

## Elastic buckling analysis of uniaxially compressed CCSS Kirchhoff plate using single finite Fourier sine integral transform method

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### ABSTRACT

In this study, the single Fourier sine integral transform method was used to solve the elastic buckling problem of Kirchhoff rectangular plates simply supported at two opposite edges  $x = 0$ , and  $x = a$  and clamped at the other two edges  $y = 0$ , and  $y = b$ . The problem considered was that for uniaxial uniform compressive load in the  $x$  coordinate direction. The single finite Fourier sine integral transformation was applied to the governing fourth order partial differential equation of Kirchhoff plates under uniaxial uniform in-plane compressive loads to convert the problem to a fourth order ordinary differential equation in terms of the finite Fourier sine transform space variables. Solution of the ordinary differential equations yielded the buckling modal shape functions in the Fourier sine transform variables. Enforcement of boundary conditions along the  $y$  direction at  $y = 0$ , and  $y = b$  yielded an algebraic eigenvalue eigenvector problem which was solved to obtain non-trivial solutions. The characteristic buckling equation was obtained by requiring the vanishing of the matrix of coefficients as the transcendental equation involving the buckling load. The buckling load was obtained by solving the transcendental equation for various assumed values of the plate aspect ratios. Critical buckling loads for various values of the plate aspect ratio were found to be identical with classical solutions obtained in the technical literature. The present study thus yielded exact solution for the buckling loads and buckling modes of uniaxially compressed Kirchhoff plates; illustrating the effectiveness of the analytical tool.

## 1. INTRODUCTION

Navier and Saint Venant presented equations for the elastic buckling analysis of rectangular thin plates. Navier's equations included twisting forces while Saint Venant improved on Navier's equation by further incorporating the axially applied compressive edge forces and shearing forces. The differential equations derived variously by Navier and Saint Venant provided the foundations for the research work on the elastic stability of thin rectangular plates under uniaxial, biaxial and shear buckling. The basic problem of elastic buckling of plates is thus to determine the minimum loads (buckling loads) and the associated shapes (buckling modal shapes) at which the plate would collapse (fail) when subjected to uniaxial, biaxial or shear compressive forces for given edges support conditions [1-3]. Elastic buckling problems can be solved using classical analytical method or approximate numerical methods. Ibearugbulem and Ezeh [4] used algebraic shape functions derived from a truncation of the infinite Taylor – Maclaurin's series for the particular case of clamped ends of thin beams in the Ritz variational method to solve the elastic buckling problem of uniaxially uniformly compressed rectangular Kirchhoff plates with clamped edges. Ezeh et al [5] used the Galerkin's indirect variational method to solve the elastic buckling problem of rectangular Kirchhoff plates with clamped edges under uniform axial compression in one axis only (uniaxial uniform compression case).

Ibearugbulem et al. [6] presented the elastic buckling analysis of uniaxially compressed simply supported thin

rectangular plate using the truncated Taylor – Maclaurin shape function by using the one parameter Ritz variational method. They obtained solutions that approximated exact solutions from the technical literature.

Nwoji et al. [7] presented the elastic buckling analysis of simply supported thin plates using the double finite Fourier sine integral transform method. They considered two cases of uniaxial uniform compressive loading and biaxial compressive loading; and obtained solutions that agreed with the solutions from literature.

Timoshenko [8] solved the elastic buckling problems of thin rectangular plates under uniaxial uniform compression load for various edge support conditions. Timoshenko presented solutions using the Navier method and using energy minimization techniques. He obtained characteristic buckling loads and characteristic buckling modal shapes that agreed with laboratory findings presented by Bridget et al [9].

In this work, the single finite Fourier sine transform method is used to solve the elastic buckling problem of uniaxially compressed Kirchhoff plate with simply supported edges  $x = 0$ , and  $x = a$  and clamped edges  $y = 0$ , and  $y = b$ .

