_0^2\left(\int_0^y (u_0)dy\right)\right\}dy = A(say) \tag{21}$$

A similar solution of the equation (20) under the present boundary and integral conditions is possible if we take

$$\Psi = (Ax)^{1/4} f(\eta), \eta = \left(\frac{A}{1}\right)^{1/4} y x^{-\frac{3}{4}} \tag{22}$$

$$and\ u_0 = \frac{\partial\Psi}{\partial y} = (\frac{A}{x})^{\frac{1}{2}} f'(\eta), v_0 = -\frac{\partial\Psi}{\partial x} = \frac{1}{4}(\frac{A}{x^3})^{\frac{1}{4}}\{3\eta f'(\eta) - f(\eta)\} \qquad (23)$$

Where a prime denotes differentiation w.r.t. 'η', and the equation of continuity is satisfied identically.

Substituting in the equation (20), we get

$$4f''' + ff'' + 2\,f'^2 = 0 \qquad (24)$$

and the boundary conditions are

$$\eta = 0; f = 0, f' = 0\ ; \quad \eta = \infty : f' = o \qquad (25)$$

and integral condition

$$\int_0^\infty ff'^2\, d\eta = 1 \qquad (26)$$

Multiplying by $f(\text{Integrating factor})$ throughout and integrating the equation (24) gives,

$$4ff'' - 2f'^2 + f^2 f' = 0 \qquad (27)$$

Where the constant of integration is zero by using boundary condition (25).

The differential equation (27) can be linearized if we substitute $f' = \phi$, and considering the function f as the independent variable, we get $f'' = \phi \frac{d\phi}{df}$ and the linearized form of the equation (27) is

$$\frac{d\phi}{df} - \frac{1}{2f}\,\phi = -\frac{f}{4}\ \text{, as} \quad \phi \neq 0 \qquad (28)$$

The solution of (28) is given by

$$\phi = f' = C\sqrt{f} - \frac{1}{6}f^2 \qquad (29)$$

Where C is arbitrary constant to be determined.

Assuming at $\eta = \infty, f = f_\infty,$ then in view of boundary condition (25) we get

$$C = \frac{1}{6} f_\infty^{\frac{3}{2}} \qquad (30)$$

The value of f_∞ is yet to be determined and for this we use the integral condition (26) which may be written as

$$\int_0^{f_\infty,} ff'df = 1 \qquad (31)$$

From (31) , we get

$$\int_0^{f_\infty,} f\left(C\sqrt{f} - \frac{f^2}{6}\right) df = 1$$

Or, $f_\infty = 40^{\frac{1}{4}} = 2.515$ (32)

Now to solve the differential equation (28) we substitute

$$F=\frac{f}{f_{\infty,}} \tag{33}$$

So that it becomes $\frac{dF}{d\eta}=\frac{f_{\infty,}}{6}\left(\sqrt{F}-F^2\right)$

Solving we get

$$\eta=\frac{2}{f_{\infty,}}\left(ln\frac{1+\sqrt{F}+F}{\left(1-\sqrt{F}\right)^2}+2\sqrt{3}arctg\ \frac{\sqrt{3F}}{2+\sqrt{F}}\right) \tag{34}$$

To develop a computational algorithm with non-uniform-grid, finite difference expressions are introduced for the various terms in equations (16) and (17/14) as,

$$\frac{\partial W}{\partial x}=\frac{1.5\,W_j^{n+1}-2W_j^n+0.5\,W_j^{n-1}}{\Delta x}+o(\Delta x^2) \tag{35}$$

$$\frac{\partial W}{\partial y}=\frac{W_{j+1}^{n+1}-(1-r_y^2)W_j^{n+1}-r_y^2\,W_{j-1}^{n+1}}{r_y\left(r_y+1\right)\Delta y}+o(\Delta y^2) \tag{36}$$

$$\frac{\partial^2 W}{\partial y^2}=2\,\frac{W_{j+1}^{n+1}-(1+r_y)W_j^{n+1}+r_y\,W_{j-1}^{n+1}}{r_y\left(r_y+1\right)\Delta y^2}+o(\Delta y^2) \tag{37}$$

$$W_j^{n+1}=2W_j^n-W_j^{n-1} \tag{38}$$

$$\text{and}\quad y_{j+1}-y_j=r_y\left(y_j-y_{j-1}\right)=r_y\Delta y_j \tag{39}$$

Where W stands for either u_{p_0} or ρ_{p_0}.

Now the equations (16) and (17/14) reduced to

$$a_j^*{u_{p_0}}_{j-1}^{n+1}+b_j^*{u_{p_0}}_j^{n+1}+c_j^*{u_{p_0}}_{j+1}^{n+1}=d_j^* \tag{40}$$

Where $a_j^*=\frac{1}{\Delta x}\left[-p_{v_{p_0}}r_y-\epsilon q\right]$

$$b_j^*=\frac{1}{\Delta x}\left[1.5\left(2{u_{p0}}_j^n-{u_{p0}}_j^{n-1}\right)+p_{v_{p_0}}\left(r_y-\frac{1}{r_y}\right)+\left(1+\frac{1}{r_y}\right)\epsilon q+\frac{FL}{U}\Delta x\right]$$

$$c_j^*=\frac{1}{\Delta x}\left[\frac{1}{r_y}\left(p_{v_{p_0}}-\epsilon q\right)\right]$$

$$d_j^*=\frac{1}{\Delta x}\left[\left(2{u_{p0}}_j^n-0.5{u_{p0}}_j^{n-1}\right)\left(2{u_{p0}}_j^n-{u_{p0}}_j^{n-1}\right)+\frac{FL}{U}\,{u_0}_j^{n+1}\Delta x+M\,\Delta x\right]$$

$$p_{v_{p_0}}={v_{p_0}}_j^{n+1}\left(2{v_{p_0}}_j^n-{v_{p_0}}_j^{n-1}\right)\frac{\Delta x}{(1+r_y)\Delta y},\qquad q=\frac{2\,\Delta x}{(1+r_y)\Delta y^2}$$

and for diffusion equation (17)

$$a_j^{\blacksquare}{\rho_{p_0}}_{j-1}^{n+1}+b_j^{\blacksquare}{\rho_{p_0}}_j^{n+1}+c_j^{\blacksquare}{\rho_{p_0}}_{j+1}^{n+1}=d_j^{\blacksquare} \tag{41}$$

Where $a_j^{\blacksquare}=\left[-{v_{p0}}_j^{n+1}{r_y}^2\Delta y-2\epsilon r_y\right]$

$$b_j^{\blacksquare} = \left[\frac{1.5\, p^{\blacksquare}\, u_{p0_j}^{\ n+1}}{\Delta x} - v_{p0_j}^{\ n+1}\left(1 - r_y^2\right)\Delta y + 2\epsilon\left(1 + r_y\right)\right], \quad c_j^{\blacksquare} = \left[\, v_{p0_j}^{\ n+1}\Delta y - 2\epsilon\right]$$

$$d_j^{\blacksquare} = p^{\blacksquare}\, u_{p0_j}^{\ n+1}\left(\frac{2\rho_{p_{0_j}}^{\ n} - 0.5\rho_{p_{0_j}}^{\ n-1}}{\Delta x}\right), \qquad p^{\blacksquare} = r_y\left(1 + r_y\right)\Delta y^2$$

