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Analytical investigation of heat transfer of solar air collector by Adomian decomposition method

Mohamed Kezzar[1], Ismail Tabet[2]*, Meriem Chieul[3], Noureddine Nafir[3], Abdelkader Khentout[4]

[1]Department de Génie Mécanique, Faculté de Technologie, Université 20 Août 1955 Skikda , Algérie
[2]Department de Physique, Faculté des Sciences , Université 20 Août 1955 Skikda, Algérie
[3]Département de Génie Electrique, Faculté de Technologie, Université 20 Août 1955 Skikda, Algérie
[4] Département de Forage, Faculté d'hydrocarbure, Université de Ouargla, Algérie

Corresponding Author Email: tabet21@yahoo.fr

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ABSTRACT

The Adomian decomposition method (ADM) is applied in this paper to investigate the heat transfer in solar air collector. Results obtained using Adomian decomposition method (ADM) and the numerical Runge–Kutta fourth-order method are compared. The Adomian decomposition method (ADM) is effective for finding exact solutions of differential equations. Several parameters, such as air mass flow, width, and length of the collector, the effect of efficiency of the solar air collector. The increase in air mass flow improves the efficiency of solar air collector, whereas the dimensions of the collector are a negatively effect on the efficiency of the solar air collector.

1. INTRODUCTION

Renewable energy will replace fossil fuels in the coming years. Solar energy is one of the most significant renewable energy sources that the world needs; solar energy utilization can be classified into two categories: solar thermal system, which converts solar energy into heat, and photovoltaic system, which converts solar energy into electricity [1].The essential element to exploiting solar energy is the solar collector. The solar collector is a heat exchanger that converts the energy of solar radiation to internal energy of the transport medium. A flat plate thermal collector is a solar collector and consists of two types, namely, thermal water collector and thermal air collector. The solar air collector is widely used for applications in low and medium temperatures, such as heating of air, drying of agricultural products, timber seasoning, and other industrial applications [2]. Several parameters affect the thermal efficiency of solar air collector. Numerous studies have been conducted on the solar air collector [3] to improve its low thermal conductivity and low thermal capacity [4]. An effective way to increase the convective heat transfer rate is to increase the heat transfer surface and increase the turbulence inside the canal by using ailerons or corrugated surfaces [5–6].

Nonlinear equations in engineering can be solved using many methods [7–8]. The Adomian decomposition method (ADM) was applied to obtain the solutions of various kinds of problems for ODEs and PDEs [9-10]. Babolian et al. [11] compared the performance between the ADM method and the Homotopy perturbation method (HPM) to solve the nonlinear differential equation by theoretical analysis. Result shows that the ADM method is equivalent to HPM with a specific convex Homotopy for nonlinear differential equation. Arslanturk [12] used the ADM method to evaluate the efficiency of straight fins with thermal conductivity depending on the temperature and to determine the temperature distribution inside the fin.Vasile Marinca et al. [13] used Optimal Homotopy Asymptotic Method (OHAM), to solve nonlinear equations arising in heat transfer,Results obtained by OHAM, which does not need small parameters are compared with numerical results and a very good agreement was found

D. Lesnic. [14-15] used Adomian's decomposition approach is employed for solving some inverse boundary value problems in heat conduction another work make by D. Lesnic. [16] That he used The Adomian decomposition method (ADM) to determine the temperature distribution within a single fin with temperature heat transfer.

The aim of this work is to study heat transfer in solar air collector, using an analytical Adomian decomposition method (ADM). To verify the accuracy of the proposed method, the results were compared to those obtained by the numerical solution of Runge Kutta method. The effect of certain parameters on the efficiency of the solar air collector is also treated

2. MATHEMATICAL MODELING

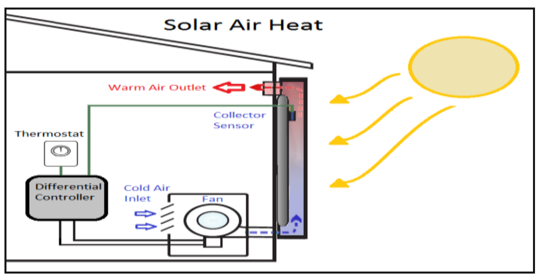

Figure1. Simple solar air heater [17]

