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Composition Operator from Weighted Bergman Space to Logarithmic Bloch Space on the Unit Polydisc

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Abstract

This paper introduces the logarithmic Bloch space $B_{\log}^{q}(D^n)$, a new space of analytic functions on the unit polydisc, investigates the composition operator C_φ from weighted Bergman space to logarithmic Bloch space on the unit polydisc, and provides the sufficient and necessary conditions to ensure the boundedness and compactness of the composition operator C_φ from weighted Bergman space to logarithmic Bloch space.

Key words

Composition operator, Weighted Bergman space, Logarithmic Bloch space, Boundedness, Compactness.

1. Introduction

Let $D=\{z\in C:|z|<1\}$ be the open unit disk in C, and we denote by D^n the open unit polydisc in C^n:

$$D^n = D\times D\times\cdots\times D=\{z=(z_1,z_2,\cdots,z_n)\in C^n:|z_k|<1,1\le k\le n\}$$

and by ∂D^n the full topological boundary of D^n. Then, let $H(D^n)$ denote the space of all holomorphic functions on D^n.

For $0 < p < +\infty$ and $\alpha > -1$, the weighted Bergman space $A_\alpha^p(D^n)$ consists of all functions $f \in H(D^n)$ such that

$$\| f\|_{\alpha,p}^p = \int_{D^n} |f(z)|^p dm_\alpha(z) < +\infty$$

where

$$dm_\alpha(z_1,\cdots,z_n) = dv_\alpha(z_1)\cdots dv_\alpha(z_n) = (\alpha+1)^n \prod_{k=1}^{n} (1-|z_k|^2)^\alpha dv(z_1)\cdots dv(z_n)$$

Here

$$dv_\alpha(z) = (\alpha+1)^\alpha (1-|z|^2)^\alpha dv(z)$$

is a weighted measure of area on D^n with $dv(z)$ being the normalized Lebesgue measure of area on D. When $1 \le p < +\infty$, $A_\alpha^p(D^n)$ is a Banach space with respect to the norm $\| \ \|_{\alpha,p}$.

For $q > 0$, $f \in H(D^n)$ is said to belong to the q-Bloch space $B^q(D^n)$, provided that

$$\sup_{z\in D^n} \sum_{k=1}^{n} (1-|z_k|^2)^q \left| \frac{\partial f}{\partial z_k}(z) \right| < +\infty$$

It is well known that B^q is a Banach space with respect to the norm:

$$\| f\|_{B^q} = f(0) + \sup_{z\in D^n} \sum_{k=1}^{n} (1-|z_k|^2)^q \left| \frac{\partial f}{\partial z_k}(z) \right| < +\infty$$

When $q = 1, B^1 = B$ is a classical Bloch space.

According to Li Haiying[1], the logarithmic Bloch space $B_{\log}^{q}(D^n)$ is defined as follows.

For $q>0$, $f\in H(D^n)$ is said to belong to the logarithmic Bloch space $B_{\log}^{q}(D^n)$, if there exist some $M\geq 0$ such that

$$\sum_{k=1}^{n}(1-|z_k|^2)^q \log\frac{2}{1-|z_k|^2}\left|\frac{\partial f}{\partial z_k}(z)\right|<M$$

for any $z\in D^n$. Its norm in the $B_{\log}^{q}(D^n)$ is defined as

$$\|f\|_{B_{\log}^{q}} = f(0)+\sup_{z\in D^n}\sum_{k=1}^{n}(1-|z_k|^2)^q \log\frac{2}{1-|z_k|^2}\left|\frac{\partial f}{\partial z_k}(z)\right|.$$

It is easy to see that the logarithmic Bloch space $B_{\log}^{q}(D^n)$ is a Banach space with respect to the above norm.

Let $\varphi=(\varphi_1,\cdots,\varphi_n)$ be a holomorphic self-map of D^n. The composition operator C_φ is defined by $C_\varphi f = f\circ\varphi$, $f\in H(D^n)$.

