The Identification of Critical Nodes in Ad Hoc Network Based on Node Degree and Clustering Coefficient

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Abstract

Although the nodes in the wireless ad hoc network are designed to have similar function and equal status, there still exist some critical nodes that may cause the partition of network topology due to the mobility of nodes and the harsh environment. In this paper, we propose that such critical nodes can be detected by neighbor degree and loop degree. Using this method, the critical nodes can be identified quickly from paths without multihop paths by clustering coefficient. Finally, this paper uses weighted coupling method of node degree and clustering coefficient to analyze the importance of critical nodes, which is helpful to sort and protect the critical nodes.

Key words

Wireless ad hoc network, Critical nodes, Node degree, Clustering coefficient.

1. Introduction

As a kind of wireless communication network without support from fixed infrastructure such as base station, the wireless ad hoc network is integrated with mobile terminal as nodes. These network nodes are both signal transmitters and signal receivers, each of which is in the same status, naturally makes up a network by spaces between nodes and by relevance among each other [1]. The wireless ad hoc network can quickly establish a communication network in a sudden occasion without fixed base station and other infrastructures involved. It has the features of rapid deployment and free mobility. As the network nodes behave with the same status, which avoids the single point failure, the network has much higher robustness.
Although the network nodes are designed to have similar feature, due to the mobility of the nodes and the harsh communication environment, some nodes will have a great impact on the network topology, such as network partition caused by failure. It is also likely that the network latency and the average path length will increase [2]. This paper defines the nodes with this character as critical nodes. If such nodes can be identified in wireless ad hoc network and protected with a certain extent, it is of great importance to enhance the robustness of the network.

Based on the algorithm of Depth First Search (DFS) in graph theory, STÉPHANE D proposed a network topology partition detection algorithm. This algorithm utilizes the network topology information, and the required average complexity is $O(n^2)$ [3]. In the case of single point failure and multi-point failure, Tae-Hoon K proposed a method to discover the critical nodes based on algebraic graph theory. This method can further explore the node density and the relationship between critical nodes. The three local topology control methods are used to reduce the importance of critical nodes, thus to improve the network connectivity [4]. According to greed and competition mechanism, Hadi N proposed a method to predict the network survivability time and possible partitions of the network using the law of node mobility [5]. Based on gray prediction model, Sha Y proposed an algorithm to predict the stability of nodes, which not only considers the stability of maintenance route, but also enhances the accuracy of the prediction method compared with the Prediction algorithm based on Midpoint Range Circle (PMRC) [6].

All of the above studies aim to judge whether the critical node exists. If there are multiple critical nodes in the network with limited protection capacity, it is necessary to sort the critical nodes in accordance with their importance. The priority must be given to more important critical nodes in the protection process. In this paper, the critical nodes are identified based on neighbor degree and loop degree, and sorted by their importance using the weighted coupling method. It is useful to discover those critical nodes with greater contributions to topology.

### 2. Method

#### 2.1 Detection of Critical Nodes

We use the method of Li J.D [7] to define critical nodes as follows:

$$N_i - M_i \geq 2,$$

(1)

where $N_i$ and $M_i$ are neighbor degree and basic loop degree of node $i$. The neighbor degree $N_i$ is the number of neighbor nodes of the node $i$. If equation (1) is satisfied, node $i$ is the critical node.
Literature [7] proves that equation (1) is necessary and sufficient condition for the existence of critical nodes.

If there is a pair of neighbor nodes $(j,k)$ of node $i$, which makes up the path $P_t(j,k) = j, i, k$. There is another path $P_t(j,k)$, which can transmit information from node $j$ to node $k$ via some relay nodes (instead of the node $i$). No matter how many paths $P_t(j,k)$ exist, they are logically equivalent to a path $P_t'(j,k)$. Then there is a loop $C_t(j,k)$, which consists of $P_t'(j,k)$ and $P_t(j,k)$.

The basic loop degree is the total number of basic loops where the node $i$ passes through different pair of neighbor nodes, and $M_i = \sum C_t(j,k)$ (see Fig. 1).

![Fig.1. Neighbor Pairs and Basic Loops](image)

### 2.2 Judgement Method For Critical Nodes Based on Weighted Coupling

As described in section 2.1, the critical nodes can be detected in the network. As shown in Fig. 2, nodes 4 and 7 satisfy equation (1), thus they are the critical nodes defined in the paper.

