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On the Numerical Treatment of Magneto-Hydro Dynamics Free Convection with Mixed Boundary Conditions

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ABSTRACT

In this paper, Lattice Boltzmann method (LBM) is proposed to simulate Magneto-hydrodynamic (MHD) free convection in a two-dimensional open cavity with mixed boundary conditions (BCs). The cavity is getting under a uniform transverse magnetic field. The proposed numerical scheme solved the flow field and the temperature field using D2Q9 lattice model. So, the main aim of this study is to highlight the effectiveness of this mesoscopic model to predict the effects of pertinent parameters such as the Hartmann number varying from 0 to 150 and the Prandtl number altering in a wide range of Pr=0.025 and 0.71. Rayleigh number is fixed at moderate value of 10^5. This in-house numerical code used in this paper is ascertained and a good agreement with literature is highlighted. The appropriate validation with previous numerical investigations demonstrated that this attitude is a suitable method and a powerful approach for engineering MHD problems. Findings and results show the alterations of Hartman number that influence the isotherms and the streamlines widely at different Rayleigh and Prandtl numbers simultaneously. Moreover, heat transfer declines with the increment of Hartmann number. The effect of the magnetic field on the average Nusselt number at Liquid Gallium (Pr=0.025) is also highlighted.

1. INTRODUCTION

Free convection in closed and open cavities has many engineering applications such as: cooling systems of solar collectors, electronic components, building and thermal insulation systems, nuclear reactor systems, food storage industry and geophysical fluid mechanics. Convection under the influence of a magnetic field received a considerable attention in crystal growth in fluids, metal casting, fusion reactors and geothermal energy extractions, natural convection is under the influence of a magnetic field [1-20]. Recent attention has been intensively focused on the cases with mixed boundary conditions on the walls of an open cavity [21-24].

Over the last decades, LBM was an applicable method for simulating fluid flow and heat transfer successfully [25-32]. It becomes a powerful, an effective and easy numerical method, it is used in simulating complex flow problems with different boundary conditions [33, 34].

This surge in interest of the D2Q9 LBM (Figure 1-c) model is mainly attributed to its computational simplicity, direct discretization, ability and efficiency. It is known that the conventional computational fluid dynamics (CCFD) solvers, namely the volume finite element method (FVM), the control volume finite element method (CVFEM), the finite difference method (FDM) are macroscopic models but the LBM is a recent mesoscopic approach describing and capturing engineering physics better [30]. This mesoscopic approach includes simple calculations procedure, efficient implementation for a parallel architecture and robustness for handling complex geometry

Engineering applications including linear, sinusoidal, convective, Dirichlet, open, Neumann Boundary Conditions (BC), or a mixture [35] of these conditions in different walls of the MHD cavity is a tricky task for numerical heat and mass transfer in physical engineering simulations.

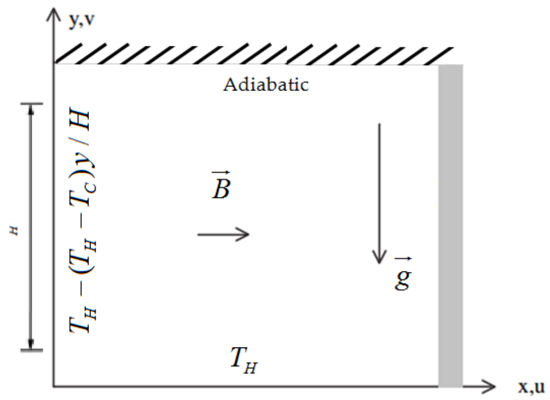
This is a typical problem with mixed boundary conditions and should not be confused with the considerably simpler problem when the temperature is prescribed over certain complete sides of the rectangle, while the temperature gradient is prescribed over the remaining sides. As far as the writer is aware no analytical solution of the mixed boundary value problem above formulated (or of the analogous problem for the cylinder) is to be found in the literature. We must therefore (if interested in numerical answers) resort to the alternative of substituting for the differential equation of heat conduction and for the equations expressing the initial and boundary conditions their appropriate difference analogs, and solving the resulting system.

