

Investigation of electronic and elastic properties of YNi_2-xMx (M: Fe, Co, Cu and Zn): Ab initio calculations analyzed with Data mining approach

Mostafa K. Benabadji^{1,2*}, Ammaria Mahmoudi¹, Djazia Bouabdallah¹, Fatiha Saidi^{1,2}, Houda I. Faraoun¹, Ghouti Merad¹

¹ Division Etude et Prédiction des Matériaux (DEPM), Unité de Recherche Matériaux et Energies Renouvelable (URMER), Université Abou Bekr Belkaid, B.P 119 Fg. Pasteur, Tlemcen 13000, Algérie

² Ecole Supérieure en Génie Electrique et Energétique (ESGEE), B.P 64, CH2 ACHABA Hanifi, Technopôle USTO 31000 Oran, Algérie

Corresponding Author Email: kbenabadji@yahoo.fr

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ABSTRACT

We investigated Structural, electronic and mechanical properties of pure YNi_2 and YNi_2-xMx (M: Fe, Co, Cu and Zn) Laves phases using first principles calculations. Density functional theory is considered within framework of both pseudo-potentials and plane wave's basis using VASP (Vienna ab initio Software Package). The optimized structural parameters were in good agreement with experiment. We calculated formation heat for pure YNi_2 and showed that the cubic C15- YNi_2 Laves phase are more stable than C14 and C36 hexagonal phases. We evaluated and discussed Electronic density of states (DOSs) and charge density distribution. The elastic properties were calculated, discussed and analyzed with data mining approach.

1. INTRODUCTION

Laves phases are a class of materials presenting an excellent physical and chemical properties [1-10]. These compounds with AB_2 chemical formula, were selected for many large and attractive applications, such as superconducting and hydrogen storage materials, high-temperature and pressure structural materials [11]. $RETM_2$ compounds (RE: Rare Earth, TM: Transition Metals) crystallize for most in the cubic phase C15 type $MgCu_2$. Easily absorb hydrogen forming stable hydrides [12]. However, few combinations present some problems regarding their elastic properties such as low hardness and stiffness or high ductility for specific industrial applications. To overcome these issues we can use a partial replacement or alloying with different type of elements.

The YNi_2 compound crystallizes in the C15 structure [13]. A Rare Earth Y (d^1s^2 , space group $P6_3/mmc$, $a=3.65 \text{ \AA}$, $c=5.73 \text{ \AA}$) and a transition metal Ni (d^8s^2 , space group $Fm\bar{3}m$, $a=3.53 \text{ \AA}$) form it. It is a material with industrial requirements making it a good candidate for hydrogen storage. Thus, we were interested to explore these physical properties. Several theoretical works based on first principles calculations, report that Laves phases compounds exhibit a good electronic and mechanical properties such as polar covalent bonding (responsible for the intrinsic stiffness) and a B/G ratio (B : bulk modulus and G : shear modulus) for some compounds close to 1.75, indicating significant ductility and high hardness as well [14-25].

In this paper, we studied the effect of alloying with (M: Fe, Co, Zn and Cu) on YNi_2 properties. Therefore, we used ab initio calculations to examine the structural, elastic and electronic properties of YNi_2-xMx in $MgCu_2$ cubic phase for three concentrations ($x=0.125$, 0.25 and 0.375). After relaxation, we obtained optimized lattice parameters and we discussed formation enthalpies, elastic properties and densities

of states.

1.2 Computational methods

To determine the physical properties, we performed calculations using the Vienna Ab initio Simulation Package VASP [26-28] code based on the density functional theory (DFT) [29-30]. We used Ultrasoft Vanderbilt type pseudopotentials [31] to describe the interactions between ions and electrons. We applied the generalized gradient approximation GGA (PW91) of Perdew et al. [32] to evaluate the exchange-correlation energies of all examined structures.