## 2. RESEARCH AIM AND OBJECTIVES

The general aim of this study is to use the single finite Fourier sine transform method to solve the elastic buckling problem of uniaxially compressed Kirchhoff plate with simply

supported edges  $x = 0$ ,  $x = a$ , and clamped edges  $y = 0$  and  $y = b$ . The specific objectives include:

- (i) to apply the single finite Fourier sine integral transformation to the governing fourth order partial differential equation of Kirchhoff plates under uniaxial uniform compressive load.
- (ii) to show that the boundary value problem simplifies to an algebraic eigenvalue – eigenvector problem in terms of the Fourier sine transform variable.
- (iii) to solve the resulting algebraic eigenvalue – eigenvector problem, and hence determine the characteristic buckling equations for the elastic buckling problem of CCSS plate considered.
- (iv) to determine the characteristic buckling loads and the buckling modes.

### 3. THEORETICAL FRAMEWORK/GOVERNING PARTIAL DIFFERENTIAL EQUATIONS

The governing partial differential equations PDE for the elastic buckling of Kirchhoff plates is given by:

$$D \left( \frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right) + N_{xx} \frac{\partial^2 w}{\partial x^2} + N_{yy} \frac{\partial^2 w}{\partial y^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} = 0 \quad (1)$$

where  $N_{xx}$  is the uniform compressive force in the  $x$  direction  
 $N_{yy}$  is the uniform compressive force in the  $y$  direction  
 $N_{xy}$  is the twist,  $w$  is the deflection,  $x$  and  $y$  are the inplane Cartesian coordinates  
 $D$  is the flexural rigidity of the plate given by

$$D = \frac{Eh^3}{12(1-\mu^2)} \quad (2)$$

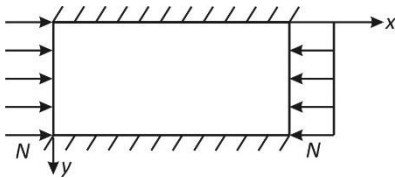


Figure 1. Kirchhoff plate under uniform compression

$E$  is the Young's modulus of elasticity of the plate material,  
 $h$  is the plate thickness

$\mu$  is the Poisson's ratio of the plate material.

For the elastic buckling problem of Kirchhoff plate under uniaxial compression along the  $x$  axis which is shown in Figure 1, we have  $N_{xx} = N_0$ ,  $N_{yy} = 0$ ,  $N_{xy} = 0$ , and the governing PDE simplifies to:

$$D \left( \frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right) + N_0 \frac{\partial^2 w}{\partial x^2} = 0$$

where  $0 \leq x \leq a$ ,  $0 \leq y \leq b$

The boundary conditions for simply supported edges  $x = 0$ ,  $x = a$ , and clamped edges  $y = 0$ ,  $y = b$  are:

$$w(x = 0, y) = 0 \quad (3)$$

$$w(x = a, y) = 0 \quad (4)$$

$$M_{xx}(x = 0, y) = 0 \quad (5)$$

$$M_{xx}(x = a, y) = 0 \quad (6)$$

$$-D \left( \frac{\partial^2 w}{\partial x^2} + \mu \frac{\partial^2 w}{\partial y^2} \right) \Big|_{(x=0,y)} = M_{xx} \Big|_{x=0,y} = 0 \quad (7)$$

$$-D \left( \frac{\partial^2 w}{\partial x^2} + \mu \frac{\partial^2 w}{\partial y^2} \right) \Big|_{(x=a,y)} = M_{xx} \Big|_{x=a,y} = 0 \quad (8)$$

$$w(x, y = 0) = 0 \quad (9)$$

$$w(x, y = b) = 0 \quad (10)$$

$$\frac{\partial w}{\partial y}(x, y = 0) = 0 \quad (11)$$

$$\frac{\partial w}{\partial y}(x, y = b) = 0 \quad (12)$$

### 4. METHODOLOGY

Applying the finite Fourier sine transformation to Equation (2), we have:

