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Soret and hall effect on unsteady free convection flow past an infinite vertical plate with oscillatory suction velocity and variable permeability

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ABSTRACT

An unsteady, two dimensional, free convection flow with heat and mass transfer of a viscous, incompressible, electrically conducting fluid past an infinite vertical porous plate of time dependent permeability with oscillatory suction velocity and chemical reaction in the presence of Hall current effect and Soret effect is investigated. A uniform magnetic field is applied normal to the direction of flow. The dimensionless governing equations are solved using perturbation technique in Eckert number. Numerical evaluation of the analytical solutions is carried out for the velocity field, temperature distribution and concentration distribution. The results obtained are discussed for various emerging parameters such as Prandtl number (Pr), Grashof number (Gr), Modified Grashof number (Gm), Soret number (Sr), Schmidt number (Sc), Sink strength parameter (S), Eckert number (Ec), and Hall parameter (m) encountered in the problem under investigation.

1. INTRODUCTION

Combined buoyancy-generated heat and mass transfer due to temperature and concentration variations, in porous media, have several important applications in variety of engineering processes including heat exchanger devices, petroleum reservoirs, chemical catalytic reactors, solar energy porous wafer collector systems, ceramic materials, migration of moisture through air contained in fibrous insulations and grain storage installations and the dispersion of chemical contaminants through water-saturated soil, super convecting geothermic etc. The vertical free convection boundary layer flow in porous media owing to combined heat and mass transfer has been investigated by Bejan and Khair [3].

Lai and Kulacki [11] used the series expansion method to investigate coupled heat and mass transfer in natural convection from a sphere in a porous medium. Soundalgekar [17] analyzed the effects of variable suction and the horizontal magnetic field on the free convection flow past infinite vertical porous plate and made a comparative discussion of different parameters and the free convection flow of mercury and ionized air. Many works on heat and mass transfer have focused mainly on regular geometries, the recent studies of them such as heat and mass transfer along a vertical plate with variable surface temperature and concentration in the presence of the magnetic field studied by Elbashbeshy [5]. The structure of bonding interface and the composition of research varied with different temperatures is analyzed by Yang et al. [6].

Chamkha and Khaled [3] analysed similarity solutions for hydromagnetic simultaneous heat and mass transfer by natural convection from an inclined plate with internal heat generation or absorption. Soundalgekar [19] presented an exact solution to the flow of a viscous fluid. Jashim Uddin and Fazlul Hoque [8] investigated the convective flow of nanofluids inside an isosceles triangular shaped enclosure with the uneven bottom wall using nonhomogeneous dynamic model.

Moreover, study of heat and mass transfer flow in the presence of chemical reaction has received considerable attention due to its several practical applications in chemical and hydrometallurgical industries. Usually, chemical reactions are of two types, homogeneous and heterogeneous. A chemical reaction is said to be heterogeneous or homogenous depending on whether they occur at an interface or as a single phase volume reaction. A homogenous reaction occurs uniformly throughout the given phase, whereas a heterogeneous reaction takes place in a restricted region or within the boundary of a phase. Some of the numerous important applications of heat and mass transfer flow with chemical reaction can be found in catalytic chemical reactors, food processing, polymer production, manufacture of ceramics and glassware, smelting, undergoing exothermic or endothermic chemical reaction etc. Keeping in mind the significance of such study, several researchers investigated hydromagnetic free convection heat and mass transfer flow of a viscous, incompressible and electrically conducting fluid past a vertical plate in the presence of first order chemical reaction under different conditions. Some of the relevant research studies are due to Ibrahim et al. [7], Mohamed and Abo-Dahab [12], Singh and Kumar [16], Pal and Talukdar [14], Osman et al. [13] and Barik and Dash [2].

It was discovered in1979 by Edwin Herbert Hall while working on his doctoral degree at the Johns Hopkins University in Baltimore, Maryland, USA. In fact, Hall current plays a key role in determining the flow features of the fluid flow problems because it induces secondary flow in the flow-field which is also the characteristics of Coriolis force. Therefore, it seems to be significant to compare and contrast the simultaneous effects of these two agencies on the fluid flow problems. Hall current finds applications in many scientific and engineering devices, viz. MHD power generators, Hall current accelerators, nuclear power reactors, magnetometers, underground energy storage system, Hall effect sensors, spacecraft propulsion etc. Taking into consideration the importance of combined effects of Hall current and rotation, Alam and Sattar [1] investigated unsteady hydromagnetic free convection heat and mass transfer flow with Hall effects in a rotating system in the presence of viscous dissipation and Joule heating. Recently, Seth et al. [15] studied effects of Hall current and rotation on unsteady hydromagnetic natural convection heat and mass transfer flow of an optically thick radiating fluid past an infinite vertical plate through porous medium.

It is to be noted that in all the above research studies, Soret (or thermo-diffusion) effect assumed to be neglected. But, Soret effect becomes significant when there exists a density difference in the flow regime i.e. in the areas such as hydrology, petrology and geo-sciences. This effect has various important applications in isotope separation, in mixture between gases with very light molecular weight (H_2, He) and of medium molecular weight (N_2, air) etc. Looking into the importance of such effect, Jha and Singh [9] analyzed Soret effect on unsteady free convection and mass transfer flow past an impulsively started infinite vertical plate. Kafoussias [10] investigated Soret effect on unsteady hydromagnetic free convection heat and mass transfer past an infinite moving vertical plate for two cases of practical interest, namely, (i) impulsively started plate and (ii) uniformly accelerated plate. past an impulsively started infinite isothermal vertical plate with mass transfer.

The objective of the current work, is to analyze the heat and mass transfer effects on unsteady, two dimensional free convective flow of a viscous, incompressible electrically conducting fluid along a vertical plate with suction, embedded in a porous medium in the presence of uniform magnetic field by taking into account the effect of viscous dissipation, Soret and Hall effect. The equation of continuity, motion, energy and concentration which governs the flow field is solved by using the regular perturbation method. The behavior of velocity, temperature and concentration are discussed for variations in the governing parameters.

2. MATHEMATICAL FORMULATION

Consider an unsteady free convective, viscous, incompressible, electrically conducting fluid through porous medium past an infinite vertical porous plate with oscillatory suction velocity, variable permeability and heat absorbing sinks in the presence of hall current and Soret effect. Let the x-axis be taken along the plate in the direction of the flow and y-axis normal to the plate.

A uniform magnetic field is introduced normal to the direction of flow. All the fluid properties are assumed to be constant except that of the influence of density variation with temperature and concentration. The basic flow medium is entirely due to the buoyancy force caused by temperature and concentration difference between the wall and the medium. The surface temperature and concentration level near the plate are raised uniformly.

