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MHD Viscoelastic Fluid Flow along an Infinite Oscillating Porous Plate with Heat Source and Thermal Diffusion

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Abstract

Unsteady MHD visco-elastic fluid flow has been studied with chemical reaction and thermal diffusion. The flow is governed by a coupled non-linear system of partial differential equations. The coupled non-linear partial differential equations have been discussed by usual transformation technic and the obtained dimensionless equations have been solved numerically. The governing equations of the fluid flow are solved by using perturbation technique for small elastic parameter, subject to the relevant boundary conditions. The solution for the velocity, temperature, concentration have been derived analytically and also its behaviors are discussed with reference to different flow parameters with the help of graphs.

Keywords

Heat and mass transfer, MHD flows, Porous medium, Unsteady flows and Viscoelastic fluids.

1. Introduction

Visco-elastic fluid flow has been generated in the area of heat and mass transfer of the boundary layer through a oscillating porous plate in presence of thermal diffusion in various fields like polymer processing industry such as manufacturing process of artificial film, artificial fibers. This research paper is discussed with viscoelastic fluids. There are two properties, one is viscous property and another one is elastic property and so it is named as viscoelastic fluid. The importance of viscoelastic fluid is increasing day by day due to its many engineering, biological, industrial and chemical aspects. There are some common viscoelastic fluids are engine shampoo, ointments, gels, oils, paints, honey, molten plastics, and blood. These appear in many industrial process, chemical reaction and pharmaceutical industries. Also some applications of dilute polymer solution in recent years are available. The boundary layer flow of non-Newtonian fluid in the presence of magnetic field has wide range of application in nuclear engineering industries. The major significance of the geometry of a textile structure in contributing to resistance to water penetration can be stated in the following manner. Viscoelastic properties can enhance or depress heat transfer rates, depending upon the kinematic characteristic of the flow field under consideration and the direction of heat transfer. Convective heat transfers in the flow of visco-elastic fluid in a porous medium past a stretching sheet have been studied by khan et al. (2001). The effects on MHD Viscous flow of Mass Transfer past an impulsively started infinite vertical plate with constant mass flux have been analyzed by Reddy et al. (2013). A great deal of work has been carried out to find the analytical solution of viscoelastic fluid flow of non-Newtonian fluid over impervious stretching boundary, this work has been analyzed by Chamkha and Ahmed (2011). The visco-elasticity on the flow and heat transfer in a porous medium over a stretching sheet have been discussed by Abel and Veena (1998) Heat Transfer in MHD Viscoelastic Fluid Flow over a Stretching Sheet with Variable Thermal Conductivity have been investigated by Dash et al. (2004). The heat and mass transfer under a chemical reaction with a heat source have been studied by Kandasamy et al. (2005). The chemical reaction and radiation absorption on an unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical permeable in a porous medium with heat source and suction has been studied by Kesavaiah et al. (2011). An analytical study of MHD heat and mass transfer oscillatory flow fluid over a oscillating plate in a porous medium has been studied by Choudhury and Das (2012). The effect of heat and mass transfer over a stretched vertical surface in a porous medium filled with a viscoelastic fluid under Soret effect in the presence of magnetic field has been investigated by Gbadeyan et al. (2011). MHD Viscoelastic fluid flow through a vertical flat plate with soret effect has been analyzed by Hossain and Alam (2015). Viscous dissipative heat on two-dimensional unsteady free convective flow past an infinite vertical porous plate with temperature oscillates in time and constant suction at the plate has been analyzed by Raju and Varma (2011). The effects thermal conductivity and heat source/sink on MHD flow is studied by Sharma and Singh (2009). The effects on the accelerated flow of a viscous incompressible fluid past an infinite vertical porous plate has many important technological applications in the astrophysical, geophysical and engineering problem. Such type of work has been studied by Jha et al. (1991). Free convection effects on the flow past an accelerated vertical plate in an incompressible dissipative fluid have been analyzed by Alharbi et al. (2010).

The objective of the work is to analyze the heat and mass transfer effects on MHD visco elastic fluid flow through an oscillating porous plate with viscous dissipation heat transfer under the influence of a uniform transverse magnetic field and heat source in the presence of chemical reaction

2. Mathematical Formulation

Consider unsteady flow of an incompressible visco-elastic fluid through porous medium with heat and mass transfer near an oscillating infinite porous plate with heat source, chemical reaction and thermal diffusion. According to the co-ordinate system x-axis is taken along the upward direction of floe and z-axis is normal to it. Let u and w be the velocity component of x and z directions respectively. If the plate is extended to infinite length, then all the physical variables in the problem are functions of z and t alone. Initially the plate and fluid are at rest then the plate is set to an oscillatory motion. Also it is considered that the fluid and the plate is at rest after that the plate is to be moving with a constant velocity U_o in its own plate instantaneously at time $t > 0$, the species temperature of the plate is raised to $T_W\left(> T_\infty\right)$.Where T_w is species temperature at the plate and T_∞ be the temperature species far away from the plate. The physical model of the study is shown in Figure 1.

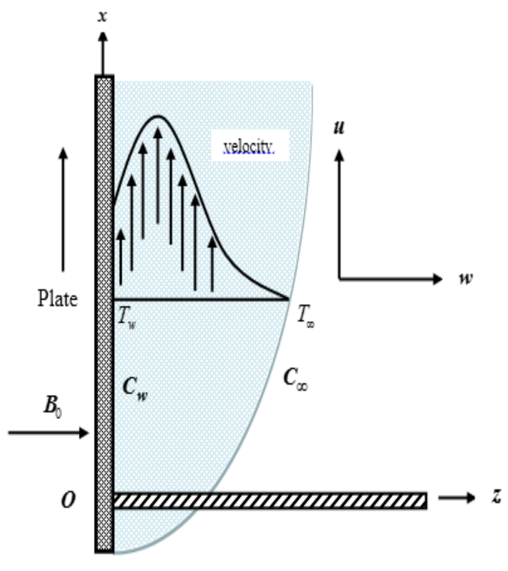


Fig. 1. Physical Model of Boundary Layer

The governing equation for the unsteady incompressible visco-elastic fluid flow with simultaneous heat and mass transfer near an oscillating porous plate are as follows;

$$\frac{\partial w}{\partial t} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} - w_o \frac{\partial u}{\partial z} = \upsilon \frac{\partial^2 u}{\partial z^2} - \frac{\kappa_o}{\rho}\left(\frac{\partial^3 u}{\partial z^2 \partial t} + w_0 \frac{\partial^3 u}{\partial z^3}\right) - \frac{\sigma \beta_0{}^2 u}{\rho} - \frac{\upsilon}{\kappa} u + g\beta\left(T - T_\infty\right) + g\beta^*\left(C - C_\infty\right) \tag{2}$$

$$\frac{\partial v}{\partial t} - w_0 \frac{\partial v}{\partial z} = \upsilon \frac{\partial^2 v}{\partial z^2} - \frac{\kappa_0}{\rho}\left(\frac{\partial^3 v}{\partial z^2 \partial t} + w_0 \frac{\partial^3 v}{\partial z^3}\right) - \frac{\sigma \beta_0{}^2}{\rho} v - \frac{\upsilon}{\kappa} v \tag{3}$$

$$\frac{\partial T}{\partial t} - w_0 \frac{\partial T}{\partial z} = \alpha \frac{\partial^2 T}{\partial z^2} - S\left(T - T_\infty\right) \tag{4}$$

$$\frac{\partial C}{\partial t} - w_0 \frac{\partial C}{\partial z} = D \frac{\partial^2 C}{\partial z^2} - \kappa_1\left(C - C_\infty\right) + D_T \frac{\partial^2 T}{\partial z^2} \tag{5}$$

Combining the equation (2) and (3), Let $q = u + iv$

$$\frac{\partial q}{\partial t} - w_0 \frac{\partial q}{\partial z} = \upsilon \frac{\partial^2 q}{\partial z^2} - \frac{\kappa_0}{\rho}\left(\frac{\partial^3 q}{\partial z^2 \partial t} + w_0 \frac{\partial^3 q}{\partial z^3}\right) - \frac{\sigma \beta_0{}^2}{\rho} q - \frac{\upsilon}{\kappa} q + g\beta\left(T - T_\infty\right) + g\beta^*\left(C - C_\infty\right) \tag{6}$$

The first order velocity slip boundary condition of the problem and the plate executes linear harmonic oscillations in its own plane are given by,

$$q = U_0 e^{i\omega t} + L_1 \frac{\partial q}{\partial z}, T = T_w, C = C_w \qquad \text{at} \rightarrow z = 0 \tag{7}$$

$$q = 0, T \rightarrow T_\infty, C \rightarrow C_\infty \qquad \text{at} \rightarrow z \rightarrow \infty$$

where, $L_1 = \left(2 - m_1\right)\left(L / m_1\right)$, $L = \mu\left(\pi / 2p\rho\right)^{\frac{1}{2}}$ is the mean free path and m_1 is Maxwell reflection coefficient.