We used $13 \times 13 \times 13$ (cubic C15 structure) and $13 \times 13 \times 11$ (C14 and C36 structure) sampling of the Brillouin zone generated according to the Monkhorst-Pack scheme [33] and set the cut-off energy (limiting the number of plane waves in the basis) to 320 eV for YNi_2 .

To obtain the equilibrium unit cell volumes, we performed the total energy calculations for each structure with a set of volumes, with all atoms occupying their ideal lattice sites [11]. We fitted curve of total energies versus volumes using murnaghan's equation of state [34].

1.3 Data mining techniques

For materials design, not only is the creation of data whether through calculation or experiment important, but a way to analyze the data in a comprehensive and robust manner is also necessary. Some of the challenges in searching through discrete data include the difficulty of analyzing large amounts of data, understanding the correlations among various properties, and using the correlations to better understand the underlying physics of the system. Utilizing a multivariate analysis, the data can be examined so that trends and correlations become apparent.

Data mining is employed in this work in order to fully uncover inter-correlations in the data. Additionally, the mathematical relationship between descriptors must be developed before descriptor reduction is possible. Once the relationships are known then the model can be accurately extended to new data so that a virtual system can be fully described with limited data. The data mining method used here is principal component analysis (PCA) [35], which is a powerful statistical approach for the analysis of materials properties and has been used to address a variety of physics and materials science issues [36-37].

PCA is a classification method, which projects the spatial data into a set of principal components (PC) and maps the data on a dimensionally reduced space. The PC capturing the most information is associated with the eigenvalue corresponding with the largest eigenvalue of the covariance matrix of the original dataset. All PC's are orthogonal to each other, and thus each captures unique information. The advantage of PCA is that typically, a few PC's are sufficient for describing a system, and a dataset of n-dimensions can be reduced to a few dimensions with minimal loss of information [38]. The PC's do not necessarily have an obvious physical meaning, but rather are a combination of variables, which explain the largest variation in the data. The reduction in dimensionality makes trends and correlations, which are "hidden" in the data to become easily visualized and described in PC space. PCA decomposes the original data matrix into the scores and loadings matrices, where the scores values classify the samples and the loadings values classify the descriptors in terms of their separation of the samples. The correlations among the descriptors become obvious in a PCA analysis, and by defining the correlations in the data, we can then reduce the number of descriptors to a minimum to permit a more convenient data analysis.

2. RESULTS AND DISCUSSIONS

2.1 Structural stability of pure YNi₂

These calculations allowed us to determine the most stable structure among hexagonals (C14 and C36) and cubic C15 Laves phases.

Table 1. Equilibrium values of the three units cells: the lattice constants (a, b, c in Å) and bulk modulus (GPa) for both phases calculated with GGA, compared with other values listed in parentheses

	a (Å)	b (Å)	c (Å)	B ₀	B' ₀
C14	5.08	-	8.34	117.3	4.26
Space group : P6 ₃ /mmc				0	8
C36	5.08	-	16.67	118.3	4.09
Space group: P6 ₃ /mmc				9	8
C15	7.18	-	-	119.2	4.55
Space group: Fd3m	(7.22) a	-	-	0	3

^aRef. [13]

We obtained the cell parameters and bulk modulus (Table 1) for C14, C15 and C36 Laves phases after fitting the curve of total energy versus volume (Figure1).

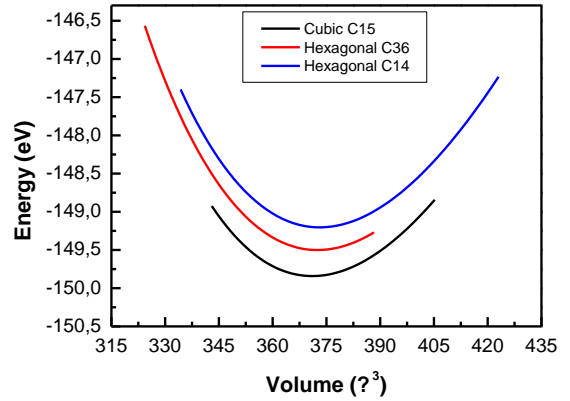


Figure 1. Total energy versus volume for YNi₂ compound crystallized in C14, C15 and C36 Laves phases

In Figure 1, we noticed that the equilibrium volumes of the three studied phases were almost equal indicating a possibility of observing a phase transition according to the temperature. We found that the cubic C15 phase is the most stable structure.

