Bioconvection of Couple Stress Fluid in a Channel with Expanding or Contracting Walls

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ABSTRACT

This paper deals with the flow of an incompressible couple-stress fluid containing gyrotactic microorganisms in a channel with expanding or contracting porous walls. The bio-convection in couple stress fluid is advantageous to study various kinds of physical problems since couple stress fluid model can explain the rheology of various complex fluids, such as polymeric suspension, lubricants, liquid crystals, and blood. The governing equations are reduced to a system of non-linear ordinary differential equations using similarity transformations, linearized using successive linearization method and then solved using the Chebyshev collocation method. The influence of couple stress fluid parameters and other pertinent physical parameters on the motile microorganism distribution and density number of motile microorganism is examined.

1. INTRODUCTION

The couple stress fluid theory pioneered by Stokes [1], is one among the fluid theories that take into account couple stresses in addition to the classical Cauchy stress. It is the generalization of the classical theory of viscous fluids with distinct features, such as the presence of couple stresses, body couples and non-symmetric stress tensor. In this theory, curvature twist rate tensor is proposed based on pure kinematic aspects of rotation vector and couple stress is defined in terms of this curvature twist rate tensor. Stress tensor at any point contains symmetric and antisymmetric parts. The anti-symmetric part is due to the curvature twist rate tensor. The second-order gradient of the velocity vector is introduced into stress constitutive equations. The couple stress fluid theory presents models for fluids whose microstructure is mechanically significant. The effect of very small microstructure in a fluid can be felt if the characteristic geometric dimension of the problem considered is of the same order of magnitude as the size of the microstructure [2]. The main effect of couple stresses is to introduce a size-dependent effect that is not present in the classical viscous theories. Couple stress fluid model has several industrial and scientific applications. The couple stress fluids are capable of describing various types of lubricants, blood, suspension fluids, etc. A large number of theoretical investigations dealing with the flow of couple stress fluid in different geometries in bounded and unbounded domains have appeared during the last few decades. Stokes [1] discussed the hydromagnetic steady flow of a fluid with a couple stress effect. Stokes [2] presented a long list of problems discussed by researchers about this theory. The important fields where couple stress fluids have applications are Bio-fluid mechanics [3-6], squeezing [7] and lubrication of synovial joints [8], Synthetic and Plastic industries. Islam and Zhou [9] presented analytical solutions for two-dimensional flows of couple stress fluids. Khan et al. [10] obtained exact solutions for MHD flow of couple stress fluid with heat transfer.

On the other hand, the fluid flow in a channel with deformable/expanding/contracting porous walls has gained significance because of their uses in the perspiration cooling or heating, modelling of pulsating diaphragms, separation of isotopes, paper manufacturing, purification, and the grain regression during solid propellant combustion. Several authors have considered the flow of a couple stress fluid in a two-dimensional channel with porous walls. Srinivasacharya et al. [11] analyzed the flow of a couple stress fluid in a channel with expanding or contracting porous walls. Khan et al. [12] examined the couple stress fluid flow in a semi-infinite rectangular channel with uniformly expanding or contracting porous walls. Odelu and Naresh [13] analyzed the effects of Hall and ion slip currents, chemical reaction, Soret and Dufour on the heat and mass transfer characteristics of an electrically conducting couple stress fluid through channels with expanding or contracting porous walls. Shennawy and Elkhair [14] investigated the effect of slip boundary condition and magnetic field on the couple stress fluid flow in a porous channel with expanding or contracting walls.

Bioconvection is the name given to spontaneous pattern formation in suspensions of microorganisms due to swimming of the microorganisms. Bioconvection has various applications in engineering, biotechnology and biological systems. Pedley et al. [15] developed a continuum model for a suspension of swimming gyrotactic microorganisms by extending the model of Childress et al. [16] for geotactic microorganisms with an equation for conservation of motile microorganisms. Shaw and Sibanda et al. [17] considered the bioconvection near an inclined permeable plate embedded in a water-based nanofluid saturated porous medium containing motile microorganisms. Xu et al. [18] investigated the interaction of both nanoparticles and gyrotactic microorganisms in the mixed convection flow of a nanofluid over a stretching surface. Raees et al. [19] analyzed the bioconvection flow of a nanofluid in a horizontal channel with mixed convection. Raju [20] studied the effects of nonlinear thermal radiation and chemical reaction on the bioconvection

flow towards a rotating cone/plate in a rotating fluid. Makinde and Animasaun [21] examined the effect of chemical reaction of quartic autocatalytic nature on bioconvection of nanofluid over an upper horizontal surface of a paraboloid of revolution by including gyrotactic microorganisms, Brownian motion and thermophoresis effects in the flow. Mosayebidorcheh et al. [22] investigated the interaction of nanoparticles and gyrotactic microorganisms in the flow of nanofluid flow through a horizontal channel. Zhao et al. [23] examined the bioconvection flow of nanofluid between two infinite parallel plate in the presence of the magnetic field and a first-order chemical reaction. Bin-Mohsin et al. [24] studied the bioconvection in a channel filled with a nanofluid containing gyrotactic microorganisms. Khan et al. [25] considered the bioconvection flow of couple stress nanofluid over a stretched porous surface in the presence of activation energy and gyrotactic micro-organisms.

In this paper, the bio-convection boundary layer flow of a couple stress fluid containing gyrotactic microorganisms in a channel with expanding or contracting porous walls is analyzed. A set of nonlinear coupled ordinary differential equations are constructed using the similarity transformation. Numerical solutions to these nonlinear differential equations by applying the Chebyshev spectral collocation method.

2. MATHEMATICAL FORMULATION

Consider the couple stress fluid flow containing motile microorganisms between two infinite porous parallel plates. The plates are contracting and expanding uniformly at a time-dependent rate $\dot{H}(t)$. The flow is unsteady and fluid is incompressible. The coordinate system is chosen such that the x-axis is along the line parallel to two plates and the y-axis is perpendicular to the plates. The two plates are placed at the distance $y = 2H(t)$, which is smaller than the width and length of the channel. The fluid is aspirated or injected into the channel with a uniform speed through the channel walls. Let the temperature, concentrations and concentration of microorganisms at the lower and upper walls be T_1 and T_2, C_1 and C_2 and ϑ_1 and ϑ_2, respectively.

With the above assumptions, the equations governing the unsteady incompressible couple stress fluid the flow, in the absence of body forces and body couple, are

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 \tag{1}$$

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\nabla^2 U - \frac{\eta_1}{\rho}\nabla^4 U \tag{2}$$

$$\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial x} + V\frac{\partial V}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{\mu}{\rho}\nabla^2 V - \frac{\eta_1}{\rho}\nabla^4 V \tag{3}$$

$$\frac{\partial T}{\partial t} + U\frac{\partial T}{\partial x} + V\frac{\partial T}{\partial y} = \alpha\,\nabla^2 T \tag{4}$$

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial x} + V\frac{\partial C}{\partial y} = D_B\nabla^2 C \tag{5}$$

$$\frac{\partial \vartheta}{\partial t} + U\frac{\partial \vartheta}{\partial x} + V\frac{\partial \vartheta}{\partial y}$$
$$+ \frac{b_c w_c}{C_1 - C_2}\left[\frac{\partial}{\partial x}\left(\vartheta\frac{\partial C}{\partial y}\right) + \frac{\partial}{\partial y}\left(\vartheta\frac{\partial C}{\partial y}\right)\right] = D_n\nabla^2\vartheta \tag{6}$$

where, $(U, V, 0)$ is the velocity vector, T and C is the temperature and concentration and ϑ is the density of the motile microorganism. ρ is the density, μ is the viscosity, α is the thermal diffusivity, D is the mass diffusivity and D_n is the microorganism diffusivity, w_c is the maximum cell swimming speed and b_c is the Chemotaxis constant.