For continuity equation for particle phase (14)

$$a_j^{"}\rho_{p_{0_{j-1}}}^{\ n+1} + b_j^{"}\rho_{p_{0_j}}^{\ n+1} + c_j^{"}\rho_{p_{0_{j+1}}}^{\ n+1} = d_j^{"}$$

$$a_j^{"} = \frac{1}{\Delta x}\left[-p_{v_{p_0}} r_y\right]$$

$$b_j^{"} = \frac{1}{\Delta x}\left[1.5 u_{p_{0_j}}^{\ n+1} + p_{v_0}\left(r_y - \frac{1}{r_y}\right) + (DUPX + DVPY)\Delta x\right], \quad c_j^{"} = \frac{1}{\Delta x}\left[\frac{p_{v_{p_0}}}{r_y}\right]$$

$$d_j^{"} = \frac{1}{\Delta x}\left[u_{p_{0_j}}^{\ n+1}\left(2\rho_{p_{0_j}}^{\ n} - 0.5\rho_{p_{0_j}}^{\ n-1}\right)\right]$$

$$DUPX = \left(\frac{\partial u_{p_0}}{\partial x}\right)_j^{n+1} = \frac{1.5 u_{p_{0_j}}^{\ n+1} - 2\, u_{p_{0_j}}^{\ n} + 0.5 u_{p_{0_j}}^{\ n-1}}{\Delta x}$$

$$DVPY = \left(\frac{\partial v_{p_0}}{\partial y}\right)_j^{n+1} = \frac{1.5 v_{p_{0_j}}^{\ n+1} - 2\, v_{p_{0_j}}^{\ n} + 0.5 v_{p_{0_j}}^{\ n-1}}{\Delta x}$$

$$p_{v_{p_0}} = \frac{v_{p_{0_j}}^{\ n+1}\Delta x}{(r_y+1)\Delta y}, \quad q = \frac{2\,\Delta x}{(r_y+1)\Delta y^2}$$

2nd set and its solution:

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0 \tag{42}$$

$$u_{p_1}\frac{\partial \rho_{p_0}}{\partial x} + \rho_{p_0}\frac{\partial u_{p_1}}{\partial x} + u_{p_0}\frac{\partial \rho_{p_1}}{\partial x} + \rho_{p_1}\frac{\partial u_{p_0}}{\partial x} + v_{p_1}\frac{\partial \rho_{p_0}}{\partial y}$$
$$+ \rho_{p_0}\frac{\partial v_{p_1}}{\partial y} + v_{p_0}\frac{\partial \rho_{p_1}}{\partial y} + \rho_{p_1}\frac{\partial v_{p_0}}{\partial y} = 0 \tag{43}$$

$$u_0\frac{\partial u_1}{\partial x} + u_1\frac{\partial u_0}{\partial x} + v_0\frac{\partial u_1}{\partial y} + v_1\frac{\partial u_0}{\partial y}$$
$$= \frac{\partial^2 u_1}{\partial y^2} - \frac{1}{1-\varphi}\frac{FL}{U}\alpha\,\rho_{p_1}\left(u_0 - u_{p_0}\right) - \frac{1}{1-\varphi}\frac{FL}{U}\alpha\,\rho_{p_0}\left(u_1 - u_{p_1}\right) + \frac{1}{1-\varphi}\alpha M\rho_{p_1} \tag{44}$$

$$u_{p_0}\frac{\partial u_{p_1}}{\partial x} + u_{p_1}\frac{\partial u_{p_0}}{\partial x} + v_{p_0}\frac{\partial u_{p_1}}{\partial y} + v_{p_1}\frac{\partial u_{p_0}}{\partial y} = \epsilon\frac{\partial^2 u_{p_1}}{\partial y^2} + \frac{FL}{U}\left(u_1 - u_{p_1}\right) \tag{45}$$

$$u_{p_0}\frac{\partial \rho_{p_1}}{\partial x} + u_{p_1}\frac{\partial \rho_{p_0}}{\partial x} + v_{p_0}\frac{\partial \rho_{p_1}}{\partial y} + v_{p_1}\frac{\partial \rho_{p_0}}{\partial y} = \epsilon\frac{\partial^2 \rho_{p_1}}{\partial y^2} \tag{46}$$

Subjected to the boundary condition

$$y = 0:\ u_1 = 0, \quad u_{p_1} = u_{p_{w1}},\ \rho_{p_0} = \rho_{p_{w1}}$$

$$y = \infty:\ u_1 = u_{p_1} = 0,\ \rho_{p_1} = 0 \tag{47}$$

and the integral condition

$$\frac{\partial}{\partial x}\int_0^\infty\{u_0^2(\int_0^y(u_1)dy)\}\,dy+\frac{\partial}{\partial x}\int_0^\infty\{2u_0u_1(\int_0^y(u_1)dy)\}\,dy$$

$$+\frac{1}{1-\varphi}\frac{FL}{U}\alpha\begin{bmatrix}\int_0^\infty\left\{u_0\int_y^\infty\rho_{p_0}(u_1-u_{p_1})dy\right\}dy\\+\int_0^\infty\left\{u_0\int_y^\infty\rho_{p_1}(u_0-u_{p_0})dy\right\}dy\\+\int_0^\infty\left\{u_1\int_y^\infty\rho_{p_0}(u_0-u_{p_0})dy\right\}dy\end{bmatrix}-\frac{1}{1-\varphi}\alpha M\int_0^\infty u_1(\int_y^\infty\rho_{p_0}dy)dy-$$

$$\frac{1}{1-\varphi}\alpha M\int_0^\infty u_0(\int_y^\infty\rho_{p_1}dy)dy \qquad = 0 \quad (48)$$

is identically satisfied

Using equations (35) to (39) in (42) and (44) to (46), we get

$$v_{1j}^{n+1}=v_{1j-1}^{n+1}-0.5\frac{\Delta y}{\Delta x}\begin{bmatrix}\left(1.5\,u_{1j}^{n+1}-2\,u_{1j}^{n}+0.5\,u_{1j}^{n-1}\right)+\\\left(1.5\,u_{1j-1}^{n+1}-2\,u_{1j-1}^{n}+0.5\,u_{1j-1}^{n-1}\right)\end{bmatrix} \quad (49)$$

$$a_ju_{1_{j-1}}^{n+1}+b_ju_{1_j}^{n+1}+c_ju_{1_{j+1}}^{n+1}=d_j \quad (50)$$

$$a_j^{***}u_{p_{1_{j-1}}}^{n+1}+b_j^{***}u_{p_{1_j}}^{n+1}+c_j^{***}u_{p_{1_{j+1}}}^{n+1}=d_j^{***} \quad (51)$$

$$a_j^{**}\rho_{p_{1_{j-1}}}^{n+1}+b_j^{**}\rho_{p_{1_j}}^{n+1}+c_j^{**}\rho_{p_{1_{j+1}}}^{n+1}=d_j^{**} \quad (52)$$

Where $a_j=\frac{1}{\Delta x}[-p_{v_0}r_y-q]$

$$b_j=\frac{1}{\Delta x}\left[1.5u_{0_j}^{n+1}+p_{v_0}\left(r_y-\frac{1}{r_y}\right)+q\left(1+\frac{1}{r_y}\right)+\frac{1}{1-\varphi}\frac{FL}{U}\alpha\,\rho_{p0_j}^{n+1}\Delta x+\left(\frac{\partial u_0}{\partial x}\right)_j^{n+1}\Delta x\right]$$

$$c_j=\frac{1}{\Delta x}\left[\frac{1}{r_y}\left(p_{v_0}-q\right)\right]$$

$$d_j=\frac{1}{\Delta x}\begin{bmatrix}u_{0_j}^{n+1}\left(2u_{1_j}^{n}-0.5u_{1_j}^{n-1}\right)-\left(2v_{1_j}^{n}-v_{1_j}^{n-1}\right)\left(\frac{\partial u_0}{\partial y}\right)_j^{n+1}\Delta x\\-\frac{1}{1-\varphi}\frac{FL}{U}\alpha\Delta x\begin{Bmatrix}\left(2\rho_{p1_j}^{n}-\rho_{p1_j}^{n-1}\right)\left(u_{0_j}^{n+1}-u_{p_{0_j}}^{n+1}\right)\\-\rho_{p_{0_j}}^{n+1}\left(2u_{p1_j}^{n}-u_{p1_j}^{n-1}\right)\end{Bmatrix}+\\\frac{\alpha}{1-\varphi}M\left(2\rho_{p1_j}^{n}-\rho_{p1_j}^{n-1}\right)\Delta x\end{bmatrix}$$