2.2 Formation enthalpies and volume

In order to investigate the alloying ability and stability, we calculated the formation energy (ΔE_f) of AB₂ type Laves phase by [15]:

$$\Delta E_f = E_{\text{tot}}^{\text{AB}_2} - (E_{\text{solid}}^{\text{A}} + 2E_{\text{solid}}^{\text{B}}) \quad (1)$$

We calculated the formation energy ($\Delta E_f^{\text{alloy}}$) of AB_{2-x}C_x alloy by [39]:

$$\Delta E_f^{\text{alloy}} = E_{\text{tot}}^{\text{AB}_{2-x}\text{C}_x} - (E_{\text{solid}}^{\text{A}} + (2-x)E_{\text{solid}}^{\text{B}} + xE_{\text{solid}}^{\text{C}}) \quad (2)$$

Table 2. Equilibrium volume and heats of formation of the YNi₂ intermetallic compound and their alloys (per formula unit, other value given in parentheses)

Compounds		Volume (Å ³ /f.u.)	Enthalpies ΔH_{form} (eV/f.u.)
C14	YNi ₂	46.59	-1.405
C36	YNi ₂	46.56	-1.442
C15	YNi ₂	46.26	-1.476
		(47.18) ^a	(-1.45) ^a
		(45.90) ^b	(-1.21) ^c
C15	YNi _{1.875} Zn _{0.125}	46.99	-1.442
	YNi _{1.75} Zn _{0.25}	47.68	+1.103
	YNi _{1.875} Cu _{0.125}	46.64	-1.438
	YNi _{1.75} Cu _{0.25}	46.95	-1.376
	YNi _{1.625} Cu _{0.375}	47.12	+1.010
	YNi _{1.875} Co _{0.125}	46.28	-1.454
	YNi _{1.75} Co _{0.25}	46.21	-1.415
	YNi _{1.625} Co _{0.375}	46.10	-1.376
	YNi _{1.875} Fe _{0.125}	46.34	-1.419
	YNi _{1.75} Fe _{0.25}	46.28	-1.358
YNi _{1.625} Fe _{0.375}	46.19	-1.299	

^aRef. [13]

^bRef. [36]

^cRef. [37]

E_{solid}^A , E_{solid}^B and E_{solid}^C represent the energy per atom of A, B and C in solid states.

At zero Kelvin and under pressure zero Pa, the enthalpy equals to the energy, that is $\Delta E_f(\text{AB}_2) = \Delta H_f(\text{AB}_2)$ and $\Delta E_f^{\text{alloy}} = \Delta H_f^{\text{alloy}}$ [40].

From table 2, we can see that the heats of formation for YNi_2 are -1.405, -1.442 and -1.476 eV/atom for C14, C36 and C15 structures, respectively. These values showed that the lowest heat of formation for YNi_2 correspond to the C15 structure Laves phase. Relaxed systems with different transition metals (Fe, Co, Cu and Zn) studied for three concentrations 0.125, 0.25 and 0.375 showed negative ΔH_{form} values except for $\text{YNi}_{1.75}\text{Zn}_{0.25}$ and $\text{YNi}_{1.625}\text{Cu}_{0.375}$ (+1.103 and +1.010 eV/atom respectively). They crystallized in cubic C15 structure with a little volume distortion compared to the pure system. Concentration of Zn or Cu atoms above 0.25 in YNi_2 -C15 matrix imply an important deformation. So the loss of the structural symmetry.

2.3 Elastic properties

The elastic properties can give valuable information about the nature of binding between neighboring atomic planes. Usually, the C_{ij} elastic constants can define significantly the anisotropic binding characteristics and structural stability. These constants have been frequently associated to the shear modulus G and Young's modulus E. Ab initio calculations of these constants requires accurate methods. A popular approach is used to evaluate the elastic constants from their known structure [16, 41-45].