The conditions on the boundary of the walls are:

$$u = 0,\ v = -A\dot{H},\ v_x = u_y,\ T = T_1,\ C = C_1,\ \vartheta = \vartheta_1\ \text{at}\ y = -H$$
$$u = 0,\ v = A\dot{H},\ v_x = u_y,\ T = T_2,\ C = C_2,\ \vartheta = \vartheta_2\ \text{at}\ y = H \tag{7}$$

where, A represent the wall permeability.

We now introduce the following similarity variables

$$\eta = \frac{y}{H(t)},\ u = -\nu H^{-2}xF'(\eta,t),\ v = \nu H^{-1}F(\eta,t),$$
$$\theta = \frac{T - T_0}{T_1 - T_0},\ \phi = \frac{C - C_0}{C_1 - C_0},\ \vartheta = \vartheta_1\chi(\eta) \tag{8}$$

Making use of the above similarity variables in Eqns. (1)-(7), we obtain

$$S^2 F^{VI} - F^{IV} - (\beta_1\eta + F)F''' - (3\beta_1 + F')F'' = 0 \tag{9}$$

$$\theta'' - P_r\left(F - \eta\beta_1\right)\theta' = 0 \tag{10}$$

$$\phi'' - P_r L_e\left(F - \eta\beta_1\right)\phi' = 0 \tag{11}$$

$$\frac{P_e}{S_c}\left[\chi\phi'' + \chi'\phi'\right] - \frac{1}{S_c}\chi'' + \left(F - \eta\beta_1\right)\chi' = 0 \tag{12}$$

$$F'(-1) = F''(-1) = 0,\ F(-1) = -R_e,\ \theta(-1) = 1,$$
$$\phi(-1) = 1,\ \chi(-1) = 1$$
$$F'(1) = F''(1) = 0,\ F(1) = R_e,\ \theta(1) = \delta\theta,$$
$$\phi(1) = \delta\phi,\ \chi(1) = \delta\chi \tag{13}$$

where, the prime denotes differentiation with respect to η and $s = \sqrt{\frac{\eta_1}{\mu}}$ is the couple-stress parameter, $\beta_1 = H\dot{H}/\nu$ wall expansion ratio, where $\beta_1 < 0$ corresponds to the case when the channel walls are contracting and $\beta_1 > 0$ corresponds to the case when the channel walls are expanding, $Le = \alpha/D$ is the Lewis number, $Pr = \nu/\alpha$ is the Prandtl number, $Sc = \nu/D_n$ is the bioconvection Schmidt number, $Re = AH\dot{H}/\nu$ is the Permeability Reynolds number with Re greater than 0 is for suction and $Pe = b_c w_c/D_n$ is the bioconvection Peclet number.

3. METHOD OF SOLUTION

The system of differential Eqns. (9)-(12) along with the

boundary conditions (13) are linearized using the successive linearization method. The linearized equations are solved numerically by Chebyshev spectral collocation method.

The successive linearisation method (SLM) is proposed and developed by Makukula et al. [26]. This method linearizes the nonlinear equations. To linearize the nonlinear boundary value problem in an unknown function $z(\eta)$ (i.e. $f(\eta)$ or $\theta(\eta)$ or $\phi(\eta)$ or $\chi(\eta)$ in this problem) using SLM, it is approximated as

$$z(\eta) = z_k(\eta) + \sum_{m=0}^{k-1} z_m(\eta) \tag{14}$$

where, $z_k(\eta)$ is an unknown function and $z_0(\eta)$, $z_1(\eta)$, ... $z_{k-1}(\eta)$ are known approximate solutions. Substituting (14) in the given nonlinear differential equation and ignoring the non-linear terms of $z_k(\eta)$ gives the linearized differential equation in $z_k(\eta)$. Hence, the subsequent approximations $z_k(\eta)$, $r=1, 2, ...$ are obtained by solving the linearized differential equations in $z_k(\eta)$, $r = 1, 2, ...$ successively, given that the previous guess $z_m(\eta)$, $m = 0, 1, 2, ... , k-1$ are known. The initial guess $z_0(\eta)$, is chosen such that it satisfy the given boundary conditions.

Hence, using the succesive linearization method, the linearized version of the Eqns. (9)-(12) is given by

$$S^2 F_k^{VI} - F_k^{IV} + a_{1,k-1} F_k^{///} + a_{2,k-1} F_k^{//}$$
$$+ a_{3,k-1} F_k^{/} + a_{4,k-1} F_k = z_{1,k-1} \tag{15}$$

$$b_{1,k-1} F_k + \theta_k^{//} + b_{2,k-1} \theta_k^{/} = z_{2,k-1} \tag{16}$$

$$c_{1,k-1} F_k + \phi_k^{//} + c_{2,k-1} \phi_k^{/} = z_{3,k-1} \tag{17}$$

$$d_{1,k-1} F_k + d_{2,k-1} \phi_k^{//} + d_{3,k-1} \phi_k^{/} + d_{4,k-1} \chi_k^{//}$$
$$+ d_{5,k-1} \chi_k^{/} + d_{6,k-1} \chi_k = z_{4,k-1} \tag{18}$$

where, the coeeficients $a_{s,k-1}$, $z_{s,k-1}$, $s= 1,2,3,4$, $b_{l,k-1}$, $c_{l,k-1}$, $l = 1,2$ and $d_{i,k-1}$, $i = 1,2,3,4,5,6$ are interms of the approximations $F_m(\eta)$, $\theta_m(\eta)$, $\phi_m(\eta)$, $\chi_m(\eta)$, $m = 1, 2, 3, ... k-1$ and their derivatives and are known at the previous stage.

The Chebyshev spectral collocation method [27, 28] is based on the Chebyshev polynomials defined on the interval [-1,1]. To solve a differential equation using this method, for an unknown function $z_k(\eta)$, on [-1,1], first, we discretize the interval [-1,1] using the following $N+1$ Gauss-Lobatto collocation points

$$\xi_j = \cos\frac{\pi j}{N}, \quad j = 0,1,2,......,N \tag{19}$$

The unknown function $F_k(\eta)$, $\theta_k(\eta)$, $\phi_k(\eta)$, $\chi_k(\eta)$, are approximated at the above collocatopn points by

$$f_k(\xi) \approx \sum_{j=0}^{N} f_k(\xi_j) T_j(\xi_j), \theta_k(\xi) \approx \sum_{j=0}^{N} \theta_k(\xi_j) T_j(\xi_j),$$
$$\phi_k(\xi) \approx \sum_{j=0}^{N} \phi_k(\xi_j) T_j(\xi_j), \chi_k(\xi) \approx \sum_{j=0}^{N} \chi_k(\xi_j) T_j(\xi_j) \tag{20}$$

where, T_j is the kth Chebyshev polynomial defined by $T_j(\xi) = cos(k \cos^{-1} \xi)$.