The main aim of this paper is to identify the ability of Lattice Boltzmann Method (LBM) for solving magnetic field simultaneously in the presence of a non isothermal boundary condition. It is endeavored to express the best situation for heat transfer and fluid flow with the MHD and mixed BCs parameters. The effects of Rayleigh number on streamlines, isotherms and the Nusselt number are investigated.

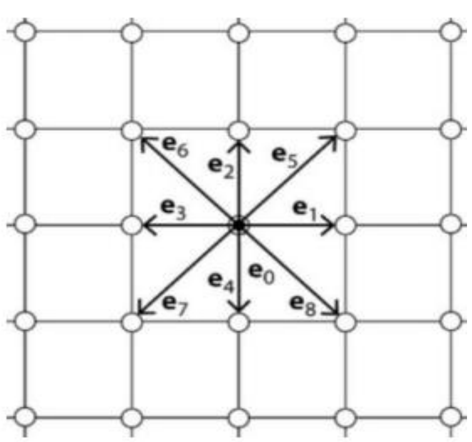
2. MATHEMATICAL FORMULATION

Plotting of considered model is shown in Figure 1. It displays a two-dimensional open cavity with side length of H. At first case the left vertical is maintained at high temperature (TH). Whereas at the second case, the vertical left wall is linearly heated. An external cold air enters into the enclosure from the east opening boundary, while the fluid is correlated with the opening boundary at constant temperature (TC). The horizontal walls are insulated and impermeable to mass transfer. The open cavity is filled with liquid gallium with Prandtl number of 0.025. The gravitational acceleration acts downward. The uniform external magnetic field with a constant magnitude B is applied in the x-direction (transverse field). It is assumed that the induced magnetic field produced by the motion of an electrically conducting fluid is negligible compared to the applied magnetic field. Thermo-physical properties of the fluid are assumed to be constant, and the density variation in the buoyancy force term is handled by the Boussinesq approximation. The flow is two-dimensional, laminar and incompressible; in addition, it is assumed that the viscous dissipation and Joule heating are neglected.

An open cavity is considered for the present study with the physical dimensions as shown in Figure 1. The left vertical is linearly heated. An external cold air enters into the enclosure from the east opening boundary, while the fluid is correlated with the opening boundary at constant temperature (Tc). The horizontal walls are insulated and impermeable to mass transfer. The open cavity is filled with liquid gallium. The gravitational acceleration acts downward.



(a)



(b)

Figure 1. Geometries (a) and D2Q9 velocities (b)

The uniform external magnetic field with a constant magnitude B0 is applied in the x-direction. The density variation in the fluid is approximated by the standard Boussinesq model. It is assumed that the induced magnetic field produced by the motion of an electrically conducting fluid is negligible compared to the applied one. Furthermore, it is assumed that Joule heating and the viscous dissipation are neglected. Therefore, standard D2Q9 for flow and for temperature can be written in non dimensional form as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\begin{aligned}&\rho(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y})=-\frac{\partial p}{\partial x}+\mu(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2})\\&+\frac{Ha^2\mu}{H^2}(v\sin\gamma\cos\gamma-u\sin^2\gamma)\end{aligned} \tag{2}$$

$$\begin{aligned}&\rho(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y})=-\frac{\partial p}{\partial y}+\mu(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2})\\&+\frac{Ha^2\mu}{H^2}(u\sin\gamma\cos\gamma-v\cos^2\gamma)+\rho g\beta(T-T_m)\end{aligned} \tag{3}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}) \tag{4}$$