The following non-dimensional quantities have been used as;

$$u^* = \frac{u}{U}, v^* = \frac{v}{V}, t^* = \frac{tU_0{}^2}{\upsilon}, \; z^* = \frac{zU_0}{\upsilon},$$

$$w^* = \frac{v}{U_0{}^2} w \;, w_0{}^* = \frac{w_0}{U_0}, \theta = \frac{T - T_\infty}{T_w - T_\infty}, \; C^* = \frac{C - C_\infty}{C_w - C_\infty}$$

The obtained governing equations are as follow;

$$\frac{\partial q}{\partial t}-w_0\frac{\partial q}{\partial t}=\frac{\partial^2 q}{\partial z^2}-Rc\left(\frac{\partial^3 q}{\partial z^2\partial t}+w_0\frac{\partial^3 q}{\partial z^3}\right)-\left(M^2+\frac{1}{K}\right)q+G_r\theta+G_mC \tag{8}$$

$$P_r\frac{\partial \theta}{\partial t}-w_0P_r\frac{\partial \theta}{\partial z}=\frac{\partial^2\theta}{\partial z^2}-P_rS\theta \tag{9}$$

$$S_c\frac{\partial C}{\partial t}-w_0S_c\frac{\partial C}{\partial z}=\frac{\partial^2 C}{\partial z^2}-K_cS_cC+S_0S_c\frac{\partial^2\theta}{\partial z^2} \tag{10}$$

The corresponding boundary conditions are as follows;

$$q=U_0e^{i\omega t}+L_1\frac{\partial q}{\partial z},\ T=T_w, C=C_w,\ \text{at}\quad z=0 \tag{11}$$

$$q=0, T\to T_\infty, C\to C_\infty\quad \text{at}\quad z\to\infty$$

where, $M^2=\frac{\sigma\beta_0^2\upsilon}{\rho U_0^2}$ is the Hartmann number, $K=\frac{\kappa U_0^2}{\upsilon^2}$ is the permeability parameter, $R=\frac{L_1w_0}{\upsilon}$ is the Rarefaction parameter, $P_r=\frac{\upsilon}{\alpha}$ is the Prandtl number, $S_c=\frac{\upsilon}{D}$ is the Schmidt number , $R_c=\frac{U_0^2K_0}{\rho\upsilon^2}$ is the elastic parameter , $K_c=\frac{\upsilon K_1}{U_0^2}$ is the chemical reaction parameter, $S=\frac{\upsilon_s}{\rho\upsilon^2}$ is the Heat Source parameter, $G_r=\frac{g\beta\upsilon}{U_0^3}(T_w-T_\infty)$ is the Grasshof number, $w_0=\frac{w_0}{U_0}$ is the Suction velocity and $G_m=\frac{g\beta\upsilon}{U_0^3}(C_w-C_\infty)$ is the mass Grasshof number, $S_0=\frac{D_T}{\upsilon}\left(\frac{T_w-T_\infty}{C_w-C_\infty}\right)$ is the soret number.

Equation (8) is third order and the two boundary conditions are available. Due to inadequate boundary condition, a perturbation method has been applied with $R_c<1$ as the perturbation parameter.This assumption is quite consistent as the model under consideration is valid only for slightly elastic fluid.

$$q=q_0+R_cq_1+O(R_c)^2 \tag{12}$$

$$\theta=\theta_0+R_c\theta_1+O(R_c)^2 \tag{13}$$

$$C=C_0+R_cC_1+O(R_c)^2 \tag{14}$$

Substituting equations (12) to (14) in equations (8) to (10) and equating like powers of R_c , The following equations have been obtained as follows;

Zeroth order:

$$\frac{\partial q_0}{\partial t}-w_0\frac{\partial q_0}{\partial z}=\frac{\partial^2 q_0}{\partial z^2}-(M^2+\frac{1}{K})q_0+G_r\theta_0+G_mC_0 \tag{15}$$

$$P_r\frac{\partial \theta_0}{\partial t}-w_0P_r\frac{\partial \theta_0}{\partial z}=\frac{\partial^2 \theta_0}{\partial z^2}-P_rS\theta_0 \tag{16}$$

$$S_c\frac{\partial C_0}{\partial t}-w_0S_c\frac{\partial C_0}{\partial z}=\frac{\partial^2 C_0}{\partial z^2}-K_cS_cC_0+S_cS_0\frac{\partial^2 \theta_0}{\partial z^2} \tag{17}$$

First order:

$$\frac{\partial q_1}{\partial t}-w_0\frac{\partial q_1}{\partial z}=\frac{\partial^2 q_1}{\partial z^2}-\left(\frac{\partial^3 q_0}{\partial z^2\partial t}+w_0\frac{\partial^3 q_0}{\partial z^3}\right)-\left(M^2+\frac{1}{K}\right)q_1+G_r\theta_1+G_mC_1 \tag{18}$$

$$P_r\frac{\partial \theta_1}{\partial t}-w_0P_r\frac{\partial \theta_1}{\partial z}=\frac{\partial^2 \theta_1}{\partial z^2}-P_rS\theta_1 \tag{19}$$

$$S_c\frac{\partial C_1}{\partial t}-w_0S_c\frac{\partial C_1}{\partial z}=\frac{\partial^2 C}{\partial z^2}-K_cS_cC_1+S_cS_0\frac{\partial^2 \theta_1}{\partial z^2} \tag{20}$$

With corresponding boundary conditions are as follows;

$$\begin{aligned}&q_0=e^{i\omega t}+R\frac{\partial q_0}{\partial z},q_1=R\frac{\partial q_1}{\partial z},\theta_0=1,\theta_1=0,C_0=1,C_1=0 \quad \text{at} \quad z=0\\&q_0=0,q_1=0,\theta_0=0,\theta_1=0,C_0=0,C_1=0 \quad \text{at} \quad z\to\infty\end{aligned} \tag{21}$$

In order to reduce the system of partial differential equation into a system of ordinary differential equation, the following equations have been used as follows;

$$q_0(z,t)=q_{00}(z)+q_{01}(z)e^{i\omega t} \tag{22}$$

$$q_1(z,t)=q_{10}(z)+q_{11}(z)e^{i\omega t} \tag{23}$$

$$\theta_0(z,t)=\theta_{00}(z)+\theta_{01}(z)e^{i\omega t} \tag{24}$$

$$\theta_1(z,t)=\theta_{10}(z)+\theta_{11}(z)e^{i\omega t} \tag{25}$$

$$C_0(z,t)=C_{00}(z)+C_{01}(z)e^{i\omega t} \tag{26}$$

$$C_1(z,t)=C_{10}(z)+C_{11}(z)e^{i\omega t} \tag{27}$$

Substituting equation (22) to (27) into equation (15) to (20) and equating the harmonic and non-harmonic terms, The following equations have been obtained as follows;

$$q''_{00} + w_0 q'_{00} - \left(M^2 + \frac{1}{K}\right) q_{00} = -G_r \theta_{00} - G_m C_{00} \tag{28}$$

$$q''_{01} + w_0 q'_{01} - \left(M^2 + \frac{1}{K} + i\omega\right) q_{01} = -G_r \theta_{01} - G_m C_{01} \tag{29}$$

$$q''_{10} + w_0 q'_{10} - \left(M^2 + \frac{1}{K}\right) q_{10} = -G_r \theta_{10} - G_m C_{10} + w_0 q'''_{00} \tag{30}$$

$$q''_{11} + w_0 q'_{11} - \left(M^2 + \frac{1}{K} + i\omega\right) q_{11} = -G_r \theta_{11} - G_m C_{11} + w_0 q'''_{01} + i\omega q''_{01} \tag{31}$$

$$\theta''_{00} + w_0 P_r \theta'_{00} - P_r S \theta_{00} = 0 \tag{32}$$

$$\theta''_{01} + w_0 P_r \theta'_{01} - P_r \left(S + i\omega\right) \theta_{01} = 0 \tag{33}$$

$$\theta''_{10} + w_0 P_r \theta'_{10} - P_r S \theta_{10} = 0 \tag{34}$$

$$\theta''_{11} + w_0 P_r \theta'_{11} - P_r \left(S + i\omega\right) \theta_{11} = 0 \tag{35}$$

$$C''_{00} + w_0 S_c C'_{00} - K_c S_c C_{00} = -S_c S_0 \theta''_{00} \tag{36}$$

$$C''_{01} + w_0 S_c C'_{01} - \left(i\omega + K_c\right) S_c C_{01} = -S_c S_0 \theta''_{01} \tag{37}$$

$$C''_{10} + w_0 S_c C'_{10} - K_c S_c C_{10} = -S_c S_0 \theta''_{10} \tag{38}$$

$$C''_{11} + w_0 S_c C'_{11} - \left(i\omega + K_c\right) S_c C_{11} = -S_c S_0 \theta''_{11} \tag{39}$$

Corresponding boundary Conditions are as follows;

$$q_{00} = R\frac{\partial q_{00}}{\partial z}, q_{01} = 1 + R\frac{\partial q_{01}}{\partial z}, q_{01} = R\frac{\partial q_{10}}{\partial z}, q_{11} = R\frac{\partial q_{11}}{\partial z}$$

$$\theta_{00} = 1, \theta_{01} = 0, \theta_{10} = 0, \theta_{11} = 0, C_{00} = 1, C_{01} = 0, C_{10} = 0, C_{11} = 0 \text{ at } z = 0 \tag{40}$$

$$q_{00} = 0, q_{01} = 0, q_{10} = 0, q_{11} = 0, \theta_{00} = 0, \theta_{01} = 0, \theta_{10} = 0, \theta_{11} = 0, C_{00} = 0,$$

$$\theta_{01} = 0, \theta_{10} = 0, \theta_{11} = 0 \text{ at } z \to \infty \tag{41}$$