The derivatives of the unknown functions at the collocation points are represented as

$$\frac{d^a f_k}{d\eta^a} = \sum_{j=0}^{N} \mathbf{D}_{jk}^a f_k(\xi_j), \frac{d^a \theta_k}{d\eta^a} = \sum_{j=0}^{N} \mathbf{D}_{jk}^a \theta_k(\xi_j),$$
$$\frac{d^a \phi_k}{d\eta^a} = \sum_{j=0}^{N} \mathbf{D}_{jk}^a \phi_k(\xi_j), \frac{d^a \chi_k}{d\eta^a} = \sum_{j=0}^{N} \mathbf{D}_{jk}^a \chi_k(\xi_j) \tag{21}$$

where, $\mathbf{a}$ is the order of differentiation and $\mathbf{D}$ being the Chebyshev spectral differentiation matrix. Substituting Eqns. (20)-(21) into the Eqns. (15)-(18), we get the following system of the algebraic equation

$$\mathbf{A}_{k-1} \mathbf{X}_k = \tilde{\mathbf{R}}_{k-1}, \tag{22}$$

In Eq.(19), A_{k-1} is a $(4N + 4) \times (4N + 4)$ square matrix and X_k and $\bar{R}_{k-1}$ are $(4N + 4) \times 1$ column vectors

$$\mathbf{A}_{k-1} = \begin{bmatrix} A_{11} & 0 & 0 & 0 \\ A_{21} & A_{22} & 0 & 0 \\ A_{31} & 0 & A_{33} & 0 \\ A_{41} & 0 & A_{43} & A_{44} \end{bmatrix}, \mathbf{X}_k = \begin{bmatrix} \mathbf{F}_k \\ \Theta_k \\ \Phi_k \\ \Lambda_k \end{bmatrix}, \tilde{\mathbf{R}}_{k-1} = \begin{bmatrix} \tilde{z}_{1,k-1} \\ \tilde{z}_{2,k-1} \\ \tilde{z}_{3,k-1} \\ \tilde{z}_{4,k-1} \end{bmatrix} \tag{23}$$

where,

$$\mathbf{F}_k = [F_k(\xi_0), F_k(\xi_1), F_k(\xi_2),...,F_k(\xi_{N-1}), F_k(\xi_N)]^T$$
$$\Theta_k = [\theta_k(\xi_0), \theta_k(\xi_1), \theta_k(\xi_2),...,\theta_k(\xi_{N-1}), \theta_k(\xi_N)]^T$$
$$\Phi_k = [\phi_k(\xi_0), \phi_k(\xi_1), \phi_k(\xi_2),...,\phi_k(\xi_{N-1}), \phi_k(\xi_N)]^T$$
$$\Lambda_k = [\chi_k(\xi_0), \chi_k(\xi_1), \chi_k(\xi_2),...,\chi_k(\xi_{N-1}), \chi_k(\xi_N)]^T$$
$$\tilde{Z}_{m,r-1} = [z_{m,r-1}(\xi_0), z_{m,r-1}(\xi_1), z_{m,r-1}(\xi_2),...,$$
$$z_{m,r-1}(\xi_{N-1}), z_{m,r-1}(\xi_N)], m = 1,2,3,4,5$$
$$A_{11} = S^2 \mathbf{D}^6 - D^4 + a_{1,r-1}\mathbf{D}^3 + a_{2,r-1}\mathbf{D}^2 + a_{3,r-1}\mathbf{D} + a_{4,r-1}\mathbf{I},$$
$$A_{21} = b_{1,r-1}\mathbf{D} + b_{2,r-1}\mathbf{I}, \quad A_{22} = \mathbf{D}^2 + b_{3,r-1}\mathbf{D},$$
$$A_{31} = c_{1,r-1}\mathbf{I}, \quad A_{33} = \mathbf{D}^2 + c_{2,r-1}\mathbf{D},$$
$$A_{41} = d_{1,r-1}\mathbf{I}, \quad A_{43} = d_{2,r-1}\mathbf{D}^2 + d_{3,r-1}\mathbf{D},$$
$$A_{44} = d_{4,r-1}\mathbf{D}^2 + d_{5,r-1}\mathbf{D} + d_{6,r-1}\mathbf{I}$$

where, $\mathbf{I}$ is the identity matrix, $\mathbf{0}$ is the zero matrix and $\mathbf{D}$ is the Chebyshev derivative matrix.

Incorporating the conditions on the boundary in the matrix Eq. (20), the solution is attained as

$$\mathbf{X}_k = \mathbf{A}_{k-1}^{-1} \mathbf{R}_{k-1} \tag{24}$$

4. RESULTS AND DISCUSSION

In this section, the variation of density of motile microorganisms and density number of motile microorganisms with various values of non-dimensional parameters are presented.

To validate the accuracy of the present numerical procedure, SLM, the values of velocity, microrotation and temperature at η=0.5 are calculated by using shooting method and the comparison is displayed in Table 1. It is noticed from this table

that the error between these two numerical results is negligible so that the numerical calculations which we made by SLM are appropriate.

Table 1. Correlation of f, θ, ϕ, and χ at $\eta = 0.5$ using SLM and shooting method for various values of β_1 with Pr = 0.71; Le =1; Pe = 1; Sc = 0.22; Re =4; $\delta\chi$ = 1; $\delta\varphi$ = 0; $\delta\theta$= 0.2; S = 0.1

	f	θ	ϕ	χ
SLM				
β_1= -15	2.8629	0.5553	0.4437	1.2858
β_1= 15	3.2642	0.3301	0.1626	0.9125
Shooting Method				
β_1= -15	2.8629	0.5551	0.4437	1.2858
β_1= 15	3.2642	0.3301	0.1625	0.9124

(a)

(b)
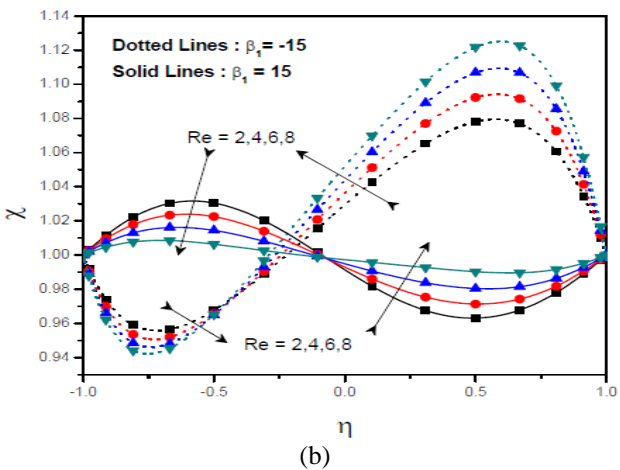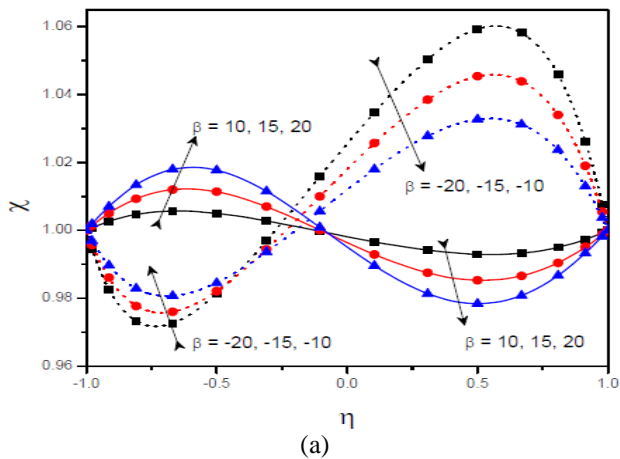
Figure 1. Effect of (a) β_1, (b) *Re* on $\chi(\eta)$