$$p_{v_0}=v_{0_j}^{n+1}\frac{\Delta x}{(1+r_y)\Delta y}, \qquad q=\frac{2\Delta x}{(1+r_y)\Delta y^2}$$

$$a_j^{***}=\frac{1}{\Delta x}\left[-p_{v_{p_0}}r_y-\epsilon q\right]$$

$$b_j^{***}=\frac{1}{\Delta x}\begin{bmatrix}1.5u_{p_{0_j}}^{n+1}+DUPX.\Delta x+v_{p_{0_j}}^{n+1}\frac{\Delta x}{(1+r_y)\Delta y}\left(r_y-\frac{1}{r_y}\right)\\+\epsilon\frac{2\Delta x}{(1+r_y)\Delta y^2}\left(1+\frac{1}{r_y}\right)+\frac{FL}{U}\Delta x\end{bmatrix}$$

$$c_j^{***}=\frac{1}{\Delta x}\left[\frac{1}{r_y}\left(p_{v_{p_0}}-\epsilon q\right)\right]$$

$$d_j^{***} = \frac{1}{\Delta x}\left[u_{p_{0j}}^{\ n+1}\left(2u_{p_{1j}}^{\ n} - 0.5u_{p_{1j}}^{\ n-1}\right) - S_{v_{p_1}} DUPY.\Delta x + \frac{FL}{U} u_{1j}^{\ n+1}\Delta x\right]$$

$$S_{v_{p_1}} = 2v_{p_{1j}}^{\ n} - v_{p_{1j}}^{\ n-1}, \ p_{v_{p_0}} = v_{p_{0j}}^{\ n+1}\frac{\Delta x}{(1+r_y)\Delta y}, \quad q = \frac{2\Delta x}{(1+r_y)\Delta y^2}$$

$$DUPX = \left(\frac{\partial u_{p_0}}{\partial x}\right)_j^{n+1}\left[\frac{1.5u_{p_0j}^{\ n+1} - 2u_{p_0j}^{\ n} + 0.5u_{p_0j}^{\ n-1}}{\Delta x}\right],$$

$$DUPY = \left(\frac{\partial u_{p_0}}{\partial y}\right)_j^{n+1} = \left[\frac{u_{p_0j+1}^{\ n+1} - (1-r_y^2)u_{p_0j}^{\ n+1} - r_y^2 u_{p_0j-1}^{\ n+1}}{r_y(1+r_y)\Delta y}\right]$$

$$a_j^{**} = \frac{1}{\Delta x}\left[-r_y p_{v_0} - \epsilon q\right] \qquad b_j^{**} = \frac{1}{\Delta x}\left[1.5u_{p0_j}^{\ n+1} + p_{v0}\left(r_y - \frac{1}{r_y}\right) + \epsilon q\left(1 + \frac{1}{r_y}\right)\right],$$

$$c_j^{**} = \frac{1}{\Delta x}\left[\frac{1}{r_y}\left(p_{v_{p_0}} - \epsilon q\right)\right] \qquad d_j^{**} = \frac{1}{\Delta x}\left[\begin{array}{c}\left(2\rho_{p1_j}^{\ n} - 0.5\rho_{p1_{0j}}^{\ n-1}\right)u_{p_{0j}}^{\ n+1} \\ -u_{p_{1j}}^{\ n+1}\left(\frac{\partial \rho_{p_0}}{\partial x}\right)_j^{n+1}\Delta x - v_{1j}^{\ n+1}\left(\frac{\partial \rho_{p_0}}{\partial y}\right)_j^{n+1}\Delta x\end{array}\right]$$

Solution for the temperature distribution

By taking a perturbation on the Schlichting [9] model by writing

$$u = u_0 + u_1, v = v_0 + v_1, u_p = u_{p_0} + u_{p_1}, \ v_p = v_{p_0} + v_{p_1}, \rho_p = \rho_{p_0} + \rho_{p_1},$$

$$T = T_0 + T_1, \ T_p = T_{p_0} + T_{p_1}$$

Where $u_1, v_1, u_{p_1}, v_{p_1}, \rho_{p_1}$, T_1 and T_{p_1} are perturbation quantities,

and substituting in equations (6) and (7) we get the following two sets of equations.

Set-1 and its solution

$$u_0\frac{\partial T_0}{\partial x} + v_0\frac{\partial T_0}{\partial y} = \frac{1}{Pr}\frac{\partial^2 T_0}{\partial y^2} - \frac{2}{3}\frac{1}{1-\varphi}\frac{FL}{U}\frac{\alpha}{Pr}\rho_{p_0}\left(T_0 - T_{p_0}\right) + Ec\left(\frac{\partial u_0}{\partial y}\right)^2$$
$$+\frac{1}{1-\varphi}\frac{FL}{U}\alpha\, Ec\,\rho_{p_0}\left(u_0 - u_{p_0}\right)^2 + \frac{1}{1-\varphi}\alpha\, EcM\rho_{p_0}u_{p_0} \tag{53}$$

$$u_{p_0}\frac{\partial T_{p_0}}{\partial x} + v_{p_0}\frac{\partial T_{p_0}}{\partial y} = \frac{\epsilon}{Pr}\frac{\partial^2 T_{p_0}}{\partial y^2} + \frac{FL}{U}\left(T_0 - T_{p_0}\right) - \frac{3}{2}\frac{FL}{U}PrEc\left(u_0 - u_{p_0}\right)^2$$
$$+\frac{3}{2}Pr.Ec\,\epsilon\left[u_{p_0}\frac{\partial^2 u_{p_0}}{\partial y^2} + \left(\frac{\partial u_{p_0}}{\partial y}\right)^2\right] + \frac{3}{2}Pr\,Ec\,Mu_{p_0} \tag{54}$$

Subjected to the boundary condition

$$y = 0: \ T_0 = 0, \ \ T_{p_0} = T_{p_{wo}}; \ y = \infty: \ T_0 = 0, \ T_{p_0} = 0 \tag{55}$$

and the integral condition

$$\frac{\partial}{\partial x}\int_0^\infty\left\{u_0 T_0\int_0^y u_0\, dy\right\}dy = \frac{1}{1-\varphi}\frac{2\,\alpha}{3\,Pr}\frac{FL}{U}\int_0^\infty\left\{u_0\int_y^\infty \rho_{p_0}\left(T_{p_0} - T_0\right)dy\right\}dy$$

$$+\frac{1}{1-\varphi}\frac{FL}{U}\alpha\, Ec\int_0^\infty\left\{u_0\int_y^\infty \rho_{p_0}\left(u_0 - u_{p_0}\right)^2 dy\right\}dy + \ Ec\int_0^\infty\left\{u_0\int_y^\infty\left(\frac{\partial u_0}{\partial y}\right)^2 dy\right\}dy$$

$$+\frac{1}{1-\varphi}\alpha M Ec\int_0^\infty u_0\left(\int_y^\infty \rho_{p_0}u_{p_0}\, dy\right)dy \tag{56}$$

Since we are considering the case of a dilute suspension of particles, following Soo [11], the temperature distribution in the fluid is not significantly affected by the presence of the particles. Therefore, the 2nd, 4th and 5th term in the R.H.S. of equation (53), and 1st and 2nd term of R.H.S. in equation (56) are dropped.