The solutions are as follows;

$$q_{00} = a_3 e^{-m_2 z} + a_4 e^{-m_2 z} + a_5 e^{-m_6 z} + a_6 e^{-m_{10} z} \tag{42}$$

$$q_{01} = a_7 e^{-m_{12} z} \tag{43}$$

$$q_{10} = a_8 e^{-m_2 z} + a_9 e^{-m_2 z} + a_{10} e^{-m_6 z} + a_{11} e^{-m_{10} z} + a_{12} e^{-m_{10} z} \tag{44}$$

$$q_{11} = a_{15} e^{-m_{12} z} \tag{45}$$

$$\theta_{00} = e^{-m_2 z}, \theta_{01} = \theta_{10} = \theta_{11} = 0 \tag{46}$$

$$C_{00} = a_1 e^{-m_2 z} + a_2 e^{-m_6 z}, C_{01} = C_{10} = C_{11} = 0 \tag{47}$$

Hence the solution of velocity, Temperature and Concentration of the flow field are as follows;

$$\begin{aligned} q = {} & a_3 e^{-m_2 z} + a_4 e^{-m_2 z} + a_5 e^{-m_6 z} + a_6 e^{-m_{10} z} + a_7 e^{-m_{12} z} e^{i\omega t} + \\ & R_c (a_8 e^{-m_2 z} + a_9 e^{-m_2 z} + a_{10} e^{-m_6 z} + a_{11} e^{-m_{10} z} + a_{12} e^{-m_{10} z} + a_{15} e^{-m_{12} z}) \end{aligned} \tag{48}$$

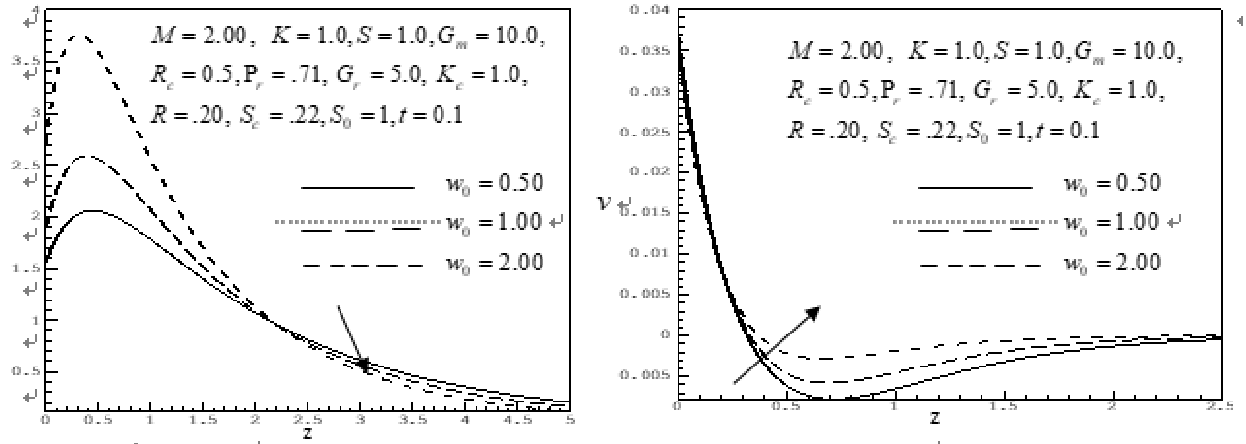
$$\theta = e^{-m_2 z} \tag{49}$$

$$C = a_1 e^{-m_2 z} + a_2 e^{-m_6 z} \tag{50}$$

3. Results and Discussion

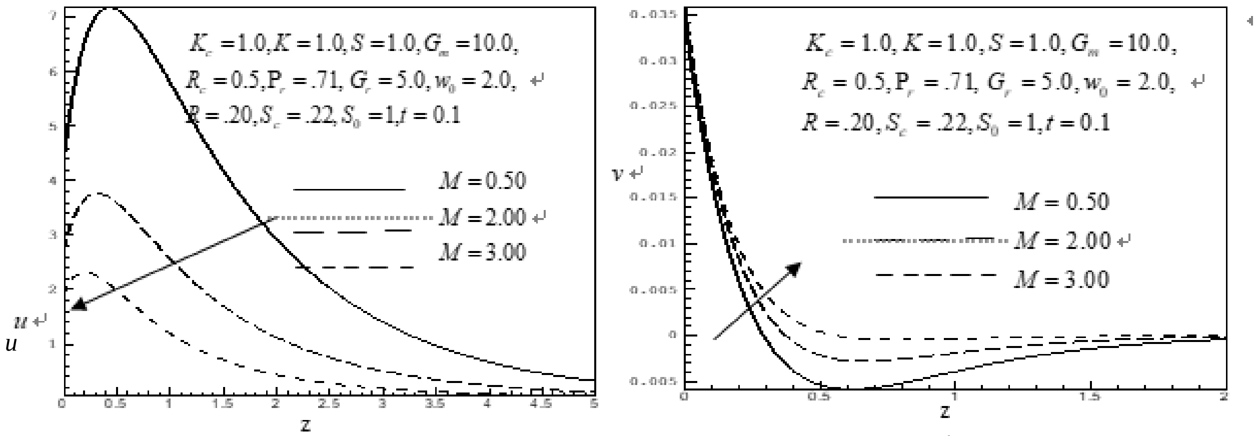
Case i:

For the purpose of discussion the results of the problem, the approximate solutions are obtained for various parameters. In order to analyze the physical situation of the model, it has been computed for the analytical solution of the non-dimensional velocity, temperature and concentration distributions with the boundary layer for different values of Prandlt number, Hartmann number, Porosity parameter, the elastic parameter, Chemical reaction parameter, Heat source, Scmidt number, the suction parameter and Soret number with time $t = 0.1$.The effect of the suction parameter (w_0), the Hartmann number (M) and the permeability parameter (K) on the velocity distribution u and v are represented in Figure 2. It is noticed that the velocity distribution u decreases with the increase of (w_0), (M) and (K). While the velocity distribution v increase with the increase of (w_0) and (M). And the velocity distribution v decreases with the increase of (K). In Figure 3, the effect of the velocity distributions u and v for the elastic parameter (R_c), the Heat source parameter (S) and Soret number (S_0) are represented. It is found that the velocity distribution u increases with the increase of R_c and S_0 and the velocity distribution u decreases with the increase of S. There is no effect on velocity distribution v with the increase of (R_c), (S) and (S_0).The effect of the chemical reaction parameter (K_c) and the prandlt number (p_r) on the velocity distributions u and v are represented in Figure 4. The velocity distribution u decreases with the increase of (K_c) and (P_r), and there is no effect on velocity distribution v with the increase of (K_c) and (P_r). Figures 5-7 depict the effect of the Prandlt number (p_r), the Heat source parameter (S) and the suction parameter (w_0) on the temperature distributions. It is observed that the temperature distribution decreases with the increase of (P_r), (S) and (w_0). Figures 8-10 depict the effect of the chemical reaction parameter (K_c), the Schmidt number (S_c) and the suction parameter (w_0) on the concentration distribution. It is noticed that the concentration distribution decreases with the increase of $(\mathrm{K_c})$, $(\mathrm{S_c})$ and $(\mathrm{w_0})$.



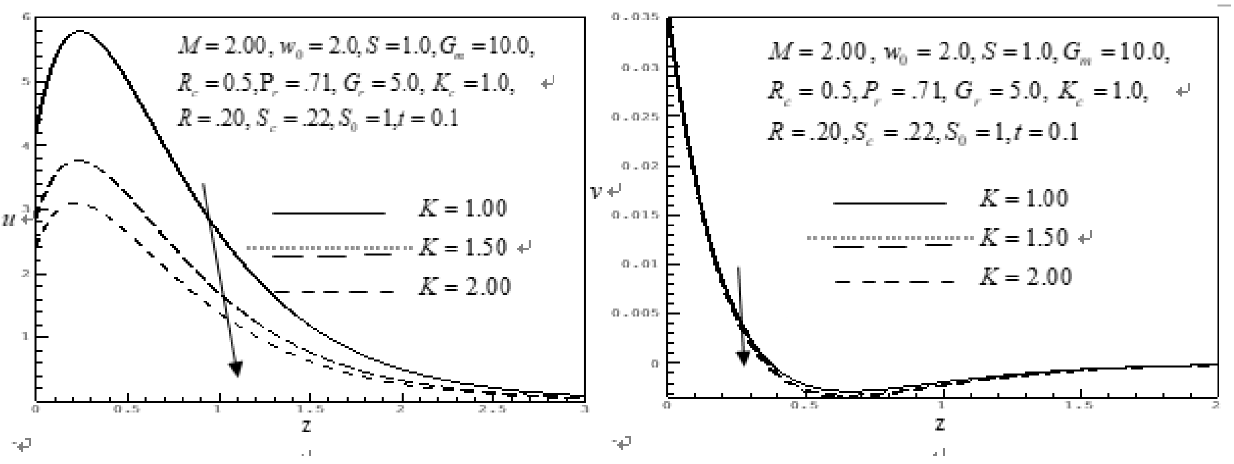
(a)

(b)