A solar collector transforms the energy of the sun into thermal energy extracted by the flowing air in the collector. It has several practical advantages; indeed, the use of air for the heating of the premises makes it possible to make remarkable energy savings. Figure 1 shows a building heating system using a solar collector placing on the side of the building

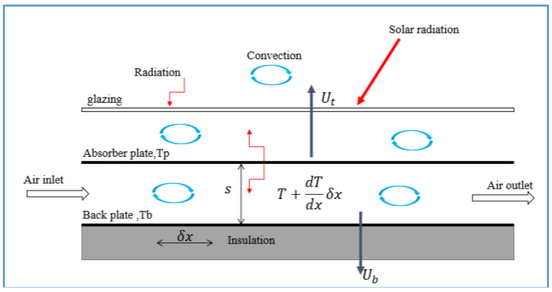

Figure 2. Control volume for energy balance of solar air collector

Figure 2 illustrate the diagram of solar air collector, this collector constituting two absorbent plates and a cover in glass, the air circulate between two plates, the different heat exchanges of each component of the solar air collector are cited [2].

The energy balance of absorber plate is given as follows:

$$\mathrm{S(dx)} = U_t(\mathrm{T}_p - T_a) + h^c_{p-a}(dx)(\mathrm{T}_p - T) + h^r_{p-b}(\mathrm{dx})(T_p - T_b) \quad (1)$$

with $S = (\alpha_p \tau).G_t$

The air energy balance of fluid is given as follows:

$$\frac{\dot{m}}{W} c_p (T' dx) = h^c_{p-a}(\mathrm{dx})(T_p - T) + h^c_{b-a}(\mathrm{dx})(T_b - T) \quad (2)$$

The energy balance on the surface of the back plate (1 × dx) gives

$$h^r_{p-a}(dx)(T_p - T_a) = h^c_{p-a}(dx)(T_p - T) + U_b(dx)(T_b - T_a) \quad (3)$$

Given that, U_b is much UT, $U_L \approx U_t$. Therefore, U_b is neglected and Eq. (3) is solved for T_b:

$$T_b = \frac{h^r_{p-b} T_p + h^c_{b-a} T}{h^r_{p-b} + h^c_{b-a}} \quad (4)$$

Substitution of eq. (4) in eq. (1) gives

$$T_a(U_L + h) = S + U_L T_a + hT \quad (5)$$

With

$$h = h^c_{b-a} + \frac{1}{1/h^c_{b-a} + 1/h^r_{p-b}} \quad (6)$$

Substitution of eq. (4) into eq. (2) gives;

$$hT_p = \frac{\dot{m}}{W} c_p T' + hT \quad (7)$$

Considering Eqs. (5) and (7), the following equations are obtained:

$$\frac{\dot{m}}{W} c_p T' = F'[S - U_L(T - T_a)] \quad (8)$$

$$F' = \frac{1/U_L}{1/U_L + 1/h} = \frac{h}{h + U_L} \quad (9)$$

In all previous mathematical models, we assume that air behaves for an ideal gas with a constant specific heat, which would produce an error in the thermal analysis. In reality, the specific heat of air varies with temperature and this will significantly influences the thermal performance of the collector. A model was presented by a team of researchers called the polynomial equation of NASA [7]. This model has the following formula for the ambient temperature between 200 K and 1000K:

$$\frac{C_p}{R_g} = 3.56839 - 6.788729\times10^{-4}T + 1.5537\times10^{-6}T^2 - 3.29937\times10^{-12}T^3 - 466.395\times10^{-15}T^4 \quad (10)$$

$$C_p = a + bT \quad (11)$$

where Rg is the gas constant (J/kg K). For low temperatures (275 K–400 K), the specific heat is almost linear, that is, a close approximation.