Let $\bar{B} = (0, B_0, 0)$ and $\bar{V} = (u', v', 0)$. Under Boussenesq's approximation, the fundamental equations which governs the flow are

Equation of continuity:

$$\frac{\partial V'}{\partial y'} = 0 \tag{1}$$

That is

$$V' = -V_0'\left(1 + \varepsilon e^{i\omega' t'}\right) \tag{2}$$

Equation of momentum:

$$\frac{\partial u'}{\partial t'} + V'\frac{\partial u'}{\partial y'} = g\,\beta\,(T' - T_\infty') + g\beta'(C' - C_\infty') + \gamma\frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{P(1+m^2)}u' - \frac{\gamma}{K'}u' \tag{3}$$

Equation of Energy:

$$\frac{\partial T'}{\partial t'} + V'\frac{\partial T'}{\partial y'} = \frac{K}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} + S'(T' - T_\infty') + \frac{\gamma}{C_p}\left(\frac{\partial u'}{\partial y'}\right)^2 \tag{4}$$

On disregarding the Joule's heating

Equation of Concentration:

$$\frac{\partial C'}{\partial t'} + V'\frac{\partial C'}{\partial y'} = D\,\frac{\partial^2 C'}{\partial y'^2} + K_1(C' - C_\infty') + D_m\left(\frac{\partial^2 T'}{\partial y'^2}\right) \tag{5}$$

The corresponding boundary conditions are given by

$$\begin{aligned} &U' = 0, V' = -V_0'\left(1 + \varepsilon e^{i\omega' t'}\right) \\ &, T' = T_w' + \varepsilon(T_w' - T_\infty')e^{i\omega' t'} \\ &C' = C_w' + \varepsilon(C_w' - C_\infty')e^{i\omega' t'} \quad \text{at } y' = 0 \\ &u' \to 0, T' \to T_w', C' \to C_w' \quad \text{as } y' \to \infty \end{aligned} \tag{6}$$

Introduce the non dimensional parameters

$$y = \frac{y' V_0'}{\upsilon};\ t = \frac{t' V_0'^2}{4\upsilon};\ \omega = \frac{4\upsilon\omega'}{V_0'^2};\ u = \frac{u'}{V_0'};\ \upsilon = \frac{\mu}{\rho};\ V = \frac{V'}{V_0'};$$
$$T = \left(\frac{T' - T\infty'}{Tw' - T\infty'}\right) \quad C = \left(\frac{C' - C\infty'}{Cw' - C\infty'}\right) \tag{7}$$

Substitute the non dimensional parameters into (2) – (5) it reduces to

$$\frac{1}{4}\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y}(1 + \varepsilon e^{i\omega t}) = GrT + \text{Gm}C + \frac{\partial^2 \text{u}}{\partial \text{y}^2} - M_1 u - \frac{u}{K_0(1+\varepsilon e^{i\omega t})} \tag{8}$$

$$\frac{Pr}{4}\frac{\partial T}{\partial t} - (1 + \varepsilon e^{i\omega t})\frac{\partial T}{\partial y} = \frac{\partial^2 \text{T}}{\partial \text{y}^2} + \frac{Pr}{4}ST + PrEc\left(\frac{\partial u}{\partial y}\right)^2 \tag{9}$$

where $M_1 = \frac{M}{1+m^2}$

$$\frac{1}{4}\frac{\partial c}{\partial t} - (1 + \varepsilon e^{i\omega t})\frac{\partial c}{\partial y} = \frac{1}{SC}\frac{\partial^2 \text{c}}{\partial \text{y}^2} + K_2 C + Sr\frac{\partial^2 T}{\partial y^2} \tag{10}$$

The non dimensionalized boundary conditions are

$$\begin{aligned} &u = 0,\ T = 1 + \varepsilon e^{i\omega t},\ C = 1 + \varepsilon e^{i\omega t} \text{ at } y = 0 \text{ and} \\ &u \to 0,\ T \to 0,\ C \to 0 \quad \text{as } y \to \infty \end{aligned} \tag{11}$$

To solve equation (8), (9) and (10), we assume from V.M. Soundalgekar, J.P Bhat [19] that

$$u(y,t) = u_0(y) + \varepsilon e^{i\omega t}u_1(y) \tag{12}$$

$$T(y,t) = T_0(y) + \varepsilon e^{i\omega t}T_1(y) \tag{13}$$

$$C(y,t) = C_0(y) + \varepsilon e^{i\omega t} C_1(y) \tag{14}$$

Substituting (12) – (14) in equation (8) - (10), equating like terms and neglecting the co-efficient of higher powers of ε, we get

$$\frac{\partial^2 u_0}{\partial y^2} + \frac{\partial u_0}{\partial y} - A_1 u_0 = -GrT_0 - GmC_0 \tag{15}$$

$$\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial u_1}{\partial y} - A_2 u_1 = -GrT_1 - GmC_1 \frac{\partial u_0}{\partial y}' - \frac{1}{K_0} u_0 \tag{16}$$

$$\frac{\partial^2 T_0}{\partial y^2} + Pr\frac{\partial T_0}{\partial y} + \frac{PrS}{4} T_0 = -PrEc\left(\frac{\partial u_0}{\partial y}\right)^2 \tag{17}$$

$$\frac{\partial^2 T_1}{\partial y^2} + Pr\frac{\partial T_1}{\partial y} + \frac{Pr}{4}[S - i\omega]T_1 = -2PrEc\frac{\partial u_0}{\partial y}\frac{\partial u_1}{\partial y} - Pr\frac{\partial T_0}{\partial y} \tag{18}$$

$$\frac{\partial^2 C_0}{\partial y^2} + Sc\frac{\partial C_0}{\partial y} + ScK_2 C_0 = -ScSrT_0'' \tag{19}$$

$$\frac{\partial^2 C_1}{\partial y^2} + Sc\frac{\partial C_1}{\partial y} + ScK_1 C_1 = -Sc\frac{\partial C_0}{\partial y} - SrSc\frac{\partial^2 T_1}{\partial y^2} \tag{20}$$

The associated boundary conditions will become as

$u_0 = 0$, $u_1 = 0$, $T_0 = 1$, $T_1 = 0$, $C_0 = 1$, $C_1 = 0$ at $y = 0$
$u_0 \to 0$, $u_0 \to 0$, $T_0 \to 0$, $T_1 \to 0$, $C_0 \to 0$, $C_1 \to 0$ as $y \to \infty$ (21)

Using multi parameter perturbation technique and assuming $Ec << 1$, we write

$u_0 = u_{00} + Ecu_{01}$: $T_0 = T_{00} + Ec\,T_{01}$: $C_0 = C_{00} + Ec\,C_{01}$ (22)

$u_1 = u_{10} + Ecu_{11}$: $T_1 = T_{10} + Ec\,T_{11}$: $C_1 = C_{10} + Ec\,C_{11}$ (23)