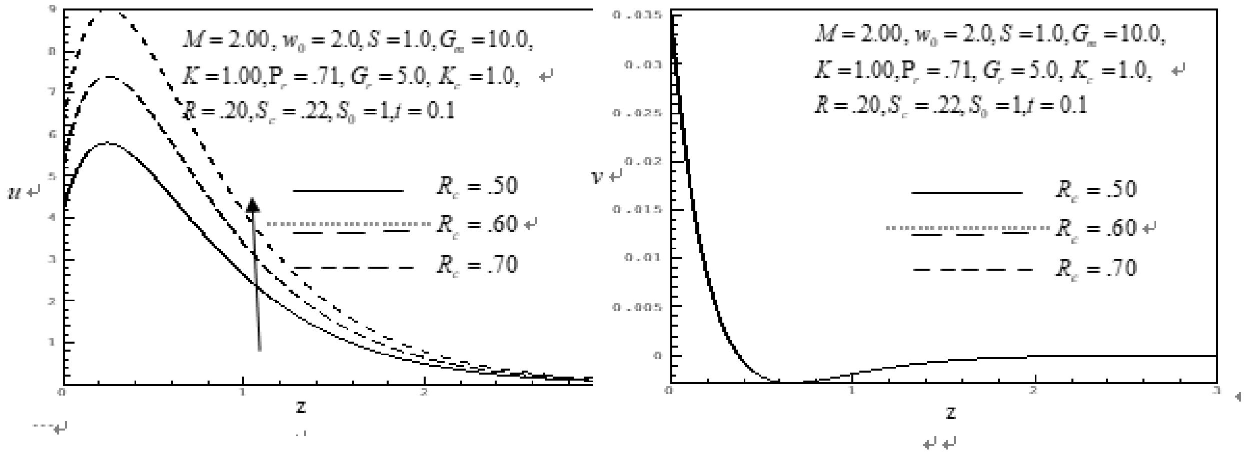
(a)

(b)

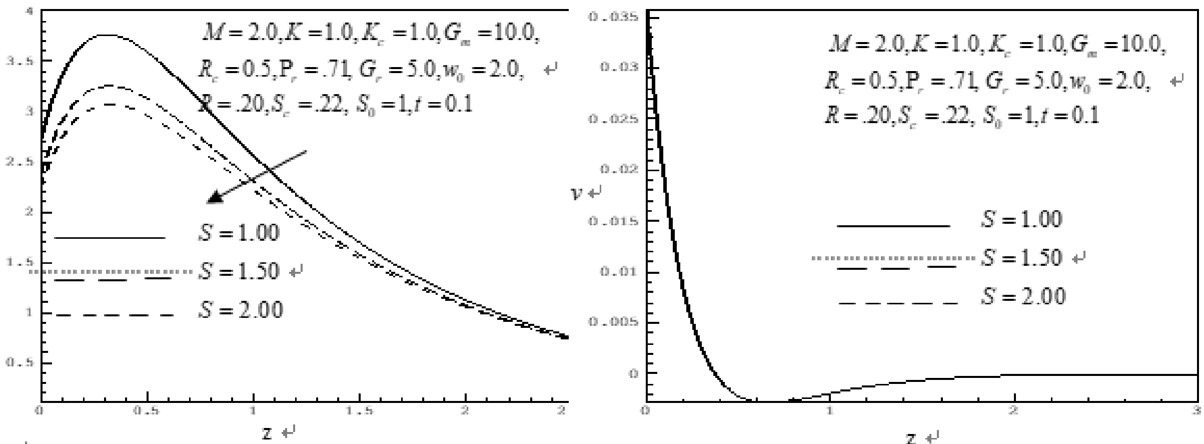


(a) (b)

Fig. 2. The Velocity Distribution for Suction Velocity (w_0), Hartmann Number (M) and Permeability Parameter (K) in Figures (a), (b) with t=0.1.



(a) (b)



(a) (b)

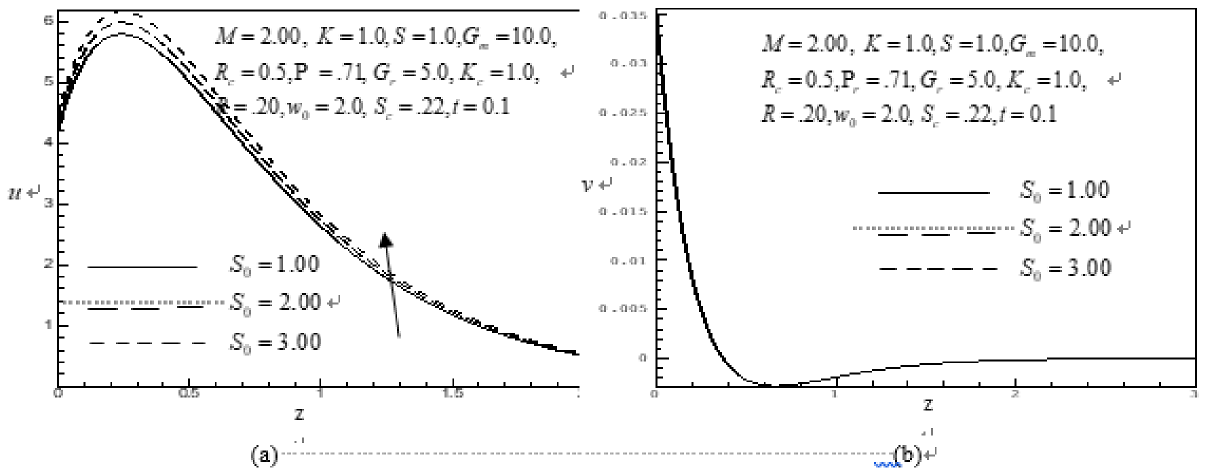


Fig. 3. The Velocity Distribution for the Elastic Parameter (R_c), the Heat Source Parameter (S) and Soret Number (S_0) in Figures (a), (b) with t=0.1.

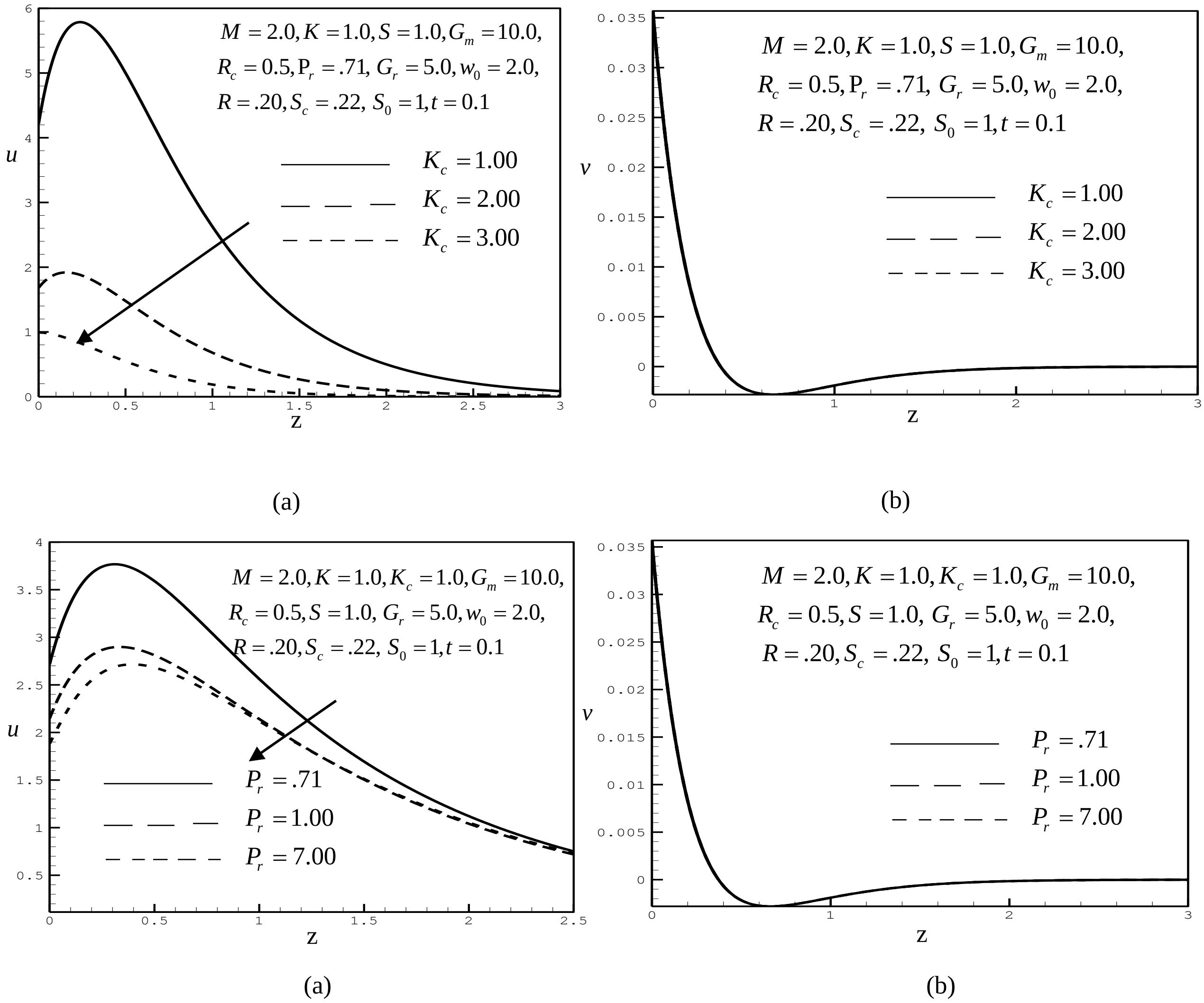


Fig. 4. The Velocity Distribution for the Chemical Reaction Parameter (K_c) and Prandlt Number (p_r) in Figures (a), (b) with t=0.1