Figure 1 presents the effect of wall expansion ratio β_1, Reynolds Number *Re* on the density of the motile microorganisms $\chi(\eta)$ for the wall expansion ratio β_1= -15 and β_1 = 15. Figure 1a depicts the variation of $\chi(\eta)$ for different values of β_1. From this figure, it is noticed that, as β_1 increases, the density of the motile microorganisms is also increasing in the first half of the observed region. In other words, near to the lower plate, whether wall contracts or expands, the density of the motile microorganisms is observed to be increasing. Similarly, near to the upper plate, the density of the motile microorganisms is decreasing with an increase in the wall expansion ratio. If β_1 is positive, i.e. the plates are expanding, the density of the motile microorganisms is increased up to a

point close to the middle of the channel and exhibits reverse pattern in the upper half of the channel. The reverse trend is observed for the negative values of β_1. The influence of the parameters *Re* on the density of motile microorganisms is displayed in Figure 1b. If β_1 is positive, the density of the motile microorganisms is growing in the lower half of the channel and lessening in the upper half region of the channel as *Re* is increasing. Similarly, for β_1 negative, a reverse pattern is observed for increasing values of *Re*.

(a)

(b)

(c)
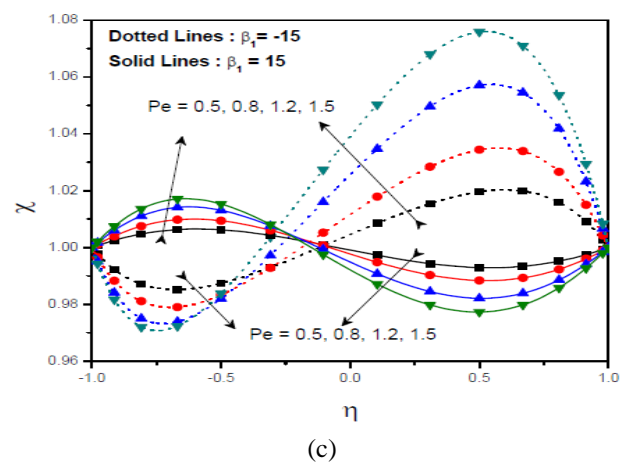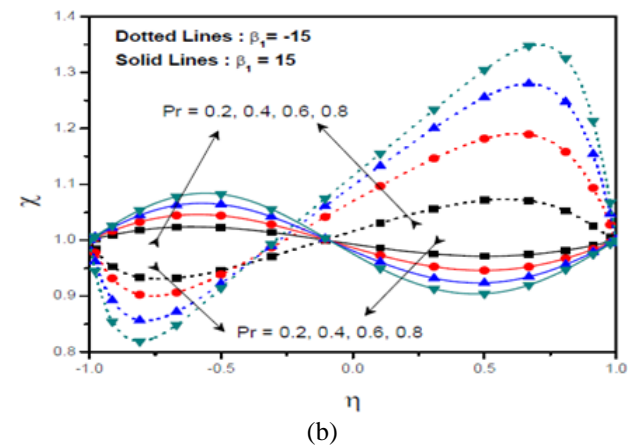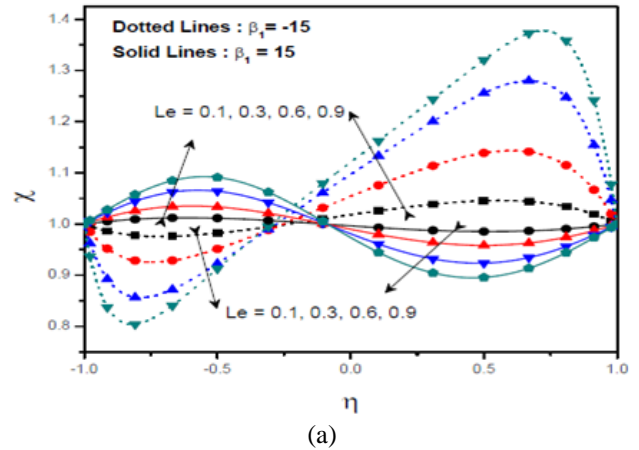
Figure 2. Effect of (a) Le (b) Pr, (c) Pe on $\chi(\eta)$

The effect of Lewis number *Le*, and bioconvection Peclet Number *Pe* on the density of the motile microorganisms χ for the wall expansion ratio β_1 = -15 and β_1 = 15 is depicted in Figure 2. Figure 2a displays the variation of the density of motile microorganisms with the Lewis number *Le*. If the plates

are expanding, i.e. for β_1 is positive, the density of the motile microorganisms is increasing in the lower half of the channel and decreasing in the upper half region of the channel as the Lewis number is increasing. As Le is increasing, a decreasing pattern is observed in the lower half region and an increasing pattern in the upper half of the channel is noticed when the plates are contracting. The influence of the parameters Pr and Pe on the density of motile microorganisms is displayed in Figures 2b and 2c. If β_1 is positive, the density of the motile microorganisms is increasing in the lower half of the channel and decreasing in the upper half region of the channel as Re and Pe are increasing. Similarly, for β_1 negative, a reverse pattern is observed for increasing values of Pr and Pe.


(d)
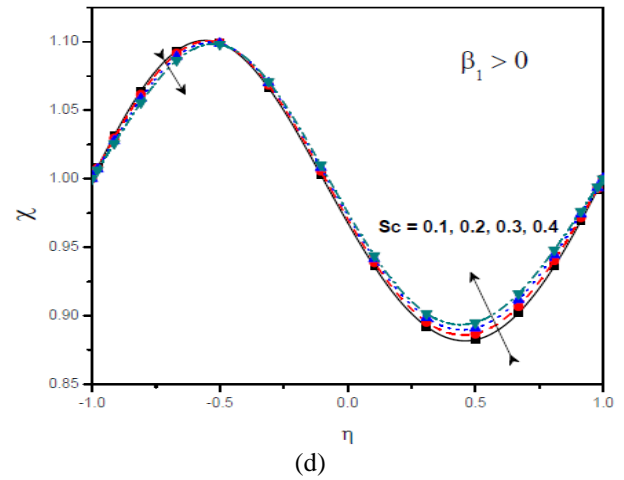
Figure 3. Effect of (a) S, (b) β_1, (c), (d) Sc with β_1 on $\chi(\eta)$

Figure 3 represents the variation of the density of motile microorganisms χ with couple stress parameter S, Schmidt Number Sc. From Figure 3a and Figure 3b, it is interesting to note that the influence of couple stress parameter on the density of motile microorganisms is not significant irrespective of the plates are expanding or contracting. Figure 3c presents the variation of χ with Sc in β_1 is negative, χ is diminishing in the lower half of the channel and rising in the upper half region of the channel as Sc is enhanced. The reverse pattern is noticed when the plates are expanding. Figure 3d reveals that the impact of Sc on the density of motile microorganisms is minimal when the plates are expanding while the Schmidt Number has its effect on the fluid when the plates are contrasting.
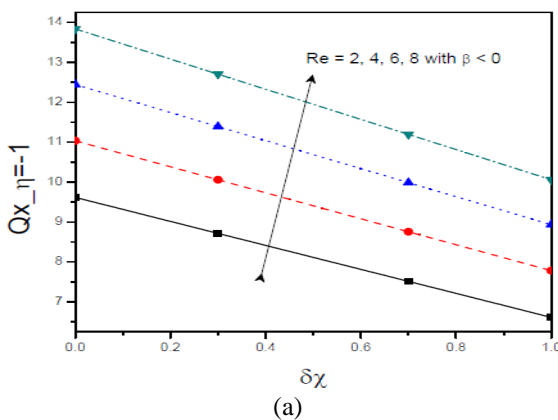
The effects of wall expansion ratio β_1 on the density number of motile microorganisms Q_x at the lower and upper plates is presented in Figure 4. When the plates are expanding, the density number of motile microorganisms is increasing at the lower plate as displayed in Figures 4a-4b and the upper plate displayed in Figures 4c-4d. Further, the density number of motile microorganisms is decreasing at the lower plate and increasing at the upper plate when the plates are contracting. Also, Q_x is decreasing for increasing values of slip parameter $\delta\chi$ at the lower plate and increasing at the upper plate except for $\beta_1 > 3$ in which it is decreasing.