Using Equation (22) & (23) in the equation (15) – (20) and equating the co-efficient of Ec° and Ec^1 only, we get the following sets of differential equations for u_{00}, u_{01}, u_{10}, u_{11}, and T_{00}, T_{01}, T_{10}, T_{11} and C_{00}, C_{01}, C_{10}, C_{11}

$$u_{00}'' + u_0' - A_1 u_{00} = -GrT_{00} - Gm\,C_{00} \tag{24}$$

$$u_{01}'' + u_{01}' - A_1 u_{01} = -GrT_{01} - Gm\,C_{01} \tag{25}$$

$$u_{10}'' + u_{10}' - A_2 u_{10} = -GrT_{10} - Gm\,C_{10} - u_{00}' - \frac{1}{k_0}u_{00} \tag{26}$$

$$u_{11}'' + u_{11}' - A_2 u_{11} = -GrT_{11} - Gm\,C_{11} - u_{01}' - \frac{1}{k_0}u_{01} \tag{27}$$

$$T_{00}'' + PrT_{00}' + \frac{PrS}{4}T_{00} = 0 \tag{28}$$

$$T_{01}'' + PrT_{01}' + \frac{PrS}{4}T_{01} = -Pr(u_{00}')^2 \tag{29}$$

$$T_{10}'' + PrT_{10}' + \frac{Pr}{4}(iw - s)T_{10} = -PrT_{00}' \tag{30}$$

$$T_{11}'' + PrT_{11}' - \frac{Pr}{4}(iw - s)T_{11} = -2Pru_{00}'u_{10}' - PrT_{01}' \tag{31}$$

$$C_{00}'' + ScC_{00}' + ScK_2 C_{00} = -ScSrT_{00}'' \tag{32}$$

$$C_{01}'' + ScC_{01}' + ScK_2 C_{01} = -ScSrT_{01}'' \tag{33}$$

$$C_{10}'' + ScC_{10}' + ScK_1 C_{10} = -ScC_{00}' - ScSrT_{10}'' \tag{34}$$

$$C_{11}'' + ScC_{11}' + ScK_1 C_{11} = -ScC_{01}' - ScSrT_{11}'' \tag{35}$$

and the corresponding boundary conditions are

$u_{00} = 0$, $u_{01} = 0$, $T_{00} = 1$, $T_{01} = 0$, $C_{00} = 1$, $C_{01} = 0$
$u_{10} = 0$, $u_{11} = 0$, $T_{10} = 1$, $T_{11} = 0$, $C_{10} = 1$, $C_{11} = 0$ at $y = 0$
$u_{00} \to 0$, $u_{01} \to 0$, $T_{00} \to 0$, $T_{01} \to 0$, $C_{00} \to 0$, $C_{01} \to 0$
$u_{01} \to 0$, $u_{11} \to 0$, $T_{01} \to 0$, $T_{11} \to 0$, $C_{01} \to 0$, $C_{11} \to 0$ as $y \to \infty$
(36)

Solving these differential equation (24) - (35) with the help of the corresponding boundary conditions and then substitute the values in the relations (22) and (23), we obtain the mean velocity, mean temperature and mean concentration as,

$$\begin{aligned} u_0 = &\left((A_3 + A_4)e^{-a_8 y} - A_3 e^{-a_2 y} - A_4 e^{-a_6 y}\right) \\ &+ Ec(D_{36}e^{-a_8 y} - D_{28}e^{-a_2 y} - D_{29}e^{-a_6 y} + D_{30}e^{-2a_8 y} \\ &+ D_{31}e^{-2a_2 y} + D_{32}e^{-2a_6 y} - D_{33}e^{-(a_2+a_8)y} \\ &+ D_{34}e^{-(a_2+a_6)y} - D_{35}e^{-(a_6+a_8)y}\Big) \end{aligned} \tag{37}$$

$$\begin{aligned} u_1 = &(D_{19}e^{-a_{10}y} - D_{14}e^{-a_4 y} - D_{15}e^{-a_2 y} - D_{16}e^{-a_{12}y} \\ &- D_{17}e^{-a_8 y} - D_{18}e^{-a_6 y}) + Ec\,(D_{93}e^{-a_{10}y} \\ &+ D_{73}e^{-a_4 y} + D_{74}e^{-(a_8+a_{10})y} + D_{75}e^{-(a_4+a_8)y} \\ &+ D_{76}e^{-(a_2+a_8)y} + D_{77}e^{-(a_8+a_{12})y} + D_{78}e^{-2a_8 y} \\ &+ D_{79}e^{-(a_6+a_8)y} + D_{80}e^{-(a_2+a_{10})y} + D_{81}e^{-(a_2+a_4)y} \\ &+ D_{82}e^{-2a_2 y} + D_{83}e^{-2a_6 y} + D_{84}e^{-(a_2+a_{12})y} \\ &+ D_{85}e^{-(a_2+a_6)y} + D_{86}e^{-(a_6+a_{10})y} \\ &+ D_{87}e^{-(a_4+a_6)y} + D_{88}e^{-(a_6+a_{12})y} + D_{89}e^{-a_2 y} \\ &+ D_{90}e^{-a_{12}y} + D_{91}e^{-a_6 y} + D_{92}e^{-a_8 y}) \end{aligned} \tag{38}$$

$$\begin{aligned} T_0 = &(e^{-a_2 y}) + Ec(D_9 e^{-a_2 y} - D_3 e^{-2a_8 y} \\ &- D_4 e^{-2a_2 y} - D_5 e^{-2a_6 y} + D_6 e^{-(a_2+a_8)y} \\ &- D_7 e^{-(a_2+a_6)y} + D_8 e^{-(a_6+a_8)y}\Big) \end{aligned} \tag{39}$$

$$\begin{aligned} T_1 = &((1 - D_1)e^{-a_4 y} + D_1 e^{-a_2 y}) + Ec(D_{53}e^{-a_4 y} \\ &+ D_{37}e^{-(a_8+a_{10})y} + D_{38}e^{-(a_4+a_8)y} + D_{39}e^{-(a_2+a_8)y} \\ &+ D_{40}e^{-(a_8+a_{12})y} + D_{41}e^{-2a_8 y} + D_{42}e^{-(a_6+a_8)y} \\ &+ D_{43}e^{-(a_2+a_{10})y} + D_{44}e^{-(a_2+a_4)y} + D_{45}e^{-2a_2 y} \\ &+ D_{46}e^{-2a_6 y} + D_{47}e^{-(a_2+a_{12})y} + D_{48}e^{-(a_2+a_6)y} \\ &+ D_{49}e^{-(a_6+a_{10})y} + D_{50}e^{-(a_4+a_6)} \\ &+ D_{51}e^{-(a_6+a_{12})y} + D_{52}e^{-a_2 y}\Big) \end{aligned} \tag{40}$$

$$\begin{aligned} C_0 = &((1 - D_2)e^{-a_6 y} + D_2 e^{-a_2 y}) + Ec(D_{27}e^{-a_6 y} \\ &+ D_{20}e^{-a_2 y} - D_{21}e^{-2a_8 y} - D_{22}e^{-2a_2 y} \\ &- D_{23}e^{-2a_6 y} + D_{24}e^{-(a_2+a_8)y} \\ &- D_{25}e^{-(a_2+a_6)y} + D_{26}e^{-(a_6+a_8)y}\Big) \end{aligned} \tag{41}$$

$$\begin{aligned} C_1 = &(D_{13}e^{-a_{12}y} + D_{10}e^{-a_6 y} - D_{11}e^{-a_2 y} \\ &+ D_{12}e^{-a_4 y}) + Ec(D_{72}e^{-a_{12}y} + D_{54}e^{-a_6 y} \\ &+ D_{55}e^{-a_2 y} + D_{56}e^{-a_4 y} + D_{57}e^{-2a_8 y} \\ &+ D_{58}e^{-2a_2 y} + D_{59}e^{-2a_6 y} \\ &+ D_{60}e^{-(a_2+a_8)y} + D_{61}e^{-(a_2+a_6)y} \\ &+ D_{62}e^{-(a_6+a_8)y} + D_{63}e^{-(a_8+a_{10})y} \\ &+ D_{64}e^{-(a_4+a_8)y} + D_{65}e^{-(a_8+a_{12})y} \\ &+ D_{66}e^{-(a_2+a_{10})y} + D_{67}e^{-(a_2+a_4)y} \\ &+ D_{68}e^{-(a_2+a_{12})y} + D_{69}e^{-(a_6+a_{10})y} \\ &+ D_{70}e^{-(a_4+a_6)} + D_{71}e^{-(a_6+a_{12})y}\Big) \end{aligned} \tag{42}$$

where the constants a_2, a_4, a_6, a_8, a_{10}, D1, D_2, D_3, D_4,… used above are function of the physical parameters involved in the problem given in *Appendix*.