Hence equation (53) and (56) reduces to,

$$u_0\frac{\partial T_0}{\partial x}+v_0\frac{\partial T_0}{\partial y}=\frac{1}{Pr}\frac{\partial^2 T_0}{\partial y^2}+Ec\left(\frac{\partial u_0}{\partial y}\right)^2 \tag{57}$$

And $\frac{\partial}{\partial x}\int_0^\infty\{u_0T_0\int_0^y u_0\, dy\}dy = Ec\int_0^\infty\left\{u_0\int_y^\infty\left(\frac{\partial u_0}{\partial y}\right)^2 dy\right\}dy$ (58)

The equation (57) is a linear differential equation in T_0. So we can solve the equation by the principle of superposition of solutions T_{00} and T_{01} such that T_{00} is the solution of the equation

$$u_0\frac{\partial T_{00}}{\partial x}+v_0\frac{\partial T_{00}}{\partial y}=\frac{1}{Pr}\frac{\partial^2 T_{00}}{\partial y^2} \tag{59}$$

And T_{01} is the solution of the equation

$$u_0\frac{\partial T_{01}}{\partial x}+v_0\frac{\partial T_{01}}{\partial y}=\frac{1}{Pr}\frac{\partial^2 T_{01}}{\partial y^2}+Ec\left(\frac{\partial u_0}{\partial y}\right)^2 \tag{60}$$

So that $T_0 = T_{00}+T_{01}$ (61)

T_{00} and T_{01} satisfies the boundary and the integral conditions are given by

$$y=0: T_{00}=0,\ T_{01}=0\ ;\ \ y=\infty: T_{00}=0,\ T_{01}=0 \tag{62}$$

And $\int_0^\infty T_{00}u_0(\int_0^y u_0 dy)\, dy = const = I\ (say)$ (63)

Furthermore, $\frac{\partial}{\partial x}\int_0^\infty T_{01}u_0(\int_0^y u_0 dy)\, dy = Ec\int_0^\infty\left\{u_0\int_y^\infty\left(\frac{\partial u_0}{\partial y}\right)^2 dy\right\}dy$ (64)

is identically satisfied.

It implies the constancy of the product of volume and heat flux, for a given prandtl number, through any cross-section of the boundary layer perpendicular to the wall.

For arbitrary value of Pr, it is assumed that

$$T_{00} = \frac{l}{E}\left(\frac{A}{x}\right)^{1/2} h(\eta) \tag{65}$$

Substituting u_0 and v_0 from (23) and T_{00} from (65) in equation (59), the function $h(\eta)$ satisfies the differential equation

$$4h'' + Pr(fh' + 2f'h) = 0 \tag{66}$$

With the boundary conditions

$$\eta=0;\ \ h=0; \qquad \eta \to \infty, h = 0 \tag{67}$$

and the Integral condition $\int_0^\infty hff'\, d\eta = 1$ (68)

Introducing the transformation $s = [F(n)]^{3/2}$ and $H(s) = \frac{2}{3} f_\infty^2\, h(\eta)$ in the equation (66).

The transformed equation is

$$s(1-s)\frac{d^2H}{ds^2} + \left\{\frac{2}{3} - \left(\frac{2}{3} - Pr + 1\right)s\right\}\frac{dH}{ds} + \frac{4}{3}\ PrH = 0 \tag{69}$$

With the boundary condition $s = 0: H = 0\ ;\ s \to 1: H = 0$ (70)

and the integral condition $\int_0^1 Hs^{1/3}\ ds = 1$ (71)

Equation (69) is a hyper-geometric equation, whose solution is given by

$$H(s) = A\ \ {}_2F_1(a,b,c;s) + +Bs^{1/3}\ \ {}_2F_1(a-c+1, b-c+1; 2-c; s) \tag{72}$$

Where $a + b = \frac{2}{3} - Pr, ab = -\frac{4}{3}\mathrm{Pr},$ and $c = \frac{2}{3}$ (73)

And ${}_2F_1$ (a,b,c;s)= $\sum_{r=0}^{\infty} \frac{a_r b_r}{C_r} \cdot \frac{s^r}{r!} = \frac{a_0 b_0}{c_0}\frac{s^0}{0!} + \frac{a_1 b_1}{c_1}\frac{s^1}{1!} + \cdots \ldots \ldots \ldots \ldots \ldots \ldots$

Since the Prandtl number Pr of a fluid is always positive integer so the series is absolutely converges.

Now by boundary condition (62), for $s = 0$, A= 0 (74)

and H(s)=$Bs^{1/3}$ ${}_2F_1(a-c+1, b-c+1; 2-c; s)$ (75)

In (67) 'B' is still an unknown constant which will be determined by the integral condition (63) which is given by

$$B = \frac{5}{3}\left\{ \;{}_3F_2\left(a-c+1, b-c+1, \frac{5}{3}, 2-c, \frac{8}{3}; 1\right)\right\}^{-1} \tag{76}$$

The solution of the equation (60) can be obtained by using finite difference technique as follow.

$$a_j^{\oplus} T_{01_{j-1}}^{n+1} + b_j^{\oplus} T_{01_j}^{n+1} + c_j^{\oplus} T_{01_{j+1}}^{n+1} = d_j^{+} \tag{77}$$

Where $a_j^{\oplus} = \frac{-r_y v_{0_j}^{n+1}}{(1+r_y)\Delta y} - \frac{2}{Pr(1+r_y)\Delta y^2} = \frac{1}{\Delta x}\left[-p_T r_y - \frac{1}{Pr} q\right]$

$b_j^{\oplus} = = \frac{1}{\Delta x}\left[1.5 u_{0_j}^{n+1} + p_T\left(r_y - \frac{1}{r_y}\right) + \frac{1}{Pr} q\left(1 + \frac{1}{r_y}\right)\right]$, $\quad c_j^{\oplus} == \frac{1}{\Delta x}\left[\frac{1}{r_y}\left(p_T - \frac{1}{Pr} q\right)\right]$

$d_j^{\oplus} = \frac{1}{\Delta x}\left[Ec\left(\frac{u_{0_{j+1}}^{n+1} - u_{0_j}^{n+1}}{\Delta y}\right)^2 \Delta x + u_{0_j}^{n+1}\left(2T_{01_j}^{n} - 0.5T_{01_j}^{n-1}\right)\right]$

$p_T = \frac{v_{0_j}^{n+1}\Delta x}{(1+r_y)\Delta y}$ and $q = \frac{2\,\Delta x}{(1+r_y)\Delta y^2}$

The solution of the equation (54) is obtained by using finite difference technique.

Using equations (35) to (39) in equation (54), we get

$$a_j^{\blacksquare\blacksquare} T_{p0_{j-1}}^{n+1} + b_j^{\blacksquare\blacksquare} T_{p0_j}^{n+1} + c_j^{\blacksquare\blacksquare} T_{p0_{j+1}}^{n+1} = d_j^{\blacksquare\blacksquare} \tag{78}$$

Where $a_j^{\blacksquare\blacksquare} = \frac{1}{\Delta x}\left[-r_y\, p_{v0} - \frac{\epsilon}{Pr} q\right]$