Over the years, researchers have dug deep into the theory of composition operators in various classical spaces [2-5]. Recent years has seen some of them exploring the composition operator on different spaces of analytic functions. For example, Tang and Hu [6] characterized the bounded or compact composition operators between weighted Bergman space and q-Bloch space on the unit disk D. Focusing on high-dimensional cases, Zhang [7] represented the boundedness or compactness of the composition operator from Bergman space to μ-Bloch space in the unit sphere. Wu and Xu [8] summarised the conditions to ensure the boundedness or compactness of weighted composition operators from Bergman to Bloch space on the unit polydisc. Reference [9] probed into the boundedness and compactness of composition operators between the weighted Bergman and q-Bloch space. References [10-12] provided the conditions for composition operators to be bounded and compact on the logarithmic Bloch spaces.

In light of the above, this paper aims to disclose the conditions that ensure C_φ is a bounded or compact operator from weighted Bergman space $A_\alpha^p(D^n)$ to logarithmic Bloch space $B_{\log}^{q}(D^n)$ on the unit polydisc. Denoted by C, the constants in this paper are positive and different from one occurrence to the other.

2. Auxiliary Results

Several auxiliary results were introduced to serve as proofs of the main theorems. The following lemmas were particularly useful in the course of proof.

Lemma 2.1 [9] Let $0 < p < +\infty$ and $-1 < \alpha < +\infty$, then

$$|f(z)| \le \frac{C\|f\|_{\alpha,p}}{\prod_{k=1}^{n}(1-|z_k|^2)^{\frac{2+\alpha}{p}}}$$

for all $f \in A_\alpha^p(D^n)$ and $z_k \in D^n$.

Proof. We denote by $\beta(z,w)$ the Bergman metric on D^n. For any $z \in D^n$ and $R > 0$, we use

$$D(z,r) = \{w \in D^n; \beta(z,w) < R\}$$

to depict the Bergman metric sphere at z with radius R. It is well known that for any fixed $R > 0$, we have

$$m_\alpha(D(z,R)) \sim \prod_{k=1}^{n}(1-|z_k|^2)^{2+\alpha}$$

Now, let any $f \in A_\alpha^p(D^n)$, then $f \in H(D^n)$ and $|f|^p$ is the subharmonic. By the sub-mean-value property for $|f|^p$,we have

$$|f(z)|^p \le \frac{C}{m_\alpha(D(z,R))}\int_{D(z,R)}|f(w)|^p dm_\alpha(w) \sim \frac{C}{\prod_{k=1}^{n}(1-|z_k|^2)^{2+\alpha}}\int_{D^n}|f(w)|^p dm_\alpha(w) = \frac{C\|f\|_{\alpha,p}}{\prod_{k=1}^{n}(1-|z_k|^2)^{2+\alpha}}$$

The result echoes with that of Lemma 2.1.

Lemma 2.2[8] Suppose $0 < p < +\infty$ and $-1 < \alpha < +\infty$, then

$$\left|\frac{\partial f}{\partial z_k}(z)\right| \le \frac{C\|f\|_{\alpha,p}}{(1-|z_k|^2)\prod_{l=1}^{n}(1-|z_l|^2)^{\frac{2+\alpha}{p}}}, z\in D^n, k=1,2,\cdots,n.$$

for any $f \in A_\alpha^p(D^n)$.

Please refer to [8] for the proof of the lemma.

Lemma 2.3. Let $0 < p,q < +\infty$, $-1 < \alpha < +\infty$ and φ be a holomorphic self-map of D^n. Then, C_φ is a compact operator from $A_\alpha^p(D^n)$ to $B^q(D^n)$ if and only if $\|C_\varphi f_j\|_{B^q} \to 0$ as $j \to \infty$ for any bounded sequence $\{f_j\}_{j=1}^{\infty}$ in $A_\alpha^p(D^n)$ that uniformly converges to 0 on compact subset of D^n. The proof is omitted because the lemma can be proved in a standard way (e.g. Proposition 3.11 in [13]) using the Montel's theorem and the definition of compact operator.

3. Main Results and Proofs

Based on the above lemmas, this section discusses the boundedness and compactness of the composition operator $C_\varphi : A_\alpha^p(D^n) \to B_{\log}^q(D^n)$.