3. SKIN FRICTION, RATE HEAT AND MASS TRANSFER

Skin friction co-efficient (τ) at the plate is

$$\tau = \left(\frac{\partial c}{\partial y}\right)_{y=0} = u_0^{'}(0) + \varepsilon e^{i\omega t}u_1^{'}(0) \qquad (43)$$

Heat transfer coefficient (Nu) at the plate is

$$N_u = -\left(\frac{\partial T}{\partial y}\right)_{y=0} = T_0^{'}(0) + \varepsilon e^{i\omega t}T_1^{'}(0) \qquad (44)$$

Mass transfer coefficient (S_h) at the plate is

$$S_h = -\left(\frac{\partial C}{\partial y}\right)_{y=0} = C_0^{'}(0) + \varepsilon e^{i\omega t}C_1^{'}(0) \qquad (45)$$

4. RESULT AND DISCUSSION

The formulation of the problem that accounts for the viscous dissipation effect on the flow of a viscous incompressible fluid along a vertical infinite plate embedded in a porous medium in the presence of uniform magnetic field with Hall and Soret effect is accomplished. The governing equations of the flow field were solved analytically, using a perturbation method, and the expressions for the velocity, temperature, concentration were obtained. In order to get a physical insight of the problem, the above physical quantities are computed numerically for different values of the governing parameters viz. the thermal Grashof number (*Gr*), the solutal Grashof number (*Gc*), the Prandtl number (*Pr*), the Schmidt number (*Sc*), the Eckert number (E), the Hall parameter (m) and the Porosity parameter (K_0). (Pr = 0.71, Ec = 0.001, Sc = 0.22, K_2 = 1.0, K_1 = 1.0, K_0 =10, M = 2.0, ε = 0.1, ωt = π/2, Gr = 5.0, Sr = 1.0, Gm = 2.0, S = 1, m=1.0).

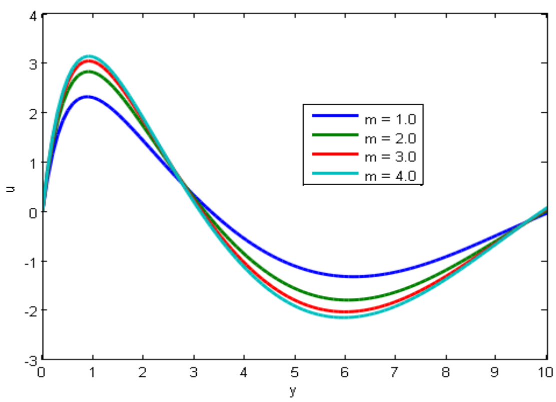

Figure 1. Velocity profile for various m

Fig. 1 shows the velocity profiles in the boundary layer for various values of m. Fig.1 illustrates the influence of Hall parameter on the velocity u for the case of air (*Pr* = 0.7). It is observed that the velocity increases rapidly near to the wall of the porous plate, reaches a maximum and decays then shows the reverse behavior for various m. This is because of the fact that the application of the transverse magnetic field to an electrically conducting fluids gives rise to a respective type of force known as Lorentz force. This force has the tendency to slowdown the motion of the fluid in the boundary layer.

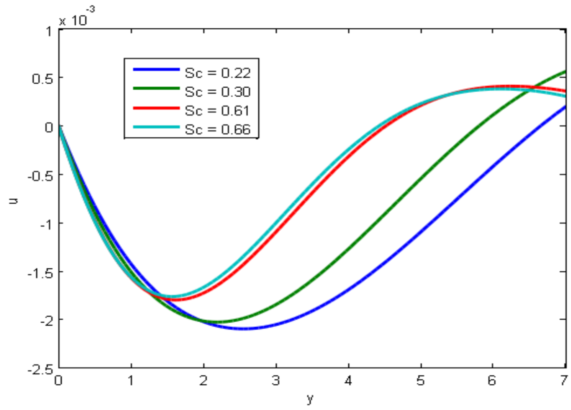

Figure 2. Velocity profile for various Sc

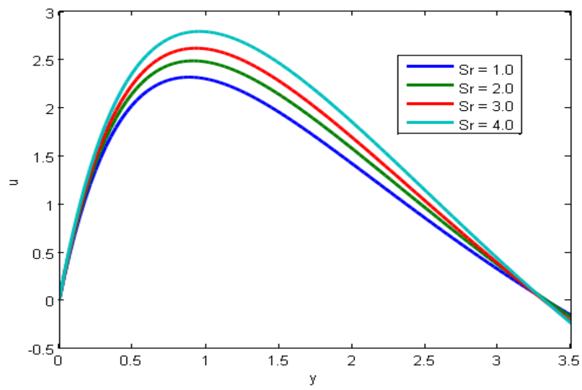

Figure 3. Velocity profile for various Sr

Fig. 2 presents the typical velocity profiles in the boundary layer for various values of the Schmidt number (Sc). The velocity distribution decreases ant then showing the increasing profile when Sc increases. Fig.3 exhibits the typical velocity profiles in the boundary layer for various values of the Soret number (Sr). The velocity distribution attains a distinct maximum value in the vicinity of the plate and then decreases properly to approach a free stream value of y.

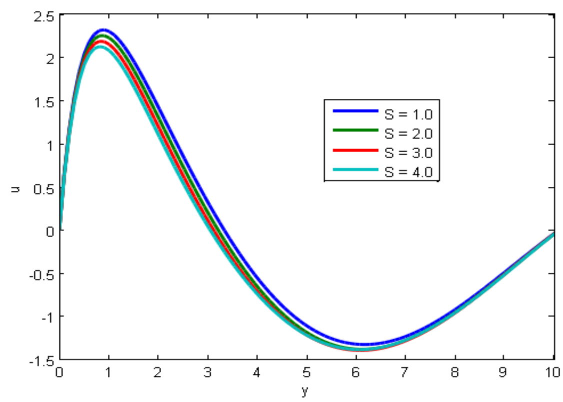

Figure 4. Velocity profile for various S

Fig. 4 represents the velocity profile for various values of S. As expected, velocity increases with the increase in heat source parameter. For the case of different values of the thermal Grashof number, the velocity profiles in boundary layer are shown in fig. 5 for air (Pr = 0.71) and water (Pr = 7.1). It is observed that an increase in *Gr* leads to a rise in the values of velocity due to enhancement in buoyancy force.