$b_j^{\blacksquare\blacksquare} = \frac{1}{\Delta x}\left[1.5\left(u_{p0_j}^{n+1}\right) + \left(r_y - \frac{1}{r_y}\right)p_{v0} + \left(1 + \frac{1}{r_y}\right)\frac{\epsilon q}{Pr} + \frac{FL}{U}\Delta x\right]$, $c_j^{\blacksquare\blacksquare} = \frac{1}{\Delta x}\left[\frac{1}{r_y}\left(p_{v0} - \frac{\epsilon q}{Pr}\right)\right]$

$$d_j^{\blacksquare\blacksquare} = \frac{1}{\Delta x}\left[\begin{array}{c} \left(2T_{p0_j}^{n} - 0.5T_{p0_j}^{n-1}\right)\left(u_{p0_j}^{n+1}\right) + \frac{FL}{U} T_{0_j}^{n+1}\Delta x \\ -\frac{3}{2}\frac{FL}{U} Pr Ec\left(u_{0_j}^{n+1} - u_{p0_j}^{n+1}\right)^2 \Delta x \\ +\frac{3}{2} Pr Ec\,.\,\epsilon\left\{\left(\frac{\partial u_{p0}}{\partial y}\right)^2 + u_{p0}\frac{\partial^2 u_{p0}}{\partial y^2}\right\}\Delta x \\ \frac{3}{2} Pr Ec M u_{p0_j}^{n+1}\Delta x \end{array}\right]$$

2nd set and its solution:

$$u_0\frac{\partial T_1}{\partial x} + u_1\frac{\partial T_0}{\partial x} + v_0\frac{\partial T_1}{\partial y} + v_1\frac{\partial T_0}{\partial y} = \frac{1}{Pr}\frac{\partial^2 T}{\partial y^2}$$
$$+\frac{2}{3}\frac{\alpha}{1-\varphi}\frac{1}{Pr}\frac{FL}{U}\left\{\rho_{p0}\left(T_{p_1} - T_1\right) + \rho_{p1}\left(T_{p_0} - T_0\right)\right\} + 2Ec\frac{\partial u_0}{\partial y}\frac{\partial u_1}{\partial y}$$

$$+\frac{\alpha}{1-\varphi}\frac{FL}{U}Ec\ \left\{\rho_{p_1}\left(u_0-u_{p_0}\right)^2+2\rho_{p_0}\left(u_0-u_{p_0}\right)\left(u_1-u_{p_1}\right)\right\}$$

$$+\frac{1}{1-\varphi}\alpha\ EcM\rho_{p_0}u_{p_1}+\frac{1}{1-\varphi}\alpha\ EcM\rho_{p_1}u_{p_0} \tag{79}$$

$$u_{p_0}\frac{\partial T_{p_1}}{\partial x}+u_{p_1}\frac{\partial T_{p_0}}{\partial x}+v_{p0}\frac{\partial T_{p_1}}{\partial y}+v_{p_1}\frac{\partial T_{p_0}}{\partial y}=\frac{\epsilon}{Pr}\frac{\partial^2 T_{p_1}}{\partial y^2}-\frac{FL}{U}\left(T_{p_1}-T_1\right)$$

$$+\frac{3}{2}\mathrm{Pr}\,Ec\,\epsilon.\left(u_{p_0}\frac{\partial^2 u_{p_1}}{\partial y^2}+u_{p_1}\frac{\partial^2 u_{p_0}}{\partial y^2}+2\frac{\partial u_{p_0}}{\partial y}\frac{\partial u_{p_1}}{\partial y}\right)$$

$$-3\frac{FL}{U}\ PrEc\left(u_0-u_{p_0}\right)\left(u_1-u_{p_1}\right)+\frac{3}{2}\mathrm{Pr}\,Ec\,Mu_{p_1} \tag{80}$$

Using equations (35) to (39) in equation (79) and (80), we get

$$a_j^{+}T_{1_{j-1}}^{n+1}+b_j^{+}T_{1_j}^{n+1}+c_j^{+}T_{1_{j+1}}^{n+1}=\ d_j^{+} \tag{81}$$

$$a_j^{++}T_{p1_{j-1}}^{n+1}+b_j^{++}T_{p1_j}^{n+1}+c_j^{++}T_{p1_{j+1}}^{n+1}=\ d_j^{++} \tag{82}$$

Where $a_j^{+}=\frac{1}{\Delta x}\left[-p_{v_0}\,r_y-\frac{q}{Pr}\right]$

$$b_j^{+}=\frac{1}{\Delta x}\begin{bmatrix}1.5u_{0_j}^{n+1}+p_{v_0}\left(r_y-\frac{1}{r_y}\right)\\ +\left(1+\frac{1}{r_y}\right)\frac{q}{Pr}+\frac{2}{3}\frac{\alpha}{1-\varphi}\frac{1}{Pr}\frac{FL}{U}\rho_{p0_j}^{n+1}\Delta x\end{bmatrix}$$

$$c_j^{+}=\frac{1}{\Delta x}\left[\frac{1}{r_y}\left(p_{v_0}-\frac{q}{Pr}\right)\right]$$

$$d_j^{+}=\frac{1}{\Delta x}\begin{bmatrix}\left(2T_{1_j}^{n}-0.5T_{1_j}^{n-1}\right)u_{0_j}^{n+1}-u_{1_j}^{n+1}DTX\Delta x-S_{v_1}DTY\Delta x+\\ \frac{2}{3}\frac{\alpha}{1-\varphi}\frac{1}{Pr}\frac{FL}{U}\left\{\rho_{p_{0j}}^{n+1}S_{T_{p_1}}+S_{\rho_{p_1}}\left(T_{p_{0j}}^{n+1}-T_{0_j}^{n+1}\right)\right\}\Delta x\\ +\frac{1}{1-\varphi}\frac{FL}{U}\alpha\ Ec\begin{Bmatrix}S_{\rho_{p_1}}\left(u_{0_j}^{n+1}-u_{p_{0j}}^{n+1}\right)^2\\ +2\rho_{p_{0j}}^{n+1}\left(u_{0_j}^{n+1}-u_{p_{0j}}^{n+1}\right)\left(u_{1_j}^{n+1}-S_{u_{p_1}}\right)\end{Bmatrix}\Delta x\\ +2Ec\ DUY.DUY1\Delta x+\frac{1}{1-\varphi}\ \alpha\ EcM\left(\rho_{p_{0j}}^{n+1}S_{u_{p_1}}+S_{\rho_{p_1}}\ u_{p_{0j}}^{n+1}\right)\Delta x\end{bmatrix}$$

$S_{v_1}=2v_{1_j}^{n}-v_{1_j}^{n-1}$, $S_{u_{p_1}}=2u_{p_{1j}}^{n}-u_{p_{1j}}^{n-1}$, $S_{T_{p_1}}=2T_{p_{1j}}^{n}-T_{p_{1j}}^{n-1}$,

$S_{\rho_{p_1}}=2\rho_{p_{1j}}^{n}-\rho_{p_{1j}}^{n-1}$

$$DTX=\left(\frac{\partial T_0}{\partial x}\right)_j^{n+1}=\left[\frac{1.5T_{0_j}^{n+1}-2T_{0_j}^{n}+0.5T_{0_j}^{n-1}}{\Delta x}\right],\quad DTY=\left(\frac{\partial T_0}{\partial y}\right)_j^{n+1}=\left[\frac{T_{0_{j+1}}^{n+1}-\left(1-r_y^2\right)T_{0_j}^{n+1}-r_y^{\,2}T_{0_{j-1}}^{n+1}}{r_y\left(1+r_y\right)\Delta y}\right]$$

$$DUY = \left(\frac{\partial u_0}{\partial y}\right)_j^{n+1} = \left[\frac{u_{0\,j+1}^{\;n+1} - (1-r_y^2)u_{0\,j}^{\;n+1} - r_y{}^2 u_{0\,j-1}^{\;n+1}}{r_y(1+r_y)\Delta y}\right] ,$$

$$DUY1 = \left(\frac{\partial u_1}{\partial y}\right)_j^{n+1} = \left[\frac{u_{1\,j+1}^{\;n+1} - (1-r_y^2)u_{1\,j}^{\;n+1} - r_y{}^2 u_{1\,j-1}^{\;n+1}}{r_y(1+r_y)\Delta y}\right]$$