Theorem 3.1 Let $0 < p,q < +\infty$, $-1 < \alpha < +\infty$ and φ be a holomorphic self-map of D^n. Then, $C_\varphi : A_\alpha^p(D^n) \to B_{\log}^q(D^n)$ is a bounded composition operator if and only if the following is satisfied:

$$\sup_{z\in D^n} \frac{\sum_{k,j=1}^{n} \frac{(1-|z_k|^2)^q}{(1-|\varphi_j(z)|^2)} \bullet \log\frac{2}{1-|z_k|^2} \left|\frac{\partial \varphi_j}{\partial z_k}(z)\right|}{\prod_{l=1}^{n}(1-|\varphi_l(z)|^2)^{\frac{2+\alpha}{p}}} < +\infty \qquad (1)$$

Proof. Suppose (1) holds and denote any positive constant as M . Let

$$M = \sup_{z\in D^n} \frac{\sum_{k,j=1}^{n} \frac{(1-|z_k|^2)^q}{(1-|\varphi_j(z)|^2)} \bullet \log\frac{2}{1-|z_k|^2} \left|\frac{\partial \varphi_j}{\partial z_k}(z)\right|}{\prod_{l=1}^{n}(1-|\varphi_l(z)|^2)^{\frac{2+\alpha}{p}}} < +\infty$$

For any $f \in A_\alpha^p(D^n)$, the following equation holds by Lemmas 2.1 and 2.2:

$$\left\|C_\varphi f\right\|_{B_{\log}^q} = \left|f(\varphi(0))\right| + \sup_{z\in D^n}\sum_{k=1}^{n}(1-\left|z_k\right|^2)^q \log\frac{2}{1-\left|z_k\right|^2}\left|\frac{\partial(f\circ\varphi)}{\partial z_k}(z)\right|$$

It is clear that

$$\left|f(\varphi(0))\right| \le \frac{C\left\|f\right\|_{\alpha,p}}{\prod_{k=1}^{n}(1-\left|\varphi_k(0)\right|^2)^{\frac{2+\alpha}{p}}} \tag{2}$$

holds. Then, we have

$$\begin{aligned}
&\sum_{k=1}^{n}(1-\left|z_k\right|^2)^q \log\frac{2}{1-\left|z_k\right|^2}\left|\frac{\partial(f\circ\varphi)}{\partial z_k}(z)\right| \\
&\quad \le \sum_{k,j=1}^{n}(1-\left|z_k\right|^2)^q \log\frac{2}{1-\left|z_k\right|^2}\left|\frac{\partial f}{\partial w_j}(\varphi(z))\right|\left|\frac{\partial\varphi_j}{\partial z_k}(z)\right| \\
&\quad \le C\frac{\sum_{k,j=1}^{n}\frac{(1-\left|z_k\right|^2)^q}{1-\left|\varphi_j(z)\right|^2}\log\frac{2}{1-\left|z_k\right|^2}\left|\frac{\partial\varphi_j}{\partial z_k}(z)\right|}{\prod_{l=1}^{n}(1-\left|\varphi_l(z)\right|^2)^{\frac{2+\alpha}{p}}}\left\|f\right\|_{\alpha,p} \\
&\quad \le MC\left\|f\right\|_{\alpha,p}
\end{aligned} \tag{3}$$

According to (2) and (3), it is possible to derive that C_φ is a bounded composition operator from $A_\alpha^p(D^n)$ to $B_{\log}^q(D^n)$.

Conversely, suppose C_φ is a bounded composition operator from $A_\alpha^p(D^n)$ to $B_{\log}^q(D^n)$. Then, it is easy to obtain $\varphi_j \in B_{\log}^q(D^n)$ and

$$\sup_{z\in D^n}\sum_{k=1}^{n}(1-\left|z_k\right|^2)^q \log\frac{2}{1-\left|z_k\right|^2}\left|\frac{\partial\varphi_j}{\partial z_k}(z)\right| < +\infty$$

by taking $f(z)=z_j$ $(j=1,\cdots,n)$ in $A_\alpha^p(D^n)$, respectively. To prove (1), we take