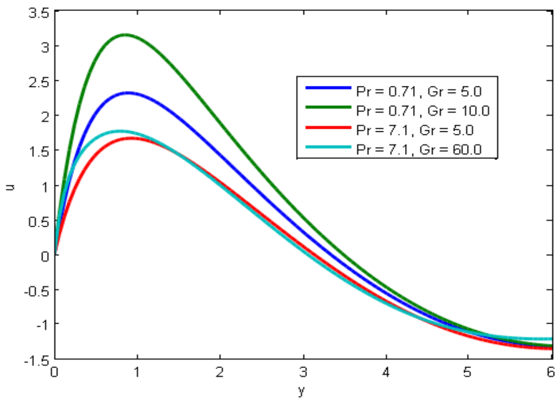

Figure 5. Velocity profile for various Gr

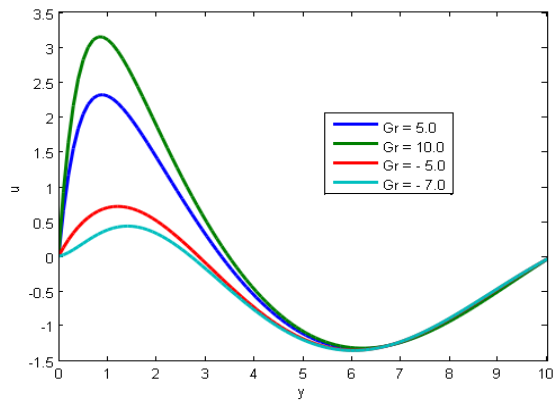

Figure 6. Velocity profile for various Gr > 0 and Gr < 0

The effects of thermal Grashof number on the velocity are shown in Fig. 6. When Gr > 0 the velocity increases rapidly and decreases when Gr < 0. Here, the positive values of *Gr* correspond to heating of the plate and negative values of Gr correspond to cooling of the plate. In addition, it is observed that the velocity increases sharply near the wall of the porous plate as *Gr* increases and then decays to the free stream value. Fig. 7, shows velocity profiles for different values of the porosity parameter K_0. It is observed that the fluid velocity increases sharply, attains a peak value near to the plate and decays continuously till y = 3.0 and then the velocity decreases.

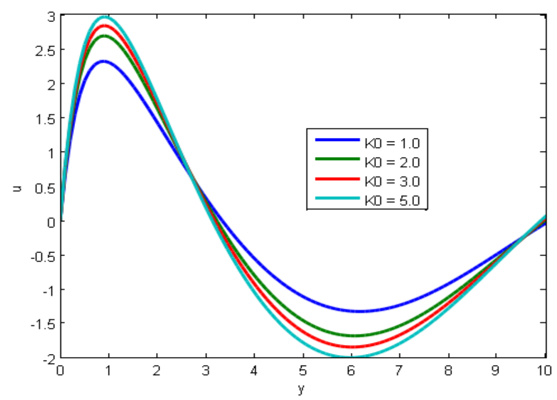

Figure 7. Velocity profile for various K_0

For the case of different values of the solutal Grashof number, the velocity profiles in boundary layer are shown in Fig. 8. As expected, the fluid velocity increases and the peak value becomes more distinctive due to increase in the buoyancy force represented by *Gc*. The effects of the viscous dissipation parameter i.e. the Eckert number on temperature is shown in Fig. 9. It is observed that the temperature increases near the plate, then decays continuously with increasing y.

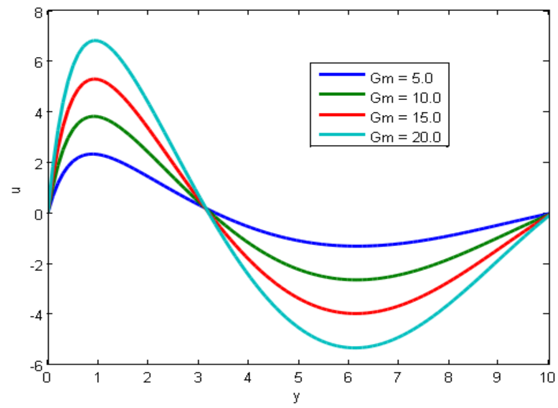

Figure 8. Velocity profile for various Gm

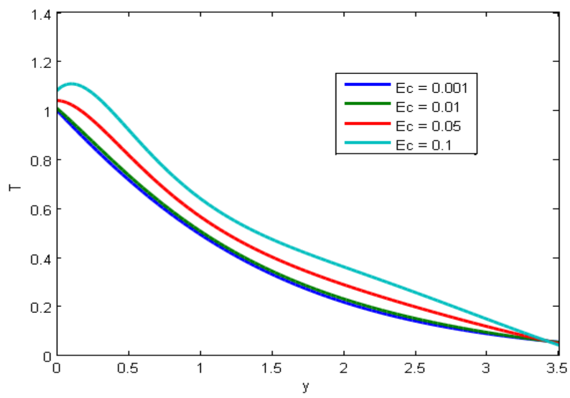

Figure 9. Temperature profile for various Ec

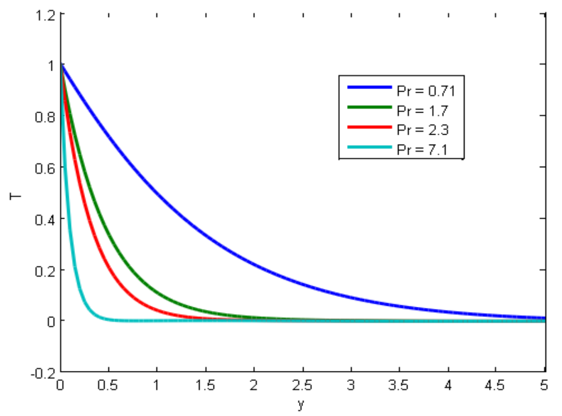

Figure 10. Temperature profile for various Pr

Fig. 10. shows the behavior of temperature for different values of Prandtl number. It is observed that an increase in the Prandtl number results a decrease of the thermal boundary layer thickness. The reason is that smaller values of *Pr* are equivalent to increase in the thermal conductivity of the fluid and therefore, heat is able to diffuse away from the heated surface more rapidly for higher values of *Pr*. Fig. 11 depicts the effect of sink strength parameter *S* on the temperature profiles of the flow field. Curve with S > 0 represent the presence of heat sink in the flow field. From figure, it is seen

that a growing sink strength parameter decreases the temperature of the flow field.