And $a_j^{++} = \frac{1}{\Delta x}\left[-r_y p_{v0} - \frac{\epsilon q}{Pr}\right]$

$$b_j^{++} = \frac{1}{\Delta x}\left[1.5(u_{p0})_j^{n+1} + p_{v0}\left(r_y - \frac{1}{r_y}\right) + \left(1 + \frac{1}{r_y}\right)\frac{\epsilon}{Pr} + \frac{FL}{U}\Delta x\right] ,$$

$$c_j^{++} = \frac{1}{\Delta x}\left[\frac{1}{r_y}\left(p_{v0} - \frac{q}{Pr}\right)\right]$$

$$d_j^{++} = \frac{1}{\Delta x}\left[\begin{array}{c} u_{p_{0j}}^{\;n+1}\left(2T_{p_{1j}}^{\;n} - 0.5T_{p_{1j}}^{\;n-1}\right) - u_{p_{1j}}^{\;n+1}\left(\frac{\partial T_{p_0}}{\partial x}\right)_j^{n+1}\Delta x - v_{p_{1j}}^{\;n+1}\left(\frac{\partial T_{p_0}}{\partial y}\right)_j^{n+1}\Delta x + \frac{FL}{U}T_{1j}^{\;n+1}\Delta x \\ +\frac{3}{2}\Pr Ec\,\epsilon\Delta x\left\{u_{p_{0j}}^{\;n+1}\left(\frac{\partial^2 u_{p_1}}{\partial y^2}\right)_j^{n+1} + u_{p_{1j}}^{\;n+1}\left(\frac{\partial^2 u_{p_0}}{\partial y^2}\right)_j^{n+1} + 2\left(\frac{\partial u_{p_0}}{\partial y}\right)_j^{n+1}\left(\frac{\partial u_{p_1}}{\partial y}\right)_j^{n+1}\right\} \\ -3\frac{FL}{U}\,PrEc\left(u_{0j}^{\;n+1} - u_{p_{0j}}^{\;n+1}\right)\left(u_{1j}^{\;n+1} - u_{p_{1j}}^{\;n+1}\right)\Delta x + \frac{3}{2}\Pr EcMu_{p_{1j}}^{\;n+1}\Delta x \end{array}\right]$$

4. Discussion of the Results

We choose the following parameters involved

$\rho = 0.913\ kg/m^3$; $\rho_p = 8010kg/m^3$; $\alpha = 0.1$; $D = 50\mu m, 100\mu m$; $U = 0.45m/sec$,

$L = 0.044\ m;$

$Ec = 0.1$; $Pr = 0.71, 1.0, 7.0$; $\mu = 22.26 \times 10^{-6}$ kg/m sec; $\nu = 2.43 \times 10^{-5}\ m^2/sec$

Fig -2 Shows the perturbed velocity (u_1) distribution against y for different value of M. The figure is Blasius type near the plate. The magnitude of u_1 increases with the increase of M.

Fig- 3(a) and 3(b) shows the perturbed velocity profile u_1 without and with electrification of particles respectively. In both the cases the velocity distribution near the plate is of Blasius type and away from the plate it resembles with the distribution of plane free jet. It is observed that the numerical value of u_1 is greater in case of electrification of particles.

Fig-4 and 5(a), 5 (b) show the temperature distribution T for initial and viscous heating with and without charged SPM respectively. In all the cases the distribution near the plate is of Blasius type and of free jet type away from the plate. Further, the numerical value of T1 with electrification is less than that of the value without electrification.

FIGURES:

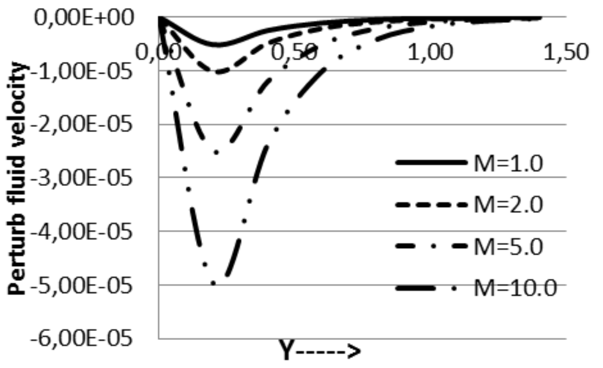


Fig. 2:Variation of Perturb fluid velocity

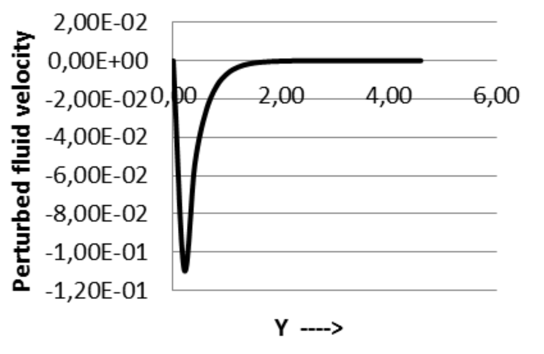


Fig. 3(a): Perturbed fluid velocity without electrification

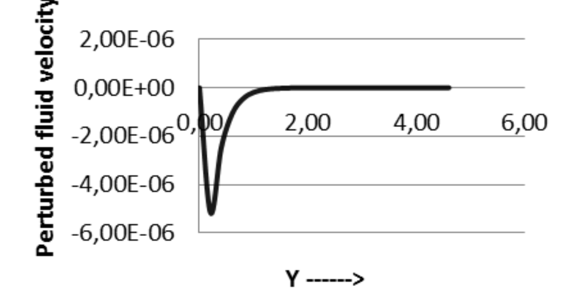


Fig.3(b):Perturbed fluid velocity with electrification

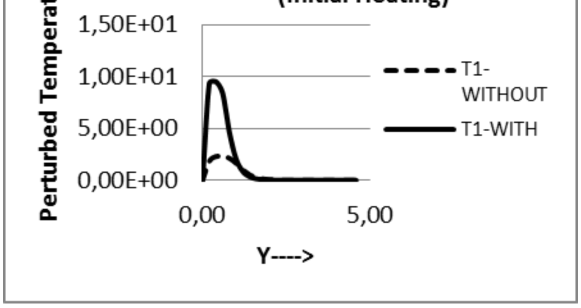


Fig.4:Perturbed Temperature with & without electrification (Initial Heating)

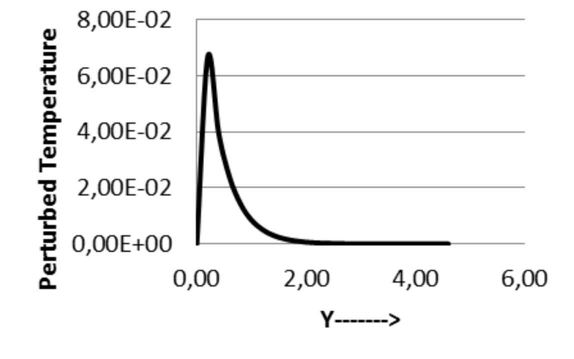


Fig-5(a):Perturbed Temperature without electrification(Viscous heating)