$$f_{w,j}(z)=\frac{z_j-\varphi_j(w)}{1-\overline{\varphi_j(w)}z_j}\prod_{l=1}^{n}[\frac{1-|\varphi_l(w)|^2}{(1-\overline{\varphi_l(w)}z_l)^2}]^{\frac{2+\alpha}{p}}$$

for any $w\in D^n$. It is easy to prove that $f_{w,j}\in A_\alpha^p$ and $\|f\|_{\alpha,p}\le C$, $f_{w,j}(\varphi(w))=0$. Without loss of generality, some j $(j=1,\cdots,n)$ can be fixed to obtain

$$\frac{\partial f_{w,j}}{\partial \xi_j}(\varphi(w))=\frac{1}{1-|\varphi_j(w)|^2}\prod_{l=1}^{n}[\frac{1}{(1-|\varphi_l(w)|^2)}]^{\frac{2+\alpha}{p}}$$

When $l\ne j$, there is $\frac{\partial f_{w,j}}{\partial \xi_j}(\varphi(w))=0$.Thus, we have

$$C\|C_\varphi\|\ge\|C_\varphi\|\|f_w\|_{\alpha,p}\ge\|C_\varphi f_w\|_{B^q_{\log}}\ge\sup_{z\in D^n}\sum_{k=1}^{n}(1-|w_k|^2)^q\log\frac{2}{1-|w_k|^2}\left|\frac{\partial f_w}{\partial w_j}(\varphi(w))\right|\left|\frac{\partial\varphi_j}{\partial z_k}(w)\right|$$

$$=\sup_{z\in D^n}\sum_{k=1}^{n}\frac{(1-|w_k|^2)^q}{1-|\varphi_j(w)|^2}\log\frac{2}{1-|w_k|^2}\left|\frac{\partial\varphi_j}{\partial z_k}(w)\right|\frac{1}{\prod_{l=1}^{n}(1-|\varphi_l(w)|^2)^{\frac{2+\alpha}{p}}}$$

Then, there is

$$\sup_{z\in D^n}\frac{\sum_{k=1}^{n}\frac{(1-|z_k|^2)^q}{1-|\varphi_j(z)|^2}\log\frac{2}{1-|z_k|^2}\left|\frac{\partial\varphi_j}{\partial z_k}(z)\right|}{\prod_{l=1}^{n}(1-|\varphi_l(z)|^2)^{\frac{2+\alpha}{p}}}\le n\|C_\varphi\|<\infty$$

Q.E.D.

Theorem 3.2. Let $0<p,q<+\infty$, $-1<\alpha<+\infty$ and φ be a holomorphic self-map of D^n. Then , $C_\varphi:A_\alpha^p(D^n)\to B^q_{\log}(D^n)$ is a compact composition operator if and only if both of the following are satisfied:

(a) $\varphi_j \in B^q_{\log}(D^n)$ for all $j \in \{1, \cdots, n\}$

(b) $$\lim_{\varphi(z)\to\partial D^n} \sup_{z\in D^n} \frac{\sum_{k,j=1}^{n} \frac{(1-|z_k|^2)^q}{(1-|\varphi_j(z)|^2)} \bullet \log\frac{2}{1-|z_k|^2} \left|\frac{\partial\varphi_j}{\partial z_k}(z)\right|}{\prod_{l=1}^{n}(1-|\varphi_l(z)|^2)^{\frac{2+\alpha}{p}}} = 0$$

Proof. Suppose both (a) and (b) hold. Then, there exists $0<\delta<1$ for any $\varepsilon>0$ such that

$$\frac{\sum_{k,j=1}^{n} \frac{(1-|z_k|^2)^q}{(1-|\varphi_j(z)|^2)} \bullet \log\frac{2}{1-|z_k|^2} \left|\frac{\partial\varphi_j}{\partial z_k}(z)\right|}{\prod_{l=1}^{n}(1-|\varphi_l(z)|^2)^{\frac{2+\alpha}{p}}} < \varepsilon \tag{4}$$

as $|\varphi_j(z)|^2 > 1-\delta$.