The afore mentioned intermetallic compounds have very appealing mechanical properties. To show that, we were interested to calculate the elastic constants for YNi_2 and their alloys for C15 structure type Laves phase.

The three independent elastic constants C_{11} , C_{12} and C_{44} characterize the cubic C15 structure. The obtained values for studied systems are presented in Table 3. We report that the elastic constants of these compounds are not determined experimentally.

Table 3. The calculated elastic constants, bulk modulus B (GPa), shear modulus G (GPa), Young's modulus E (GPa), Poisson's ratio ν , ratio B/G, Cauchy pressure C_{12} - C_{44} and Anisotropy factor A

Compounds	C_{11}	C_{12}	C_{44}	B	G	E	ν	B/G	C_{12} - C_{44}	A
YNi_2	164.15	96.72	34.14	119.20	33.97	93.07	0.370	3.51	62.58	1.012
$\text{YNi}_{1.875}\text{Zn}_{0.125}$	225.01	57.31	86.28	113.21	85.30	204.53	0.190	1.33	-28.97	1.028
$\text{YNi}_{1.875}\text{Cu}_{0.125}$	221.87	61.59	81.61	115.02	81.02	196.83	0.210	1.42	-20.02	1.018
$\text{YNi}_{1.75}\text{Cu}_{0.25}$	221.86	58.08	84.87	112.67	83.66	201.20	0.200	1.35	-26.79	1.036
$\text{YNi}_{1.875}\text{Co}_{0.125}$	221.39	66.59	78.95	118.19	78.33	192.46	0.228	1.51	-12.36	1.020
$\text{YNi}_{1.75}\text{Co}_{0.25}$	221.58	69.44	77.19	120.15	76.74	189.81	0.236	1.57	-7.75	1.015
$\text{YNi}_{1.625}\text{Co}_{0.325}$	222.416	68.25	80.16	119.64	78.91	194.06	0.230	1.52	-11.91	1.036
$\text{YNi}_{1.875}\text{Fe}_{0.125}$	220.03	66.91	79.73	118.06	78.44	192.67	0.228	1.51	-12.82	1.041
$\text{YNi}_{1.75}\text{Fe}_{0.25}$	209.87	75.55	57.58	120.32	61.24	157.06	0.282	1.96	17.96	0.857
$\text{YNi}_{1.625}\text{Fe}_{0.375}$	218.59	72.74	68.35	121.36	70.15	176.44	0.258	1.73	4.39	0.940

For the C15 cubic phase, the independent single-crystal elastic constants (C_{11} and C_{12}) can be derived from the bulk modulus B and tetragonal shear modulus C' defined by the following equations $B = (C_{11} + 2C_{12}) / 3$ and $C' = (C_{11} - C_{12}) / 2$. The C_{44} constant is the trigonal shear modulus. We calculated these constants using the scheme described in detail by Mehl et al. [42]. Polycrystalline elastic modulus is obtainable by taking the average of single elastic constants in several schemes. Both Voigt G_V [46] and Reuss G_R [47] components of shear modulus are the upper and lower theoretical limits of the real shear modulus, respectively. They are described as [48]:

$$G_V = \frac{C_{11} - C_{12} + 3C_{44}}{5} \quad (3)$$

$$G_R = \frac{5(C_{11} - C_{12})C_{44}}{3(C_{11} - C_{12}) + 4C_{44}} \quad (4)$$

The real shear modulus G and the Young's modulus E can be written respectively as:

$$G = \frac{1}{2}(G_V + G_R) \quad (5)$$

$$E = \frac{9BG}{3B+G} \quad (6)$$

The Poisson's ratio ν and elastic anisotropy A are obtained by the following expressions:

$$\nu = \frac{3B-2G}{2(3B+G)} \quad (7)$$

$$A = 2 \frac{C_{44}}{C_{11} - C_{12}} \quad (8)$$

The bulk B, shear G and Young's E modulus, Poisson's ν and B/G coefficients and anisotropy factor A for the three compounds are listed in Table 3.