By using the table of physical properties for air at different temperatures in ref. [7], the coefficients “a” and “b” obtained 980.52 and 0.083, respectively. By substituting Eq. (11) intoEq. (8) and rearranging the equation, we obtained the following:

$$\frac{\dot{m}}{W} aT'(x) + \frac{\dot{m}}{W} bT'(x)T(x) + F'U_L T(x) - F'(S + T_a U_L) \quad \text{(12-a)}$$

The initial state of the eq. (12) is

$T=Ti$, $at.$ $x=0$ (12-b)

Reynolds number should be determined.

$$R_e = \frac{\rho V D}{\mu} = \frac{\dot{m} D}{A_c \mu} \quad (13)$$

where A_c = s×W and D is the hydraulic diameter of the channel.

$$D = 4\frac{Ws}{2(W+s)} = 2\frac{Ws}{(W+s)} \quad (14)$$

After determination of the Reynolds number, the coefficients of heat transfer by convection of turbulent flow will be calculated as follows:

$$h_{p-a}^{c} = h_{b-a}^{c} = \frac{K}{D} 0.0158 R_e^{0.8} \quad (15)$$

The following relation expresses the thermal efficiency of the sensor:

$$\eta_T = \frac{Q_u}{A_s G_t} = \frac{\dot{m} \int_{T_i}^{T_f} (a + bT)\, dT}{(W.L) G_t} \quad (16)$$

3. DECOMPOSITION ADOMIAN METHOD ADM

3.1. Principle

The principle of this method is as follows:
Consider the following equation [10]:

$$F_u(t) = g(t) \quad (17)$$

where F is the ordinary or partial differential operator including linear and nonlinear terms. Eq. (17) can also be written as follows:

$$Lu + Ru + Nu = g(t) \quad (18)$$

where L is the easy reversible linear operator, A is the residue of the linear operator, N is the non-linear operator, and g is the source (a known function).

As already mentioned,L is the invertible linear operator; thus, Eq. (17)

$$L^{-1}Lu + L^{-1}Ru + L^{-1}Nu = L^{-1}g(t) \quad (19)$$

where $L^{(-1)}$ constitutes an integral of order N (double, triple...), depending on the problem studied.
The solution of equation (19) given by:

$$u = \varphi + L^{-1}g - L^{-1}Ru - L^{-1}Nu \quad (20)$$

where φ is a function that will determine from the boundary conditions and/or initials.The principle of ADM method is to find a general solution as a finite series. The components of the latter are given by:

$$u = \sum_{n=0}^{+\infty} u_n \quad (21)$$

The non-linear term "Nu" of Eq. (18) can take the following form:

$$Nu = \sum_{n=0}^{+\infty} A_n \left(u_0\ u_1, u_2, \ldots\ldots\ldots\ldots, u_n\right) \quad (22)$$

where $A_n$represents Adomian polynomials, which can be calculated by the following formula:

$$A_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n}\left[N\left(\sum_{n=0}^{\infty}\lambda^i\ u_i\right)\right]\right]_{\lambda=0},\ n = 0,1,2,\ldots.., n \quad (23)$$

where $\lambda$is a real parameter.
Substitution of Eqs. (21) and (22) into Eq. (20)

$$\sum_{n=0}^{\infty} u_n = \varphi + L^{-1}g - L^{-1}R\sum_{n=0}^{\infty} u_n - L^{-1}\sum_{n=0}^{\infty} A_n \quad (24)$$

From eq. (20), the recurrence formula, which defines the components of the solution, is given as follows:

$$u_0 = \varphi + L^{(-1)}g,\ u_{(n+1)} = -L^{(-1)}(Ru_n + A_n) \quad (25)$$

Once Adomian polynomials are determined, the solution of Eq. (17) is expressed by:

$$u_{n+1} = \sum_{n=0}^{+\infty} u_n \quad (26)$$

Eq. (26) represents the solution of the equation studied as a rapidly converging series.