Hartmann number is written as $Ha = HB\sqrt{\sigma/\mu}$.
The macroscopic velocity and temperature are [25-34]:

$$\mathrm{u}(\mathrm{r},t)=\sum_k e_k f_k(\mathrm{r},t)/\sum_k f_k(\mathrm{r},t) \tag{5}$$

$$T=\sum_k g_k(\mathrm{r},t) \tag{6}$$

Nusselt number Nu is one of the most important dimensionless parameters in describing the convective heat transport. The local Nusselt number and the average value at the bottom wall are calculated as;

$$NU_y=-\frac{H}{\Delta T}\frac{\partial T}{\partial x} \tag{7}$$

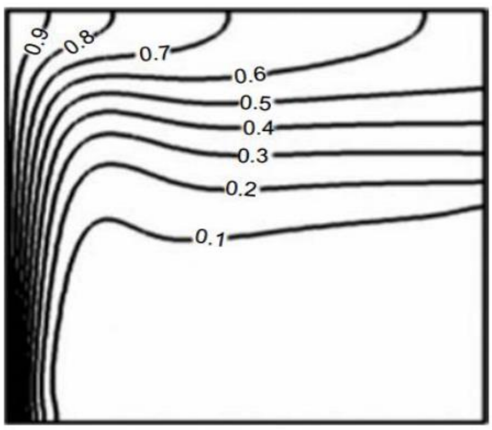
$$NU_{avg}=\frac{1}{H}\int_0^H NU_y dy \tag{8}$$

3. RESULTS AND DISCUSSIONS

In order to check the accuracy of the present results, the present code is validated against published works in the literature, in Figure 2 we compare the steady state isotherms at Pr=0.71 for Ra=10^5 in the absence of a magnetic field (Hartmann number, Ha=0) with reference [20], a good agreement is observed. A second validation is highlighted, for a different Prandtl number of Pr=0.025 (liquid gallium), the steady state isotherms of linearly heated side walls MHD cavity with a moderate Hartmann number of 50, Pr=0.025 and Ra=10^5. The obtained numerous investigations have been compared with reference [7] and present work. After grid assessment and validation with previous literature (Figure 2), we illustrate the effect of magnetic field in an open cavity with linearly heated west Boundary which is filled with liquid gallium for Ha=50 and Ra=10^5.

We seek to provide the behavior of the Nusselt bottom wall Nub in the case of two linear vertical walls. Figure 3 shows clearly that the local Nusselt number at the bottom wall Nub exhibits an oscillatory behavior with the horizontal distance x/X and that it is exactly symmetric about the centerline of the bottom wall. The Variation of local Nusselt number with distance at bottom wall in the case of linearly heated side walls (Pr= 0.025 and Ra=10^5) is depicted for different Hartman numbers.

In Figure 4 (a-b), the outlet section of flow on the open boundary moves downward and the movement of the flow gets limited with the increment of Hartmann number that it can influences heat transfer from the linearly heated wall to the cold open boundary. The effect of the presence of the magnetic field is clear in the counter of the isotherms where the isotherms recede from the linearly heated left wall slowly and their gradient on the left wall declines extremely which it exposes the decrease in heat transfer in the open cavity. Figure 4c highlight the variation of local Nusselt number Nub with horizontal distance X at bottom wall in the case of linearly heated side wall for a MHD open cavity (Pr=0.025 and Ra=10^5). We notice an increasing trend when the position vary from to 0 to left (west)-edge of the bottom side wall because of the linearly heated left boundary. Nub reach the maximum at the right (east)side of the bottom edge due to the cooled right (east) wall.



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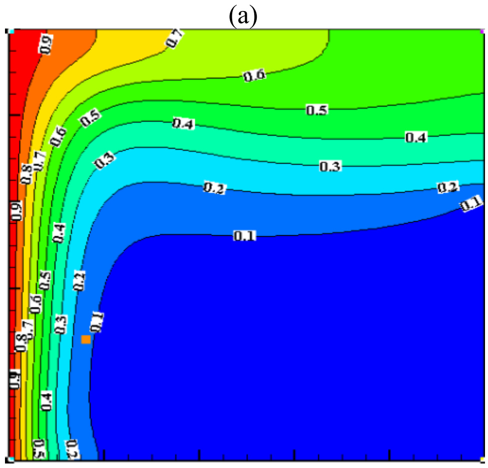