The ductility of the material is indicated by the B/G ratio. If $B/G \geq 1.75$, the compound should have important ductility. For pure C15- YNi_2 compound and $\text{YNi}_{2-x}\text{M}_x$ systems ($x=0.125, 0.25$ and 0.375), $A \approx 1$ indicating isotropic elasticity except for $\text{YNi}_{1.75}\text{Fe}_{0.25}$ where $A=0.857$.

In general, If the Poisson's ratio ν : $-1 \leq \nu \leq 0.5$, the polar covalent bonding is determined. If $\nu \approx 1/3$, the compound is ductile. If $\nu < 1/3$ the material becomes brittle.

As seen in Table 3, the pure YNi_2 has a B/G ratio equal to 3.51 indicating a high ductility due high compression modulus

B and low shear modulus G. This latter is confirmed by Poisson's ratio with a value of $0.37 \approx 1/3$. Whereas, for all alloys, the mechanical behavior is changed by M-atoms addition, so the B/G ratio took values between 1.33 and 1.57, due to the high value of shear modulus G with a slight change in the value of the bulk modulus B, except for $\text{YNi}_{1.75}\text{Fe}_{0.25}$ and $\text{YNi}_{1.625}\text{Fe}_{0.375}$ (1.93 and 1.73 respectively). Otherwise, The Young's modulus E has a very high values compared to the pure YNi_2 , revealing a decreasing in ductility and increasing in stiffness for the some alloys. Moreover, the Poisson's ratio decreased under $1/3$.

2.4 Electronic properties

Structural stability and mechanical properties of YNi_2 and its alloys $\text{YNi}_{2-x}\text{Zn}_x$ are related to their chemical bonding nature so it was necessary to perform electronic densities of state (DOS) calculation.

2.5 Pure YNi_2

Total and partial DOS and electron charge densities distribution of pure YNi_2 are presented in Figs. 2 and 3, respectively.

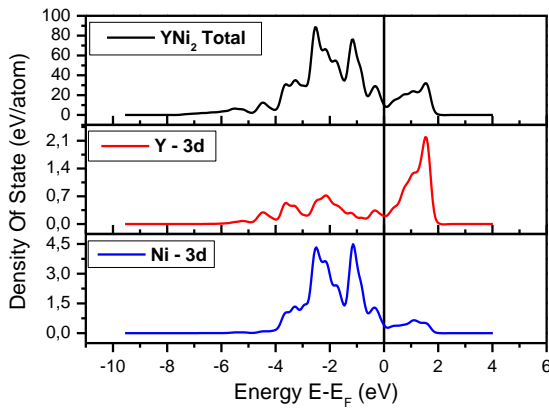


Figure 2. Total and partial density of state of pure YNi_2

As shown in Fig. 2, before Fermi level and between -4 to 0 eV, the total DOS is dominated by Ni d states. The metallic character is due to the presence of electronic states of Ni atoms. The effect of Y atoms is considered negligible compared to that of Ni atoms at Fermi level.

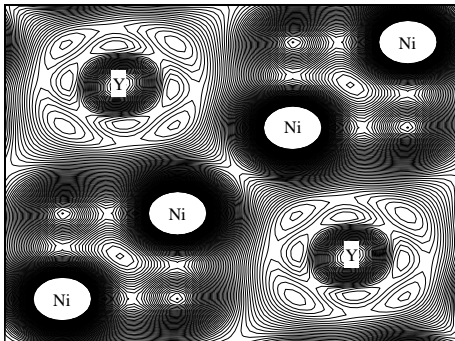


Figure 3. The contour plots of charge densities. The (010) plan for cubic-C15 structure of pure YNi_2