$$\int_0^a \left\{ D \left( \frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right) + N_0 \frac{\partial^2 w}{\partial x^2} \right\} \sin \frac{m\pi x}{a} dx = 0 \quad (13)$$

$$\int_0^a \left( \frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} + \frac{N_0}{D} \frac{\partial^2 w}{\partial x^2} \right) \sin \frac{m\pi x}{a} dx = 0 \quad (14)$$

$$\int_0^a \frac{\partial^4 w}{\partial x^4} \sin \frac{m\pi x}{a} dx + 2 \int_0^a \frac{\partial^4 w}{\partial x^2 \partial y^2} \sin \frac{m\pi x}{a} dx + \int_0^a \frac{\partial^4 w}{\partial y^4} \sin \frac{m\pi x}{a} dx + \frac{N_0}{D} \int_0^a \frac{\partial^2 w}{\partial y^2} \sin \frac{m\pi x}{a} dx = 0 \quad (15)$$

$$\left( \frac{m\pi}{a} \right)^4 \int_0^a w(x, y) \sin \frac{m\pi x}{a} dx - 2 \left( \frac{m\pi}{a} \right)^2 \frac{\partial^2}{\partial y^2} \int_0^a w(x, y) \sin \frac{m\pi x}{a} dx + \frac{\partial^4}{\partial y^4} \int_0^a w(x, y) \sin \frac{m\pi x}{a} dx - \left( \frac{m\pi}{a} \right)^2 \frac{N_0}{D} \int_0^a w(x, y) \sin \frac{m\pi x}{a} dx = 0 \quad (16)$$

$$\text{Let } \int_0^a w(x, y) \sin \frac{m\pi x}{a} dx = W(k, y) \quad (17)$$

where  $W(k, y)$  is the finite Fourier sine transform of  $w(x, y)$

$$\text{let } \frac{m\pi}{a} = \beta_m \quad (18)$$

Then,

$$\begin{aligned} \beta_m^4 W(k, y) - 2\beta_m^2 \frac{\partial^2}{\partial y^2} W(k, y) + \frac{\partial^4}{\partial y^4} W(k, y) - \\ \beta_m^2 \frac{N_0}{D} W(k, y) = 0 \end{aligned} \quad (19)$$

$$\begin{aligned} \frac{\partial^4}{\partial y^4} W(k, y) - 2\beta_m^2 \frac{\partial^2}{\partial y^2} W(k, y) + \\ \left( \beta_m^4 - \beta_m^2 \frac{N_0}{D} \right) W(k, y) = 0 \end{aligned} \quad (20)$$

Due to the geometric restraints of fixed supports on the edges  $y = 0$ , and  $y = b$ , the buckling load  $N_0$  is such that  $N_0 > D\beta_m^2$ .

The solution to the fourth order ordinary differential equation (ODE) is, using the method of  $D$  – operators:

$$W(k, y) = c_1 \cosh \alpha_1 y + c_2 \sinh \alpha_1 y + c_3 \cos \alpha_2 y + c_4 \sin \alpha_2 y \quad \dots(21)$$

where

$$\alpha_1^2 = \left( \beta_m^2 \frac{N_0}{D} \right)^{1/2} + \beta_m^2 \quad (22)$$

$$\alpha_2^2 = \left( \frac{\beta_m^2 N_0}{D} \right)^{1/2} - \beta_m^2 \quad (23)$$

and  $c_1(k)$ ,  $c_2(k)$ ,  $c_3(k)$  and  $c_4(k)$  are the four constants of integration corresponding to the fourth order of the ODE.