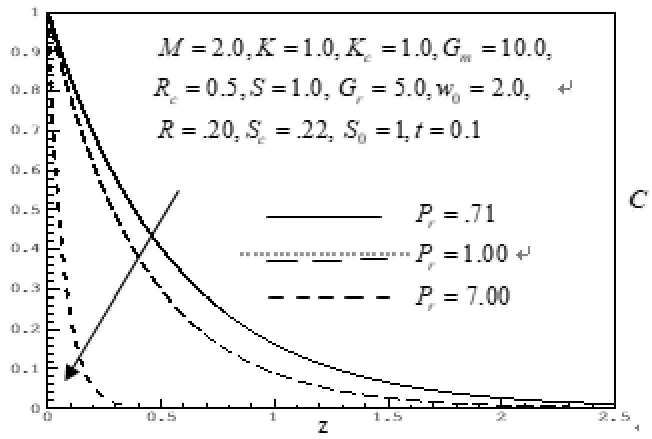


Fig. 5. Temperature Distribution for Different Values of Prandlt Number (P_r)

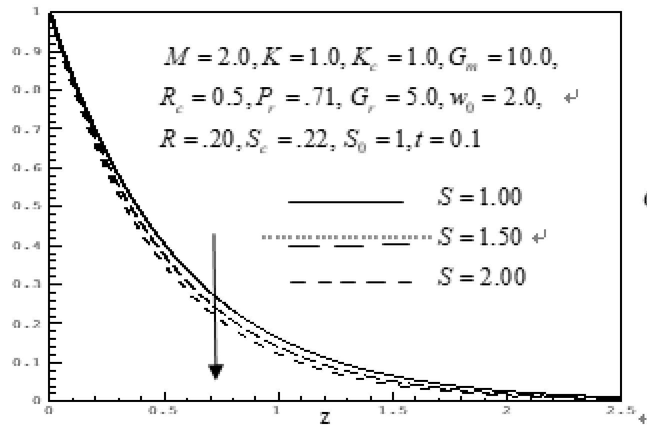


Fig. 6. Temperature Distribution for Different Values of Heat Source Parameter (S)

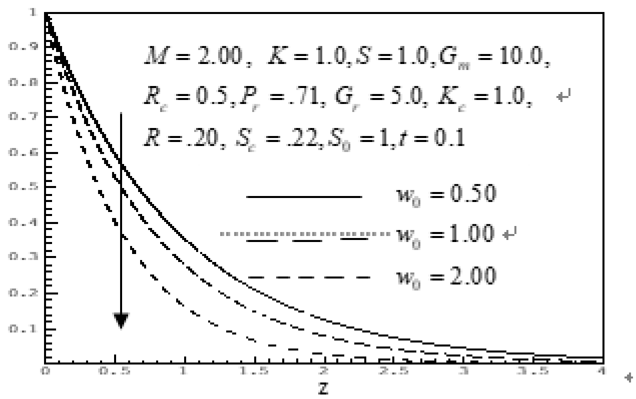


Fig. 7. Temperature Distribution for Different Values of Suction Velocity (w_0)

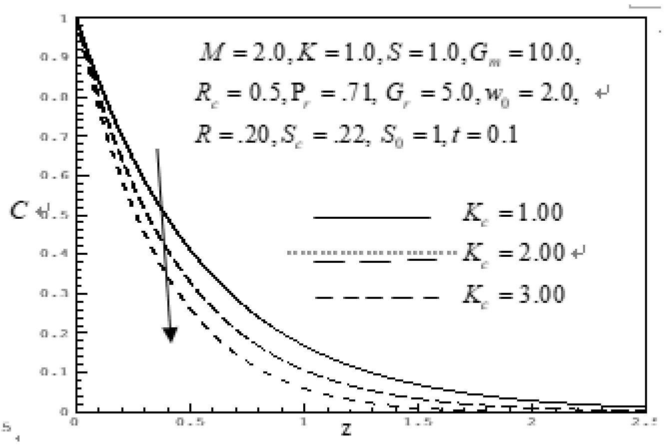


Fig. 8. Concentration Distribution for Different Values of the Chemical Reaction Parameter (K_c)

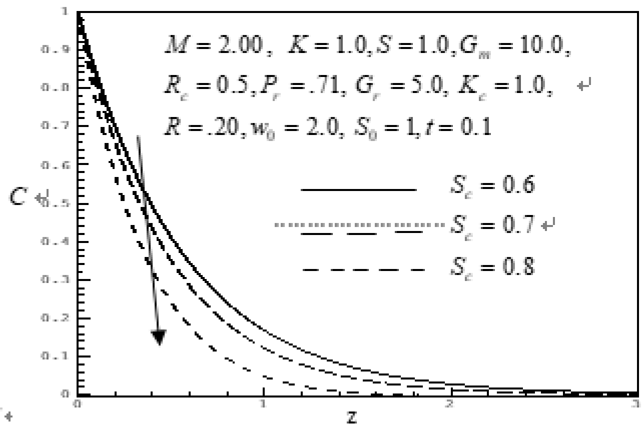


Fig. 9. Concentration Distribution for Different Values of the Schmidt Number (S_c)

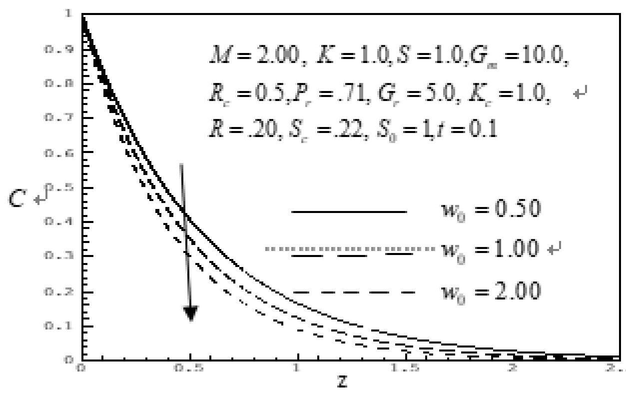


Fig. 10. Concentration Distribution for Different Values of the Suction Parameter (w_0)

Case ii: Comparison

The unsteady flow of an incompressible visco-elastic fluid flow with heat and mass transfer along an oscillating porous plate under the influence of uniform transvers magnetic field has been discussed. The effect of non-dimensional parameters on the governing flow such as Prandlt number (P_r), Hartmann number (M), Elastic parameter (R_c), Chemical reaction parameter (K_c), Heat source parameter (S), the Schmidt number (S_c), Suction parameter (w_0) on the velocity, temperature and concentration distributions have been studied analytically and presented with the help of Figures (11-16). Qualitative comparisons have been shown in Figures (11-16). The effect of Heat source parameter (S) on the velocity distributions u and v of Raghunath et al. (2016) are represented in Figures $(a,\ b)$ of (11) and present work are represented in Figures $(\mathrm{c},\ \mathrm{d})$ of (11). In Figure 11(c), it is noticed that the velocity distribution u decreases with the increase of the value of $(S\ )$ and in Figure 11(d), there is no effect on velocity distribution v with the increase of (S). In Figures $(a,\ b)$ of (12-13), the effect of Hartmann number (M) and the suction parameter (w_0) on the velocity distributions u and v of Raghunath et al. (2016) are represented and the effect of the Hartmann number (M) and the suction parameter (w_0) on the distributions u and v of present work are represented in Figures $(\mathrm{c},\ \mathrm{d})$ of (12-13). In Figure(c) of (12-13), the velocity distribution u decreases with the increase of (M) and (w_0). And in Figure (d) of (12-13), the velocity distribution v increases with the increase of (M) and (w_0).The velocity distributions u and v of Raghunath et al. (2016) for the elastic parameter (R_c) are showed in Figures $(a,\ b)$ of (14) and the effect of present work are represented in Figures $(\mathrm{c},\ \mathrm{d})$ of (14). In Figures 14(c), it is found that the velocity distribution u increases with the increase of (R_c) and in Figures 14(d), there is no effect on velocity distribution v with the increase of (R_c). Figures $(a,\ b, c)$ of (15) depict the effect of Prandlt number (P_r), Heat source parameter (S) and the suction parameter (w_0) on the temperature distribution of Raghunath et al. (2016) and Figures $(\mathrm{d},\ \mathrm{e}, f)$ of (15) depict the effect of the Prandlt number (P_r), the Heat source parameter (S) and the suction parameter (w_0) on the temperature distribution of present work. It is observed that the temperature distribution decreases with the increase of $(P_r),\ (S)$ and (w_0).Figures $(a,\ b, c)$ of (16) depict the effect of the chemical reaction parameter(K_c), the Schmidt number (S_c) and the suction parameter (w_0) on the concentration distribution of Raghunath et al. (2016) and Figures $(\mathrm{d},\ \mathrm{e},\ f)$ of (16) depict the effect of the chemical reaction parameter (K_c), the Schmidt number (S_c) and the suction parameter (w_0) on the temperature distribution of present work. It is observed that the concentration distribution decreases with the increase of (Kc), (Sc) and (w_0).