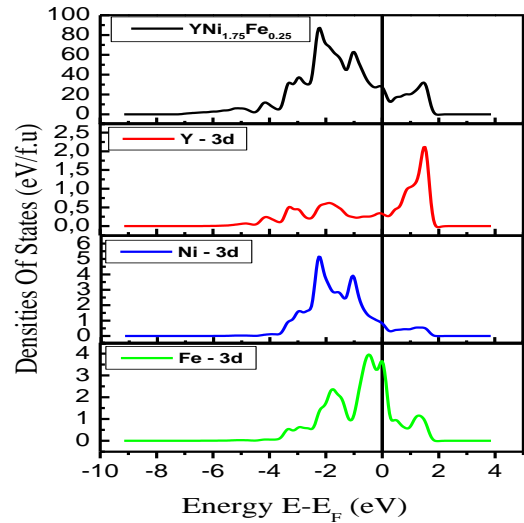


Figure 4. Calculated total and partial DOS of $\text{YNi}_{1.75}\text{Fe}_{0.25}$ compound

In Fig.3, we displayed the contour maps of the electron densities in the (010) plane for C15- YNi_2 phase.

To determine the nature of existing chemical bonds, it was interesting to plot the electronic charges density contours of C15- YNi_2 compound (Fig.3). We noticed that the distribution of charges is maximum between the Ni atoms, confirming the strong metallic character, validating what was observed earlier in the YNi_2 DOS (Fig.2).

The $\text{YNi}_{2-x}\text{Fe}_x$ alloys:

Electronic densities of state of $\text{YNi}_{1.875}\text{Fe}_{0.25}$ and $\text{YNi}_{1.625}\text{Fe}_{0.375}$ alloys are presented in Figs. 4 and 5. Where some effects of incorporated Fe atoms in pure compound are discussed.

As seen in Fig.4 and Fig.5 the effect of Fe atom in the YNi_2 -DOS. Fermi level was shifted towards the negative values. This displacement is accompanied by increase in the metallic character compared to the pure system.

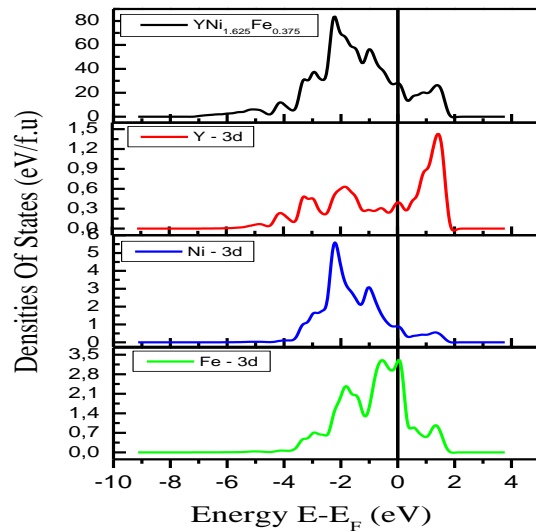


Figure 5. Calculated total and partial DOS of $\text{YNi}_{1.625}\text{Fe}_{0.375}$ compound

The Fe-3d states are localized and form two peaks between -2 eV and 0 eV. They strengthened the polar covalency in the

Ni-3d-Fe-3d bonds for both alloys compared to pure YNi₂, which explain the Young modulus E increase (Table 3). So the previous analysis of the elastic properties are validated.

2.6 Data mining results

In order to identify the trends or clustering in materials property data, we construct a database for several intermetallic YNi_{2-x}M_x [(M: Fe, Co, Cu and Zn), (x:0, 0.125, 0.25 and 0.375)] including the, elastic constants, bulk and shear modulus, young modulus, Poisson's ratio ν , the B/G ratio, the Cauchy pressure and Anisotropy ratio A. Table 3 contains a portion of the dataset used.

The first analysis done was to examine the general trends in different intermetallics. The resulting score plot of this analysis is shown in Fig.6.