(a)
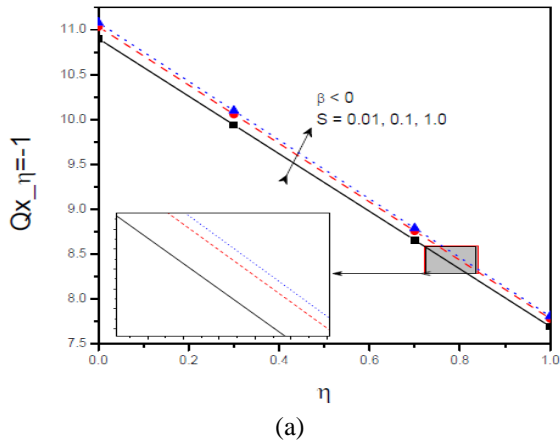

(b)

(c)

(b)

(b)

(c)

(c)

(d)

(d)
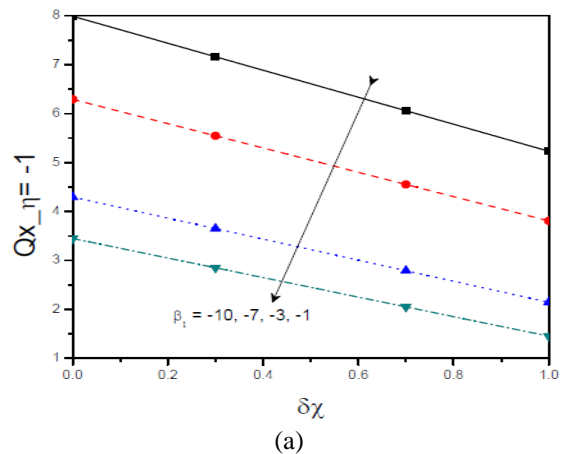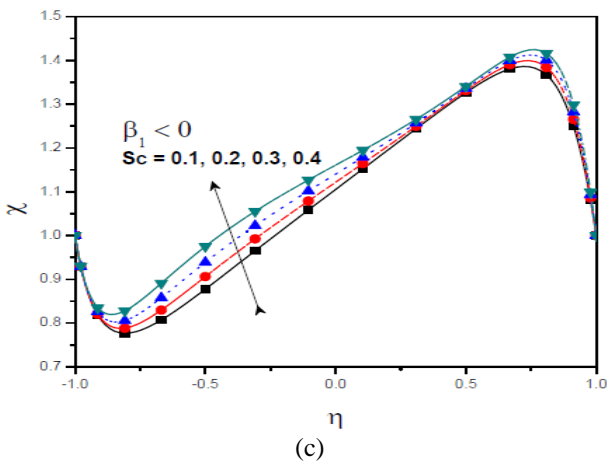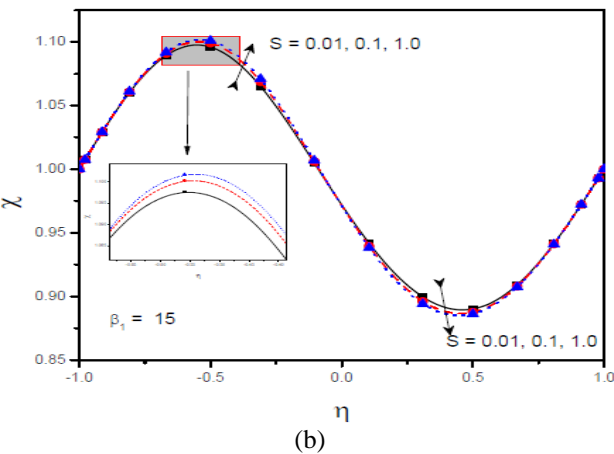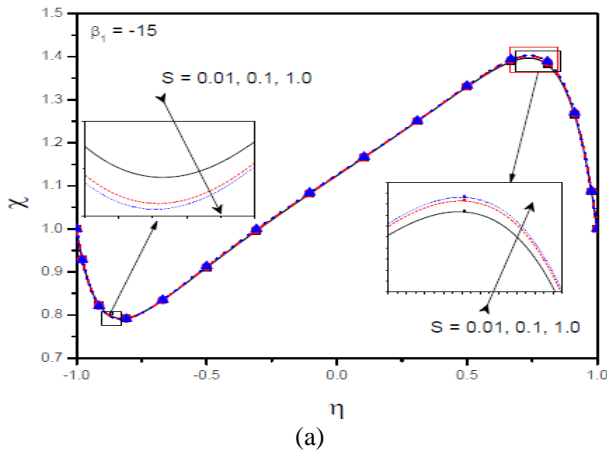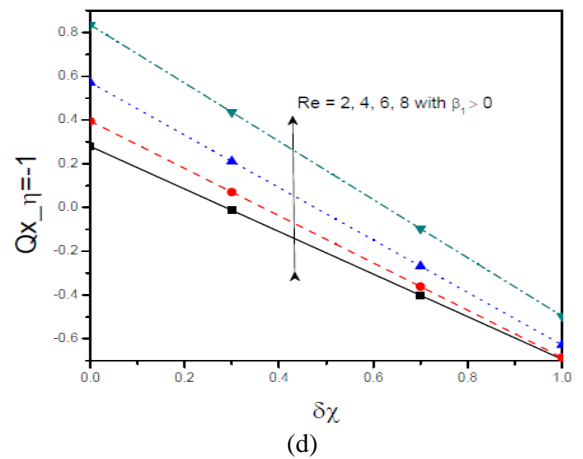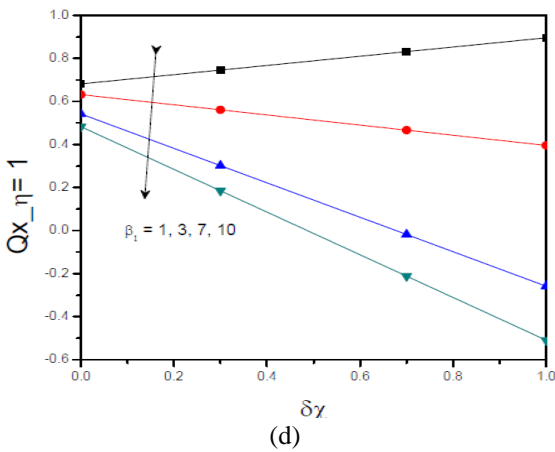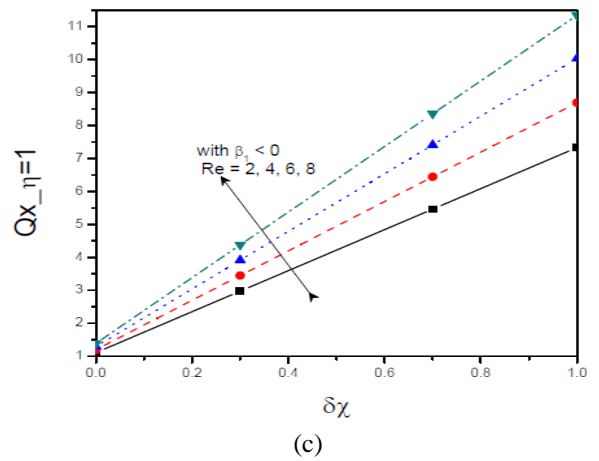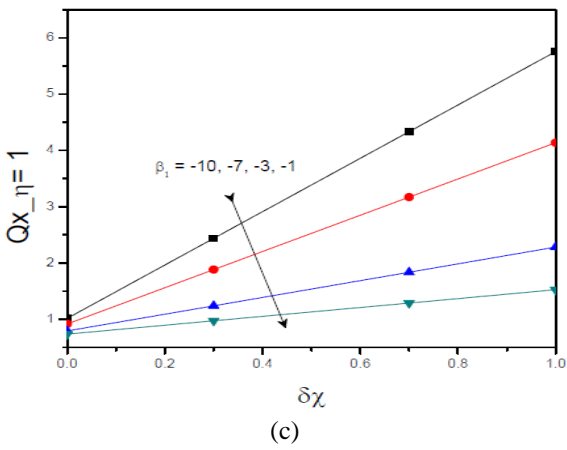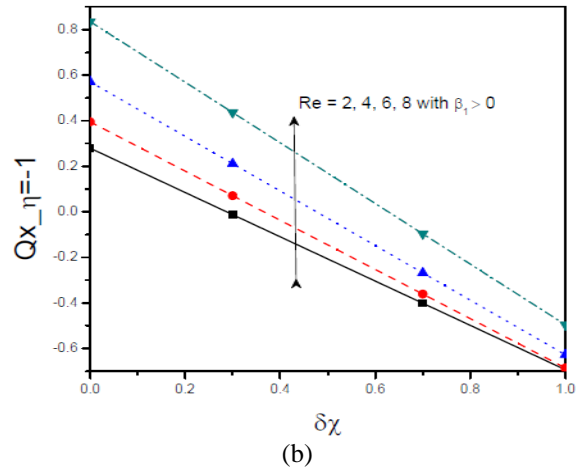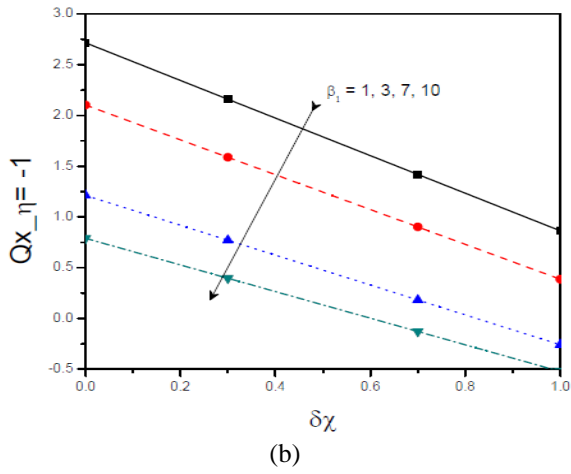
Figure 4. Effect of the parameter β_1 with $\delta\chi$ variation on Q_x at $\eta = -1$ (Lower Plate) and $\eta = 1$ (Upper Plate)
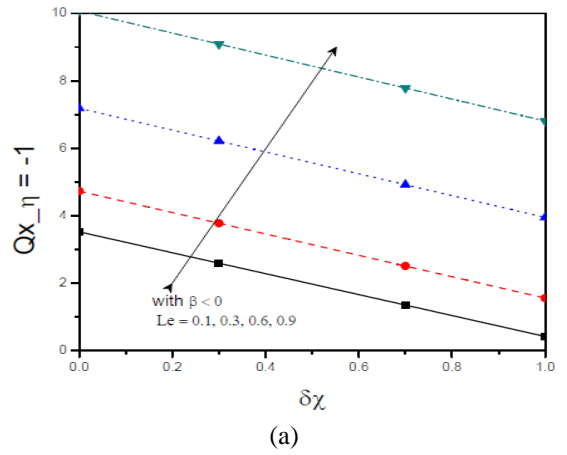
Figure 5. Effect of the parameter Re with $\delta\chi$ variation on Q_x at $\eta = -1$ (Lower Plate) and $\eta = 1$ (Upper Plate)