Let $\{f_m\}$ be any sequence $\{f_m\}$ in $A^p_\alpha(D^n)$ that converges to 0 on compact subset of D^n and satisfies $\|f_m\|_{\alpha,p} \le C$. Then, $\{f_m\}$ and $\left\{\frac{\partial f_m}{\partial z_k}\right\}$ uniformly converges to 0 on $\Omega = \left\{w : |w|^2 \le 1-\delta\right\}$, where Ω is any compact subset of D^n.

(i) If $dist(\varphi(z), \partial D^n) < \delta$, then the following can be deduced from (4) and Lemma 2.2

$$\sup_{z\in D^n} \sum_{k=1}^{n} (1-|z_k|^2)^q \log\frac{2}{1-|z_k|^2} \left|\frac{\partial(f_m\circ\varphi)}{\partial z_k}(z)\right|$$

$$\le \sup_{z\in D^n} \sum_{k,j=1}^{n} (1-|z_k|^2)^q \log\frac{2}{1-|z_k|^2} \left|\frac{\partial\varphi_j}{\partial z_k}(z)\right| \left|\frac{\partial f_m}{\partial w_j}(\varphi(z))\right|$$

$$\le \sup_{z\in D^n} \sum_{k,j=1}^{n} (1-|z_k|^2)^q \log\frac{2}{1-|z_k|^2} \left|\frac{\partial\varphi_j}{\partial z_k}(z)\right| \frac{C\|f_m\|_{\alpha,p}}{(1-|\varphi_k(w)|^2)\prod_{l=1}^{n}(1-|\varphi_l(w)|^2)^{\frac{2+\alpha}{p}}}$$

$$= C\sup_{z\in D^n} \frac{\sum_{k,j=1}^{n} \frac{(1-|z_k|^2)^q}{1-|\varphi_j(z)|^2} \log\frac{2}{1-|z_k|^2} \left|\frac{\partial\varphi_j}{\partial z_k}(z)\right|}{\prod_{l=1}^{n}(1-|\varphi_l(z)|^2)^{\frac{2+\alpha}{p}}} \|f_m\|_{\alpha,p} < C\|f_m\|_{\alpha,p} \bullet \varepsilon \tag{5}$$

(ii) Under the conditions that $dist(\varphi(z),\partial D^n)\ge\delta$, and that $\{f_m\}$ is any sequence $\{f_m\}$ in $A_\alpha^p(D^n)$ that converges to 0 on compact subset of D^n and satisfies $\|f_m\|_{a,p}\le C$. Then, $\{f_m\}$ and $\left\{\frac{\partial f_m}{\partial z_k}\right\}$ uniformly converges to 0 on $\Omega=\left\{w:|w|^2\le 1-\delta\right\}$. By condition (a), we have

$$\begin{aligned}
&\sup_{z\in D^n}\sum_{k=1}^{n}(1-|z_k|^2)^q\log\frac{2}{1-|z_k|^2}\left|\frac{\partial(f_j\circ\varphi)}{\partial z_k}(z)\right| \\
&\le \sup_{z\in D^n}\sum_{k,j=1}^{n}(1-|z_k|^2)^q\log\frac{2}{1-|z_k|^2}\left|\frac{\partial f_m}{\partial w_j}(\varphi(z))\right|\left|\frac{\partial\varphi_j}{\partial z_k}(z)\right| \\
&\le \sup_{z\in D^n}\sum_{k,j=1}^{n}(1-|z_k|^2)^q\log\frac{2}{1-|z_k|^2}\left|\frac{\partial\varphi_j}{\partial z_k}(z)\right|\sup_{z\in D^n}\left|\frac{\partial f_m}{\partial w_j}(\varphi(z))\right| \\
&\le \|\varphi_j\|_{B^q_{\log}}\sup_{z\in D^n}\left|\frac{\partial f_m}{\partial w_j}(\varphi(z))\right|\to 0\ (m\to\infty)\quad . \qquad (6)
\end{aligned}$$

It is easy to prove that $\left|f_m(\varphi(0))\right|\to 0\ \ (m\to\infty)$. According to (5) and (6), we have

$$\|C_\varphi f_m\|_{B^q_{\log}}=\|f_m\circ\varphi\|_{B^q_{\log}}\to 0\ (m\to\infty)\quad .$$

This means C_φ is a compact operator from $A_\alpha^p(D^n)$ to $B^q_{\log}(D^n)$.