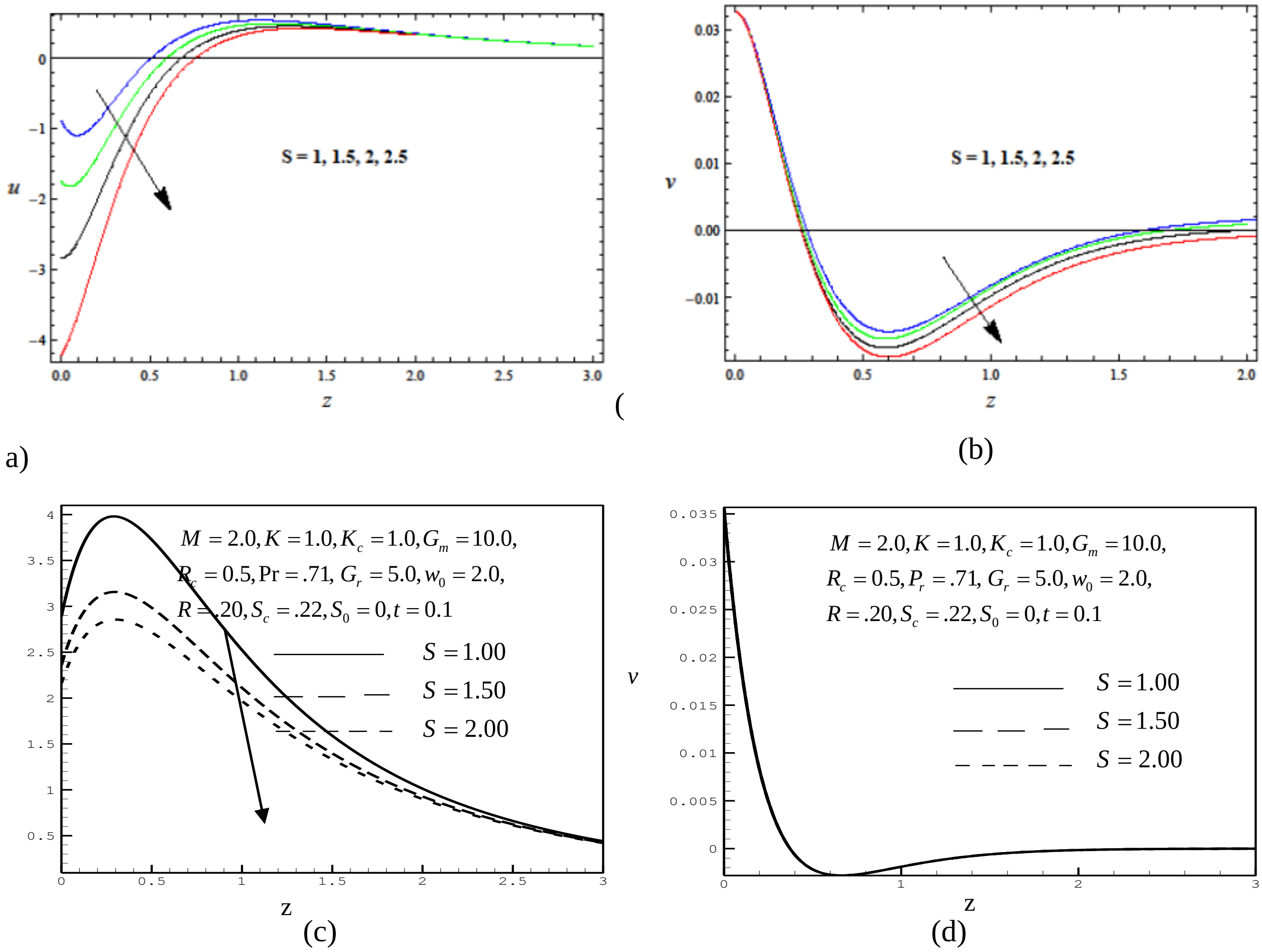
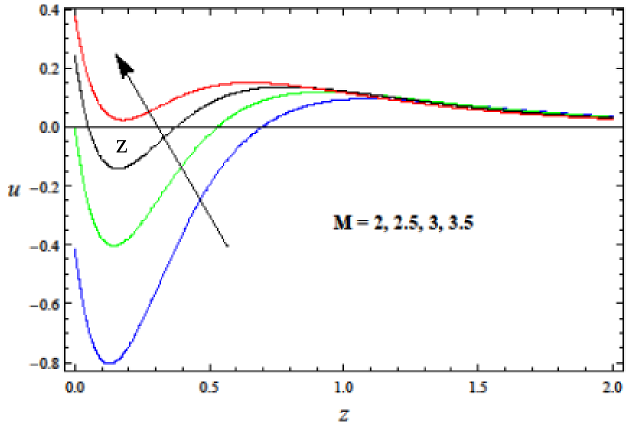
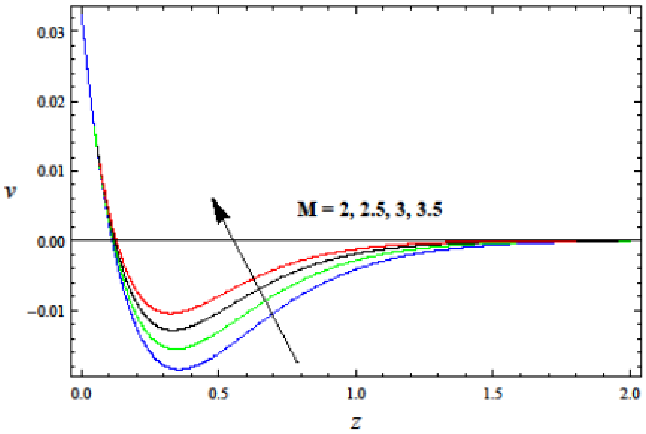


Fig. 11. The velocity distribution for the Heat source parameter (S) of Raghunath et al. (2016) in figures (a), (b) and the velocity distribution for the Heat source parameter (S) of present work in figures (c), (d) with t=0.1 and S_0=0

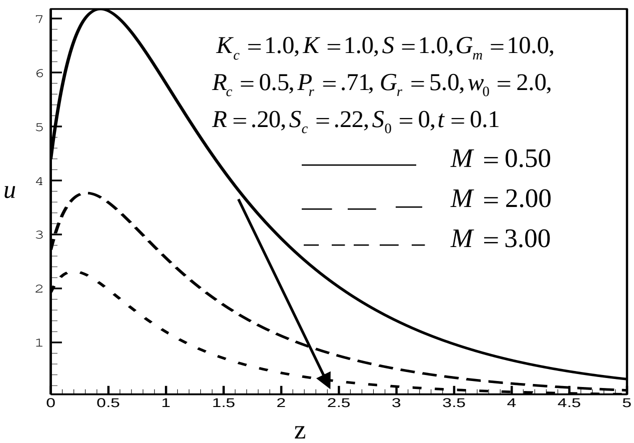
u



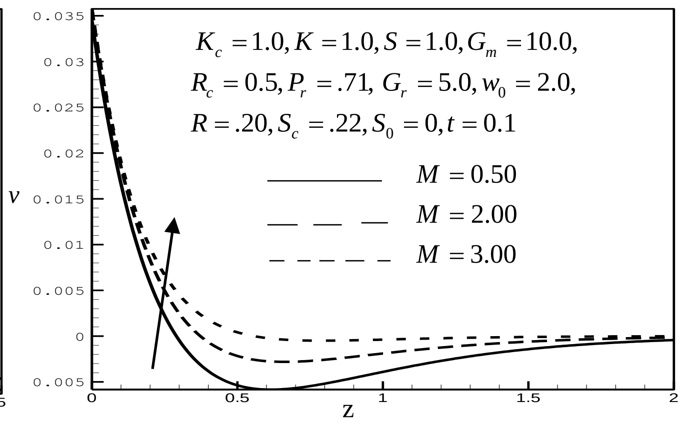
(a)



(b)

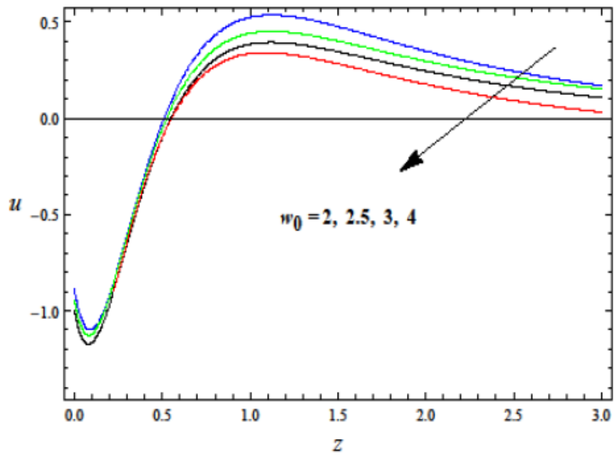


(c)

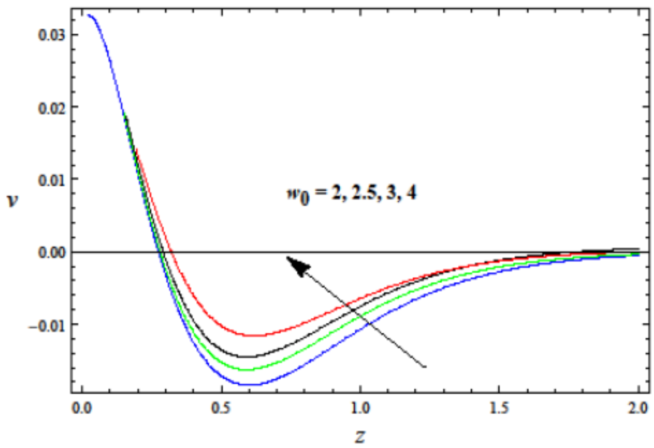


(d)