Application of the finite Fourier sine transformation to Equation (7) yields:

$$\int_0^a -D \left( \frac{\partial^2 w}{\partial x^2} + \mu \frac{\partial^2 w}{\partial y^2} \right) \sin \frac{m\pi x}{a} dx = \int_0^a M_{xx} \sin \frac{m\pi x}{a} dx = M_{xx}(k, y) \quad \dots(24)$$

$$-D \left( -\left( \frac{m\pi}{a} \right)^2 \int_0^a w(x, y) \sin \frac{m\pi x}{a} dx + \mu \frac{\partial^2}{\partial y^2} \int_0^a w(x, y) \sin \frac{m\pi x}{a} dx \right) = M_{xx}(k, y) \quad (25)$$

$$-\beta_m^2 W(k, y) + \mu \frac{\partial^2}{\partial y^2} W(k, y) = -\frac{M_{xx}}{D} \quad (26)$$

along the edge,

$$-\beta_m^2 W(k, y) + \mu \frac{\partial^2}{\partial y^2} W(k, y) = 0 \quad (27)$$

The solution for  $W(k, y)$  is made of hyperbolic functions.

## 5. RESULTS

The solutions for  $W(k, y)$  that agree with the boundary conditions is then found. Application of the finite Fourier sine transformation to the boundary conditions Equations (3 – 8) yield:

$$\begin{aligned} \frac{\partial W(k, y)}{\partial y} = c_1 \alpha_1 \sinh \alpha_1 y + c_2 \alpha_1 \cosh \alpha_1 y - \\ c_3 \alpha_2 \sin \alpha_2 y + c_4 \alpha_2 \cos \alpha_2 y \end{aligned} \quad (28)$$

$$W(k, y = 0) = c_1 + c_3 = 0 \quad (29)$$

$$c_1 = -c_3 \quad (30)$$

$$c_3 = -c_1 \quad (31)$$

$$\frac{\partial W}{\partial y}(k, y = 0) = c_2 \alpha_1 + c_4 \alpha_2 = 0 \quad (32)$$

$$c_2 = -c_4 \frac{\alpha_2}{\alpha_1} \quad (33)$$

$$c_4 = \frac{-c_2 \alpha_1}{\alpha_2} \quad (34)$$

$$\begin{aligned} W(k, y = b) = c_1 \cosh \alpha_1 b + c_2 \sinh \alpha_1 b + \\ c_3 \cos \alpha_2 b + c_4 \sin \alpha_2 b = 0 \end{aligned} \quad (35)$$

$$\begin{aligned} \frac{\partial W}{\partial y}(k, y = b) = c_1 \alpha_1 \sinh \alpha_1 b + c_2 \alpha_1 \cosh \alpha_1 b - \\ c_2 \alpha_2 \sin \alpha_2 b + c_4 \alpha_2 \cos \alpha_2 b = 0 \end{aligned} \quad (36)$$

In matrix form,

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & \alpha_1 & 0 & \alpha_2 \\ \cosh \alpha_1 b & \sinh \alpha_1 b & \cos \alpha_2 b & \sin \alpha_2 b \\ \alpha_1 \sinh \alpha_1 b & \alpha_1 \cosh \alpha_1 b & -\alpha_2 \sin \alpha_2 b & \alpha_2 \cos \alpha_2 b \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (37)$$

Then,

$$\begin{aligned} W(k, y) = c_1 \cosh \alpha_1 y - c_1 \cos \alpha_2 y + \\ c_2 \sinh \alpha_1 y - \frac{c_2 \alpha_1}{\alpha_2} \sin \alpha_2 y \end{aligned} \quad (38)$$

$$W(k, y) = c_1 (\cosh \alpha_1 y - \cos \alpha_2 y) + c_2 \left( \sinh \alpha_1 y - \frac{\alpha_1}{\alpha_2} \sin \alpha_2 y \right) \quad (39)$$

$$\frac{\partial W}{\partial y} = c_1 (\alpha_1 \sinh \alpha_1 y + \alpha_2 \sin \alpha_2 y) + c_2 (\alpha_1 \cosh \alpha_1 y - \alpha_1 \cos \alpha_2 y) \quad (40)$$