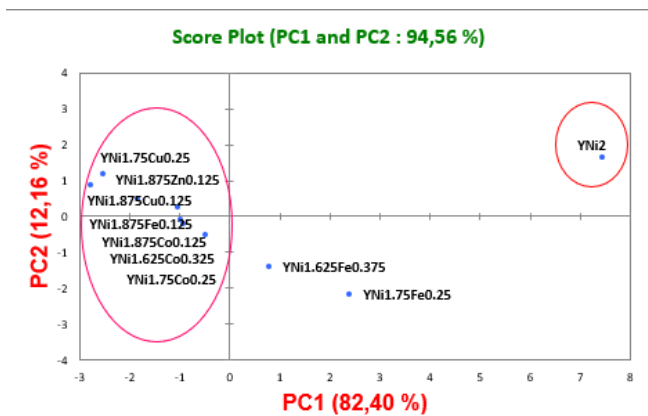


Figure 6. PCA and PLS score plot of different polar intermetallics

For this analysis, the sign of each principal component has only relational meaning. PC1 captures 82.40% of the variance in the dataset, and PC2 captures 12.16% of the variance as seen in fig.6. No other PCs are included in any of the discussions because they do not provide significant information.

From looking at this figure it appears three important clustering those with positive PC1 which we will refer to cluster 1 [YNi₂] and cluster 2 [YNi_{1.625}Fe_{0.375}, YNi_{1.75}Fe_{0.25}], and those with a negative PC1 which we will refer to cluster3 [YNi_{1.875}Zn_{0.125}, YNi_{1.875}Cu_{0.125}, YNi_{1.75}Cu_{0.25}, YNi_{1.875}Co_{0.125}, YNi_{1.75}Co_{0.25}, YNi_{1.625}Co_{0.325}, YNi_{1.875}Fe_{0.125}].

In a first insight it appears that cluster 1 and 2 includes intermetallics with high B/G ratio, high Cauchy pressure C12-C44, high bulk modulus (B), and low shear modulus G, while cluster 3 is characterized by low B/G ratio, and high G and E.

Compounds of first cluster1 is highly correlated with B/G ratio and the Cauchy pressure (C12-C44) indicating the ductility of this material.

However, Materials of the second cluster are highly correlated with bulk modulus B and B/G ratio indicating that these materials are hard with appreciable ductility, this result confirms that the addition of simple concentration of Fe can decrease ductility of YNi₂ and increase its hardness, which is in consistent with ab initio observations.

Materials of cluster 3 are highly correlated with shear modulus G and the young modulus E indicating the rigidity of these materials.

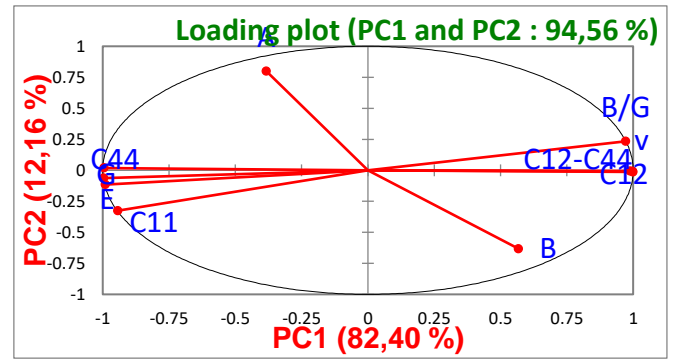


Figure 7. PCA and PLS loading plot of different elastic parameters

The loading plot corresponds with the score plot but represents the variance among descriptors. Fig.7 shows the loading plot corresponding with the samples shown in Fig. 6.

The axes of the score plot and loading plot are the same so the information in the plots can be compared directly.

According to the Fig. 6 and 7 we notice that the position of the properties G and B/G are correlated to cluster3 and cluster1 respectively. Properties with similar PC values are highly correlated, while inverse PC values indicate inverse correlations. Therefore, the properties in cluster 1 and 2 are inversely correlated with the properties in cluster 3, while the properties within clusters 2 are highly correlated with other properties within the same cluster. Many of the correlations between properties are not obvious. One correlation that is expected is the connection between the G and B/G, that means that if the hardness increases the ductility decreases which is consistent with the experimental observations and our ab initio calculations. Whereas G, C44 and E they all have a negative PC2 and PC1 values, so they are highly correlated and indicate hardness of materials of cluster 3. The ratio B/G and Cauchy pressure (C12-C44) are highly correlated, which is consistent with the fact that materials with high B/G and (C12-C44) are highly ductile.