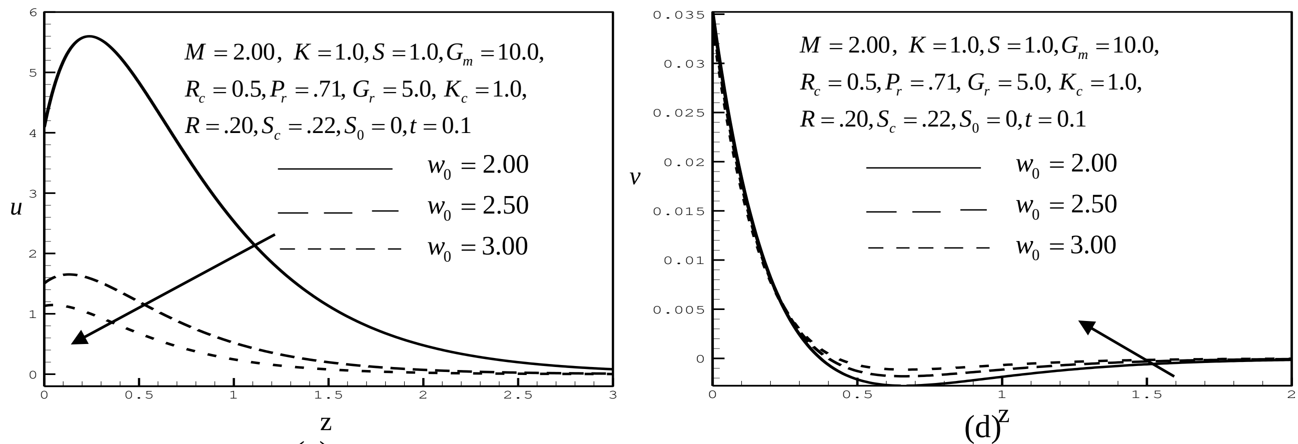
Fig. 12. The Velocity Distribution for Hartmann Number (M) of Raghunath et al. (2016) in Figures (a), (b) and the Velocity Distribution for Hartmann Number (M) of Present Work in Figures (c), (d) with t=0.1 and S_0=0



(a)

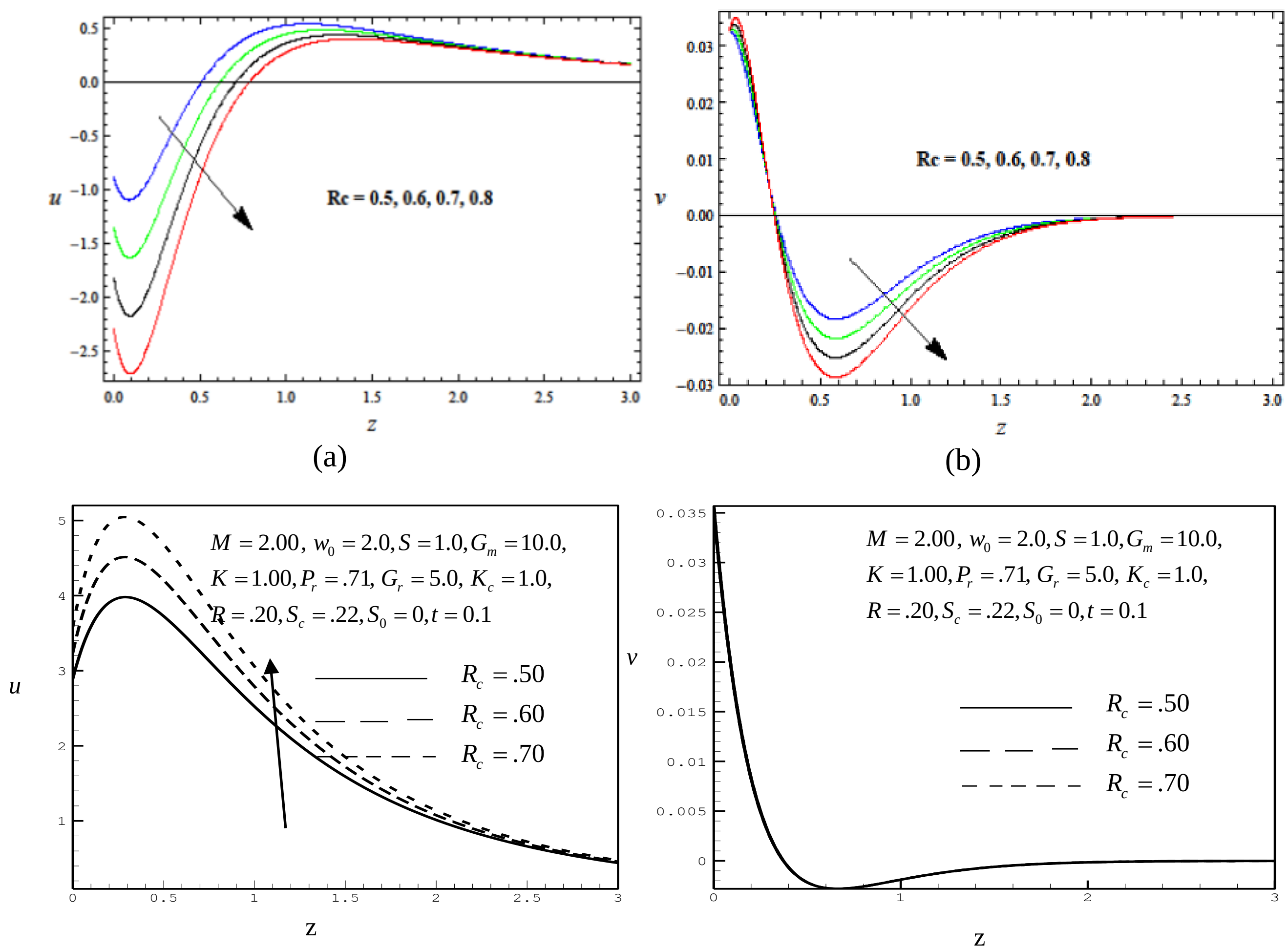


(b)



(c) (d)

Fig. 13. The Velocity Distribution for Suction Parameter (w_0) of Raghunath et al. (2016) in Figures (a), (b) and the Velocity Distribution for Suction Velocity (w_0) of Present Work in Figures (c), (d) with t=0.1 and S_0=0



(c) (d)

Fig. 14. The Velocity Distribution for Elastic Parameter (R_c) of Raghunath et al. (2016) in Figures (a), (b) and the Velocity Distribution for Elastic Parameter (R_c) of Present Work in Figures (c), (d) with t=0.1 and S_0=0

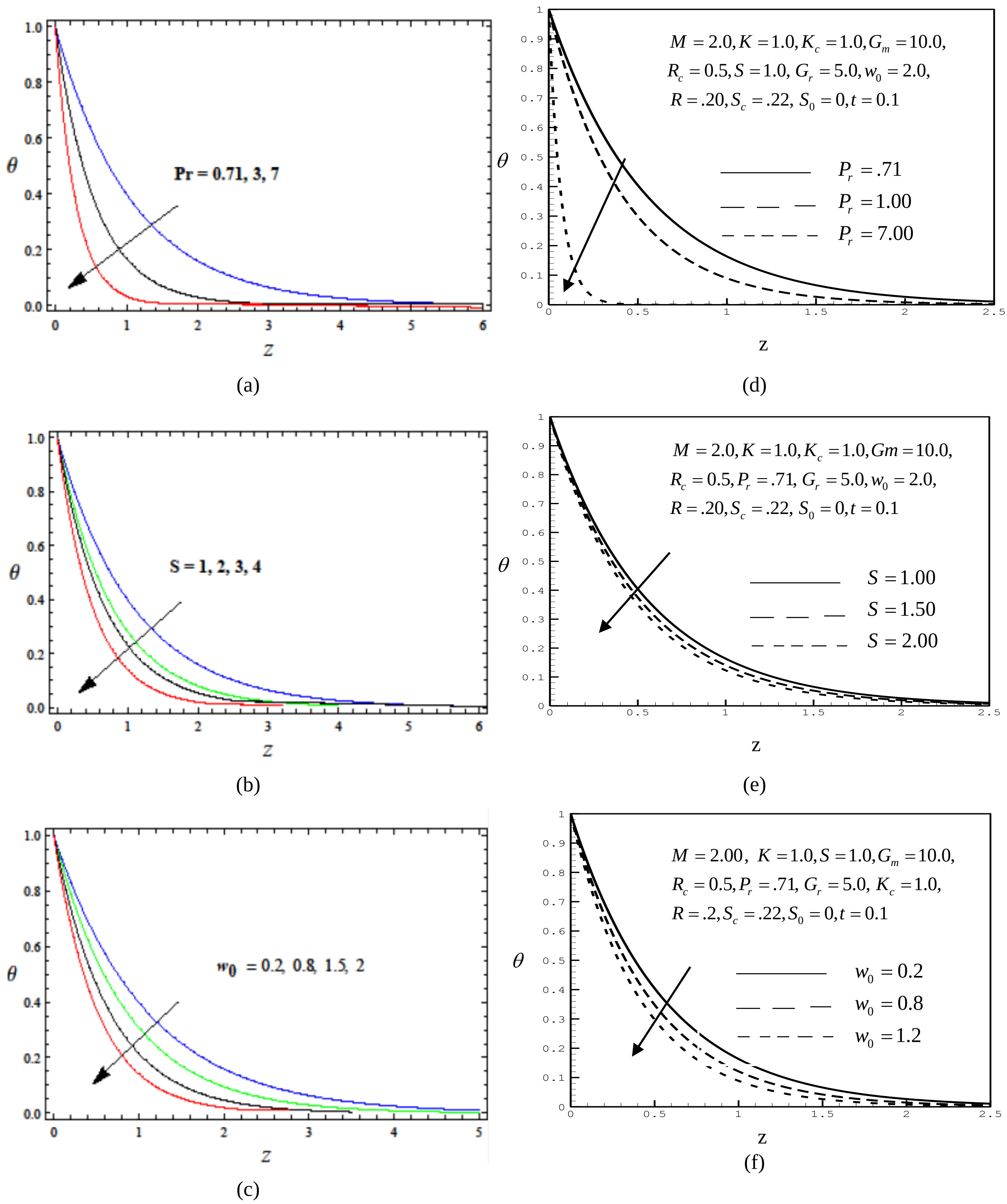


Fig. 15. Temperature Distributions for Prandlt Number (P_r), heat source parameter (S) and suction velocity (w_0) of Raghunath et al. (2016) in (a), (b), (c) and Present Work in (d), (e), (f) with t=0.1 and S_0=0