![Fig.2. Network Topology Structure](image)

However, nodes 4 and 7 are not distinct from each other in importance measure. For the purpose of saving resources, this paper detects the importance degree of critical nodes by local
topology information indexes. This method considers the number of neighbors affected by the node, namely, the greater the node degree, the more important it is [8]. The substitutability between neighbor nodes is also considered. If it is poor, the status gets more important. For this purpose, we use weighted coupling method to weigh two key attributes, i.e. node degree and clustering coefficient.

This method can adjust the parameters for the weight of node degree and clustering coefficient, and get the desired value for importance degree, as given in equation (2):

\[ NI(i) = \alpha D(i) + \beta F(C_i), \quad (2) \]

Where \( \alpha + \beta = 1 \), \( 0 < \alpha, \beta < 1 \), \( NI(i) \) is importance value of node as weighted, \( \alpha, \beta \) are the parameters, representing information importance degrees, respectively. \( D(i) \) is the degree centrality of node \( i \); \( C_i \) is clustering coefficient of node \( i \); \( F(C_i) \) is a function of clustering coefficient \( C_i \). It is obvious that for our purpose, the parameters can be adjusted with more attention to node degree or to clustering coefficient.

The degree centrality for node \( i \) can be calculated by equation (3):

\[ D(i) = \frac{N_i}{N - 1}, \quad (3) \]

where \( N_i \) is the neighbors of node \( i \); \( N \) is the number of network nodes.

\[ C_i = \frac{2E_i}{N_i(N_i - 1)}, \quad (4) \]

where \( E_i \) is the number of edges among neighbors of node \( i \).

\[ F(C_i) = e^{-C_i} \quad (5) \]

Then equation (2) can be expressed as:

\[ NI(i) = \frac{\alpha}{N - 1} + \frac{\beta}{N - 1} \frac{2E_i}{N_i(N_i - 1)} \quad (6) \]
where $0 < NI(i) < 1$.

### 2.3 Judgement Process

1. Data initialization.
2. Generate the network $G(V, E)$.
3. Identify the critical nodes.
4. Get a score on the importance degree of critical nodes.
5. Sort the critical nodes according to scores.

### 3. Case Analysis

A case analysis can be carried out to verify the proposed method. Here we choose $\alpha = \beta = 0.5$, that is, the contributions of both are regarded as equal. Then the importance degree of critical nodes can be calculated from equation (6).

<table>
<thead>
<tr>
<th>Node number</th>
<th>$D(i)$</th>
<th>$C_i$</th>
<th>$F(C_i)$</th>
<th>$NI(i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.667</td>
<td>0.067</td>
<td>0.935</td>
<td>0.801</td>
</tr>
<tr>
<td>7</td>
<td>0.444</td>
<td>0.167</td>
<td>0.846</td>
<td>0.645</td>
</tr>
</tbody>
</table>

As shown in Fig. 2, there are 10 nodes in the network. According to the method of Section 2, it can be seen that nodes 4 and 7 as the critical nodes have importance degrees 0.801 and 0.645. Therefore, node 4 is more important than node 7. If the protection measure is taken for nodes, node 4 shall be preferred.

If multihop path $P,(j,k)$ does not exist among neighbors (as shown in Fig.2), we can further deduce this from clustering coefficient. In this case, $M_i$ in the equation (1) is equivalent to $E_i$ in the equation (4). Thus we have the following equation:

$$C_i = \frac{2E_i}{N_i(N_i - 1)} = \frac{2M_i}{N_i(N_i - 1)}$$

(7)

By combination with equation (1), it is concluded:
\( C_i = \frac{2^*E_i}{N_i(N_i - 1)} \leq \frac{2^*(N_i - 2)}{N_i(N_i - 1)} \) \( (8) \)

For different degrees, we can get the clustering coefficient as required. We can quickly see whether the conditions of critical nodes are met, as shown in Tab. 2:

<table>
<thead>
<tr>
<th>Node degree</th>
<th>Clustering coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>inexistence</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0.333</td>
</tr>
<tr>
<td>4</td>
<td>0.333</td>
</tr>
<tr>
<td>5</td>
<td>0.3</td>
</tr>
<tr>
<td>6</td>
<td>0.267</td>
</tr>
<tr>
<td>7</td>
<td>0.238</td>
</tr>
<tr>
<td>8</td>
<td>0.214</td>
</tr>
<tr>
<td>9</td>
<td>0.194</td>
</tr>
<tr>
<td>10</td>
<td>0.178</td>
</tr>
</tbody>
</table>