The relationship of all intermetallics and corresponding parameters is clearly shown in the biplot of Fig.8 according to the data listed in Table 3.

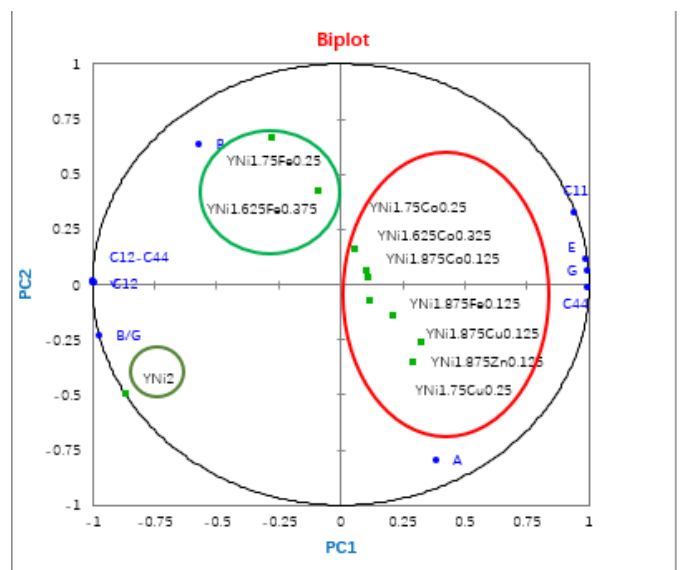


Figure 8. The biplot of different polar intermetallics and elastic parameters

This biplot obtained by PCA combines properties and criterions with different intermetallics. For this analysis, the sign of each principal component has only relative meaning. From looking at these figures it appears that materials of the first cluster are correlated with indicators of the ductility (B/G, C12–C44), those of the third cluster are highly correlated with the rigidity (E,G) indicators ,however , the compounds of the second cluster are highly correlated with hardness criterions (B) which validate our ab-initio results.

3. CONCLUSION

In this work, we analyzed the structural properties for YNi_2 compound, and we observe that YNi_2 crystallized in the cubic C15 structure. Heats of formation of $YNi_{2-x}M_x$ carried out that only $YNi_{1.75}Zn_{0.25}$ and $YNi_{1.625}Cu_{0.375}$ present positive values.

To complete the fundamental characteristics of these compounds we have analyzed their mechanical properties. A critical analysis has been carried out using data mining techniques. The major goal was to predict the better hard materials with appreciable ductility. The present results clearly demonstrate that a simple visual observation of the PCA plots, in respect to the position of different intermetallics concludes that $YNi_{1.625}Fe_{0.375}$, $YNi_{1.75}Fe_{0.25}$ intermetallics have good hardness and appreciable ductility compared to $YNi_{1.875}Zn_{0.125}$, $YNi_{1.875}Cu_{0.125}$, $YNi_{1.75}Cu_{0.25}$, $YNi_{1.875}Co_{0.125}$, $YNi_{1.75}Co_{0.25}$, $YNi_{1.625}Co_{0.325}$ and $YNi_{1.875}Fe_{0.125}$ compounds which present good rigidity. However, our results confirm that these intermetallics present very interesting mechanical properties. It was found that the YNi_2 is the most ductile, while $YNi_{1.875}Zn_{0.125}$ the most rigid structure of these series of compounds. We can explain this result that the addition of simple concentration of Fe can decrease ductility of YNi_2 and increase its hardness, which is in consistent with ab initio observations. Finally, our results reveal that Fe based systems are the most interesting by their high rigidity and appreciable ductility.

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