The boundary conditions Equations (35) and (36) yield:

$$c_1 (\cosh \alpha_1 b - \cos \alpha_2 b) + c_2 \left( \sinh \alpha_1 b - \frac{\alpha_1}{\alpha_2} \sin \alpha_2 b \right) = 0 \quad (41)$$

or,

$$c_1 (\alpha_1 \sinh \alpha_1 b + \alpha_2 \sin \alpha_2 b) + c_2 \alpha_1 (\cosh \alpha_1 b - \cos \alpha_2 b) = 0 \quad (42)$$

In matrix form, we obtain the homogeneous equations:

$$\begin{pmatrix} (\cosh \alpha_1 b - \cos \alpha_2 b) & \left( \sinh \alpha_1 b - \frac{\alpha_1}{\alpha_2} \sin \alpha_2 b \right) \\ (\alpha_1 \sinh \alpha_1 b + \alpha_2 \sin \alpha_2 b) & \alpha_1 (\cosh \alpha_1 b - \cos \alpha_2 b) \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \dots(43)$$

For non-trivial solutions,  $\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \neq 0$

The characteristic buckling equation becomes:

$$\begin{vmatrix} (\cosh \alpha_1 b - \cos \alpha_2 b) & \left( \sinh \alpha_1 b - \frac{\alpha_1}{\alpha_2} \sin \alpha_2 b \right) \\ (\alpha_1 \sinh \alpha_1 b + \alpha_2 \sin \alpha_2 b) & \alpha_1 (\cosh \alpha_1 b - \cos \alpha_2 b) \end{vmatrix} = 0 \quad (44)$$

Expanding the determinant,

$$\alpha_1 (\cosh \alpha_1 b - \cos \alpha_2 b)^2 - \left( \sinh \alpha_1 b - \frac{\alpha_1}{\alpha_2} \sin \alpha_2 b \right) \times (\alpha_1 \sinh \alpha_1 b + \alpha_2 \sin \alpha_2 b) = 0 \quad (45)$$

$$\alpha_1 (\cosh^2 \alpha_1 b - 2 \cosh \alpha_1 b \cos \alpha_2 b + \cos^2 \alpha_2 b) - \left( \alpha_1 \sinh^2 \alpha_1 b - \frac{\alpha_1^2}{\alpha_2^2} \sinh \alpha_1 b \sin \alpha_2 b + \alpha_2 \sin \alpha_2 b \sinh \alpha_1 b - \alpha_1 \sin^2 \alpha_2 b \right) = 0 \quad (46)$$

Further simplification yields the characteristic buckling equation as:

$$2(1 - \cosh \alpha_1 b \cos \alpha_2 b) + \left( \frac{\alpha_1}{\alpha_2} - \frac{\alpha_2}{\alpha_1} \right) \sinh \alpha_1 b \sin \alpha_2 b = 0 \quad \dots(47)$$

The buckling loads are obtained by solving the characteristic buckling equation for  $N_0$ .

The critical buckling stress  $(\sigma_{xx})_{cr}$  is obtained as:

$$(\sigma_{xx})_{cr} = \frac{k_b E}{12(1 - \mu^2)} \left( \frac{h}{b} \right)^2 \quad (48)$$

For  $\mu = 0.25$ , the solution for the characteristic buckling equation for various values of the plate aspect ratio ( $a/b$ ) are presented in terms of the dimensionless critical buckling load,  $\bar{N}_{cr}$ , defined as:

$$\bar{N}_{cr} = \frac{N_{cr} b^2}{\pi^2 D} \quad (49)$$

**Table 1.** Critical buckling loads  $N_{cr}$  of CCSS Kirchhoff plates subject to uniform uniaxial compressive forces ( $N_{xx} = -N_0$ )

$a/b$	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
$N_{cr} = \bar{N}_{cr} \left( \frac{\pi^2 D}{b^2} \right)$	9.448	7.055	7.304	7.691	7.055	7.001	7.304	7.055	6.972