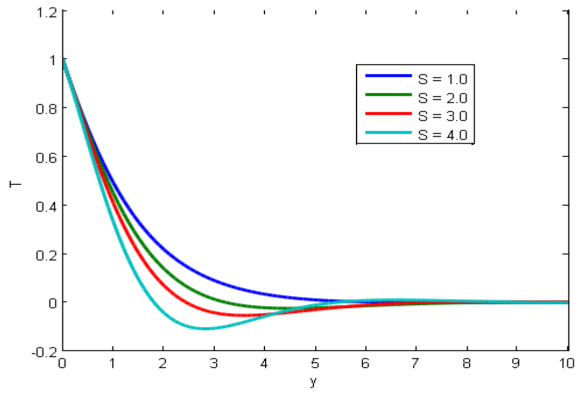

Figure 11. Temperature profile for various S

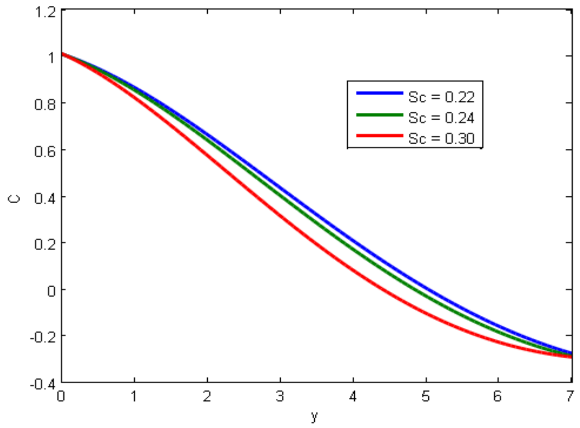

Figure 12. Concentration profile for various Sc

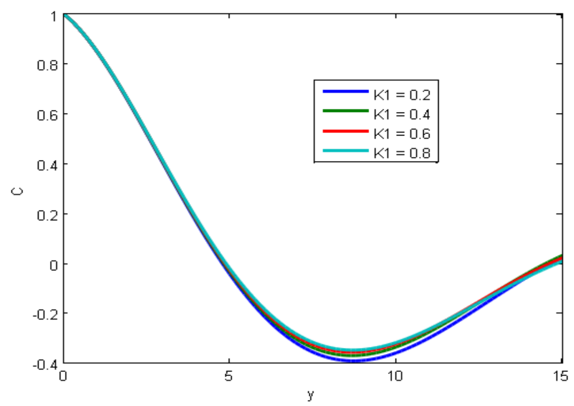

Figure 13. Concentration profile for various K_1

Figs 12 and 13. Depicts the concentration profile for various Sc and K_1. From Fig. 12 it is perceived that, species concentration decreases as the Schmidt number increases. From Fig 13, it is noticed that the concentration increases with the increase in chemical reaction parameter.

5. CONCLUSION

The governing equations for unsteady, incompressible MHD free convective heat and mass transfer flow past an infinite vertical plate embedded in a porous medium with Hall and Soret effect was formulated. Viscous dissipation effects were also included in the present work. The plate velocity is maintained at constant value and the flow was subjected to a transverse magnetic field. The resulting partial differential equations were transformed into a set of ordinary differential equations using two- term series and solved in closed form. Numerical evaluations of the closed form results were performed and graphical results were obtained to illustrate the details of the flow, heat and mass transfer characteristics and their dependence on some physical parameters. It was found that velocity increases rapidly near the wall and then decreases to free stream as Hall parameter, Schmidt number, Porosity parameter and Soret number increases. The velocity profile decreases with the increase in sink strength parameter. It was also found that when thermal and solutal Grashof numbers were increased, the thermal and concentration buoyancy effects were enhanced and thus, the fluid velocity increased. The temperature profile increases with the increase in Eckert number and decreases with the increase in Prandtl number and Sink strength parameter. Also, when the Schmidt number was increased, the concentration level gets decreased resulting in decreased fluid velocity. In addition, it is found that concentration profile increases with the increase in chemical reaction parameter.

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NOMENCLATURE

u', v'	Velocity components in x, y direction
T	Temperature of the fluid
C	Concentration of the fluid
g	acceleration due to gravity
C_P	Specific heat at constant pressure
T'_W	Temperature at the wall
C'_W	Concentration at the wall
T'_∞	Ambient Temperature
C'_∞	Ambient Concentration
K	Thermal conductivity
D	coefficient of mass diffusivity
Gr	Grashof Number for Heat Transfer
Gm	Grashof number for mass transfer
Sc	Schmidt number
Sr	Soret number
Pr	Prandtl numbr
S	Heat source parameter
Nu	Nusselt number
Sh	Sherwood number
m	Hall parameter
Ec	Eckert number
V'_0	Constant suction
B_0	Magnetic strength
S	Sink strength parameter
K_2	Chemical Reaction parameter

Greek symbols

β	coefficient of volume expansion
β'	coefficient of concentration expansion
γ	coefficient of viscosity
θ	dimensionless temperature
ϕ	dimensionless concentration
τ	Skin friction coefficient

APPENDIX

$$A_1 = M_1 + \frac{1}{K_0} \qquad A_2 = A_1 + \frac{i\omega}{4}$$

$$A_3 = \frac{Gr+GmD_2}{{a_2}^2-a_2-A_1} \qquad A_4 = \frac{Gm(1-D_2)}{{a_6}^2-a_6-A_1}$$

$$K_1 = \left[\frac{Kv}{{V_0'}^2} - \frac{i\omega}{4}\right] \qquad K_2 = \frac{Kv}{{V_0'}^2}$$

$$a_2 = \frac{Pr + \sqrt{Pr^2 - PrS}}{2}$$

$$a_4 = \frac{Pr + \sqrt{Pr^2 + (Pr)(iw - S)}}{2}$$

$$a_6 = \frac{Sc + \sqrt{Sc^2 - 4ScK_2}}{2}$$

$$a_8 = \frac{1 + \sqrt{1 + 4A_1}}{2}$$

$$a_{10} = \frac{1 + \sqrt{1 + 4A_2}}{2}$$

$$D_1 = \frac{\mathrm{Pr}\ a_2}{a_2^2 - Pra_2 - \frac{Pr}{4}(iw - s)}$$

$$D_2 = \frac{-ScSra_2^2}{a_2^2 - Sca_2 + ScK_2}$$

$$D_3 = \frac{Pra_8^2(A_3 + A_4)^2}{4{a_8}^2 - 2a_8.Pr + \frac{PrS}{4}}$$

$$D_4 = \frac{Pra_2^2A_3^2}{4{a_2}^2 - 2a_2.Pr + \frac{PrS}{4}}$$

$$D_5 = \frac{Pra_6^2A_4^2}{4{a_6}^2 - 2a_6Pr + \frac{PrS}{4}}$$

$$D_6 = \frac{2Pra_8a_2A_3(A_3 + A_4)}{(a_2 + a_8)^2 - (a_2 + a_8)Pr + \frac{PrS}{4}}$$

$$D_7 = \frac{2Pra_2a_6A_3A_4}{(a_2 + a_6)^2 - (a_2 + a_6)Pr + \frac{PrS}{4}}$$

$$D_8 = \frac{2Pra_8a_6A_4(A_3 + A_4)}{(a_6 + a_8)^2 - (a_6 + a_8)Pr + \frac{PrS}{4}}$$