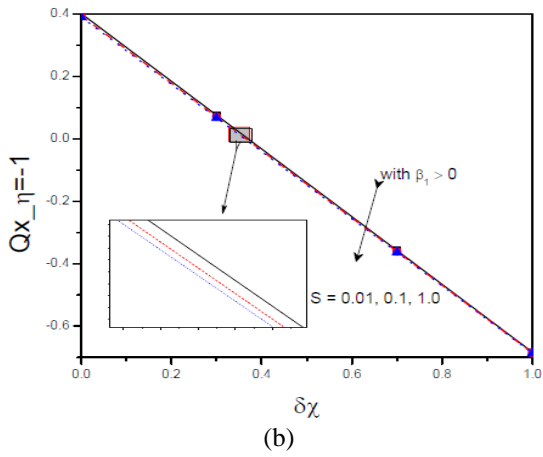
The influence of Reynolds Number *Re* on the density number of motile microorganisms at the lower and upper plates with $\delta\chi$ is portrayed in Figure 5. At the lower plate, as *Re* increases, Q_x is increasing when the plates are expanding/contrasting as shown in Figures 5a-5b. For a given *Re* as the slip parameter of motile microorganisms increases the Q_x is decreasing. Also, at the upper plate, Q_x increases with an increase in *Re*, by showing increasing pattern when β_1 is negative and decreasing in β_1 positive with the slip in motile microorganisms $\delta\chi$ as shown in Figures 5c-5d.
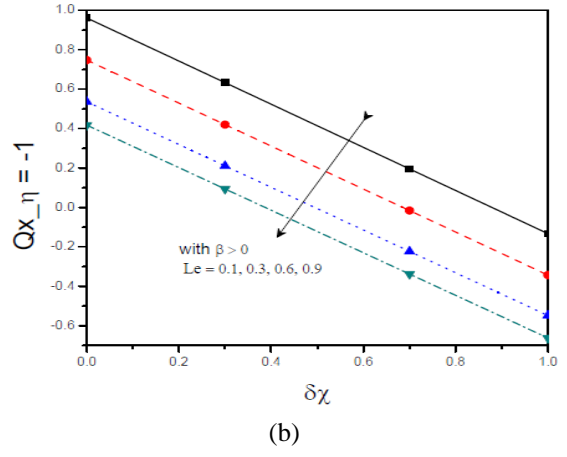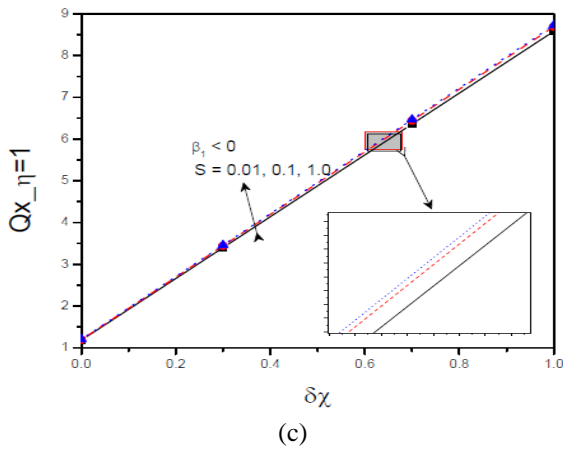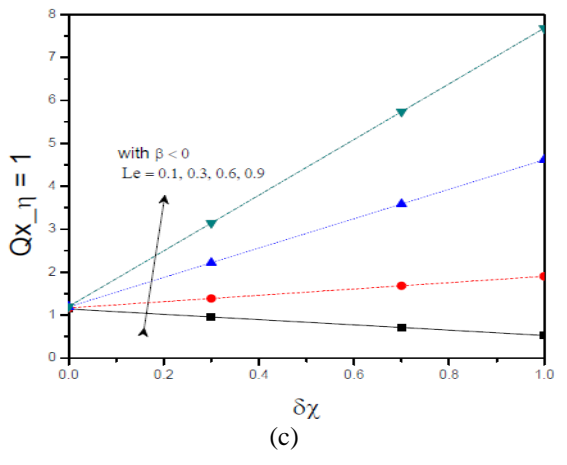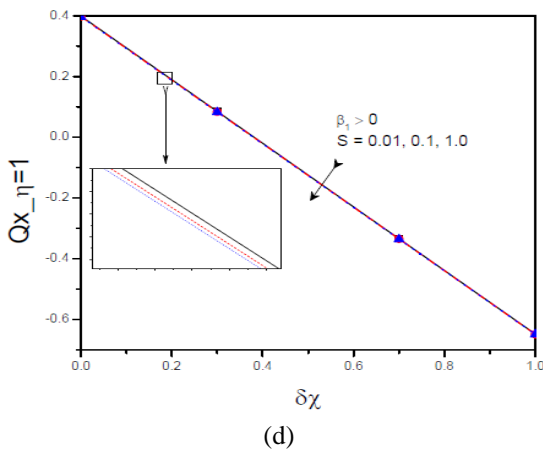