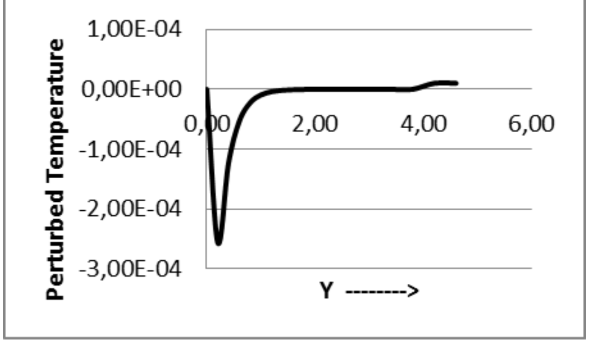
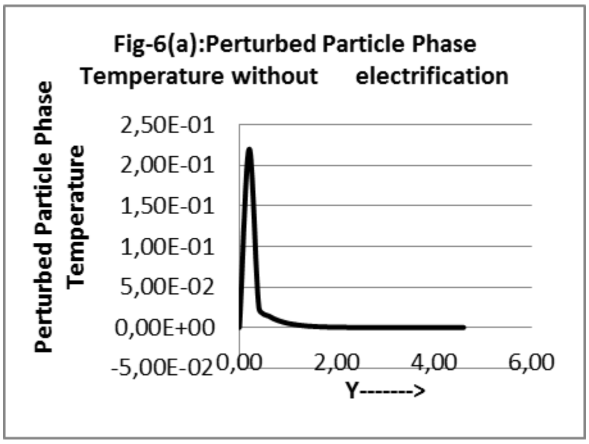
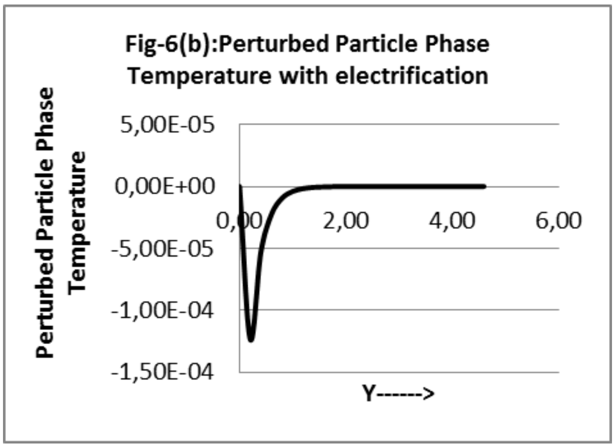


Fig-5(b):Perturbed Temperature with electrification (Viscous heating)





TABLES:

Table 1. Variation of Nu_0 with x for different Prandtl Number (Pr)

x	Initial Heating			Viscous Heating		
	$Pr=0.71$	$Pr=1.0$	$Pr=7.0$	$Pr=0.71$	$Pr=1.0$	$Pr=7.0$
1.20	-4.56E+01	-6.38E+01	-1.90E+03	-4.56E+01	-6.38E+01	-1.90E+03
1.40	-4.22E+01	-5.91E+01	-1.76E+03	-4.22E+01	-5.91E+01	-1.76E+03
1.60	-3.95E+01	-5.52E+01	-1.65E+03	-3.95E+01	-5.52E+01	-1.65E+03
1.80	-3.72E+01	-5.21E+01	-1.55E+03	-3.72E+01	-5.21E+01	-1.55E+03
2.00	-3.53E+01	-4.94E+01	-1.47E+03	-3.53E+01	-4.94E+01	-1.47E+03
2.40	-3.23E+01	-4.51E+01	-1.35E+03	-3.23E+01	-4.51E+01	-1.35E+03
2.80	-2.99E+01	-4.18E+01	-1.25E+03	-2.99E+01	-4.18E+01	-1.25E+03
3.20	-2.79E+01	-3.91E+01	-1.16E+03	-2.79E+01	-3.91E+01	-1.16E+03
3.60	-2.63E+01	-3.68E+01	-1.10E+03	-2.63E+01	-3.68E+01	-1.10E+03
4.00	-2.50E+01	-3.49E+01	-1.04E+03	-2.50E+01	-3.49E+01	-1.04E+03
4.40	-2.38E+01	-3.33E+01	-9.93E+02	-2.38E+01	-3.33E+01	-9.93E+02
4.80	-2.28E+01	-3.19E+01	-9.51E+02	-2.28E+01	-3.19E+01	-9.51E+02
5.00	-2.23E+01	-3.13E+01	-9.32E+02	-2.23E+01	-3.13E+01	-9.32E+02

Table 2. Variation of $\boldsymbol{Nu_1}$ with $\boldsymbol{x}$ for different Prandtl number $(\boldsymbol{Pr})$

x	Initial Heating			Viscous Heating		
	$Pr=0.71$	$Pr=1.0$	$Pr=7.0$	$Pr=0.71$	$Pr=1.0$	$Pr=7.0$
1.20	3.00E+00	3.03E+00	3.69E+00	-1.19E+08	-1.20E+08	-1.54E+08
1.40	9.80E+03	8.54E+03	3.38E+03	-2.60E+07	-2.63E+07	-3.29E+07

1.60	5.41E+03	4.75E+03	1.75E+03	-1.20E+06	-1.18E+06	-7.43E+05
1.80	1.19E+03	1.04E+03	4.66E+02	-9.08E+05	-9.03E+05	-8.12E+05
2.00	-6.05E+02	-4.96E+02	-1.68E+02	-1.85E+05	-1.85E+05	-1.89E+05
2.40	-6.52E+02	-5.38E+02	-1.88E+02	-1.66E+06	-1.66E+06	-1.67E+06
2.80	-6.50E+02	-5.36E+02	-1.87E+02	-1.66E+06	-1.66E+06	-1.67E+06
3.20	-6.50E+02	-5.36E+02	-1.87E+02	-1.66E+06	-1.66E+06	-1.67E+06

Table 3. Variation of c_{f_1} with x for different size of particles

x	Initial Heating			Viscous Heating		
	$D = 0.5\mu m$	$D = 50\mu m$	$D = 100\mu m$	$D = 0.5\mu m$	$D = 50\mu m$	$D = 100\mu m$
1.2	1.35E+03	3.57E+01	1.61E+01	2.61E+03	2.61E+03	2.24E+02
1.4	2.30E+02	1.28E+01	1.42E+01	2.41E+02	2.41E+02	8.74E+01
1.6	8.46E+00	5.80E+00	1.26E+01	2.91E+01	2.91E+01	8.33E+00
1.8	3.09E+00	5.63E+00	1.03E+01	1.28E+01	1.28E+01	1.96E+00
2.0	2.31E+00	5.56E+00	1.02E+01	3.17E+00	3.17E+00	1.84E+00
2.4	1.12E+00	5.31E+00	2.60E-01	9.54E-01	9.54E-01	1.54E+00
2.8	1.12E+00	5.10E+00	7.19E-02	8.99E-01	8.99E-01	1.53E+00
3.2	1.12E+00	4.97E+00	8.13E-02	8.99E-01	8.99E-01	1.53E+00
3.6	1.12E+00	4.94E+00	8.13E-02	8.99E-01	8.99E-01	1.53E+00
4.0	1.12E+00	4.93E+00	8.11E-02	8.99E-01	8.99E-01	1.53E+00
4.4	1.12E+00	4.93E+00	8.09E-02	8.99E-01	8.99E-01	1.53E+00
4.8	1.12E+00	4.93E+00	8.07E-02	8.99E-01	8.99E-01	1.53E+00
5.0	1.12E+00	4.93E+00	8.06E-02	8.99E-01	8.99E-01	1.53E+00

Table 4. Variation of c_{f_1} with x for different material density of particles (ρ_s)

x	Initial Heating			Viscous Heating		
	$\rho_s = 800$	$\rho_s = 2403$	$\rho_s = 8010$	$\rho_s = 800$	$\rho_s = 2403$	$\rho_s = 8010$
1.20	1.35E+03	3.53E+02	3.24E+02	2.61E+03	2.17E+02	1.97E+02
1.40	2.30E+02	1.27E+02	8.90E+01	2.41E+02	8.38E+01	5.90E+01
1.60	8.46E+01	1.16E+01	2.58E+01	2.91E+01	7.66E+00	1.86E+01
1.80	3.09E+01	2.82E+00	5.49E+00	1.28E+01	2.52E+00	3.46E+00
2.00	2.31E+00	2.65E+00	5.31E+00	3.17E+00	1.93E+00	3.43E+00
2.40	1.12E+00	2.54E+00	5.25E+00	8.98E-01	1.66E+00	3.37E+00
2.80	1.12E+00	2.54E+00	5.24E+00	8.98E-01	1.66E+00	3.37E+00