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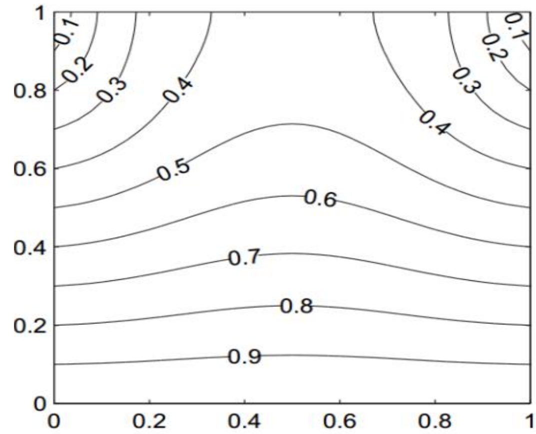

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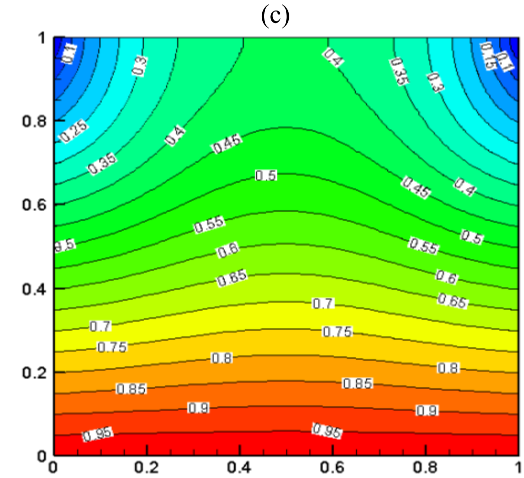


(d)

Figure 2. (a-b) Comparison of the steady state isotherms (a-b) at Pr=0.71 for Ha = 0 and Ra = 10^5 (a) ref. [20] and (b) and (c-d) steady state isotherms of linearly heated side walls MHD cavity for Ha = 50, Pr=0.025 and Ra = 10^5, (c) ref. [7] and (d) present work

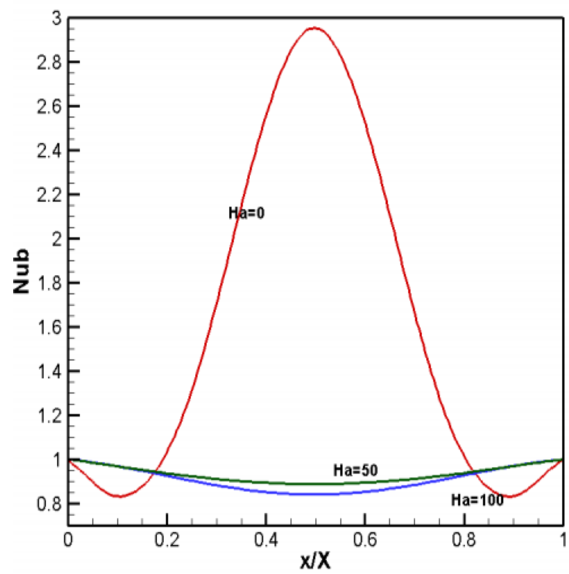
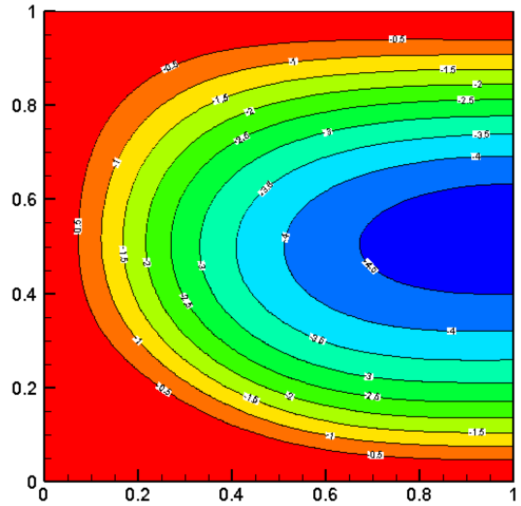
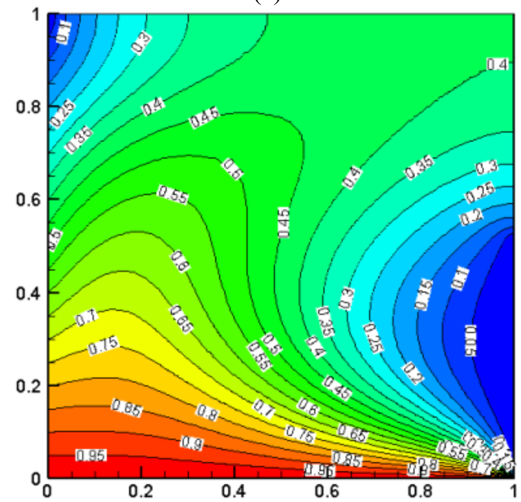