(a)

(a)

(b)

(c)

(d)

Figure 6. Effect of the parameter S with δχ variation on Q_x at η = -1 (Lower Plate) and η = 1 (Upper Plate)

Figure 7. Effect of the parameter Le with δχ variation on Q_x at η = -1 (Lower Plate) and η = 1 (Upper Plate)

The variation of the density number of motile microorganisms at the lower and upper plates with couple stress parameter S is presented in Figure 6 From Figures 6a and 6d it is understood that the impact of couple stress parameter on the density number of the motile microorganisms is insignificant.

The variation of Q_x at the lower and upper plates with $\delta\chi$ for different values of Lewis number Le is depicted in Figure 7. It is observed from these figures that Q_x is decreasing with β_l is negative and increasing with β_l is positive at the lower plate and the similar pattern holds at the upper plate. Also, for a given Le, the Q_x decreases as the $\delta\chi$ increase except at the upper plate with β_l is negative.
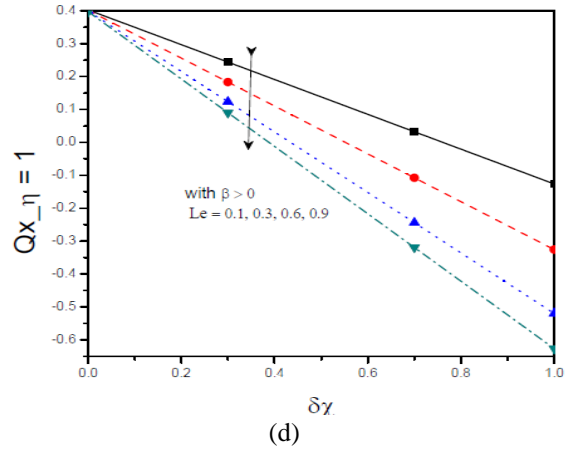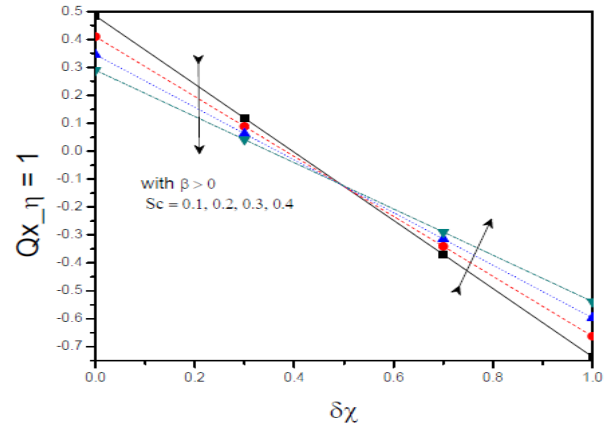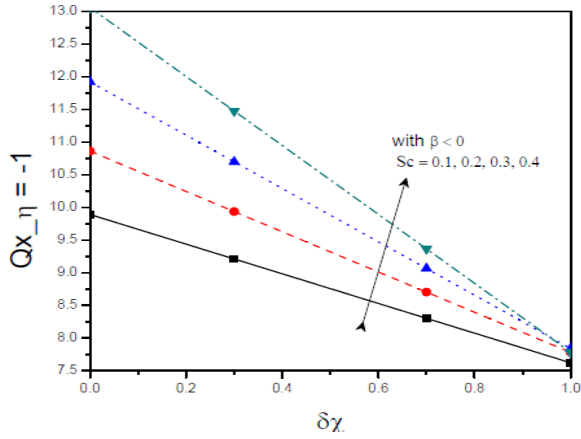

(d)

Figure 8. Effect of the parameter Sc with $\delta\chi$ variation on Q_x at $\eta=-1$ (Lower Plate) and $\eta=1$ (Upper Plate)

The impact of bioconvection Schmidt Number Sc on Q_x at the lower and upper plates with $\delta\chi$ is visualized in Figure 8. The Figures 8a and 8b reveals that Q_x is increasing as Sc increases when the plates are contrasting at both lower and upper plate and it decreases with up to a critical value and then increases with increase in the value of Sc when the plates are expanding at both the plates, as shown in Figures 8c-8d.

(a)

(b)

(c)

(a)
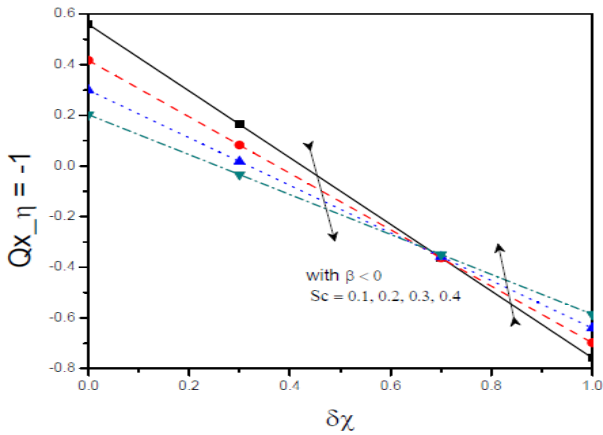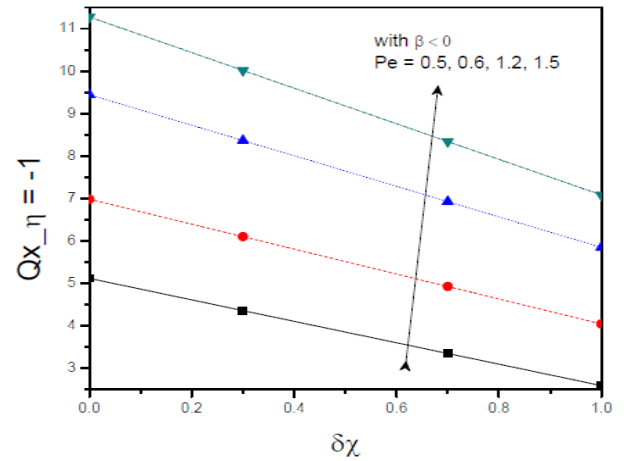
(b)