3.20	1.12E+00	2.54E+00	5.24E+00	8.99E-01	1.66E+00	3.37E+00

Table 5. Variation of c_{f_1} with x for different diffusion parameter(ϵ)

x	Initial Heating			Viscous Heating		
	$\varepsilon = 0.05$	$\varepsilon = 0.1$	$\varepsilon = 0.2$	$\varepsilon = 0.05$	$\varepsilon = 0.1$	$\varepsilon = 0.2$
1.20	1.35E+03	5.39E+02	5.39E+02	2.61E+03	3.59E+02	2.81E+02
1.40	2.30E+02	1.86E+02	1.86E+02	2.41E+02	1.23E+02	1.20E+02
1.60	8.46E+01	2.16E+01	2.16E+01	2.91E+01	1.36E+01	1.55E+01
1.80	3.09E+00	3.30E+00	3.30E+00	1.28E+01	1.60E+00	6.56E+00
2.00	2.31E+00	5.89E-01	5.89E-01	3.17E+00	9.96E-01	7.13E-01
2.40	1.12E+00	5.58E-01	5.58E-01	9.54E-01	4.45E-01	2.18E-01
2.80	1.12E+00	5.45E-01	5.45E-01	9.18E-01	4.37E-01	2.08E-01
3.20	1.12E+00	5.45E-01	5.45E-01	8.99E-01	4.37E-01	2.08E-01
3.60	1.12E+00	5.45E-01	5.45E-01	8.99E-01	4.37E-01	2.08E-01

Table 6: Variation of c_{f_1} with x

x	Without electrification	With electrification
1.2	7.80E+03	1.35E+03
1.4	6.30E+02	2.30E+02
1.6	6.25E+02	8.46E+01
1.8	5.83E+02	3.09E+01
2.0	3.66E+02	2.31E+00
2.4	3.19E+02	1.12E+00
2.8	9.56E+01	1.12E+00
3.2	9.24E+01	1.12E+00
3.6	9.24E+01	1.12E+00

Table 7. Variation of Nu_1 with x Initial Heating Viscous Heating

x	Without electrification	With electrification	Without electrification	With electrification

1.2	1.18E+04	3.00E+00	-1.89E+05	-1.19E+08
1.4	3.04E+05	9.80E+03	-4.22E+05	-2.60E+07
1.6	2.02E+06	5.41E+03	-1.21E+06	-1.20E+06
1.0	4.49E+06	-1.19E+03	-9.49E+05	-9.08E+06
2.0	1.57E+06	-6.05E+02	-6.95E+05	-1.85E+06
2.4	8.98E+03	-6.52E+02	1.13E+05	-1.66E+06
2.8	-1.03E+01	-6.50E+02	7.94E+03	-1.66E+06
3.2	-8.41E+01	-6.50E+02	7.20E+04	-1.66E+06
3.6	-4.79E+01	-6.50E+02	8.92E+04	-1.66E+06
4.0	-2.86E+01	-6.50E+02	1.03E+05	-1.66E+06
4.4	-1.78E+01	-6.50E+02	1.18E+05	-1.66E+06
4.8	-1.13E+01	-6.50E+02	1.32E+05	-1.66E+06
5.0	-9.12E+00	-6.50E+02	1.39E+05	-1.66E+06

Fig- 6(a) and 6(b) depicts particle phase temperature T_{p_1} having viscous heating with and without charged SPM. It is concluded from both the figure that both profile are Blasius type near the plate and free jet type away from the plate.

Thus we conclude that the electrification of particles reduces the numerical value of u_1 ,T_1, T_{p_1}.

To show the heat transfer in the wall jet, the Nusselts number is calculated for the case of initial heating and viscous heating without and with electrification. The value of $Nu = Nu_0 + Nu_1$, where Nu_0 is the Nusselts number not effected by electrification and Nu_1 is calculated based on perturbation temperature T_1. In Table-1 values of Nu_0 for initial heating and viscous heating for different values of Pr is calculated. Similarly, Table-2 shows the dependence of Nu_1 on Pr. From Table-1 it can be observed that Nu_0 is increasing in both for initial heating and for viscous heating. Further from Table-2, it can be observed that $Nu_1$is increasing and then decreasing towards the downstream and assuming a constant value towards the downstream of the plate.

Table-3, Table-4 and Table-5 represents variation of $c_{f_1}$with x for the parameters, size, material density and diffusion of the particles. It can be observed that c_{f_1} decreases and then remain constant towards downstream in both the cases of Initial and Viscous heating. Table-6 and Table-7 represents variation of c_{f_1} and Nu_1 with x for without and with electrification respectively. It is observed that the electrification of particles reduces the velocity and temperature gradient leading to reduction of skin friction and heat transfer.

Conclusion

In this paper the effects of electrification of suspended particulate matter on fluid flow and heat transfer is studied. It is concluded that electrification of particles reduces the numerical value of velocity of fluid and temperature of fluid and particles. So it reduces skin friction and heat transfer. Further the increase of Prandtl number (Pr) results in the decrease in temperature distribution. Physically heat is diffused away from the heated surface rapidly then for higher value of Pr as smaller value of Pr indicates an increase in thermal conductivity. Thus temperature falls more rapidly for water (Pr =7.0) then that of air (Pr = 0.71) and electrolyte solution (Pr =1.0).

References

1. L. Xie, G. Li, N. Bao, J. Zhou, Contact electrification by Collision of homogeneous Particles, 2013, Journal of Applied Physics, Vol. 113, p. 184908.
2. S. Matsusaka, Measurment and control of particle tribocharging proceeding, 2012, Joint Electrostatics Conference, Cambridge, ON, Canada.
3. S.K. Misra, P.K. Tripathy, Mathematical & numerical modeling of two phase flow & heat transfer using non-uniform grid, 2011, Far East J. Appl. Math, Vol-54, No-2, pp. 107-126.
4. J.P. Panda, N. Dash, G.C. Dash, Hydro magnetic flow and heat-transfer through porous medium of elastic viscous fluid over a porous plate in slip flow regime, 2011, AMSE-Jounals, Series 2B, Vol. 80, No. 2, pp.71-87.
5. G. Palani, P. Ganesan, Heat transfer effects on dusty-gas flow past a semi-infinite inclined plate, 2007, Forschung im Ingenieurwesen, Vol. 70, no. 3-4, p. 223.
6. K. Rietema, H.E.A van der Akker, On the momentum equations in the dispersed two-phase system, 1983, International Journal of Multiphase Flow, Vol. 9, no. 1, pp. 31-36.
7. J.L. Bansal, S.S. Tak, Approximate solutions of heat and momentum transfer in laminar plane wall jet, 1978, Appl. Sci. Res, vol. 34, pp. 299-312.
8. J.L. Bansal, Jets of conductive fluids in the presence of a transverse magnetic field, 1975, ZAMP, Vol. 55, pp. 479-489.
9. H. Schlichting, Boundary Layer Theory, 1968, 7th edition, Springer Publication, pp. 578-583.
10. S.L. Soo, Fluid dynamics of multiphase systems, 1968, Blasisdell publishing company, London, pp. 248-256.

11. S.L. Soo, Laminar and separated flow of a particulate suspension, 1965, Astronautica Acta, Vol. 11, pp. 422-431.
12. S.L. Soo, Effect of electrification on the dynamics of a particulate system, 1964, I and EC Fund, pp. 75-80.