Conversely, for any $j\in\{1,\cdots,n\}$, taking $f(z)=z_j\in A_\alpha^p$, we have $(C_\varphi f)(z)=\varphi(z_j)\in B^q_{\log}$. Thus, condition (a) must hold.

Assuming that condition (b) fails, there exists a constant $\varepsilon_0>0$ and a sequence $\{z^m\}\subset D^n$ satisfying $\varphi(z^m)\to\partial D^n$ as $m\to\infty$, such that

$$\sup_{z\in D^n}\frac{\sum_{k,j=1}^{n}\frac{(1-|z_k^m|^2)^q}{1-|\varphi_j(w^m)|^2}\left|\frac{\partial\varphi_j}{\partial z_k}(w^m)\right|}{\prod_{l=1}^{n}(1-|\varphi_l(w^m)|^2)^{\frac{2+\alpha}{p}}}\ge\varepsilon_0 \qquad (7)$$

For any $w \in D^n$, we take

$$f_m(z) = \prod_{j=1}^{n} [\frac{1-\left|w_j^m\right|^2}{(1-\overline{w_j^m} z_j^m)^2}]^{\frac{2+\alpha}{p}}$$

where $w_j = \varphi_j(z)$. Then, $\| f_m \|_{\alpha,p} = 1$ and $\{ f_m \}$ uniformly converges to 0 on compact subset of D^n. Whereas C_φ is a compact operator from $A_\alpha^p(D^n)$ to $B_{\log}^q(D^n)$, we have

$$\left\|C_\varphi f_m\right\|_{B,q} = \| f_m \circ \varphi\|_{B_{\log}^q} \to 0 \, (m \to \infty) \qquad (8)$$

However, from (7), we have

$$\left\|C_\varphi f_m\right\|_{B_{\log}^q} \geq \sup_{z\in D^n} \sum_{k=1}^{n} (1-\left|z_k\right|^2)^q \log\frac{2}{1-\left|z_k\right|^2} \left|\frac{\partial f_m}{\partial w_j}(\varphi(z))\right| \left|\frac{\partial \varphi_j}{\partial z_k}(z)\right|$$

$$= \sup_{z\in D^n} \sum_{k,j=1}^{n} (1-\left|z_k\right|^2)^q \log\frac{2}{1-\left|z_k\right|^2} \left|\frac{\partial \varphi_j}{\partial z_k}(z)\right| (\frac{2+\alpha}{p})^n \prod_{l=1}^{n} \frac{1}{(1-\left|\varphi_l(z)\right|^2)^{\frac{2+\alpha}{p}}} \bullet \frac{2\left|\varphi_j(z)\right|}{1-\left|\varphi_j(z)\right|^2}$$

$$= \frac{2^n(2+\alpha)^n}{p^n} \left|\varphi_j(z^m)\right| \sup_{z\in D^n} \frac{\sum_{k,j=1}^{n} \frac{(1-\left|z_k^m\right|^2)}{1-\left|\varphi_j(z^m)\right|^2}^q \log\frac{2}{1-\left|z_k^m\right|^2} \left|\frac{\partial \varphi_j}{\partial z_k}(z^m)\right|}{\prod_{l=1}^{n} (1-\left|\varphi_l(z^m)\right|^2)^{\frac{2+\alpha}{p}}}$$

$$= \frac{2^n(2+\alpha)^n}{p^n} \left|\varphi_l(z^m)\right| \sup_{z\in D^n} \sum_{k,l=1}^{n} \frac{(1-\left|z_k^m\right|^2)^q}{\prod_{l=1}^{n} (1-\left|\varphi_l(z^m)\right|^2)^{\frac{2+\alpha+p}{p}}} \left|\frac{\partial \varphi_l}{\partial z_k}(z^m)\right|$$

$$\geq \frac{2^n(2+\alpha)^n}{p^n} \left|\varphi_j(z^m)\right| \varepsilon_0$$

This contradicts with (8) and indicates that (b) holds.

Q.E.D.

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