3.2 Analytical solution by ADM method

In this study, ADM is applied to solve the first-order nonlinear equation governing the temperature distribution (Eq. 12-a).

$$T'(\eta) + \frac{\dot{b}}{a} T'(\eta) T(\eta) + \frac{W}{\dot{m}a} F_p U_l T(\eta) = \frac{W}{\dot{m}a} F_p \left(S + T_a U_l\right) \quad (27)$$

To adapt Eq. (12) at the analytical processing, it is written as follows:

$$LT = \frac{W}{\dot{m}a} F_p \left(S + T_a U_l\right) - \frac{\dot{b}}{a} 燗T - \frac{W}{\dot{m}a} F_p U_l T \quad (28)$$

The differential operator L is given by:

$$L = \frac{d}{d\eta} \quad (29)$$

In addition, the inverse operator L^{-1} is expressed by:

$$L^{-1} = \int_0^{\eta} (\bullet)\, d\eta \quad (30)$$

By application of Eq. (30), Eq. (28) becomes the following form:

$$T(\eta) = T(0) + \int_0^{\eta} \frac{W}{\dot{m}a} F_p \left(S + T_a U_l\right) d\eta + L^{-1}\left(NT\right) \quad (31)$$

with

$$NT = -\frac{\dot{b}}{a} 燗T - \frac{W}{\dot{m}a} F_p U_l T \quad (32)$$

By including the limit conditions (Eq. 12-a), Eq. (31) becomes

$$T(\eta) = \sum_{n=0}^{\infty} T_n = T_0 + L^{-1}\left(NT\right) \quad (33)$$

where the function T_0 is expressed by:

$$T_0 = \alpha + \frac{W}{\dot{m}a} F_p \left(S + T_a U_l\right)\eta \tag{34}$$

The constant α mainly depends on limit conditions.

Thereafter, application of the Adomian algorithm (23) provides polynomials (A_0, A_1...A_n). These are expressed by:

$$A_0 = \frac{F_p w}{am}\; S + U_l\; T_a - \alpha \tag{35}$$

$$A_1 = \frac{1}{a^2 m} bF_p Sw\alpha - \frac{1}{a^2 m} bF_p T_a U_l w\alpha + \frac{1}{a^2 m} bF_p U_l w\alpha^2 - \frac{1}{a^2 m^2} F_p^2 SU_l w^2\eta - \frac{1}{a^2 m^2} F_p^2 T_a U_l^2 w^2 \eta + \frac{1}{a^2 m^2} F_p^2 U_l^2 w^2 \alpha\eta \tag{36}$$

$$A_2 = \frac{F_p w}{2a^3 m^3}\left(S + U_l\left(T_a - \alpha\right)\right)\begin{pmatrix} 2b^2 m^2 \alpha^2 - 2bF_p mw \begin{pmatrix} S + \\ U_l\left(T_a - 3\alpha\right)\end{pmatrix}\eta \\ +F_p^2 U_l^2 w^2 \eta^2 \end{pmatrix} \tag{37}$$

$$A_3 = \frac{1}{6a^4 m^4} F_p w\,(-6b^3 m^3\left(S + U_l\left(T_a - \alpha\right)\right)\alpha^3 + 18b^2 F_p m^2 w\alpha \begin{pmatrix} S^2 + SU_l\left(2T_a - 3\alpha\right) + \\ U_l^2\left(T_a^2 - 3T_a\alpha + 2\alpha^2\right)\end{pmatrix}\eta + 3bF_p^2 mU_l w^2 (4S^2 + SU_l\left(8T_a - 11\alpha\right) + U_l^2(4T_a^2 - 11T_a\alpha + 7\alpha^2))\eta^2 - F_p^3 U_l^3 w^3 (S + U_l(T_a - \alpha))\eta^3) \tag{38}$$

The components of the solution according to the ADM method are expressed by:

$$T_1 = \frac{F_p}{am}\left(Sw + T_a U_l w - U_l w\alpha\right)\eta \tag{39}$$

$$T_2 = -\frac{F_p w}{2a^2 m^2}\left(S + U_l\left(T_a - \alpha\right)\right)\left(2bm\alpha + F_p U_l w\eta\right)\eta \tag{40}$$

$$T_3 = \frac{F_p w}{2a^3 m^3}\left(S + U_l\left(T_a - \alpha\right)\right)\left(2b^2 m^2 \alpha^2 \eta - bF_p mw\left(S + T_a U_l - 3U_l\alpha\right)\eta^2 + \frac{1}{3} U_l^2 U_l^2 w^2 \eta^3\right) \tag{41}$$