$D_9 = (D_3 + D_4 + D_5 - D_6 + D_7 - D_8)$

$$D_{10} = \frac{Sca_6(1 - D_2)}{a_6^2 - a_6Sc + ScK_1}$$

$$D_{11} = \frac{ScSra_2^2 D_1}{a_2^2 - a_2 Sc + ScK_1}$$

$$D_{12} = \frac{Sca_2 - ScSra_4{}^2 (1 - D_1)}{a_4^2 - a_4 Sc + ScK_1}$$

$D_{13} = - D_{10} + D_{11} - D_{12}$

$$D_{14} = \frac{Gr(1 - D_1) + GmD_{12}}{a_4^2 - a_4 - A_2}$$

$$D_{15} = \frac{GrD_1 - GmD_{11} + a_2 A_3 - \frac{1}{K_0} A_3}{a_2^2 - a_2 - A_2}$$

$$D_{16} = \frac{GmD_{13} + GmD_{10}}{a_{12}^2 - a_{12} - A_2}$$

$$D_{17} = \frac{-a_8(A_3 + A_4) + \frac{1}{K_0}(A_3 + A_4)}{a_8^2 - a_8 - A_2}$$

$$D_{18} = - \frac{a_6 A_4 - \frac{1}{K_0} A_4 + GmD_{10}}{a_6^2 - a_6 - A_2}$$

$D_{19} = D_{14} + D_{15} + D_{16} + D_{17} + D_{18}$

$$D_{20} = \frac{-ScSra_2^2 D_9}{a_2^2 - Sca_2 + Sc K_2}$$

$$D_{21} = \frac{-4ScSra_8^2 D_3}{4a_8^2 - 2Sca_8 + Sc K_2}$$

$$D_{22} = \frac{-4ScSra_2^2 D_4}{4a_2^2 - 2Sca_2 + ScK_2}$$

$$D_{23} = \frac{-4ScSra_6^2 D_5}{4a_6^2 - 2Sca_6 + Sc K_2}$$

$$D_{24} = \frac{-ScSr(a_2 + a_8)^2 D_6}{(a_2 + a_8)^2 - Sc(a_2 + a_8) + Sc K_2}$$

$$D_{25} = \frac{-ScSr(a_2 + a_6)^2 D_7}{(a_2 + a_6)^2 - Sc(a_2 + a_6) + Sc K_2}$$

$$D_{26} = \frac{-ScSr(a_6 + a_8)^2 D_8}{(a_6 + a_8)^2 - Sc(a_6 + a_8) + Sc K_2}$$

$D_{27} = - D_{20} + D_{21} + D_{22} + D_{23} - D_{24} + D_{25} - D_{26}$

$$D_{28} = \frac{GrD_9 + GmD_{20}}{a_2^2 - a_2 - A_1} \qquad D_{29} = \frac{-GmD_{27}}{a_6^2 - a_6 - A_1}$$

$$D_{30} = \frac{GrD_3 + GmD_{21}}{4a_8^2 - 2a_8 - A_1} \qquad D_{31} = \frac{GrD_4 + GmD_{22}}{4a_2{}^2 - 2a_2 - A_1}$$

$$D_{32} = \frac{GrD_5 + GmD_{23}}{4a_6{}^2 - 2a_6 - A_1}$$

$$D_{33} = \frac{GrD_6 + GmD_{24}}{(a_2 + a_8)^2 - (a_2 + a_8) - A_1}$$

$$D_{34} = \frac{GrD_5 + GmD_{23}}{(a_2 + a_6)^2 - (a_2 + a_6) - A_1}$$

$$D_{35} = \frac{GrD_8 + GmD_{26}}{(a_6 + a_8)^2 - (a_6 + a_8) - A_1}$$

$D_{36} = -[D_{28} + D_{29} - D_{30} - D_{31} - D_{32} + D_{33} - D_{34} + D_{35}]$

$$D_{37} = \frac{-2Pr\, a_8 (A_3 + A_4) a_{10} D_{19}}{(a_8 + a_{10})^2 - (a_8 + a_{10}) - \frac{Pr}{4}(iw - s)}$$

$$D_{38} = \frac{2Pr\, a_8 (A_3 + A_4) a_4 D_{14}}{(a_4 + a_8)^2 - (a_4 + a_8) - \frac{Pr}{4}(iw - s)}$$

$$D_{39} = \frac{2Pr\, a_8 (A_3 + A_4) a_{12} D_{15} - 2Pra_2 a_8 A_3 D_{17} + Pr(a_2 + a_8) D_6}{(a_2 + a_8)^2 - (a_2 + a_8) - \frac{Pr}{4}(iw - s)}$$

$$D_{40} = \frac{2Pr\, a_8 (A_3 + A_4) a_{12} D_{16}}{(a_8 + a_{12})^2 - (a_8 + a_{12}) - \frac{Pr}{4}(iw - s)}$$

$$D_{41} = \frac{2Pr\, a_8^2 (A_3 + A_4) D_{17} - 2Pr\, a_8 D_3}{4a_8^2 - 2a_8 - \frac{Pr}{4}(iw - s)}$$

$$D_{42} = \frac{2Pr\, a_8 (A_3 + A_4) D_{18} - 2Pra_6 a_8 A_4 D_{17} + Pr(a_6 + a_8)}{(a_6 + a_8)^2 - (a_6 + a_8) - \frac{Pr}{4}(iw - s)}$$

$$D_{43} = \frac{2Pr\, a_8 a_6 (A_3 + A_4) D_{17} - 2Pra_6 a_8 D_{16} A_4 + Pr(a_6 + a_8) D_8}{(a_6 + a_8)^2 - (a_6 + a_8)Pr - \frac{Pr}{4}(iw - s)}$$

$$D_{44} = \frac{2Pr\, a_2 a_{10} A_3 D_{18}}{(a_2 + a_{10})^2 - (a_2 + a_{10})Pr - \frac{Pr}{4}(iw - s)}$$

$$D_{45} = \frac{2Pr\, a_2 a_4 D_{14}}{(a_2 + a_4)^2 - (a_2 + a_4)Pr - \frac{Pr}{4}(iw - s)}$$

$$D_{46} = \frac{2Pr\, a_2 a_6 A_3 D_{17} + 2Pra_2 a_6 D_{15} A_4 + Pr(a_2 + a_6) D_7}{(a_2 + a_6)^2 - (a_2 + a_6)Pr - \frac{Pr}{4}(iw - s)}$$

$$D_{47} = \frac{2Pr\, a_6 a_{10} A_4 D_{18}}{(a_6 + a_{10})^2 - (a_6 + a_{10})Pr - \frac{Pr}{4}(iw - s)}$$

$$D_{48} = \frac{2Pr\, a_6 a_4 A_4 D_{14}}{(a_4 + a_6)^2 - (a_4 + a_6)Pr - \frac{Pr}{4}(iw - s)}$$