(c)

(d)

Figure 9. Effect of the parameter Pe with δχ variation on Q_x at η=-1 (Lower Plate) and η=1 (Upper Plate)

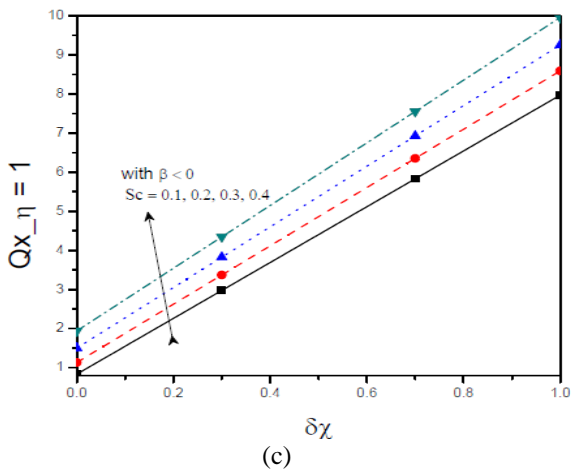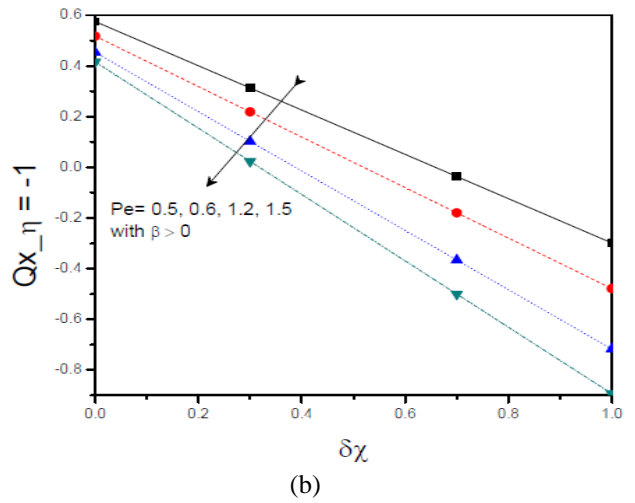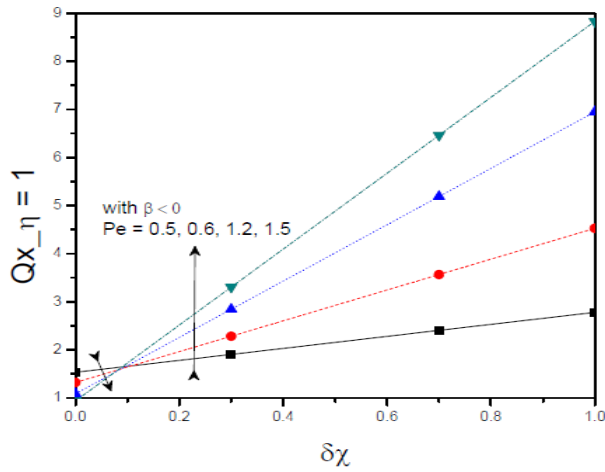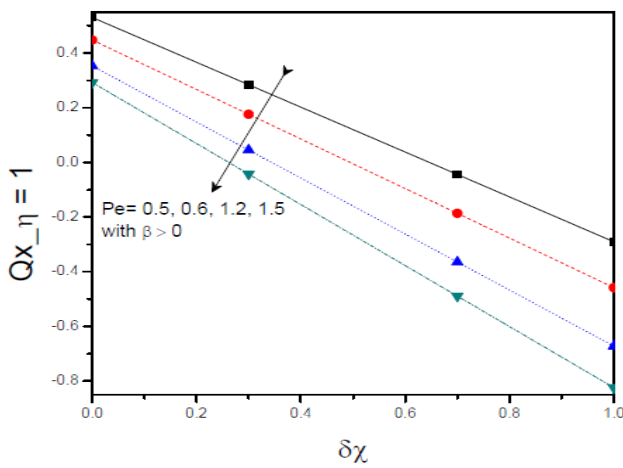The influence of bioconvection Peclet number *Pe* on the density number of motile microorganisms at the lower and upper plates with δχ is portrayed in Figure (9). At the lower plate, as P*e* increases, Q_x is increasing when the plates are contracting as shown in Figure (9a) and decreases when plates are expanding as shown in Figure (9b). As *Pe* is increasing, the density number of the motile microorganisms at the lower plate is also increasing when the plates are contracting. At the upper plate, Q_x is decreasing up to a critical value and then increasing in increasing value of *Pe* for negative β_1. In case of the plates are expanding, the density number of the motile microorganisms is decreasing with an increase in *Pe* as shown in Figures (9c-9d).

5. CONCLUSIONS

The present analysis deals with the flow of couple stress fluid containing motile microorganisms in a horizontal channel with contracting/expanding porous. The nonlinear ordinary differential equations are linearized using the successive linearization method and solved by implementing the Chebyshev collocation method. The important results are itemized below:

- The density of motile microorganisms increases near the lower wall and decreases at the upper-wall via concave-

down to concave-up for the parameter β_1, *Le, Pr* and *Pe* and shows reverse trend w.r.t. R*e*.
- The density of motile microorganisms decreases near the lower wall and increases at the upper-wall via concave-downward to concave-upward for the parameters *S* and *Sc*, but with a nominal variations
- The density number of motile microorganisms Q_x decreases as the parameters β_1, *Re*, *S*, *Sc* are increasing except at the upper plate when plates are contrasting.
- The density number of motile microorganisms were decreasing both at lower and upper plates except for *Le* and *Re* at the upper plate in the case of $\beta_1 > 0$.

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NOMENCLATURE

A	A measure of wall permeability
$H(t)$	Distance between the plates
b_c	Chemotaxis constant
C	The concentration of the fluid.
D_B	Mass diffusivity
D_n	Microorganism diffusivity
f	Dimensionless stream function
Le	Lewis Number
S	couple-stress parameter
Pe	Bio-convection Peclet number
Pr	Prandtl number
Q_x	Motile microorganism Density Number
Re	Permeability Reynolds number
Sc	Bio-convection Schmidt number
T	Fluid Temperature
w_c	Maximum cell swimming speed

Greek symbols

α	Thermal Diffusivity
β_1	Wall expansion ratio
$\delta\theta, \delta\phi, \delta\chi$	Constants
η	Non-dimensional variable
θ	Dimensionless temperature
ϑ	The density of the motile microorganism.
μ	Dynamic Viscosity of the fluid
ν	Magnetic diffusivity/kinetic viscosity
ρ	Density of the fluid
ϕ	Dimensionless concentration
χ	Dimensionless microorganism

Superscripts

$'$	Differentiation with respect to η