$$T_4 = \frac{1}{6a^4 m^4} F_p w\,(-6b^3 m^3\left(S + U_l\left(T_a - \alpha\right)\right)\alpha^3 \eta + 9b^2 F_p m^2 w\alpha (S^2 + 2ST_a U_l + T_a^2 U_l^2 - 3SU_l\alpha - 3T_a U_l^2 \alpha + 2U_l^2 \alpha^2)\eta^2 + bF_p^2 mU_l w^2 \begin{pmatrix} 4S^2 + 8ST_a U_l + 4T_a^2 U_l^2 \\ -11SU_l\alpha - 11T_a U_l^2 \alpha + 7U_l^2 \alpha^2 \end{pmatrix}\eta^3 - \frac{1}{4} F_p^3 U_l^3 w^3 (S + T_a U_l - U_l\alpha)\eta^4) \tag{42}$$

After some iterations, the solution of Eq. (12-a) by the ADM method is given by:

$$T = T_0 + T_1 + T_2 + T_3 + \ldots\ldots\ldots + T_n \tag{43}$$

4. RESULTS AND DISCUSSION

We applied the Adomian decomposition method for the analytical solution of heat transfer problem in a solar air collector.Successful results from this solution were compared with those of the numerical solution of the Runge–Kutta fourth-order method. The results of both methods matched well, as shown in Table 2, it can be seen that the results obtained using the Adomian decomposition method were well matched with the results made by the numerical solution.

A comparison between the method of Adomian decomposition and the numerical solution by Runge Kutta method by calculating the error between them, Figure 3 shows that the ADM method converges rapidly to the numerical solution.

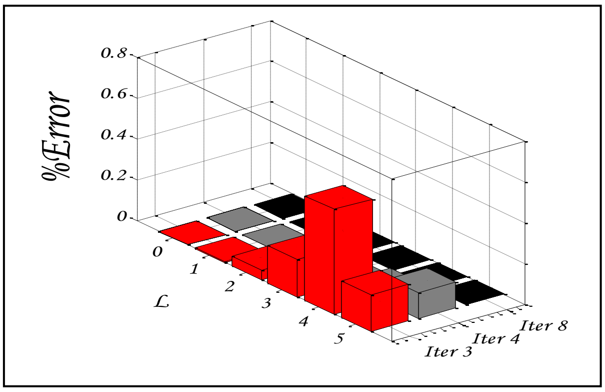

Figure 3. Convergence test of the Adomian decomposition method

Table 2. Results obtained by the ADM method

	T_{Num}	T_{ADM}	Error $= \lvert T_{Num} - T_{ADM}\rvert$
0.00	323.00	323.00	0.0000000
0.50	343.8151339846404	343.815133197579	0.78×10^{-6}
1.00	361.313685882991	361.313685746402	0.13×10^{-6}
1.50	376.0314317727599	376.031431082149	0.69×10^{-6}
2.00	388.41551394231027	388.415513081421	0.86×10^{-6}
2.50	398.83965291837495	398.839652135889	0.78×10^{-6}
3.00	407.61660914693755	407.616609915370	0.76×10^{-6}
3.50	415.0084423779446	415.008442912319	0.53×10^{-6}
4.00	421.235119677565	421.235119717379	0.03×10^{-6}
4.50	426.4811883211774	426.481188126766	0.19×10^{-6}
5.00	430.901736183969	430.901736665066	0.48×10^{-6}