As shown in Fig. 2, the degrees of nodes 3, 5, 6, 9 are all 1, so they do not satisfy the conditions for critical node identification. The degrees of nodes 1, 2, 8, and 10 are 2, but the clustering coefficients are 1, so the judgment condition of the critical node is not satisfied. The degree of node 7 is 4, while the clustering coefficient is 0.167, which is less than 0.333, thus the judgment condition of the critical node is satisfied. The degree of node 4 is 6, and the clustering coefficient is 0.067, which is less than 0.267, so the judgment condition of the critical node is satisfied. This coincides with the result of the judgment by equation (1), but it is quicker and easier. As greater limitation exists, it can only be used in the special network topology.

Fig. 3. Network Topology Structure

As shown in Fig. 3, the degree of node 3 is 3, and the neighbor nodes are directly connected with only one hop (node 1 and node 2). According to the equation of the clustering coefficient we
get $C_3 = 0.333$, the definition of Tab.2 is satisfied. However, node 3 is not the critical node that leads to the topological partition. The reason is that the multihop path exists in other two node pairs $(1, 6)$ and $(2, 6)$. It can be seen from equation (1), $N_3 - M_3 = 0$, thus the node 3 is not the critical one. So does the node 6.

The node 10 can be calculated by the equation (1), or determined according to Tab. 2. It is because that there is no multihop path between its neighbors. According to the equation (1), $N_{10} - M_{10} = 3 > 2$, $C_{10} = 0.2$, it can be known that is a critical node.

The node 7 can be calculated by the equation (1), or determined according to Tab. 2. Though there is a multihop path between its neighbors, the neighbor pairs are also linked directly. According to the equation (1), $N_7 - M_7 \geq 2$, $C_7 = 0.267$, it can be known that is a critical node.

As can be seen from the calculation, there are critical nodes 7 and 10 in Fig. 3. Their importance degrees are shown in Tab. 3:

<table>
<thead>
<tr>
<th>Node number</th>
<th>$D(i)$</th>
<th>$C_i$</th>
<th>$F(C_i)$</th>
<th>$NI(i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>0.545</td>
<td>0.267</td>
<td>0.766</td>
<td>0.656</td>
</tr>
<tr>
<td>10</td>
<td>0.455</td>
<td>0.2</td>
<td>0.819</td>
<td>0.637</td>
</tr>
</tbody>
</table>

It is clear that $NI(7) > NI(10)$, the numerical gap is not big. It can be seen only from the figure, the node 7 is much more important than node 10. This shows that the applicable scope of importance evaluation is limited. It is not likely to make an evaluation on the global position. The global topology information is required.

From the perspective of the topology of the communication network, it is easy to attack the highly important nodes. The importance of nodes is often reflected in such case that the removing of the node will damage the network. To further compare the node 7 and node 10, we measure them by removing the nodes. As given in equation (9), the importance of nodes is measured by changing the number of nodes in the giant connected component.

$$S = \frac{N - N'}{N}$$ (9)

where $N'$ is the nodes number of giant connected component after deletion, $N$ is the number of nodes in initial network. Only in the case of the single-point attack, $S$ is calculated after the nodes 7 and 10 are removed.
When the node 7 is removed, \( S_7 = \frac{6}{12} \); when the node 10 is removed, \( S_{10} = \frac{3}{12} \). The higher the value of \( S \), the more importance the node’s position. This proves that the node 7 is more importance than node 10.

**Conclusion**

Due to the characteristics of the wireless ad hoc network, it will be widely used in military communication and in civil wireless network communication in the future. This paper proposes detection method for the critical nodes. It helps find out such nodes that can easily lead to the network partition. When there are no multihop loops between neighbor nodes, a simple method is proposed to identify the critical nodes in the way that the node degree corresponds to the maximum clustering coefficient. Although there is a certain limitation in the method, it features low complexity and fast detection, provided that the network scale is small, and only the information of first-order neighbors is required. There are many critical nodes in the network. In order to better protect the critical nodes which contribute most to network topology, this paper proposes a weighted coupling method based on node degree and clustering coefficient to evaluate the importance of the network instance. The results show that the algorithm is more effective. However, some defects still exist when it faces with more complex network topology. The global topology information index shall be further considered, such as energy consumption, closeness centrality or betweenness centrality [9]. The Future study can also consider to enhance the protection ability of the preferred nodes or to improve the invulnerability of the network by increasing the redundancy [10].

**References**