$D_{53} = -[D_{37} + D_{38} + D_{39} + D_{40} + D_{41} + D_{42} + D_{43} + D_{44} + D_{45} + D_{46} + D_{47} + D_{48} + D_{49} + D_{50} + D_{51} + D_{52}]$

$$D_{54} = \frac{Sca_6 D_{27}}{a_6^2 - Sca_6 + Sc K_1}$$

$$D_{55} = \frac{Sca_2 D_{20} - ScSra_2^2 D_{52}}{a_2^2 - Sca_2 + Sc K_1}$$

$$D_{56} = \frac{- ScSra_4^2 D_{53}}{a_4^2 - Sca_4 + Sc K_1}$$

$$D_{57} = \frac{-2Sca_8 D_{21} - 4\, ScSra_8^2 D_{41}}{4a_8^2 - 2Sca_8 + Sc K_1}$$

$$D_{58} = \frac{-2Sca_2 D_{22} - 4\, ScSra_2^2 D_{45}}{4a_6^2 - 2Sca_6 + Sc K_1}$$

$$D_{59} = \frac{-2Sca_6 D_{23} - 4\, ScSra_6^2 D_{46}]}{4a_6^2 - 2Sca_6 + Sc K_1}$$

$$D_{60} = \frac{Sc(a_2 + a_8) D_{24} - ScSr\, (a_2 + a_8)^2 D_{39}}{(a_2 + a_8)^2 - Sc(a_2 + a_8) + ScK_1}$$

$$D_{61} = \frac{Sc(a_2 + a_6) D_{25} - ScSr\, (a_2 + a_6)^2 D_{48}}{(a_2 + a_6)^2 - Sc(a_2 + a_6) + ScK_1}$$

$$D_{62} = \frac{Sc(a_6 + a_8) D_{26} - ScSr\, (a_6 + a_8)^2 D_{42}}{(a_6 + a_8)^2 - Sc(a_6 + a_8) + ScK_1}$$

$$D_{63} = \frac{-ScSr(a_8 + a_{10})^2 D_{37}}{(a_8 + a_{10})^2 - Sc(a_8 + a_{10}) + Sc K_1}$$

$$D_{64} = \frac{-ScSr\, (a_4 + a_8)^2 D_{38}}{(a_4 + a_8)^2 - Sc(a_4 + a_8) + Sc K_1}$$

$$D_{65} = \frac{-ScSr\, (a_8 + a_{12})^2 D_{40}}{(a_8 + a_{12})^2 - Sc(a_8 + a_{12}) + Sc K_1}$$

$$D_{66} = \frac{-ScSr\, (a_2 + a_{10})^2 D_{43}}{(a_2 + a_{10})^2 - Sc(a_2 + a_{10}) + Sc K_1}$$

$$D_{67} = \frac{-ScSr\, (a_2 + a_4)^2 D_{44}}{(a_2 + a_4)^2 - Sc(a_2 + a_4) + Sc K_1}$$

$$D_{68} = \frac{-ScSr(a_2 + a_{12})^2 D_{47}}{(a_2 + a_{12})^2 - Sc(a_2 + a_{12}) + Sc K_1}$$

$$D_{69} = \frac{-ScSr\, (a_6 + a_{10})^2 D_{49}}{(a_6 + a_{10})^2 - Sc(a_6 + a_{10}) + Sc K_1}$$

$$D_{70} = \frac{-ScSr\, (a_4 + a_6)^2 D_{50}}{(a_4 + a_6)^2 - Sc(a_4 + a_6) + Sc K_1}$$

$$D_{71} = \frac{-ScSr\, (a_6 + a_{12})^2 D_{51}}{(a_6 + a_{12})^2 - Sc(a_6 + a_{12}) + Sc K_1}$$

$D_{72} = - [D_{50} + D_{51} + D_{52} + D_{53} + D_{54} + D_{55} + D_{56} + D_{57} + D_{58} + D_{59} + D_{60} + D_{61} + D_{62} + D_{63} + D_{64} + D_{65} + D_{66} + D_{67} + D_{68} + D_{69} + D_{70} + D_{71}]$

$$D_{73} = \frac{-GrD_{53} - GmD_{56}}{a_4^2 - a_4 - A_2}$$

$$D_{74} = \frac{-GrD_{37} - GmD_{63}}{(a_8 + a_{10})^2 - (a_8 + a_{10}) - A_2}$$

$$D_{75} = \frac{-GrD_{38} - GmD_{64}}{(a_4 + a_8)^2 - (a_4 + a_8) - A_2}$$

$$D_{76} = \frac{-GrD_{39} + \frac{1}{K_0}D_{33} - (a_2 + a_8)D_{33}}{(a_2 + a_8)^2 - (a_2 + a_8) - A_2}$$

$$D_{77} = \frac{-GrD_{40} - GmD_{65}}{(a_8 + a_{12})^2 - (a_8 + a_{12}) - A_2}$$

$$D_{78} = \frac{-GrD_{41} - GmD_{57} - \frac{1}{K_0}D_{30} + 2a_8D_{30}}{4a_8^2 - 2a_8 - A_2}$$

$$D_{79} = \frac{-GrD_{42} - GmD_{62} + \frac{1}{K_0}D_{35} - (a_6 + a_8)D_{35}}{(a_6 + a_8)^2 - (a_6 + a_8) - A_2}$$

$$D_{80} = \frac{-GrD_{43} - GmD_{66}}{(a_2 + a_{10})^2 - (a_2 + a_{10}) - A_2}$$

$$D_{81} = \frac{-GrD_{44} - GmD_{67}}{(a_2 + a_4)^2 - (a_2 + a_4) - A_2}$$

$$D_{82} = \frac{-GrD_{45} - GmD_{58} - \frac{1}{K_0}D_{31} + 2a_2D_{31}}{4a_2^2 - 2a_2 - A_2}$$

$$D_{83} = \frac{-GrD_{46} - GmD_{59} - \frac{1}{K_0}D_{32} + 2a_2D_{31}}{4a_6^2 - 2a_6 - A_2}$$

$$D_{84} = \frac{-GrD_{47} - GmD_{68}}{(a_2 + a_{12})^2 - (a_2 + a_{12}) - A_2}$$

$$D_{85} = \frac{-GrD_{48} - GmD_{61} - \frac{1}{K_0}D_{34} + (a_2 + a_6)D_{34}}{(a_2 + a_6)^2 - (a_2 + a_6) - A_2}$$

$$D_{86} = \frac{-GrD_{49} - GmD_{69}}{(a_6 + a_{10})^2 - (a_6 + a_{10}) - A_2}$$

$$D_{87} = \frac{-GrD_{50} - GmD_{70}}{(a_4 + a_6)^2 - (a_4 + a_6) - A_2}$$

$$D_{88} = \frac{-GrD_{51} - GmD_{71}}{(a_6 + a_{12})^2 - (a_6 + a_{12}) - A_2}$$

$$D_{89} = \frac{-GrD_{52} - GmD_{55} + \frac{1}{K_0}D_{25} - a_2D_{28}}{a_2^2 - a_2 - A_2}$$

$$D_{90} = \frac{-GmD_{72}}{a_{12}^2 - a_{12} - A_2}$$

$$D_{91} = \frac{-GmD_{54} + \frac{1}{K_0}D_{29} - a_6D_{29}}{a_6^2 - a_6 - A_2}$$

$$D_{92} = \frac{-\frac{1}{K_0}D_{36} + a_8D_{36}}{a_8^2 - a_8 - A_2}$$

D_{93} = -[D_{73} +D_{74} + D_{75} + D_{76} + D_{77} + D_{78} +D_{79} + D_{80} + D_{81} +D_{82} + D_{83} + D_{84} + D_{85}+ D_{86} + D_{87} + D_{88} +D_{89}+ D_{90} + D_{91} + D_{92}]