**Table 2.** Critical buckling stress for CCSS plate for  $E = 58.8 \times 10^9 \text{ N/m}^2$ ,  $h/b = 0.02$ ,  $\mu = 0.3$

$a/b$	$N_0$	$\sigma_{xx cr} (\text{N/m}^2)$
0.2	27.86	$5.927 \times 10^8$
0.4	9.49	$2.021 \times 10^8$
0.8	7.44	$1.583 \times 10^8$
1.0	7.69	$1.638 \times 10^8$
$\sqrt{2}$	7.04	$1.498 \times 10^8$
2	6.99	$1.487 \times 10^8$
$\sqrt{6}$	7.02	$1.494 \times 10^8$
3.2	6.98	$1.486 \times 10^8$
$\sqrt{12}$	6.99	$1.487 \times 10^8$
4.5	6.98	$1.485 \times 10^8$

## 6. DISCUSSION

In this work, the single finite Fourier sine transformation method was successfully used to solve the elastic buckling problem of rectangular Kirchhoff plate with two opposite

edges ( $x = 0$ , and  $x = a$ ) simply supported and the other two opposite edges ( $y = 0$  and  $y = b$ ) clamped. The elastic buckling solutions were obtained for the case when the edges  $x = 0$  and  $x = a$  are subjected to uniform uniaxial compressive load  $N_x = N_0$  and  $N_y = 0$ ,  $N_{xy} = 0$ . The elastic buckling problem solved was presented as a boundary value problem defined by the domain equation-Equation 2 – and the boundary conditions-Equations (3) – (12).

The finite sine transformation was applied to the domain partial differential equation with respect to the  $x$  – coordinate variable in Equation (13) and the linearity property of the integral transformation, and the Dirichlet boundary conditions on the simply supported edges ( $x = 0$ ,  $x = a$ ) led to the simplification of the problem to an ordinary fourth order homogeneous differential equation (Equation (20) in terms of  $W(k, y)$  the deflection in the Fourier transformed space. The solution of Equation (22) using  $D$  – operator techniques, trial function methods, variation of parameters or other methods for solving ordinary differential equations resulted in the general solution for  $W(k, y)$  as Equation (21); a solution that contains four constants of integration corresponding to the fourth order nature of the ODE in Equation (20). Enforcement of boundary conditions on the  $y = 0$ , and  $y = b$  axes on the Equation (20) yielded a set of four homogeneous equations in terms of the

unknown integration constants. The algebraic eigenvalue – eigenvector problem resulting from the enforcement of boundary conditions yielded the matrix equation – Equation (37). Further use of the boundary conditions simplified the algebraic eigenvalue – eigenvector problem as Equation (43).

The characteristic buckling equation obtained from the requirement of non – trivial solution gave the determinantal equation – Equation (44). Expansion and simplification of the determinantal equation yielded the characteristic buckling equation as Equation (47). The characteristic buckling equation is found to be a transcendental equation in terms of  $\alpha_1$  and  $\alpha_2$  which are each functions of the buckling load  $N_0$  as given in Equations (22) and (23). Solution of the buckling equation yielded the buckling loads for various values of the plate aspect ratio  $a/b$  as presented in Table 1. The critical buckling stresses as were further calculated for various plate aspect ratios for Kirchhoff plate with simply supported edges ( $x = 0$  and  $x = a$ ) and clamped edges ( $y = 0$  and  $y = b$ ) and for plate material properties given by  $E = 58.8 \times 10^9 \text{ N/m}^2$ ,  $h = 0.02b$  and  $\mu = 0.30$ , and presented in Table 2.