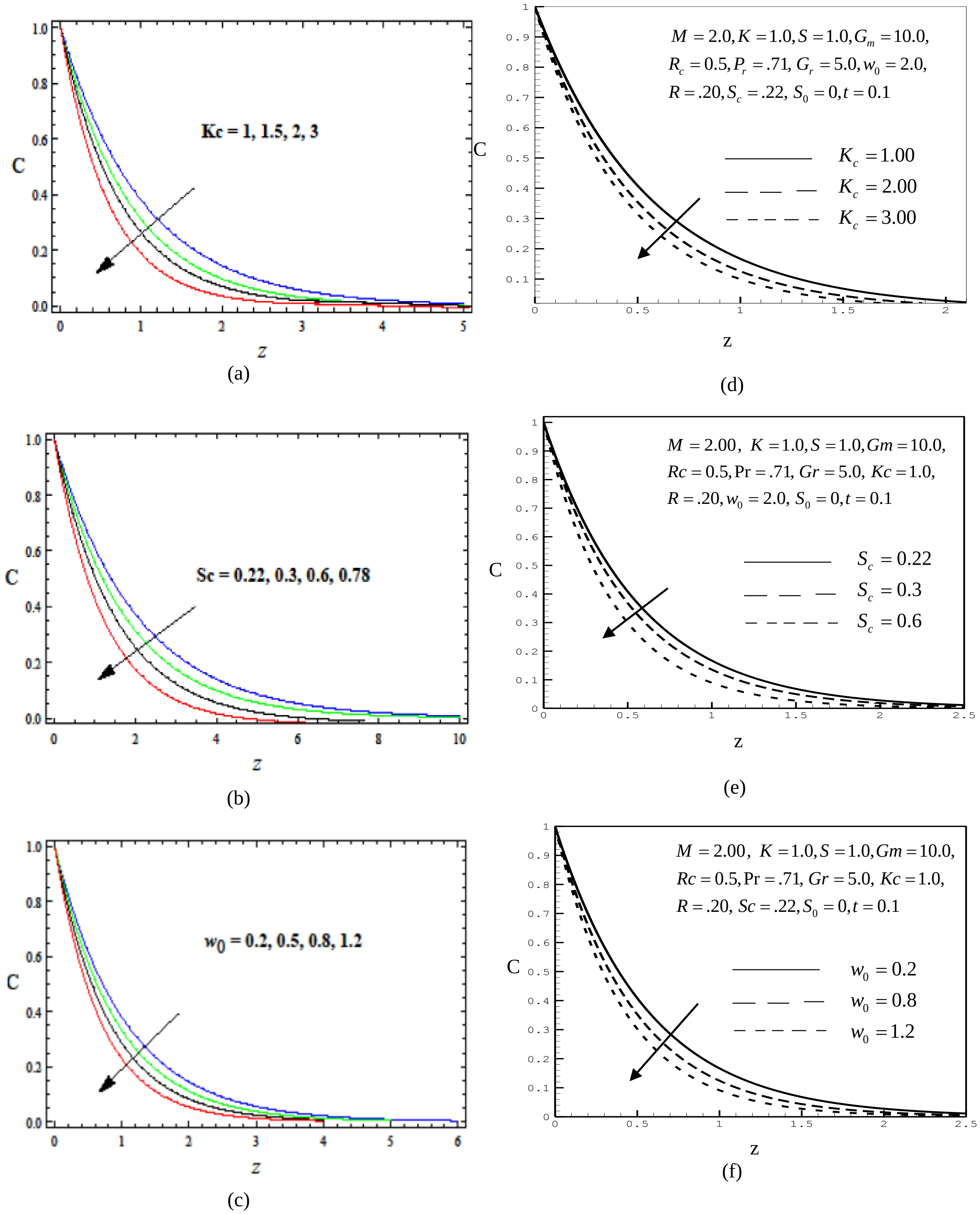


Fig. 16. Concentration Distributions for Chemical Reaction Parameter (K_c), Schmidt Number (S_c) and Suction Parameter (w_0) of Raghunath et al. (2016) in (a), (b), (c) and Present Work in (d), (e), (f) with t=0.1 and S_0=0.

z

Conclusions

MHD visco-elastic fluid have been studied for the unsteady flow of an incompressible fluid through a porous medium with simultaneously infinite oscillating plate. The resulting system of dimensionless non-linear coupled partial differential equations are analytically solved by a perturbation method. The results have been discussed for different values of parameters as Prandlt number, Hartmann number, Porosity parameter, the elastic parameter, Chemical reaction parameter, Heat source parameter, Scmidt number, the suction parameter and Soret number. The effects of different parameters on velocity, temperature and concentration distribution have been discussed graphically. The comparison between Raghunath et al. (2016) and present solution have been presented graphically for velocity profile, temperature distribution and concentration distribution.

In present work the velocity component u decreases with the increase of w_0, M and K. and the velocity component v increases with the increase of M , w_0 and the velocity component v decreases with the increase of K . In Raghunath et al. (2016) the velocity component u and v increase with the increase of M .Also the velocity component u decreases with the increase of w_0 and the velocity component v increases with the increase of w_0.

In present work the velocity component u increase with the increase of R_c and S_0 and there is no variation of velocity component v with the increase of R_c and S_0. In Raghunath et al. (2016) the velocity component u and v decrease with the increase of R_c .

In present work the velocity component u decreases with the increase of P_r, S and K_c and there is no variation of velocity component v with the increase of P_r, S and K_c. In Raghunath et al. (2016) the velocity component u and v decrease with the increase of S .

The temperature distributions decrease with the increase of the values of P_r, S and w_0 in both work.

The concentration distributions decrease with the increase of the values of K_c, S_c and w_0 in both work.

In this analysis, the effects of different parameters on the velocity, temperature distributions and concentration distributions have been discussed graphically. The analytic solution of Raghunath et al. (2016) have analysis error. The correct analysis has been shown in the field of comparison. The comparison has been drawn in graphically. From the comparison it has been found that the two solutions are qualitatively similar but not quantitatively similar and in maximum case it represents the same trend.

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Nomenclature

S_c	Schmidt number	M	Hartmann number
S_o	Soret number	K	Permeability parameter
G_r	Grashof number	β	Volumetric coefficient of thermal expansion
G_m	Modified Grashof number	w_0	Suction Velocity
D_T	Thermal diffusivity	δ	Boundary layer thickness
K_c	Chemical reaction parameter	μ	Co-efficient of viscosity
P_r	Prandtl number	υ	Co-efficient of kinematic viscosity
C	Concentration	k_1	Dimensionless permeability parameter
R_c	Elastic parameter	ρ	Boundary layer fluid density
u	Velocity components in X-direction	k_0	Dimensionless elastic parameter
v	Velocity components in Y-direction	θ	Dimensionless temperature
S	Heat Source Parameter	T	Dimensional temperature

Appendix

$$m_2=\frac{\Pr w_0+\sqrt{\Pr^2 w_0^2+4\Pr S}}{2}, m_6=\frac{w_0 Sc+\sqrt{w_0^2 Sc^2+4KcSc}}{2}$$

$$m_{10}=\frac{w_0+\sqrt{w_0^2+4\left(M^2+\frac{1}{K}\right)}}{2}, m_{12}=\frac{w_0+\sqrt{w_0^2+4\left(M^2+\frac{1}{K}+i\omega\right)}}{2}$$

$$a_1=-\frac{ScS_0m_2^2}{m_2^2+Scw_0m_2-KcSc}$$

$$a_2=1-a_1,\ a_3=\frac{-Gr}{m_2^2+w_0m_2-\left(M^2+\frac{1}{K}\right)}$$

$$a_4=\frac{-Gma_1}{m_2^2+w_0m_2-\left(M^2+\frac{1}{K}\right)}\ ,\ a_5=\frac{-Gma_2}{m_6^2+w_0m_6-\left(M^2+\frac{1}{K}\right)}$$

$$a_6=\frac{-1}{(1+Rm_{10})}\{(a_3(1+Rm_2)+a_4(1+Rm_2)+a_5(1+Rm_6)\}$$

$$a_7=\frac{1}{1+Rm_{12}}\ ,\ a_8=\frac{-w_0a_3m_2^3}{m_2^2+w_0m_2-\left(M^2+\frac{1}{K}\right)}$$

$$a_9=\frac{-w_0a_4m_2^3}{m_2^2+w_0m_2-\left(M^2+\frac{1}{K}\right)}$$

$$a_{10}=\frac{-w_0a_5m_2^3}{m_6^2+w_0m_6-\left(M^2+\frac{1}{K}\right)}$$

$$a_{11}=\frac{-w_0a_6m_{10}^3}{m_{10}^2+w_0m_2-\left(M^2+\frac{1}{K}\right)}$$

$$a_{12}=\frac{-1}{(1+Rm_{10})}\{(a_8(1+Rm_2)+a_9(1+Rm_2)+a_{10}(1+Rm_6)+a_{11}(1+Rm_{10})\}$$

$$a_{13}=\frac{\left(-m_{12}^3w_0+i\omega m_{12}^2\right)a_7}{m_{12}^2+w_0m_{12}-\left(M^2+\frac{1}{K}+i\omega\right)}$$

$$a_{14}=-a_{13}\ ,\ a_{15}=a_{13}+a_{14}$$