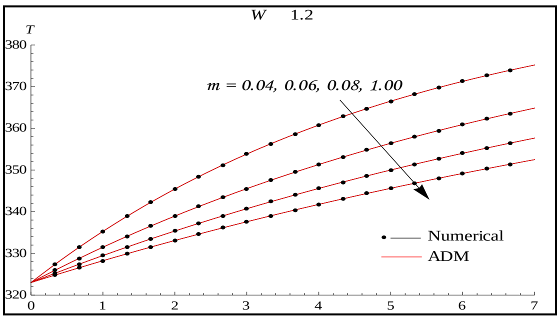

Figure 4. Effect of mass flow on air temperature

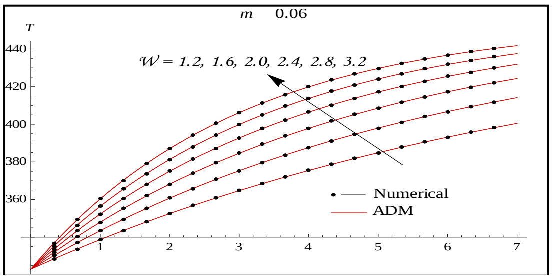

Figure 5. Effect of the collector width on air temperature

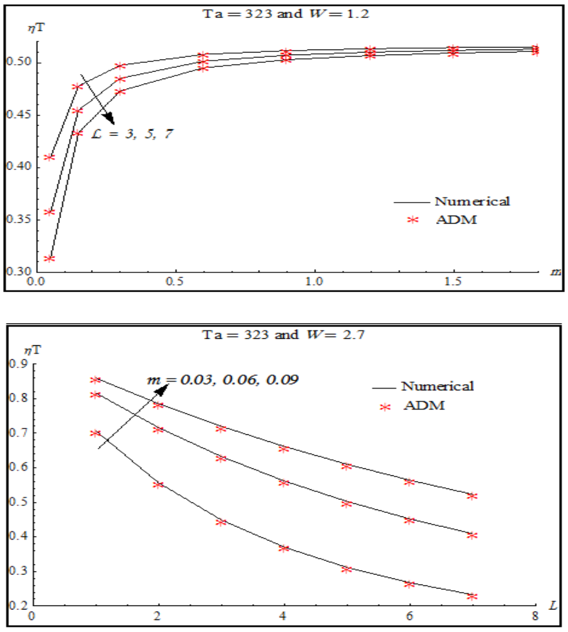

Figure 6. Effect of mass flow on the efficiency of the solar air collector

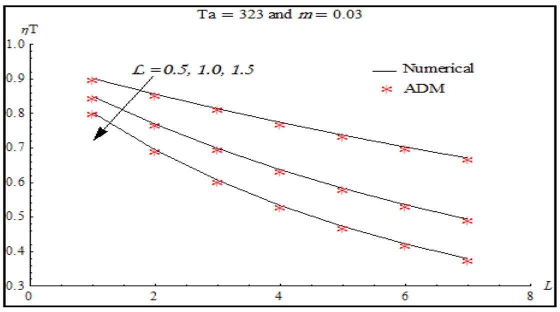

Figure 7. Effect of the collector dimensions on efficiency of solar air collectorIn the nomenclature

5. CONCLUSION

In this paper, the thermal behavior of solar air collector was analytically investigated using the ADM method. The results obtained using this method were compared with those of the numerical Runge–Kutta fourth-order method. The ADM method is confirmed to be an effective technique to find exact solutions for differential equations. The solution of the analytical method is given in the form of rapidly converging series. The air mass flow effect, as well as the width and length of the air collector, on the performance of the collector were evaluated. Increasing the air mass flow improved the collector efficiency, and the dimensions of the collector negatively influenced the performance of the collector.

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NOMENCLATURE

A_n	Adomian polynomials
A	surface (m^2)
C_P	Specific heat [J/Kg k]
h^c	convective-exchange coefficient (W m^{-2} k^{-1})
h^r	radiative-exchange coefficient (W m^{-2} k^{-1})
s	channel height (m)
S	solar intensity (w m^{-2})
l	length of the collector (m)
$\dot{m}$	mass flow (Kg.s^{-1})
m	mass (kg)
Q	heat flow (w)
t	time (s)
T	temperature (°C)
w	width of the collector (m)
U	loss coefficient (W m^{-2} k^{-1})
D	hydraulic diameter
L_i	Derivative operator
L_i^{-1}	Inverse derivative operator
a	ambient
b	back plate
p	absorber plate
g	glass cover
in	insulator
p	absorber plate
th	thermal
p	top surface of the absorber plate
R_g	gas constant J kg^{-1} K^{-1}
α_P	absorbance
η_T	Theraml efficiency
τ	transmittance
λ	thermal conductivity (W m^{-1} k^{-1})
α	Constant
δ	thickness,(m)