## 7. CONCLUSIONS

The following conclusions can be made from the study:

- (i) The single Fourier sine transform method is an ideal mathematical/analytical tool for solving the boundary value problem of elastic buckling of Kirchhoff plates with two opposite simply supported edges ( $x = 0, x = a$ ) and two opposite clamped edges ( $y = 0, y = b$ ).
- (ii) The single Fourier sine transformation applied in the  $x$ -coordinate direction simplified the problem since the Dirichlet boundary conditions are satisfied in that direction, thus simplifying the Fourier sine transformation of partial derivatives with respect to  $x$ .
- (iii) The single finite Fourier sine transformation simplifies the boundary value problem of elastic buckling to an algebraic eigenvalue – eigenvector problem, which is easier to solve.
- (iv) The characteristic buckling equations obtained by enforcement of boundary conditions are the same as the characteristic buckling equations obtained by other scholars who applied Navier’s double trigonometric series methods, and total potential energy minimization methods.
- (v) The critical buckling loads obtained were exact solutions and were the same as critical buckling loads obtained by Timoshenko and other researchers who used total potential energy minimization methods.

## REFERENCES

- [1] Bouazza M, Quinas D, Yazid A, Hamouine A. (2012). Buckling of thin plates under uniaxial and biaxial compression. *Journal of Material Science and Engineering* 32(8): 487-492.
- [2] Chattopadhyay AP. (2011). Elastic buckling behaviour of plate and tubular structures. MSc Thesis Dept of Mechanical and Nuclear Engineering. Kansas State University, Manhattan Kansas.
- [3] Dima I. (2015). Buckling of flat thin plates under combined loading. *Incas Bulletin* 7(1): 83-96.

- <https://doi.org/10.13111/2066-8201.2015.7.1.8>
- [4] Ibearugbulem OM, Ezech JC. (2013). Instability of axially compressed CCCC thin rectangular plate using Taylor – Maclaurin’s series shape function on Ritz method. *Academic Research International* 4(1): 346-351.
  - [5] Ezech JC, Owus IM, Opara HE, Oguaghamba OA. (2014). Galerkin’s indirect variational method in elastic stability analysis of all edges clamped thin rectangular plates. *International Journal of Research in Engineering (IJRET)* 3(4): 674-679.
  - [6] Ibearugbulem OM, Osadebe NN, Ezech JC, Onwuka DO. (2011). Buckling analysis of axially compressed SSSS thin rectangular plate using Taylor – Maclaurin shape function. *International Journal of Civil and Structural Engineering* 2(2): 667–672.
  - [7] Nwoji CU., Onah HN, Mama BO, Ike CC, Ikwueze EU. (2017). Elastic buckling analysis of simply supported thin plates using the double finite Fourier sine integral transform method. *Explorematics Journal of Innovative Engineering and Technology* 1(1): 37-47.
  - [8] Timoshenko S, Lessells JM. (1925). *Applied Elasticity*. Westinghouse Technical Night School Press East Pittsburg, Pa, p. 544.
  - [9] Bridget FJ, Jerome CC, Vosseller AB. (1934). Some new experiments on buckling of thin wall construction. *Trans. ASME Aero Eng.* 56: 569-578.

## NOMENCLATURE

$N_{xx}$	uniform compressive force in the $x$ – direction
$N_{yy}$	uniform compressive force in the $y$ – direction
$N_{xy}$	twist
$w(x, y)$	deflection
$x, y$	in-plane Cartesian coordinates
$D$	flexural rigidity of plate
$E$	Young’s modulus of elasticity of the plate material
$\mu$	Poisson’s ratio of plate material
$h$	plate thickness
$N_0$	uniform compressive force in the $x$ direction
$M_{xx}$	bending moment
$W(k,y)$	finite Fourier sine transform of $w(x,y)$
$k$	finite Fourier sine transform variable
$m$	integer
$\beta_m$	parameter defined to relate with $m$ and $a$
ODE	ordinary differential equation
$C$	clamped edge
$S$	simply supported edge
CCSS	plate with two opposite edges clamped and two opposite edges simply supported

## Subscripts

$cr$  critical

## Mathematical symbols

$\int$	integration sign or integral
$\frac{\partial}{\partial x}$	partial derivative with respect to $x$
$\frac{\partial^2}{\partial x \partial y}$	mixed partial derivative