Figure 3. Variation of local Nusselt number with distance at bottom wall in the case of linearly heated side walls (Pr=0.025 and Ra = 10^5)



(a)



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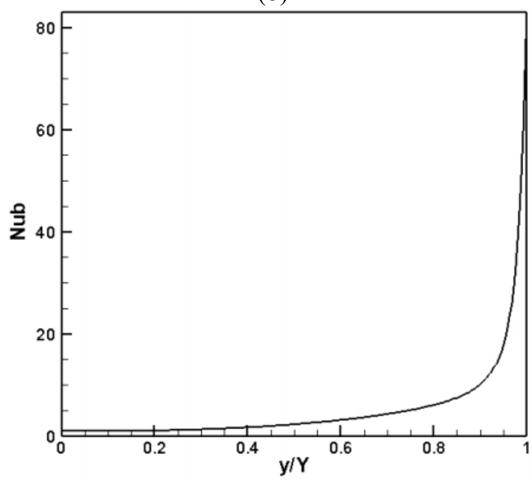

(c)

Figure 4. Isotherms (a) and streamlines (b) and Variation of local Nusselt number (c) with distance at bottom wall in the case of linearly heated side wall for a MHD open cavity

4. CONCLUSIONS

In this work, a lattice Boltzmann method is proposed to simulate MHD natural convection of two-dimensional open square cavity with a linearly heated boundary condition. A D2Q9 lattice model is used both to simulate the flow field and temperature field. Aiming to validate the proposed model, the obtained results of this study have been compared with previous numerical investigations [7, 20]. It was shown that the results predicted by the proposed method are in good agreement with other numerical results. The obtained numerical results show that the LB model is a stable, powerful approach for simulating the MHD free convection in a two-dimensional open square cavity with mixed boundary conditions and is able to study the effects of all parameter on the flow field and temperature field such as Hartmann number and Prandtl number. Besides, the implementation of the new model highlights a great ability and stability. It is noted that, the application of the magnetic field reduces the convective heat transfer rate in the open enclosure. The profiles of the local Nusselt number along the bottom wall of the MHD enclosure increased continuously for the case of linearly heated left side wall and cooled open right wall, while it exhibited an oscillatory behavior along the horizontal distance for the case of linearly heated side walls. All simulations are done for Ra=10^5 when the buoyancy-driven flow starts to dominate the heat transfer mechanism.

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NOMENCLATURE

Ha	Hartmann number
B	magnetic field intensity, T
ρ	Density,kgm^{-3}
u,v	Velocities, ms^{-1}
g	gravitational acceleration, m.s^{-2}
k	thermal conductivity, W.m^{-1}. K^{-1}
Ra	Rayleigh number
Pr	Prandtl number
H	characteristic length scale, m
e_i	discrete particle speeds, ms^{-1}
ΔT	Temperature difference, k
f	density distribution functions
g	internal energy distribution functions
T	Temperature, k
γ	Angle, rad
Nu	local Nusselt number
x,y	cartesian coordinates, m
T_m	Avearge temperature, k

Greek symbol

α	thermal diffusivity, m^2. s-^{1}
β	thermal expansion coefficient, k^{-1}
τ	Relaxation time, s
σ	electrical conductivity, Sm^{-1}
μ	dynamic viscosity, kg. m^{-1}.s^{-1}
ν	kinematic viscosity, m^{2}s^{-1}

Subscripts

avg	average
H	Hot
C	Cold